WORKSHOP
Coastal flow models and boundary conditions
Toulouse, October 24th - 27th 2022

Schedule

Monday October 24th

14h00 - 15h30	Jean-François Coulombel (CNRS, Toulouse) *Large time stability issues for finite difference schemes.* I
15h30 - 16h00	break
16h00 - 17h30	Vincent Duchêne (CNRS, Rennes) *Derivation and justification of shallow water models for surface gravity waves and stratified flows.* I

Tuesday October 25th

9h00 - 9h50	Geoffrey Beck (INRIA, Rennes) *Waves - floating structure interactions in Boussinesq regime*
9h50 - 10h40	Ali Haidar (Université de Montpellier) *Numerical simulation of shallow-water and floating object nonlinear interactions*
10h40 - 11h20	break
11h20 - 12h10	Éric Blayo (Université Grenoble Alpes) *Quelques aspects mathématiques et numériques dans la représentation des interactions océan-atmosphère*
12h15 - 14h00	lunch
14h00 - 15h30	Anne Mangeney (Université Paris Cité & Institut Universitaire de France) *Seismic waves: a unique source of information on glaciers and landslides.* I
15h30 - 16h00	break
16h00 - 17h30	Anne Mangeney (Université Paris Cité & Institut Universitaire de France) *Seismic waves: a unique source of information on glaciers and landslides.* II
19h30	social dinner at “Du plaisir à la toque”

Wednesday October 26th

9h00 - 9h50	Corentin AUDIARD (Sorbonne Université, Paris) *Sharp regularity results for solutions of boundary value problems*
9h50 - 10h40	Aric WHEELER (Indiana University, Bloomington) *Nonlinear damping vs. formation of singularities*
10h40 - 11h20	break
11h20 - 12h10	David CHIRON (Université Côte d'Azur, Nice) *Smooth branches of travelling waves for the 2D nonlinear Schrödinger equation*
12h15 - 14h00	lunch
14h00 - 15h30	Grégory FAYE (CNRS, Toulouse) *Large time stability issues for finite difference schemes.* II
15h30 - 16h00	break
16h00 - 17h30	Vincent DUCHÊNE (CNRS, Rennes) *Derivation and justification of shallow water models for surface gravity waves and stratified flows.* II

Thursday October 27th

9h00 - 9h50	Pierre LE BARBENCHON (Université de Rennes 1) *Numerical proof of stability for finite difference schemes in a finite space domain*
9h50 - 10h40	Maria KAZAKOVA (Université Savoie Mont-Blanc, Chambéry) *PML methods for mixed hyperbolic-dispersive equations*
10h40 - 11h20	break
11h20 - 12h10	Martin PARISOT (INRIA, Bordeaux) *On the "projection" structure of the Green-Naghdi equations*
12h15 - 14h00	lunch

Abstracts. Mini-courses

Jean-François COULOMBEL & Grégory FAYE (CNRS, Toulouse): *Large time stability issues for finite difference schemes*

The lectures will be devoted to proving generalized Gaussian bounds for the large time behavior of finite difference approximations of the transport equation. The goal is to review the connection with the central limit theorem in probability theory, some history of the problem and to present the general methodology leading to sharp estimates for the Green's function. In the second lecture, we shall present some sharp estimates for the Green's function of finite difference schemes that approximate the incoming transport equation on a half-line. The stability analysis relies on a precise description of the possible eigenvalues arising from the interplay between the finite difference scheme and the numerical boundary conditions.

Vincent Duchêne (CNRS, Rennes): *Derivation and justification of shallow water models for surface gravity waves and stratified flows*

We shall discuss how standard models in oceanography can be rigorously justified as asymptotic models, here in the shallow water regime, focusing on the general strategy and mathematical tools which are involved. Most of the discussion will be devoted to the justification of the Saint-Venant system for homogeneous potential flows with a free surface. Yet going through this somewhat standard procedure will allow us to unveil, by contrast, how much is yet to be understood when we venture into non-hydrostatic models or heterogeneous flows.

Anne Mangeney (Université Paris Cité & Institut Universitaire de France): *Seismic waves: a unique source of information on glaciers and landslides*

I will show here how the coupling between numerical modelling of glaciers and landslides associated with the analysis of the generated seismic waves makes it possible to recover unique information on the source processes. These waves provide a unique tool to constrain the models by providing detailed information on the dynamics of landslides and iceberg calving. This approach allows quantifying in particular the location and volume of events and relate landslide or iceberg calving activity with external forcing such as seismic, volcanic or climatic activity, with strong implication on hazard assessment.

Abstracts. Talks

Corentin Audiard (Sorbonne Université, Paris): *Sharp regularity results for solutions of boundary value problems*

We study the well-posedness of initial boundary value problems for the linear Schrödinger equations on a half space. The boundary data lie in a (allegedly optimal) Bourgain type Sobolev space, which allows to include Neuman and transparent boundary conditions in the analysis. Strichartz estimates (in L^2) are obtained thanks to an explicit solution formula. In the case of Dirichlet boundary data, the regularity of solutions is obtained provided natural compatibility conditions are satisfied. The regularity results concern fractional regularity, and include the more delicate case where the initial data are in $H^{1/2}$. The proof of regularity uses an interpolation argument that can be applied to other BVP.

Geoffrey Beck (INRIA, Rennes): *Waves - floating structure interactions in Boussinesq regime*

In the context of nearshore wave energy facilities, we have tackled, with David Lannes and Lisl Weynans, the interaction of waves with a floating structure immersed in a 2D fluid. Some difficulties come from the presence of several surfaces: the surface of the sea and the contact surface between the structure and the fluid. The horizontal plane is decomposed into two regions: the exterior region where the surface of the fluid is in contact with the air, and the interior region where it is in contact with the bottom of the object. In the exterior region, we have the standard wave equations, where the surface is free but the pressure is constrained (equal to the atmospheric pressure). In the interior region, the opposite happens: the pressure is free but the surface is constrained, which changes the structure of the equations. Finally, coupling conditions between both regions are needed. We show how to implement this program in the case where the waves are described by the nonlinear dispersive Boussinesq equations. The difficulties related to the computation of the contact surface are overcome by considering an augmented formulation: in addition to the usual equations, we find two hidden ODEs of the water column at the contact points between the waves and the structure. Finally, we propose a numerical method that exploits the added-mass effect, the dispersive boundary layer and this two hidden ODEs.

Éric Blayo (Université Grenoble Alpes): *Quelques aspects mathématiques et numériques dans la représentation des interactions océan-atmosphère*

Les interactions océan-atmosphère (OA) jouent un rôle important dans de nombreux phénomènes, tels que cyclones tropicaux ou dynamique du climat. La représentation de ces interactions au sein d'un système de modélisation OA consiste principalement à évaluer les flux échangés entre les deux milieux, et à les imposer à l'interface air-mer. Cet exposé abordera quelques questions mathématiques et numériques liées à ces deux aspects et présentera une synthèse des travaux récents menés sur ces thématiques dans notre équipe au Laboratoire Jean Kuntzmann : modélisation des couches limites de surface de part et d'autre de l'interface air-mer, schémas de discrétisation au voisinage de cette interface, algorithmique de couplage.

David Chiron (Université Côte d'Azur, Nice): *Smooth branches of travelling waves for the 2D nonlinear Schrödinger equation*

We shall present some results on the existence of smooth branches of travelling waves for the 2D Nonlinear Schrödinger equation parametrized by the speed. In the limit of small speed (joint works with E. Pacherie), the travelling wave has two well separated vortices and we prove that these are the only minimizers of the energy for fixed momentum. In the limit where the speed is close to the speed of sound, we obtain rarefaction pulses described by rational lump solutions to the KP-I equation.

Ali Haidar (Université de Montpellier): *Numerical simulation of shallow-water and floating object nonlinear interactions*

In this work, a novel numerical algorithm is introduced for the study of nonlinear interactions between free-surface shallow-water flows and a partly immersed floating object. The object's motion may be either prescribed, or computed as a response to the hydrodynamic forcing. Away from the object, the nonlinear hyperbolic shallow-water equations are used, while the description of the flow beneath the floating object reduces to a nonlinear algebraic equation for the free-surface, together with a nonlinear ordinary differential equation for the computation of the discharge. In both domains, the model accounts for the possible topography variations. When a free-motion is allowed (with heaving, surging and pitching in the horizontal one-dimensional case), this set of equations has to be supplemented with the Newton's second law for the object's motion, involving the force and torque applied over the object by the surrounding fluid, and a part of this external forcing is regarded as an added-mass effect, in order to benefit from its stabilizing influence. At the discrete level, we introduce a discontinuous Galerkin (DG) approximation relying on some arbitrary-order polynomials. This DG method is stabilized with a recent *a posteriori* Local Subcell Correction method through Finite-Volume, in the vicinity of the solution's singularities. The motion of the water-object contact-points is described with the help of an Arbitrary-Lagrangian-Eulerian description. We show that the discontinuous nature of the chosen DG approximation leads to a very natural coupling between the exterior and interior flows, resulting in a global method which ensures the preservation of the water-height positivity at the sub-cell level, preserves the class of motionless steady-states (well-balancing) and preserves the Geometric Conservation Law. Several numerical computations, involving well-balancing, prescribed motions and their impact on the surrounding fluid, or the nonlinear interactions between the object and surface waves, are investigated and highlight that our numerical model effectively allows to model the wave-floating body interactions, with a robust computation of the air-water-body contact-points evolution, as well as of the possible strong flow singularities, and retains the highly accurate sub-cell resolution of DG schemes.

Maria Kazakova (Université Savoie Mont-Blanc, Chambéry): *PML methods for mixed hyperbolic-dispersive equations*

The classical PML approach is first applied to the linearised Korteweg-de Vries equation. These equations are not always stable, the main obstruction being the classical condition found in the literature on PMLs that we recover in our analysis. We introduce two alternative strategies to design absorbing boundary conditions. We start from studying hyperbolic relaxation of the Korteweg-de Vries equation. In this case, the complete PML equations are not, again, completely stable. However, a version of the PML equations for this system derived without the source term is found to be stable and can absorb outgoing wave. Finally, we consider BBM-Boussinesq system that model bi-directional waves at the surface of an inviscid fluid layer. We show that the PML equations are always stable in this case. We illustrate numerically stability properties of diferents PML models. This talk is based on recent joint work with Christophe Besse, Sergey Gavrilyuk and Pascal Noble.

Pierre Le Barbenchon (Université de Rennes 1): *Numerical proof of stability for finite difference schemes in a finite space domain*

The goal of this talk is to study the stability of finite differences schemes for scalar hyperbolic initial boundary value problem. It is based on the GKS theory (introduced by Gustafsson, Kreiss and Sundström) and it deals with the Kreiss-Lopantinskii determinant. We will use complex analysis and geometric consideration to find zeros of this determinant and conclude on the stability of the scheme.

Martin Parisot (INRIA, Bordeaux): *On the "projection" structure of the Green-Naghdi equations*

Many reduced models of the water wave equations, in particular the Green-Naghdi equations, can be presented as a "projection" of the shallow water equations onto a set of admissible functions. We will see why we use quotation marks for this property, and how to use it to its advantage for many numerical and modeling problems (entropic stability, boundary conditions, balanced, HPC, coupling, dispersion relation).

Aric Wheeler (Indiana University, Bloomington): *Nonlinear damping vs. formation of singularities*

We explore the boundaries of damping estimates by comparing and contrasting two closely related models of combustion, the Majda and ZND models. We show that singularities form in the unweighted Lipschitz norm in finite time on both sides of the shock for both models, extending classical results of John and Liu to suitable variable coefficient systems. On the other hand, we show some energy estimates on the Majda model in exponentially weighted norms, which allows us to obtain an orbital asymptotic stability result, and also that the ZND model does not admit such estimates. This work is joint with Paul Blochas.

Practical information

• All talks will take place in Amphithéâtre Laurent Schwartz at the Institut de Mathématiques de Toulouse (building 1R3, subway station *Université Paul Sabatier* on Toulouse metro line B).

Toulouse public transportation system : https://www.tisseo.fr/

Université Paul Sabatier campus map : https://www.univ-tlse3.fr/acces-campus

• Lunches will be taken at the restaurant L'esplanade (allée Louis Lareng, at the entrance of the Université Paul Sabatier campus).

• Social dinner will take place on Tuesday evening at the restaurant Du plaisir à la toque in Toulouse, close to the place de la Daurade (9 rue des blanchers, 31000 Toulouse, 05 61 21 88 94).