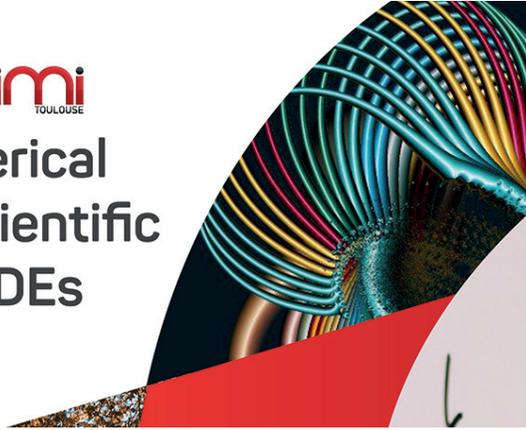


Thematic Semester **cimi**
TOULOUSE

Modeling, numerical analysis and scientific computing in PDEs



Workshop Parabolic & kinetic models in population dynamics

- Dates: 26th to 30th of September, 2022
- Location: Amphithéâtre Laurent Schwartz, building 1R3, Institut de Mathématiques de Toulouse
- Webpage: <https://indico.math.cnrs.fr/event/7589/>
- Local scientific contacts:
Grégory Faye (gfaye@math.univ-toulouse.fr)
Ariane Trescases (ariane.trescases@math.univ-toulouse.fr)
- Local administrative contacts:
Isabelle Guichard (isabelle.guichard@math.univ-toulouse.fr)
Elvire Jalran (elvire.jalran@math.univ-toulouse.fr)
Thanh Hà Lê (thanh_ha.le@math.univ-toulouse.fr)
- Organizers: Grégory Faye, Thomas Giletti, Léo Girardin, Quentin Griette, Ariane Trescases & Alessandro Zilio
- Financial support: Labex CIMI & ANR project Indyana
- Working room: 106 in building 1R1

Scientific Program

Monday 26

- 1pm30 / 1pm45: opening welcome
- 1pm45-2pm35 : Susanna Terracini
- 2pm35-3pm25 : Christopher Henderson
- 3pm25-3pm55: coffee break
- 3pm55-4pm45: Romain Ducasse
- 4pm45-5pm35: Cécile Carrère

Tuesday 27

- 9am-9am50: Laurent Desvilletes
- 9am50-10am40: Hélène Hivert
- 10am40-11h10: coffee break
- 11am10-12am: Maxime Breden
- 12am/1pm45 : lunch break @ L'Esplanade
- 1pm45-2pm35 : Hyunjoon Park
- 2pm35-3pm25 : Angela Stevens
- 3pm25-3pm55: coffee break
- 3pm55-4pm45: Elisa Affili
- 4pm45-5pm35: François Hamel

Wednesday 28

- 9am-9am50 : Catherine Choquet
- 9am50-10am40: Idriss Mazari-Fouquer
- 10am40-11h10: coffee break
- 11am10-12am: Elaine Crooks
- 12am15 : buffet
- free afternoon

Thursday 29

- 9am-9am50 : Chris Cosner
- 9am50-10am40: Noemi David
- 10am40-11h10: coffee break
- 11am10-12am: Jimmy Garnier
- 12am/1pm45 : lunch break @ L'Esplanade
- 1pm45-2pm35 : Danielle Hilhorst
- 2pm35-3pm25 : Matt Holzer
- 3pm25-3pm55: coffee break
- 3pm55-4pm45: Mingmin Zhang
- 4pm45-5pm35: Sepideh Mirrahimi
- 7pm30 : conference diner @ Côté Garonne

Friday 30:

- 9am-9am50 : Raluca Eftimie
- 9am50-10am40: Pierre Magal
- 10am40-11h10: coffee break
- 11am10-12am: Ayman Moussa
- 12am/2pm : lunch break @ L'Esplanade
- 2pm-3pm : Vincent Calvez (colloquium of IMT)

Book of abstracts

- [Elisa Affli](#) (University of Bologna)

Title: Controllability in Lotka-Volterra competitive systems with positive coefficients

Abstract: We are interested in controlling the asymptotic behaviour of two competing species in an interval by controlling the size of the populations at the boundary. In particular, we want to know if it is possible to eradicate one of the species. Since the boundary controls have to satisfy positivity constraints, classic techniques in control theory cannot be applied. In this talk, we will describe non-controllability phenomena due to the presence of barrier solutions depending on the length of the interval and the competition coefficients of the systems. This is joint work with Enrique Zuazua.

- [Maxime Breden](#) (CMAP, Ecole Polytechnique)

Title: About the many equilibria of some cross-diffusion systems in population dynamics

Abstract: Cross-diffusion is a mechanism that can be used in population dynamics to model attractive or repulsive effects between individuals. Mathematically, this corresponds to adding nonlinear coupled diffusion terms to classical reaction-diffusion systems. Cross-diffusion can generate a rich variety of patterns, showcasing concentration or segregation phenomena, but it also complicates the mathematical analysis. In this talk, I will explain how such problems can be tackled by combining numerical simulations with a posteriori estimates, to obtain computer-assisted proofs, and I will illustrate this approach on two examples: the SKT system and a specific Keller-Segel model.

- [Vincent Calvez](#) (CNRS, Institut Camille Jordan)

Title: A mathematical walk through evolutionary biology

Abstract: Evolutionary biology was formalized early on using mathematical reasoning and models, both deterministic and random. In this talk I will present some recent progress on questions of quantitative genetics, when the population undergoing evolution is described by a continuous trait (phenotype). The main thread of the talk will be the asymptotic analysis of PDE or integro-differential models that are widely studied in the evolutionary biology community, revisited with "modern" tools. In particular, I will show a fruitful analogy with the semi-classical analysis in the regime where the diversity in the population is low. This analogy allows to address a large number of study cases, for example when populations are distributed in heterogeneous habitats (from the point of view of selection), when the environment changes over time, etc...

On the mathematical side, this will be an opportunity to present results of PDE, stochastic processes, and numerical analysis.

- [Cécile Carrère](#) (Institut Denis Poisson, Université d'Orléans)

Title: How To force biphasic life cycles

Abstract: Host-microbe interactions are common in nature, but little is known about adaptive strategies for microbes that lead to host association: since bacteria evolve usually faster than their host, they would drive the evolution from monophasic life cycles to biphasic life cycles. Experimental work by N.Obeng, H.Schulenburg et al have identified microbial traits that consistently increase host-microbe association for initially monophasic bacteria. In an ongoing collaboration with F.Bansept, we present an ODE model of complex host life cycle that promotes the evolution of biphasic or host-specialist bacteria. I will also present how a PDE model of structured populations with complex host life cycles could promote biphasic bacteria.

- Catherine Choquet (LMIA, Université de La Rochelle)

Title: TBA

Abstract: TBA

- Chris Cosner (Department of Mathematics, University of Miami)

Title: Reaction-diffusion-advection models with multiple movement modes

Abstract: Classical reaction-diffusion-advection models for population dynamics with dispersal assume that all individuals move in the same way all the time. Actually, animals may switch between faster movement when searching for resources and slower movement while exploiting them, or juveniles may move differently than adults. To describe such situations requires systems of reaction-diffusion-advection equations. The resulting systems may have features different from single equations. In models based on logistic equations, the systems may be cooperative at low densities but competitive at high densities. It is well known that for a single diffusive logistic equation in a static spatially heterogeneous bounded domain, slower diffusion is advantageous. For stage structured populations where adults and juveniles have different environmental needs this is no longer always the case. This talk will describe some recent work on the theory and applications of reaction-diffusion-advection models for populations in bounded habitats where subpopulations may have different movement rates or patterns, and individuals can switch between subpopulations by behavior or contribute to them by reproduction and aging.

- Elaine Crooks (Department of Mathematics, Swansea University)

Title: Travelling waves and minimality exchange in smectic C* liquid crystals

Abstract: We consider minimality conditions for the speed of monotone travelling waves in a model of a sample of smectic C* liquid crystal subject to a constant electric field, dealing with both isotropic and anisotropic cases. Such conditions are important in understanding switching properties of a liquid crystal, and our focus is on understanding how the presence of anisotropy can affect the speed

and nature of switching. Through a study of travelling-wave solutions of a quasilinear parabolic equation, we obtain an estimate of the influence of anisotropy on the minimal speed, and sufficient conditions for linear and non-linear minimal speed selection mechanisms to hold in different parameter regimes. We also discuss sufficient conditions for so-called ‘minimality exchange?’ in a general class of parameter-dependent monostable reaction-diffusion equations with explicit travelling-wave solutions, when the minimal wave speed switches from the linearly determined value to the speed of the explicitly determined front as a parameter changes. This is joint work with Michael Grinfeld and Geoff McKay (Strathclyde).

- Noemi David (Laboratoire Jacques-Louis Lions, Sorbonne Université)

Title: Incompressible limit and rate of convergence for tumor growth models with a drift

Abstract: Both compressible and incompressible porous medium models have been used in the literature to describe the mechanical aspects of living tissues. Using a stiff pressure law, it is possible to build a link between these two different representations. In the incompressible limit, compressible models generate free boundary problems of Hele-Shaw type where saturation holds in the moving domain. In this talk, I will present the study of the incompressible limit for advection-porous medium equations motivated by tumor development. The derivation of the pressure equation in the stiff limit was an open problem for which the strong compactness of the pressure gradient is needed. To establish it, we use two new ideas: an L^3 -version of the celebrated Aronson-Bénilan estimate, also recently applied to related problems, and a sharp uniform L^4 -bound on the pressure gradient. Moreover, we provide an estimate of the convergence rate at the incompressible limit in a Sobolev negative norm.

- Laurent Desvillettes (IMJ-PRG, Université Paris Cité)

Title: Existence and regularity for cross diffusion equations coming out of population dynamics

Abstract: The SKT (Shigesada Kawasaki Teramoto) model was introduced at the end of the 70s for explaining situations of spatial heterogeneity in biology, when no heterogeneity of the data are assumed, and when “standard” Turing instability does not occur. It consists of a system of two cross-diffusion equations including logistic-type reaction terms. We will discuss the theory of weak and strong solutions for this model, focusing on the so-called “triangular” case. We will also compare it to other parabolic systems with linear and quadratic terms.

- Romain Ducasse (Laboratoire Jacques-Louis Lions, Université Paris Cité)

Title: Propagation properties in a multi-species SIR reaction-diffusion system

Abstract: We consider a multi-species reaction-diffusion system that arises in epidemiology to describe the spread of several strains, or variants, of a disease in a population. Our model is a natural spatial, multi-species, extension of the classical SIR model of Kermack and McKendrick. First, we study the long-time behavior of the solutions and show that there is a “selection via propagation”

phenomenon: starting with N strains, only a subset of them - that we identify - propagates and invades space, with some given speeds that we compute. Then, we obtain some qualitative properties concerning the effects of the competition between the different strains on the outcome of the epidemic. In particular, we prove that the dynamic of the model is not well characterized by the usual notion of basic reproduction number, which strongly differs from the classical case with one strain.

This is a joint work with S. Nordmann.

- Raluca Eftimie (Laboratoire de Mathématiques de Besançon, Université de Franche-Comté)

Title: Computational and analytical approaches for pattern formation in nonlocal hyperbolic systems for biological aggregations

Abstract: Bifurcation theory has been used for decades to investigate transitions between different states displayed by a variety of fluid dynamics models. This also led to the identification of different spatio-temporal structures: from rotating waves to heteroclinic cycles, and even snake-and-ladder structures. Here I will focus on the application of computational and analytical approaches to investigate patterns exhibited by a class of nonlocal hyperbolic models derived to describe biological aggregations: from foraging ungulates, to zigzagging flocks of birds, or rippling waves in myxobacteria. In particular, I will discuss pattern formation and transitions between different types of patterns as a result of various local and global bifurcations.

- Jimmy Garnier (CNRS, Université de Savoie Mont-Blanc)

Title: Invasion and coexistence among mutualistic community

Abstract: The arbuscular mycorrhizal fungi are a common example of mutualist community which helps plants to extend their root and thus increase their biomass. For over a decade, they have been used as organic fertilizers but what is the impact of this inoculated species on the wild type community? In this presentation, I will present a new mathematical model based on systems of partial differential equations. It has been developed in collaboration with M. Martignoni, R. Tyson and M. Hart. I will investigate the existence of coexistence steady state and provide some analytical criteria. Then I will focus on the spatial invasion of a community of fungi over another. For this purpose, I will analyze the existence of traveling wave solutions and I will try to characterize their spreading speed.

- François Hamel (Institut de Mathématiques de Marseille, Aix-Marseille Université)

Title: Spreading speeds and one-dimensional symmetry for reaction-diffusion equations in \mathbb{R}^N

Abstract: The talk will focus on the large-time dynamics of bounded solutions of reaction-diffusion equations in \mathbb{R}^N with unbounded initial support. I will discuss the existence of spreading sets and Freidlin-Gärtner formulas for the spreading speeds of the solutions in any direction, in connection

with the existence of planar traveling waves. I will also explain some results on the asymptotic one-dimensional symmetry of the elements of the Ω -limit set of the solutions, in the spirit of a famous De Giorgi conjecture on solutions of some elliptic equations in \mathbb{R}^N . The talk is based on joint works with Luca Rossi.

- Chris Henderson (Department of Mathematics, University of Arizona)

Title: The shape defect function and stability of traveling waves

Abstract: In their original paper, Kolmogorov, Petrovsky, and Piskunov demonstrated stability of the minimal speed traveling wave with an ingenious argument based on, roughly, the decreasing steepness of the profile. This proof is extremely flexible, yet entirely not quantitative being based on compactness. On the other hand, more modern PDE proofs of the stability of traveling waves solutions to reaction-diffusion equations are highly tailored to the particular equation, fairly complicated, and often not sharp in terms of the rate of convergence. In this talk, I will introduce a natural quantity, the shape defect function, that allows a simple approach to quantifying convergence to the traveling wave for a large class of reaction-diffusion equations, including both pushed, pulled, and pushmi-pullyu equations. This is a joint work with Jing An and Lenya Ryzhik.

- Danielle Hilhorst (CNRS, Laboratoire de Mathématiques d'Orsay)

Title: Convergence to a self-similar profile for a one dimensional one phase Stefan problem

Abstract: Steel corrosion plays a central role in different technological fields. We consider a simple case of a corrosion phenomenon which describes a pure iron dissolution in sodium chloride. We prove that under rather general hypotheses on the initial data, the solution of this iron dissolution model converges to a self-similar profile as time tends to infinity. To that purpose, we perform a change of coordinates to self-similar variables and apply a comparison principle together with suitable upper and lower solutions.

- Hélène Hivert (ECL, Institut Camille Jordan)

Title: Numerical schemes for concentration phenomena in Lotka-Volterra equations

Abstract: We consider a population structured in phenotypic trait, which influences the adaptation of individuals to their environment. Each individual has the trait of his parent, up to small mutations. When considered in a regime of long time and small mutation, and with appropriate hypothesis, the distribution of the population is expected to concentrate at some dominant traits. Dominant traits can also evolve in time, thanks to mutations. From a technical point of view, the concentration phenomenon is described thanks to a Hopf-Cole transform in the model. The asymptotic regime is a constrained Hamilton-Jacobi equation. Because of the lack of regularity of the constraint, it can indeed have jumps, its numerical approximation must be carefully discussed. We propose a

framework for the discretization of the limit equation, and asymptotic-preserving schemes for the original problem transformed with Hopf-Cole.

- Matt Holzer (Department of Mathematical Sciences, George Mason University)

Title: Pushed-to-pulled front transitions: continuation, speed scalings, and hidden monotonicity

Abstract: We analyze the transition between pulled and pushed fronts both analytically and numerically from a model-independent perspective. Based on minimal conceptual assumptions, we show that pushed fronts bifurcate from a branch of pulled fronts with an effective speed correction that scales quadratically in the bifurcation parameter. Strikingly, we find that in this general context without assumptions on comparison principles, the pulled front loses stability and gives way to a pushed front when monotonicity in the leading edge is lost. Our methods rely on far-field core decompositions that identify explicitly asymptotics in the leading edge of the front. We show how the theoretical construction can be directly implemented to yield effective algorithms that determine spreading speeds and bifurcation points with exponentially small error in the domain size. Example applications considered here include an extended Fisher-KPP equation, a Fisher-Burgers equation, negative taxis in combination with logistic population growth, an autocatalytic reaction, and a Lotka-Volterra model.

- Pierre Magal (Institut de Mathématiques de Bordeaux, Université de Bordeaux)

Title: Logistic equations with non-local and non-linear convection: a model for cells motion

Abstract: This presentation summarizes results on cell movement in a Petri dish. We are interested in the finite speed displacement of cells in a dish. We aim to propose a model that captures the main ingredients to reproduce the complex pattern formed by multiple islets when two types of cells are co-cultured in a Petri dish. Due to the non-local and non-linear convection term, several mathematical difficulties arise. We will start with some convergence results in the sense of Young measures topology. We will also present some Turing and Turing-Hopf bifurcations results for a single equation (i.e., a single species of cells). In the last part of the talk, we will turn to the case of the Keller-Segel hyperbolic equation. We will present some numerical simulations for the co-culture of cells. We will conclude the talk with some results about the existence of traveling waves.

- Idriss Mazari-Fouquer (CEREMADE, Paris Dauphine)

Title: Some optimal control & game theoretical problems in spatial ecology

Abstract: In this talk, we survey the qualitative conclusions of several recent studies concerning spatial heterogeneity in population dynamics. Using the heterogeneous logistic-diffusive equation as a paradigmatic model, we investigate two types of problems.

1) The first one deals with the optimal location of resources: how should resources be spread inside a domain in order to optimise certain criteria? Motivating our analysis by a study of the optimal

survival ability, we shall present some qualitative properties recently obtained that show surprising phenomena.

2) The second one focuses on a game theoretical problem in population dynamics: assume that two players are fishing from the same population (say, fishes). Is it possible for these two players to find an equilibrium strategy? We will present some results showing that there sometimes exist Nash equilibria when the fishermen are not fishing “too much”.

The talk will be mostly descriptive, and is based on collaborations with Grégoire Nadin, Yannick Privat and Domènec Ruiz-i-Balet.

- Sepideh Mirrahimi (CNRS, Institut Montpelliérain Alexander Grothendieck)

Title: Filling the gap between individual-based evolutionary models and Hamilton-Jacobi equations

Abstract: Long-term ecological or evolutionary dynamics may be strongly influenced by small sub-populations and local extinction in particular areas of (physical or phenotypical) space. Deterministic models which are derived as large population approximations of stochastic individual based models do not usually take into account such small subpopulation effects. We address this question in the study of eco-evolutionary dynamics of populations in a small mutational variance regime. In this regime, Hamilton-Jacobi equations have been emerged from asymptotic analyses of integro-differential evolutionary models, which are themselves derived from large population approximations of stochastic individual based models. In this work, we derive such a Hamilton-Jacobi equation, directly from a stochastic individual based model. This derivation allows a better understanding of the results obtained by the Hamilton-Jacobi approach and would lead to a rectification of the approach taking into account possible extinctions of sub-populations. This is a joint work with N. Champagnat, S. Méléard and V. C. Tran.

- Ayman Moussa (LJLL, Sorbonne Université)

Title: Stability, weak-strong uniqueness and derivation of the SKT system

Abstract: Several strategies are possible to derive (rigorously) the SKT system. Following an approach proposed by Daus, Desvillettes and Dietert in 2019 we will explain how this system can be rigorously linked to repulsive random walks, using a stability estimate which, as a by-product, implies a weak-strong uniqueness result for the SKT system. The main part of the talk will focus on a joint work with Vincent Bansaye and Felipe Muñoz-Hernández.

- Hyunjoon Park (Meiji University)

Title: Singular limit of stochastic Allen-Cahn equation with nonlinear diffusivity

Abstract: In this talk I give a talk about the stochastic Allen-Cahn equation with nonlinear diffusivity. I will present how the ?nonlinear diffusivity? and the ?stochastic perturbation? affect the

motion of the interface. To see this, I first give a formal asymptotic expansion to give an intuitive insight about the motion. After, with the intuition obtained during the formal expansion, I'll give a brief sketch of the result. This talk is based on a joint work with Danielle Hilhorst, Perla El Kettani and Youngjung Kim.

- Angela Stevens (University of Münster)

Title: Kermack-McKendrick models on a two-scale network, connections to the Boltzmann equations and a free boundary problem in time for the spread of Covid-19

Abstract: The Kermack-McKendrick models in epidemiology are often misinterpreted solely as the well-known SIR, ODE-system for the dynamics of susceptibles, infectious and removed. But McKendricks equations are by far more general. His systems can be adapted to cover a small world - large world scenario, with the small world having a different infection mechanism, they can be adapted to several variants of viruses competing, and even the classical Boltzmann equations can be written in McKendrick form.

In the second part of the talk two aspects of the Covid epidemic are discussed. The first is a phase change during the epidemic. The second aspect is the two scale nature of the infection network. (Joint works with Stephan Luckhaus)

- Susanna Terracini (Dipartimento di Matematica Giuseppe Peano, Università degli Studi di Torino)

Title: Rotating Spirals in segregated reaction-diffusion systems

Abstract: We give a complete characterization of the boundary traces φ_i ($i = 1, \dots, K$) supporting spiraling waves, rotating with a given angular speed ω , which appear as singular limits of competition-diffusion systems of the type

$$\begin{cases} \partial_t u_i - \Delta u_i = \mu u_i - \beta u_i \sum_{j \neq i} a_{ij} u_j & \text{in } \Omega \times \mathbb{R}^+ \\ u_i = \varphi_i & \text{on } \partial\Omega \times \mathbb{R}^+ \\ u_i(\mathbf{x}, 0) = u_{i,0}(\mathbf{x}) & \text{for } \mathbf{x} \in \Omega \end{cases}$$

as $\beta \rightarrow +\infty$. Here Ω is a rotationally invariant planar set and $a_{ij} > 0$ for every i and j . We tackle also the homogeneous Dirichlet and Neumann boundary conditions, as well as entire solutions in the plane. As a byproduct of our analysis we detect explicit families of eternal, entire solutions of the pure heat equation, parameterized by $\omega \in \mathbb{R}$, which reduce to homogeneous harmonic polynomials for $\omega = 0$.

It is a joint work with A. Salort, G. Verzini and A. Zilio

- Mingmin Zhang (Institut de Mathématiques de Toulouse, Université Paul Sabatier)

Title: Reaction-diffusion fronts in funnel-shaped domains

Abstract: We consider bistable reaction-diffusion equations in funnel-shaped domains of \mathbb{R}^N made up of straight parts and conical parts with positive opening angles. We study the large time dynamics of entire solutions emanating from a planar front in the straight part of such a domain and moving into the conical part. We show a dichotomy between blocking and spreading, by proving especially some new Liouville type results on stable solutions of semilinear elliptic equations in the whole space \mathbb{R}^N . We also show that any spreading solution is a transition front having a global mean speed, which is the unique speed of planar fronts, and that it converges at large time in the conical part of the domain to a well-formed front whose position is approximated by expanding spheres. Moreover, we provide sufficient conditions on the size R of the straight part of the domain and on the opening angle α of the conical part, under which the solution emanating from a planar front is blocked or spreads completely in the conical part. We finally show the openness of the set of parameters $(R; \alpha)$ for which the propagation is complete. This is a joint work with Prof. François Hamel.

Conference diner menu – Thursday 29th

- Restaurant: Côté Garonne
- Location: 8 avenue Maurice Hauriou, 31000 Toulouse
- Webpage: www.cotegaronne.fr
- Time: 7pm30 on Thursday 29th



Kir ou jus de fruits d'accueil et amuses bouches

Foie gras de canard mi-cuit à l'armagnac, confiture de fruits secs, bouquet de mesclun

* Cœurs de poireaux vinaigrette œuf poché et copeaux de parmesan

Le filet de canette de la maison SAMARAN, jus de viande aux cèpes, petit gratin de pommes de terre et poêlé de légumes du marché

*Le risotto aux herbes fraîches, poêlée de légumes du marché et sauce aux cèpes

La tarte fine aux pomme servie tiède, boule de crème glacée a la vanille bourbon de chez Philippe Faur

Vin de Pays d'Oc Rouge et blanc Château Fontiès « cuvée prestige »

Café

Restaurant **Côté Garonne** - 8, av. Maurice Hauriou - 31000 Toulouse
Tél : 05 61 25 42 86 - Mail : contact@cotegaronne.fr - Site web : www.cotegaronne.fr

Lunch menus

- Restaurant: Brasserie L'Esplanade
- Location: on campus, near the metro station
- Webpage: <https://www.crous-toulouse.fr/restaurant/brasserie-lesplanade/>

★ Mardi 27:

Terrine de poisson, crème d'estragon

ou

Tagliatelle de légumes assaisonnés

Rouille de seiche Sétoise garnie aux légumes d'été

ou

Légumes d'été et polenta

Tarte à la rhubarbe rouge meringuée

★ Jeudi 29:

Salade Niçoise (sans poisson pour les végétariens)

Gardiane de bœuf façon Camarguaise et riz de feria

ou

Poivrons rôtis aux olives et riz de feria

Poumpette du Tarn

★ Vendredi 30:

Carpaccio de tomate au pesto Rosso

Bravade de Nîmes et salade

ou

Lasagne aux légumes provençaux

Croustade aux pommes maison