Kinetic and hyperbolic equations: modeling, analysis and numerics

Report of Contributions

https://indico.math.cnrs.fr/e/7592

Contribution ID: **2** Type: **not specified**

Topological states in a swarmalator model

Thursday, December 15, 2022 3:50 PM (45 minutes)

Swarmalators are agents that combine the features of swarming particles and oscilla tors hence the name, contraction of ‘swarmer’ and ‘oscillator’. Each particle is endowed with a phase which modulates its interaction force with the other particles. In return, relative positions modulate phase synchronization between interacting particles. In the talk, I will present a model whree there is no force reciprocity: when a particle attracts another one, the latter repels the former. This results in a pursuit behavior. I will derive a hydrodynamic model and show that it has explicit special solutions enjoying a non-trivial topology quantified by a phase index. I will present a theoretical and numerical study of these solutions. This is joint work with Antoine Diez (Kyoto University) and Adam Walczak (Imperial College London).

Presenter: DEGOND, Pierre

Contribution ID: **3** Type: **not specified**

Vlasov-Poisson-Fokker-Planck equation in the adiabatic asymptotics

Monday, December 12, 2022 3:30 PM (45 minutes)

The main concern of this article is the study of a nonlinear Vlasov-Poisson-Fokker-Planck equation describing the electron dynamics in a thermonuclear fusion plasma, in the regime of a small electron-to-ion mass ratio ($\varepsilon \ll 1$). The first part of this work focuses on the rigorous $\varepsilon \longrightarrow 0$ asymptotic study, based on hypocoercive techniques, permitting to understand the transition from the kinetic level to the macroscopic, adiabatic electron level. The second part introduces a Hilbert-Fourier spectral method enabling to treat without too much numerical effort the above mentioned electron transition. This scheme has in particular the nice property of being Asymptotic Preserving in the sense that ε-independent meshes can be chosen, without degrading the accuracy. Some numerical tests are finally performed validating on one hand the scheme and underscoring on the other hand the mathematical results.

Presenter: LEHMAN, Etienne

Contribution ID: **4** Type: **not specified**

From collisional kinetic models to sprays: internal energy exchanges

Tuesday, December 13, 2022 4:00 PM (45 minutes)

We are interested in models describing the evolution of particles (such as solid dust particles or droplets) in a rarefied gas. We present here a derivation of a fluid-kinetic system in which the gas and the disperse phase are coupled not only by the drag force, but also by the exchange of temperature between the gas and the droplets/dust specks.
We start from a kinetic collisional model, based on the hypothesis that collisions between dust particles and gas molecules are inelastic and are given by a diffuse reflection mechanism on the surface of dust particles. We propose a model which preserves the total energy by introducing a new variable in the density function of macroscopic particles. We then derive an asymptotics to a Vlasov-Euler equation of compressible fluid when two small parameters tend to zero, namely the ratio of masses between gas molecules and dust particles on the one hand, and the Knudsen number of the gas on the other hand. We thus obtain an explicit expression of the transfer of energy between the phases.

Presenter: CHARLES, Frédérique

Contribution ID: **5** Type: **not specified**

A mass-conserving sparse grid combination technique with biorthogonal hierarchical basis functions

Monday, December 12, 2022 4:15 PM (45 minutes)

Grid-based solvers for the simulation of kinetic equations suffer from the curse of dimensionality. The sparse grid combination technique is a means to reduce the number of degrees of freedom in high dimensions, however, the hierarchical representation for the combination step with the state-of-the-art hat functions suffers from poor conservation properties and numerical instability.
We introduce two new variants of hierarchical multiscale basis functions for use with the combination technique: the biorthogonal and full weighting bases. The new basis functions conserve the total mass and are shown to significantly increase accuracy for a finite-volume solution of constant advection. Further numerical experiments based on the combination technique applied to a semi-Lagrangian Vlasov–Poisson solver show a stabilizing effect of the new bases on the simulations.

This is joint work with Theresa Pollinger, Johannes Rentrop und Dirk Pflüger.

Presenter: KORMANN, Katharina

Contribution ID: **6** Type: **not specified**

Numerical methods for the Vlasov-Poisson system under the quasi neutral and fluid scalings.

Tuesday, December 13, 2022 9:00 AM (45 minutes)

In this talk we will discuss the development and the analysis of asymptotic stable and consistent schemes in the joint quasi-neutral and fluid limits for the collisional Vlasov Poisson system.
We will first introduce the quasi neutral scaling and its formal limit and then we will discuss different numerical approaches based on splitting techniques for dealing with the stiffness of the resulting equations.
We will then propose a new scheme which is stable for arbitrary choices of the time steps independently from the small scale dynamics and we perform a linear stability analysis.

Presenter: DIMARCO, Giacomo

Contribution ID: **7** Type: **not specified**

A numerical framework for hypocoercivity in the context of kinetic equations.

Tuesday, December 13, 2022 9:45 AM (45 minutes)

We propose a numerical method for a Vlasov-Fokker-Planck model with external applied force field and prove quantitative results ensuring that it is Asymptotic-Preserving for both the macroscopic regime and the long time behavior simultaneously. We illustrate these results with various numerical experiments in which we observe, among others, transition phase between macroscopic and long time behavior as well as other finer phenomena, such as oscillations, which were not predicted by our theoretical analysis.

Presenter: BLAUSTEIN, Alain

Contribution ID: **8** Type: **not specified**

Multi-fidelity method for a class of kinetic models with uncertainties

Tuesday, December 13, 2022 11:00 AM (45 minutes)

In this talk, we will discuss some recent development on the multi-fidelity methods for solving a class of kinetic equations with multi-dimensional uncertainties and different scalings. The Boltzmann equation, linear transport equation and epidemic transport system will be particularly studied, followed by some formal error estimates. We will also briefly discuss applying deep learning approach to solve kinetic problems.

These are joint work with Giulia Bertaglia, Lorenzo Pareschi and Xueyu Zhu.

Presenter: LIU, Liu

Contribution ID: **9** Type: **not specified**

Monte Carlo Stochastic Galerkin methods for collisional plasmas

Tuesday, December 13, 2022 2:00 PM (45 minutes)

Presenter: PARESCHI, Lorenzo

Contribution ID: **10** Type: **not specified**

A kinetic model of plasma-probe interaction : theory and numerics

Tuesday, December 13, 2022 2:45 PM (45 minutes)

We are interested in the modeling of a cylindrical Langmuir probe interacting with two species electrostatic plasma. We consider the stationary Vlasov-Poisson equations written in polar coordinates with absorbing boundary conditions at the probe. We will present the general methodology based on a phase-space analysis to construct stationary solutions and explain how to deal with unpopulated orbits. Eventually, we shall discuss how to discretize the model and validate the numerical method with the so called radial sheath solutions. Two dimensional simulations will be presented, showing that some unpopulated orbits do exist when the angular momentum of the particles increase.

Presenter: BADSI, Mehdi

Contribution ID: **11** Type: **not specified**

Derivation of a two-phase flow model with surface tension

Tuesday, December 13, 2022 4:45 PM (45 minutes)

This talk deals with the derivation of a compressible two-phase flow model by means of the Least Action Principle (LAP).
The key point of this method is to provide an appropriate Lagrangian functional which depicts the behaviour of the fluid system.
It is composed of a kinetic energy, taking into account small scales features, and a potential energy.

The latter corresponds to the thermodynamical energy of the system whose derivation will be detailed.
To do so the interface separating the two phases is supposed to be sharp, with no mass, but it has a temperature.
In other words, the interface is depicted by its own internal energy, satisfying a Gibbs differential form involving surface tension.

Applying the LAP gives a partial differential equations describing the compressible behaviour of the fluid-interface system.
To finish some properties of the model will be analysed (hyperbolicity, dissipation…).

Presenter: MATHIS, Hélène

Contribution ID: **12** Type: **not specified**

Talk 11

Contribution ID: **13** Type: **not specified**

Analysis of discontinuous Galerkin/Hermite spectral methods for the Vlasov-Poisson system

Wednesday, December 14, 2022 9:00 AM (45 minutes)

In this talk, I present some recent results obtained in collaboration with Francis Filbet. We study discontinuous Galerkin approximations for the Vlasov-Poisson system written as an hyperbolic system using Hermite polynomials. We introduce a new L^2 weighted space, with a time dependent weight, allowing to prove global stability. Moreover, we prove the convergence of the proposed method by establishing error estimates between the numerical solution and the smooth solution to the Vlasov-Poisson system.

Presenter: BESSEMOULIN-CHATARD, Marianne

Contribution ID: **14** Type: **not specified**

An asymptotic preserving scheme for the full Euler equations in the low Mach limit

Wednesday, December 14, 2022 9:45 AM (45 minutes)

This work is in collaboration with Marie-Hélène Vignal, Institut de Mathématiques de Toulouse, UT3-Paul Sabatier.

In this work, we develop and study an asymptotic preserving (AP) scheme for the compressible Euler system in the low Mach number regime. For subsonic flows, the acoustic waves are very fast compared to the velocity of the fluid, we are in an incompressible regime. From a numerical point of view, when the Mach number tends to zero, classical explicit schemes present two major drawbacks : they loose consistency and impose a very restrictive constraint on the time step to guaranty the stability of the scheme since they have to follow the fast acoustic waves.

We propose a new linear asymptotic stable scheme, with a CFL condition independent of the Mach number, and asymptotically consistent, that is it degenerates into a consistent discretization of the incompressible model when the Mach number is sufficiently small.
This type of scheme has been widely studied in the literature, in particular for the isentropic case [5, 4, 3, 6] but also for the full Euler system [2, 1] with various methods. In this work we propose an AP scheme based on an IMEX (Implict-Explicit) discretization in time and cell-centered finite volume in space. I will present our AP scheme, its extension to order 2 and the MOOD procedure used to reduce the oscillations (classical problem of high order schemes). Finally, I will finish my presentation with some results on the Navier-Stokes equations.

Presenter: ALLEGRINI, Paola

Contribution ID: **15** Type: **not specified**

A Conservative Low Rank Tensor Approach with Discontinuous Galerkin discretization for Nonlinear Vlasov Equations

Wednesday, December 14, 2022 11:00 AM (45 minutes)

In this paper, we propose a novel Local Macroscopic Conservative (LoMaC) low rank tensor method with discontinuous Galerkin (DG) discretization for the physical and phase spaces for simulating the Vlasov-Poisson (VP) system. The LoMaC property refers to the exact local conservation of macroscopic mass, momentum and energy at the discrete level. The LoMaC low rank tensor algorithm (recently developed in arXiv:2207.00518) simultaneously evolves the macroscopic conservation laws of mass, momentum and energy using the kinetic flux vector splitting; then the LoMaC property is realized by projecting the low rank kinetic solution onto a subspace that shares the same macroscopic observables.

This paper is a generalization of our previous work, but with DG discretization to take advantage of its compactness and flexibility in handling boundary conditions and its superior accuracy in the long term. The algorithm is developed in a similar fashion as that for a finite difference scheme, by observing that the DG method can be viewed equivalently in a nodal fashion. With the nodal DG method, assuming a tensorized computational grid, one will be able to (1) derive differentiation matrices for different nodal points based on a DG upwind discretization of transport terms, and (2) define a weighted inner product space based on the nodal DG grid points. The algorithm can be extended to the high dimensional problems by hierarchical Tucker decomposition of solution tensors and a corresponding conservative projection algorithm. In a similar spirit, the algorithm can be extended to DG methods on nodal points of an unstructured mesh, or to other types of discretization, e.g. the spectral method in velocity direction. Extensive numerical results are performed to showcase the efficacy of the method.

Joint work with Wei Guo from Texas Tech University

Presenter: QIU, Jingmei

Contribution ID: **16** Type: **not specified**

Dynamical low-rank methods for high-dimensional collisional kinetic equations

Wednesday, December 14, 2022 2:00 PM (45 minutes)

Kinetic equations describe the nonequilibrium dynamics of a complex system using a probability density function. Despite of their important role in multiscale modeling to bridge microscopic and macroscopic scales, numerically solving kinetic equations is computationally demanding as they lie in the six-dimensional phase space. Dynamical low-rank method is a dimension-reduction technique that has been recently applied to kinetic theory, yet most of the endeavor is devoted to linear or collisionless problems. In this talk, we introduce efficient dynamical low-rank methods for Boltzmann type collisional kinetic equations, building on certain prior knowledge about the low-rank structure of the solution.

Presenter: HU, Jingwei

Contribution ID: **17** Type: **not specified**

An averaged model for thick sprays

Wednesday, December 14, 2022 2:45 PM (45 minutes)

Sprays describe particles in interaction with a surrounding gas. We are interested in the so-called "thick" sprays with a coupling through a drag force and the volume fraction of the gas.
The usual model has several problems at the level of the mathematical analysis. Those problems, and physical consideration, lead us to propose a new averaged model.
The obtained system is conservative and we prove the local-in-time existence of a strong solution.
We also propose some numerical results

Presenter: FOURNET, Victor

Contribution ID: **18** Type: **not specified**

Modified Lawson schemes for Vlasov equations

Wednesday, December 14, 2022 3:50 PM (45 minutes)

Presenter: CROUSEILLES, Nicolas

Contribution ID: **19** Type: **not specified**

Numerical approximation of 3D magnetized Vlasov equation with moment methods

Thursday, December 15, 2022 4:35 PM (45 minutes)

Moment methods for the approximation of Valsov equations regain a lot of interest, in particular with Hermite polynomials which reveal particularly adpapted to Gaussian profiles. However the application of moment methods to
Vlasov equation with a non constant strong 3D forcing magnetic field is less evident. We will review the recent advances made within the MUFFIN project.

Presenter: DESPRES, Bruno

Contribution ID: **20** Type: **not specified**

Large-magnetic field regimes and asymptotic preserving schemes for plasmas in a torus configuration

Thursday, December 15, 2022 10:30 AM (45 minutes)

Even in highly-oscillating dynamics may persist quantities that are evolving on slower scales and, at first-order, uncouple from fast oscillations. Thus asymptotically the slow dynamics obeys a closed system of uncoupled equations, which may be thought as averaged equations.

That is typically the case for plasmas subject to strong magnetic fields. In this case, the task of identifying a slow uncoupled dynamics is known as gyrokinetics. In the present talk, we shall review some contributions (mostly obtained jointly with Francis Filbet (Toulouse)), that fill some of the gaps of the analytic theory and design numerical schemes that capture accurately the slow part of the dynamics even when discretization meshes are too coarse to describe coexisting fast oscillations.

The emphasis will be on the case where the plasma is in a torus configuration, a toy-model for applications to tokamak fusion devices, and the class of numerical schemes introduced fits in the range of particle-in-cell schemes, with particle-pushers of implicit-explicit type, but are applied to a suitably augmented formulation.

Presenter: RODRIGUES, Luis Miguel

Contribution ID: **21** Type: **not specified**

An implicit, conservative, asymptotic-preserving electrostatic particle-in-cell algorithm for arbitrarily magnetized plasmas

Thursday, December 15, 2022 2:00 PM (45 minutes)

We propose a new electrostatic particle-in-cell algorithm able to use large timesteps compared to particle gyro-period under a (to begin, uniform) large external magnetic field [1]. The algorithm extends earlier electrostatic fully implicit PIC implementations [2] with a new asymptotic- preserving (AP) particle-push scheme [3] that allows timesteps much larger than particle gyroperiods. In the large-timestep limit, the AP integrator preserves all the averaged particle drifts, while recovering particle full orbits with small timesteps. The scheme allows for a seamless, efficient treatment of particles in coexisting magnetized and unmagnetized regions, conserves energy and charge exactly, and does not spoil implicit solver performance. Key to the approach is the generalization of the particle substepping approach introduced in Ref. [2] to allow for orbit segments much larger than cell sizes without spoiling conservation properties. The uniform-magnetic-field assumption allows us to use the standard Crank-Nicolson (CN) update [2] without modification [3], which is a necessary preliminary step to demonstrate viability of the approach for more general magnetic field topologies (which will otherwise require the general algorithm proposed in Ref. [3]). We demonstrate by numerical experiment with several strongly magnetized problems (diocotron instability, modified two-stream instability, and drift-wave instability) that two orders of magnitude wall-clock-time speedups are possible vs. the standard fully implicit electrostatic PIC algorithm without sacrificing solution quality and while preserving strict charge and energy conservation.

[1] G. Chen and L. Chacón, "An implicit, conservative and asymptotic-preserving electrostatic particle-in-cell algorithm for arbitrarily magnetized plasmas in uniform magnetic fields," J. Comput. Phys., submitted (2022)

[2] Chen, Guangye, Luis Chacón, and Daniel C. Barnes. "An energy-and charge-conserving, implicit, electrostatic particle-in-cell algorithm." Journal of Computational Physics 230.18 (2011): 7018-7036.

[3] Ricketson, Lee F., and Luis Chacón. "An energy-conserving and asymptotic-preserving charged-particle orbit implicit time integrator for arbitrary electromagnetic fields." Journal of Computational Physics 418 (2020): 109639.

Presenter: CHACON, Luis

Contribution ID: **22** Type: **not specified**

Sparse grid reconstructions for Particle-In-Cells methods

Thursday, December 15, 2022 2:45 PM (45 minutes)

Based on a mixed discretization (composed of mesh and particles) of the Vlasov-Maxwell system, Particle-In-Cell (PIC) methods are among the most successful models for the simulation of kinetic plasmas. Including simplicity, ease of parallelization and robustness,
PIC schemes still contain a main drawback with the statistical error associated to the
particle noise, depending on the average number of particle per cell and leading to a complexity that
grows exponentially with the dimension. Though the idea of sparse grids has been studied extensively in applied mathematics for years, Particle-In-Cell's application of the method is only at its beginnings. The use of sparse grids in the Particle-In-Cell method, through the so-called combination technique where a function is approximated on different coarser grids, allows to reduce the particle noise, thanks to the larger cells of the grids, and thus reduces the high run times of simulation.

In this presentation, we first introduce sparse grid reconstructions within PIC methods and discuss some convergence and conservation properties. Then, the performance of the method, as well as its parallelization on both shared memory architecture systems and GPUs for 3d-3v simulations are presented.

Presenter: GUILLET, Clément

Contribution ID: **23** Type: **not specified**

Global entropy stability for a class of unlimited high-order schemes for hyperbolic systems of conservation laws

Monday, December 12, 2022 2:15 PM (45 minutes)

Presenter: BERTHON, Christophe

Contribution ID: **24** Type: **not specified**

Quantum Computation of nonlinear partial differential equations and Uncertainty Quantification

Thursday, December 15, 2022 11:15 AM (45 minutes)

Nonlinear partial differential equations (PDEs) are crucial to modelling important problems in science but they are computationally expensive and suffer from the curse of dimensionality. Since quantum algorithms have the potential to resolve the curse of dimensionality in certain instances, some quantum algorithms for nonlinear PDEs have been developed. However, they are fundamentally bound either to weak nonlinearities, valid to only short times, or display no quantum advantage. We construct new quantum algorithms–based on level sets –for nonlinear Hamilton-Jacobi and scalar hyperbolic PDEs that can be performed with quantum advantages on various critical numerical parameters, even for computing the physical observables, for arbitrary nonlinearity and are valid globally in time. These PDEs are important for many applications like optimal control, machine learning, semi-classical limit of Schrodinger equations, mean-field games and many more.

Depending on the details of the initial data, it can display up to exponential advantage in both the dimension of the PDE and the error in computing its observables. For general nonlinear PDEs, quantum advantage with respect to M, for computing the ensemble averages of solutions corresponding to M different initial data, is possible in the large M limit.

We will also propose quantum algorithms for uncertainty quantification of partial differential equations with quantum advantage over M.

Presenter: JIN, Shi

Contribution ID: **25** Type: **not specified**

High order structure preserving Generalized Finite Difference schemes for Particle In Cell simulations on structured grids

Friday, December 16, 2022 9:00 AM (45 minutes)

Semi-Discrete schemes for the Vlasov-Maxwell equations based on a Particle in Cell method combined with techniques from Finite Element Exterior Calculus yield a finite dimensional hamiltonian system which after time discretization with classical geometric numerical methods have very good long time properties and preserve exactly important invariants of the system. We will propose in this talk a generalized framework which can be used in particular to derived hamiltonian systems also with an appropriate Generalized Finite Difference Framework. A performance portable implementation of this scheme aimed at very large simulations will be presented.

Presenter: SONNENDRUCKER, Eric

Contribution ID: **26** Type: **not specified**

Numerical schemes for the collisional Vlasov equation in the finite Larmor radius regime

Friday, December 16, 2022 9:45 AM (45 minutes)

This work is devoted to the construction of multiscale numerical schemes efficient in the finite Larmor radius approximation of the collisional Vlasov equation. Following the paper of Bostan and Finot (2019), the system involves two different regimes, a highly oscillatory and a dissipative regimes, whose asymptotic limits do not commute. In this work, we consider a Particle-In-Cell discretization of the collisional Vlasov system which enables to deal with the multiscale characteristics equations. Different multiscale time integrators are then constructed and analysed. We prove asymptotic properties of these schemes in the highly oscillatory regime and in the collisional regime. In particular, the asymptotic preserving property towards the modified equilibrium of the averaged collision operator is recovered. Numerical experiments are then shown to illustrate the properties of the numerical schemes.
This work has been performed in collaboration with Nicolas Crouseilles and Damien Prel.

Presenter: CRESTETTO, Anaïs

Contribution ID: **27** Type: **not specified**

Entropy stable, positivity preserving and well-balanced Godunov-type schemes for multidimensional hyperbolic system of equations

Friday, December 16, 2022 10:50 AM (45 minutes)

A class of cell centered Finite Volume schemes has been introduced to discretize the equations of Lagrangian hydrodynamics on moving mesh [4]. In this framework, the numerical fluxes are evaluated
by means of an approximate Riemann solver, based at the nodes of the mesh, which provides the nodal velocity required to move the mesh in a compatible manner. In this presentation, we describe
the generalization of this type of discretization to hyperbolic systems of conservation laws written in
Eulerian representation. The evaluation of the numerical fluxes relies on an approximate Riemann solver located at the mesh nodes. The construction of this nodal solver uses the Lagrange-to-Euler transformation introduced by Gallice [3] and revisited in [1,2] to build positive and entropic Eulerian
Riemann solvers from their Lagrangian counterparts. The application of this formalism to the case of
gas dynamics provides a positive and entropic finite volume scheme under an explicit condition on the
time step. In this work we extend this scheme to handle source terms, in particular for the Shallow Water system of equations. We show how to render the scheme well-balanced in 1D and 2D. The numerical assessment of this scheme by means of representative test cases will be presented for the first
and second orders. In particular the good behavior is illustrated by the absence of the typical numerical
pathology of traditional Finite Volume approaches for such system of PDEs.

References
[1] A. Chan, G. Gallice, R. Loubère and P.-H Maire, Positivity preserving and entropy consistent approximate Riemann solvers dedicated to the high-order MOOD-based Finite Volume discretization of Lagrangian and Eulerian gas dynamics. Computers & Fluids, 2021, 229.
[2] G. Gallice, A. Chan, R. Loubère and P.-H Maire, Entropy stable and positivity preserving Godunov-type schemes for multidimensional hyperbolic systems on unstructured grid. J. Comput. Phys, 2022, 468.
[3] G. Gallice, Positive and Entropy Stable Godunov-Type Schemes for Gas Dynamics and MHD Equations in Lagrangian or Eulerian Coordinates. Numer. Math., 2003, 94.
[4] P.-H Maire, A high-order cell-centered Lagrangian scheme for two-dimensional compressible fluid flows on unstructured meshes. J.Comput. Phys., 2009, 228.

Presenter: LOUBERE, Raphaël