Discovering the torsion bigravity world

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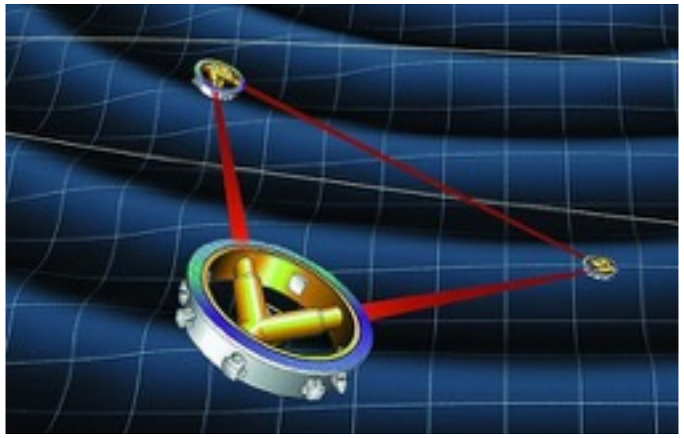
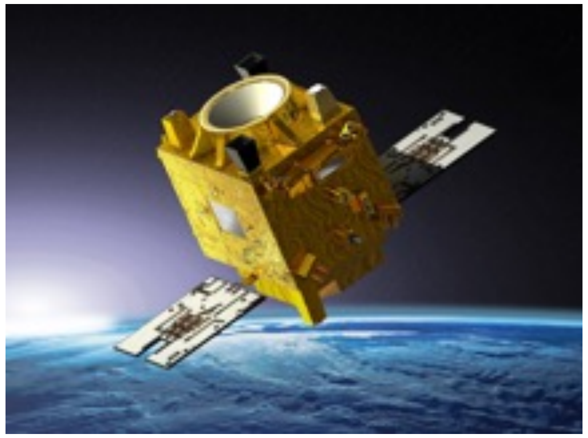
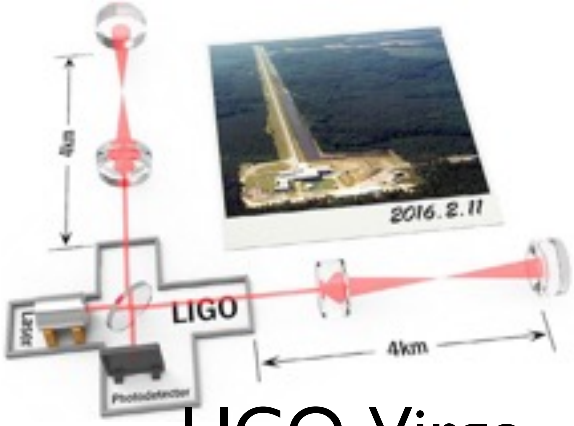
Institut des Hautes Études Scientifiques

Tours
2022

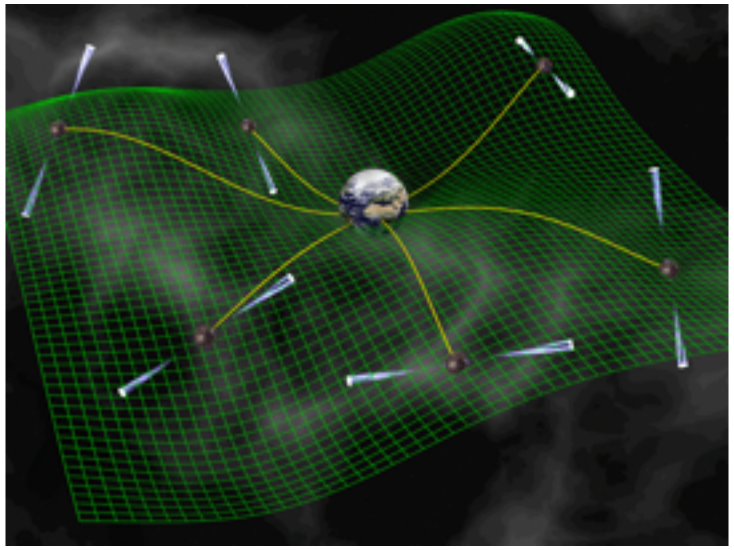
Golden Era of gravity

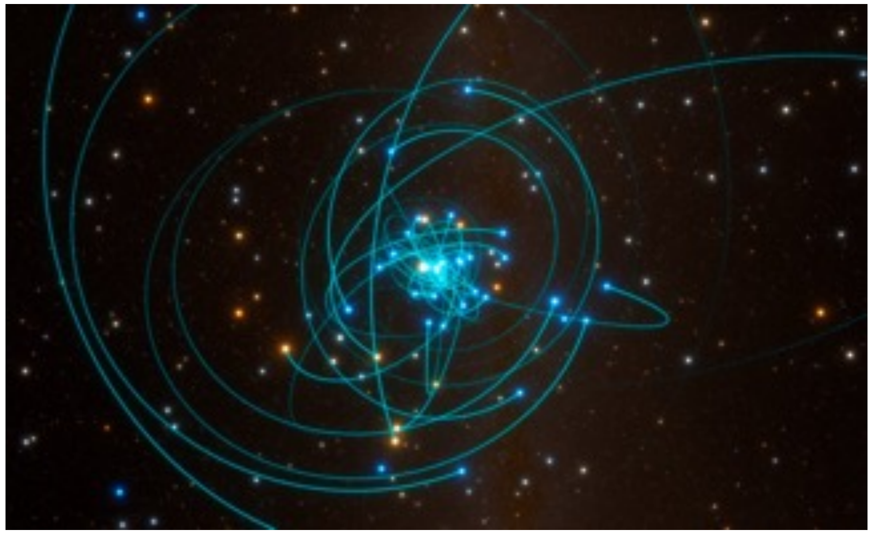

LISA

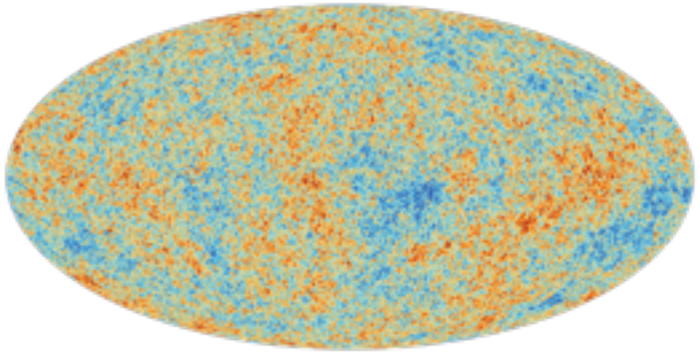

Microscope

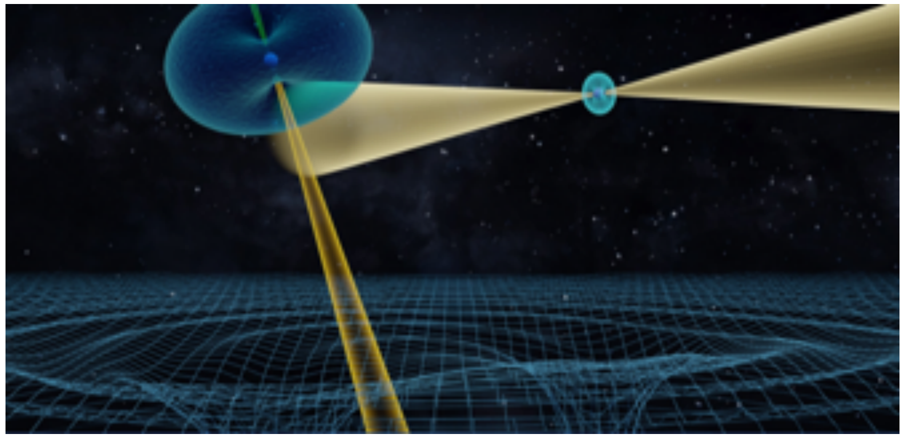


LIGO-Virgo


pulsar timing array


SgrA*


CMB


binary pulsars


EHT

Puzzles of gravity

GR? GR? GR GR GR GR GR ?GR?GR

UV

well tested

IR

distance

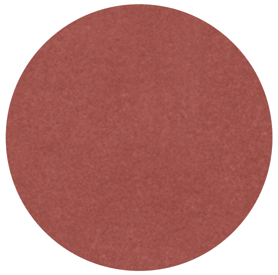
$10^{-35}m$ $1\,\mu m$ $10^{20}m$ $10^{27}m$

non-
renormalizability

dark matter

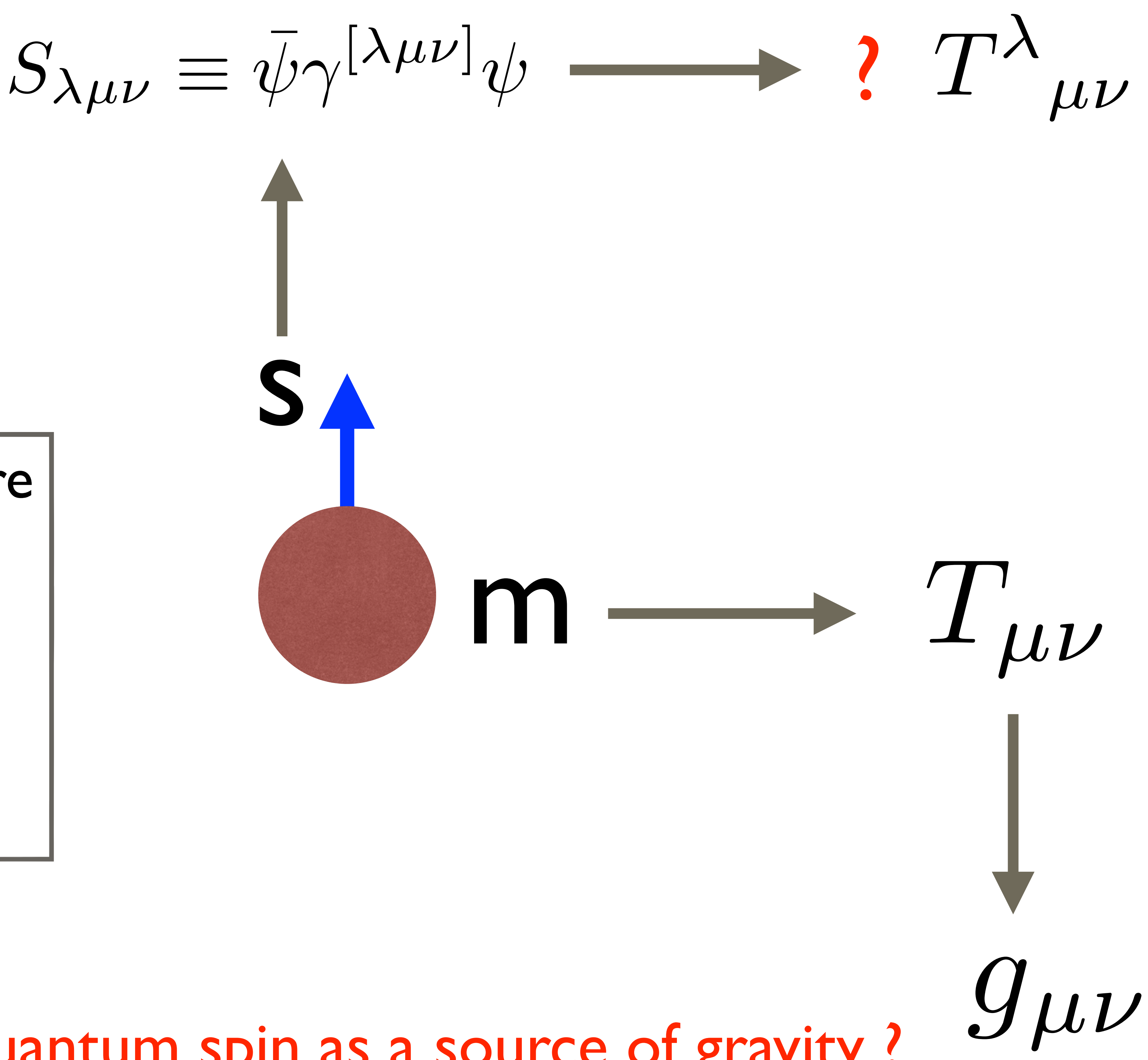
dark energy

Sources of gravity



$$\begin{tikzcd}
\mathrm{m} \arrow[r] & T_{\mu\nu} \arrow[d] \\
 & g_{\mu\nu}
\end{tikzcd}$$

Sources of gravity



quantum spin density $S_{\lambda\mu\nu} \equiv \bar{\psi}\gamma^{[\lambda\mu\nu]}\psi$

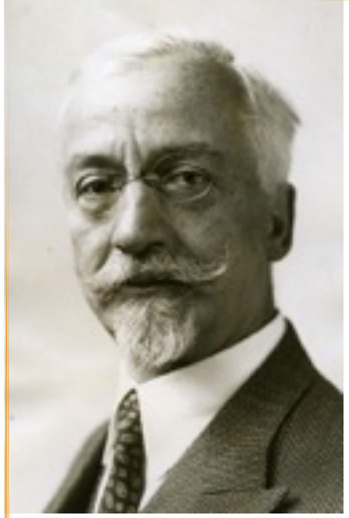
$$\begin{tikzcd} S_{\lambda\mu\nu} \arrow[r] & ? \; T^{\lambda}{}_{\mu\nu} \\ s \arrow[u] & \\ m \arrow[r] & T_{\mu\nu} \arrow[d] \\ & g_{\mu\nu} \end{tikzcd}$$

mass and spin are characteristics of a particle linked to space-time symmetries

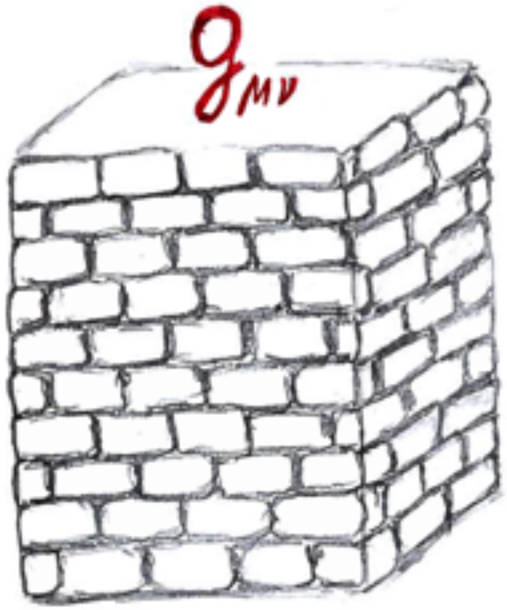
Quantum spin as a source of gravity ?

Kibble’61, Sciama’62, Hehl'70

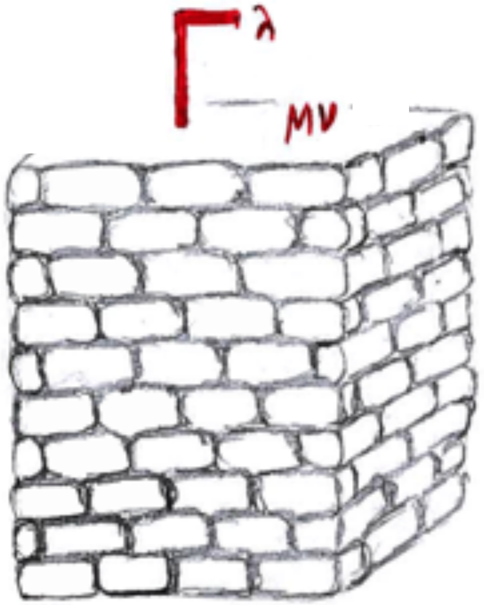
Metric and (torsionfull) connection



Elie Cartan

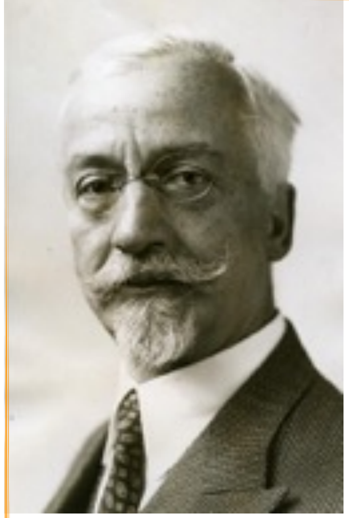


metric



connection

Metric and (torsionfull) connection



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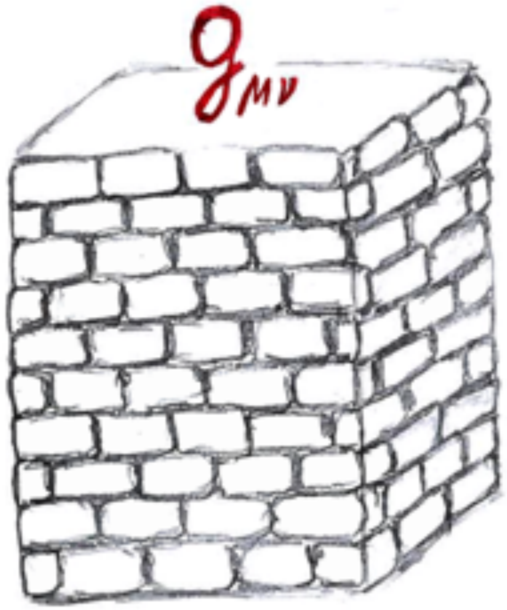
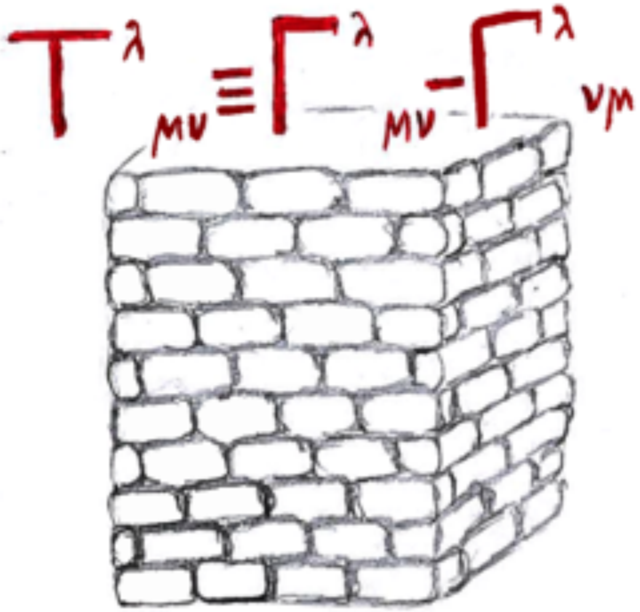


metric



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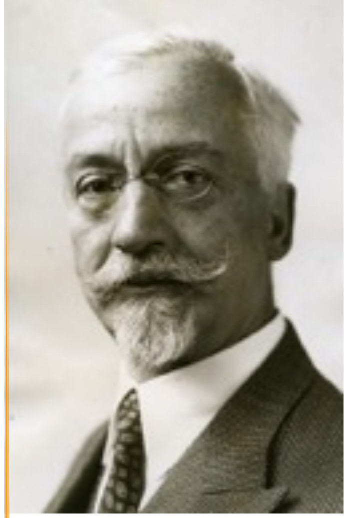

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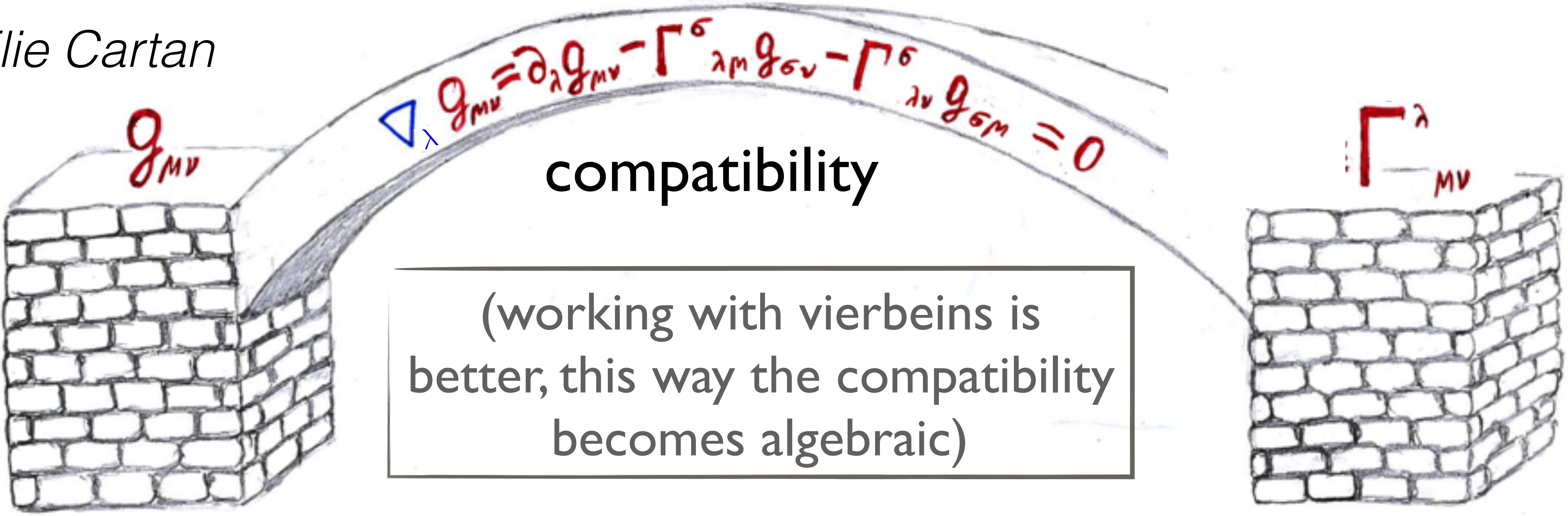


compatibility

(working with vierbeins is better, this way the compatibility becomes algebraic)

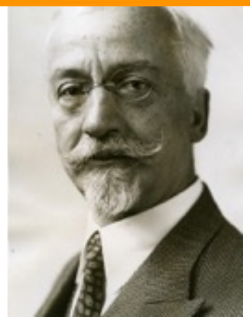
metric

connection

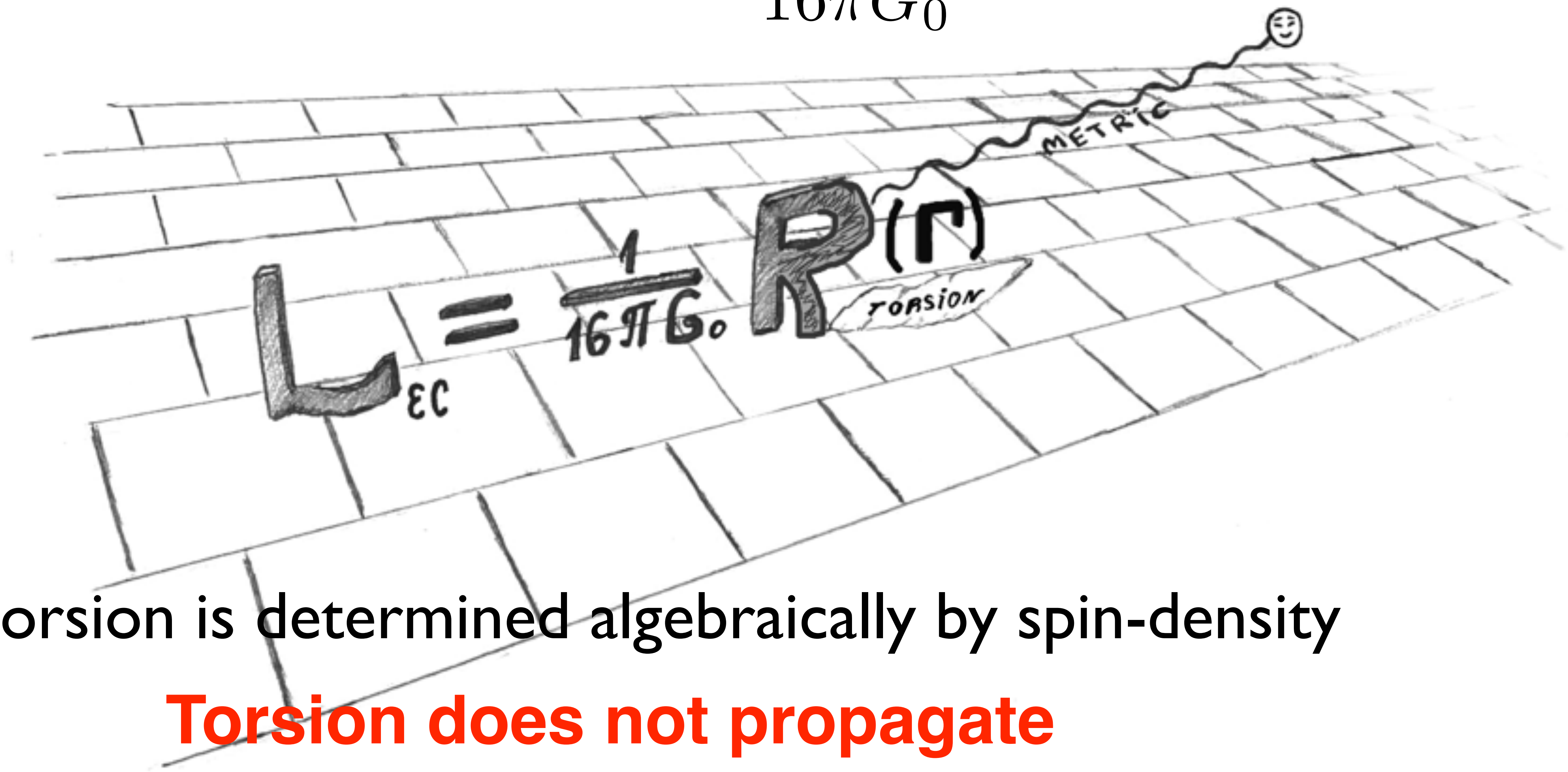
Einstein-Cartan theory

$$L_{\mathrm{Einstein-Cartan}} = \frac{1}{16\pi G_0}\mathcal{R}(\Gamma)$$

Einstein-Cartan theory



$$L_{\rm Einstein-Cartan} = \frac{1}{16\pi G_0}\mathcal{R}(\Gamma)$$

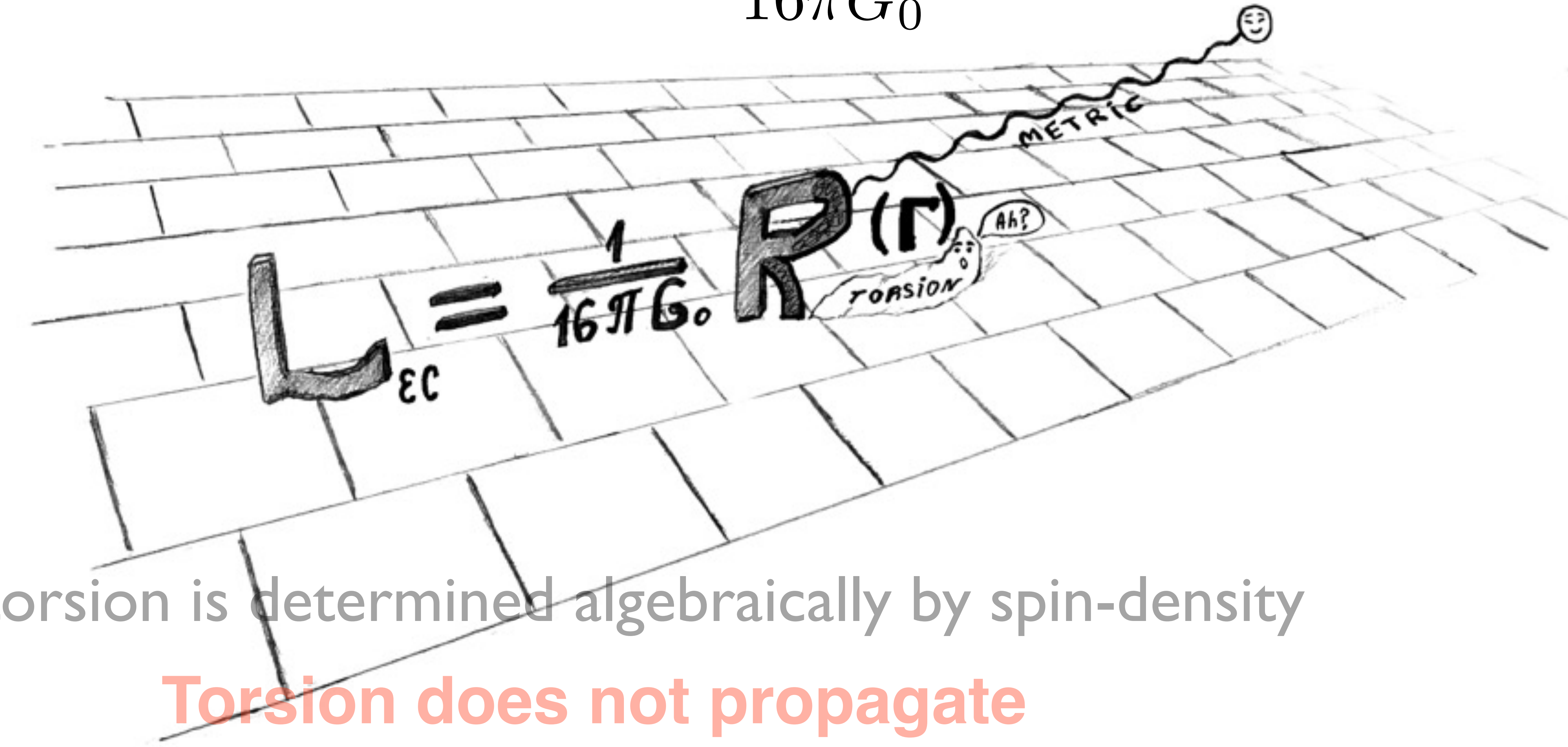


torsion is determined algebraically by spin-density

Torsion does not propagate

Einstein-Cartan theory

$$L_{\mathrm{Einstein-Cartan}} = \frac{1}{16\pi G_0}\mathcal{R}(\Gamma)$$



torsion is determined algebraically by spin-density

Torsion does not propagate

Could there be a theory of gravity containing dynamical torsion ?

Sezgin-van Nieuwenhuizen'80,
Hayashi-Shirafuji'81

Torsion bigravity

Sezgin-van Nieuwenhuizen’80,
Hayashi-Shirafuji'81

Nair
Randjbar-Daemi
Rubakov'09

Nikiforova
Randjbar-Daemi
Rubakov’09

Damour
Nikiforova'19

Torsion bigravity

Sezgin-van Nieuwenhuizen’80, Hayashi-Shirafuji'81

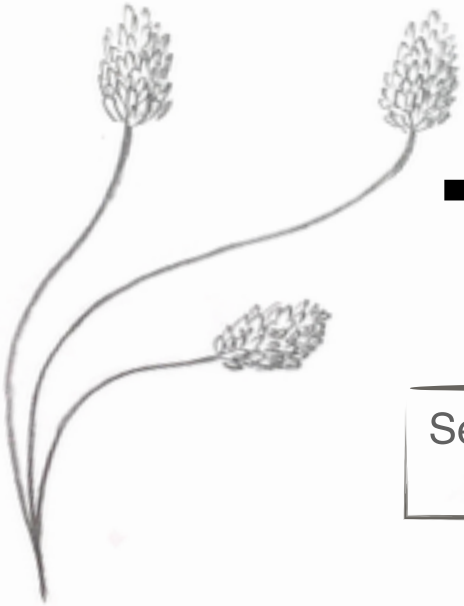
Nair Randjbar-Daemi Rubakov'09

Nikiforova Randjbar-Daemi Rubakov’09

Damour Nikiforova'19

$$L_{\mathrm{GR}} \qquad +$$

$$L = \frac{1}{16\pi G_0} R(g)$$



Torsion bigravity

Sezgin-van Nieuwenhuizen’80,
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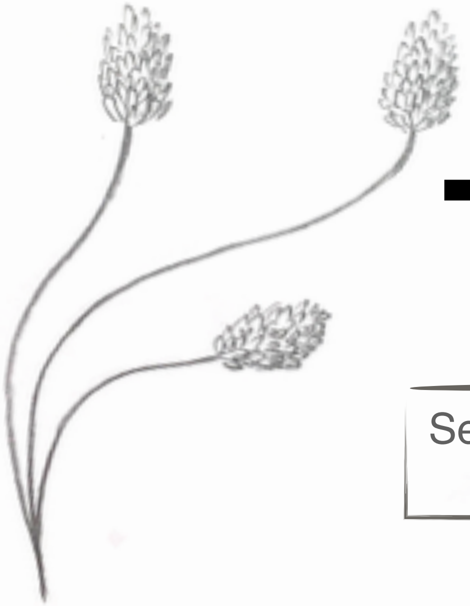
Nair
Randjbar-Daemi
Rubakov'09

Nikiforova
Randjbar-Daemi
Rubakov’09

Damour
Nikiforova'19

$$L_{\rm GR} \qquad + \qquad L_{\rm Einstein-Cartan}$$

$$L = \frac{1}{16\pi G_0} R(g) \quad + \quad \frac{1}{16\pi G_0} \mathcal{R}(\Gamma)$$



Torsion bigravity

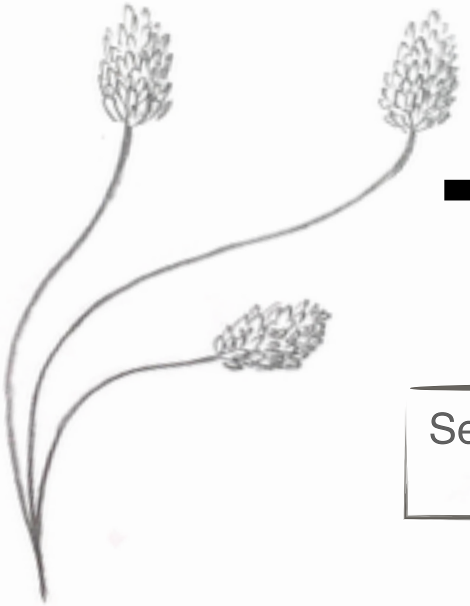
Sezgin-van Nieuwenhuizen’80, Hayashi-Shirafuji'81 → Nair Randjbar-Daemi Rubakov'09 → Nikiforova Randjbar-Daemi Rubakov’09 → Damour Nikiforova'19

$$L_{\mathrm{GR}} \quad + \quad L_{\mathrm{Einstein-Cartan}} \quad + \quad \text{quadratic terms}$$

$$L = \frac{1}{16\pi G_0} R(g) \quad + \quad \frac{1}{16\pi G_0} \mathcal{R}(\Gamma) \quad + \frac{1}{16\pi G_0} \left(\mathcal{R}_{(\mu\nu)}(\Gamma) \mathcal{R}^{(\mu\nu)}(\Gamma) - \frac{1}{3} \mathcal{R}(\Gamma)^2 \right)$$

$$+$$

$$\mathcal{R}_{[\mu\nu]}(\Gamma) \mathcal{R}^{[\mu\nu]}(\Gamma)$$



Torsion bigravity

Sezgin-van Nieuwenhuizen’80, Hayashi-Shirafuji'81 → Nair Randjbar-Daemi Rubakov'09 → Nikiforova Randjbar-Daemi Rubakov’09 → Damour Nikiforova'19

$$L_{\mathrm{GR}} \quad + \quad L_{\mathrm{Einstein-Cartan}} \quad + \quad \text{quadratic terms}$$

$$L = \frac{1}{16\pi G_0 (1+\eta)} R(g) + \frac{\eta}{16\pi G_0 (1+\eta)} \mathcal{R}(\Gamma) + \frac{\eta}{16\pi G_0 \kappa^2} \left(\mathcal{R}_{(\mu\nu)} \mathcal{R}^{(\mu\nu)} - \frac{1}{3} \mathcal{R}^2 \right)$$

$$+ \, c_{34} \mathcal{R}_{[\mu\nu]}(\Gamma) \mathcal{R}^{[\mu\nu]}(\Gamma)$$

in vierbeins

Explicit field equations

$$\frac{\eta}{(1+\eta)16\pi G_0}\left(\mathcal{R}_{ij}(\Gamma)-\frac{1}{2}\eta_{ij}\mathcal{R}(\Gamma)\right)+\frac{1}{(1+\eta)16\pi G_0}\left(R_{ij}(g)-\frac{1}{2}\eta_{ij}R(g)\right)$$

$$+\frac{\eta}{16\pi G_0\kappa^2}\left[\mathcal{R}_{ki}(\Gamma)\mathcal{R}_{kj}(\Gamma)+\mathcal{R}_{kl}(\Gamma)\mathcal{R}_{kilj}(\Gamma)-\frac{2}{3}\mathcal{R}(\Gamma)\,\mathcal{R}_{ij}(\Gamma)-\frac{1}{2}\eta_{ij}\left(\mathcal{R}_{kl}(\Gamma)\mathcal{R}_{kl}(\Gamma)-\frac{1}{3}\mathcal{R}(\Gamma)^2\right)\right]=T_{ij}$$

2nd order in connection and metric !

stress-energy tensor

$$\left[\eta_{ik}\left(D_mP_{jm}-\frac{2}{3}D_jP\right)-D_iP_{jk}\right]-\left[\eta_{jk}\left(D_mP_{im}-\frac{2}{3}D_iP\right)-D_jP_{ik}\right]$$

$$+\frac{\eta}{(1+\eta)16\pi G_0}\left(K_{ikj}-K_{jki}-K_{ill}\eta_{jk}+K_{jll}\eta_{ik}\right)$$

$$+\left(K_{mkn}-K_{nkm}-K_{mll}\eta_{nk}+K_{nll}\eta_{mk}\right)\left(\eta_{im}P_{jn}-\eta_{jm}P_{in}-\frac{2}{3}\eta_{im}\eta_{jn}P\right)=S_{ijk}$$

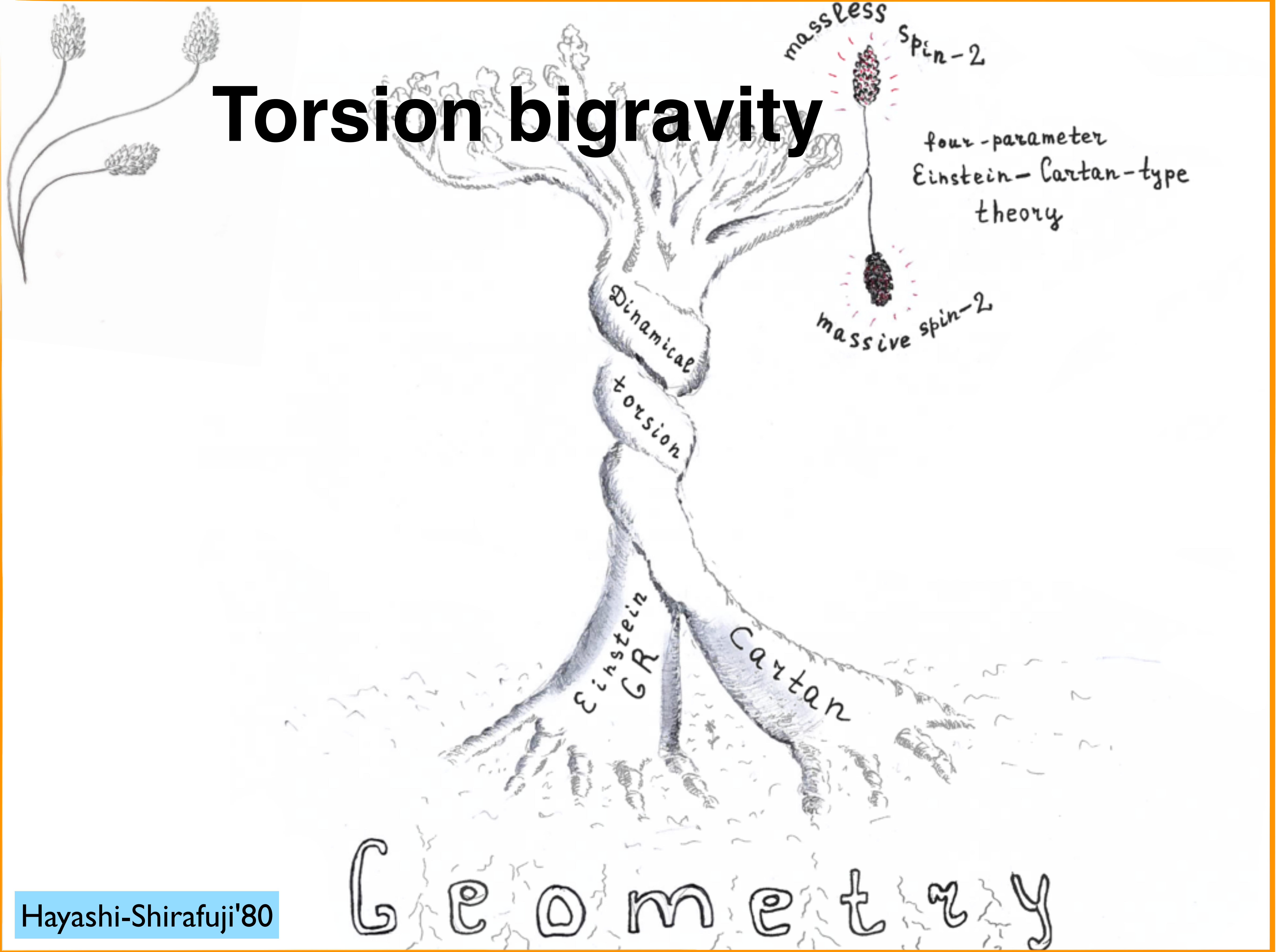
$$P_{ij}\equiv\frac{\eta}{16\pi G_0\kappa^2}\mathcal{R}_{(ij)}(\Gamma)+c_{34}\mathcal{R}_{[ij]}(\Gamma)$$

$$K_{ijk}=\frac{1}{2}\left(T_{i[jk]}+T_{j[ki]}-T_{k[ij]}\right)$$

spin density

Nikiforova-Randjbar-Daemi-Rubakov'09

Torsion bigravity



Hayashi-Shirafuji'80

Torsion bigravity

Field content around flat space:

massless spin-2

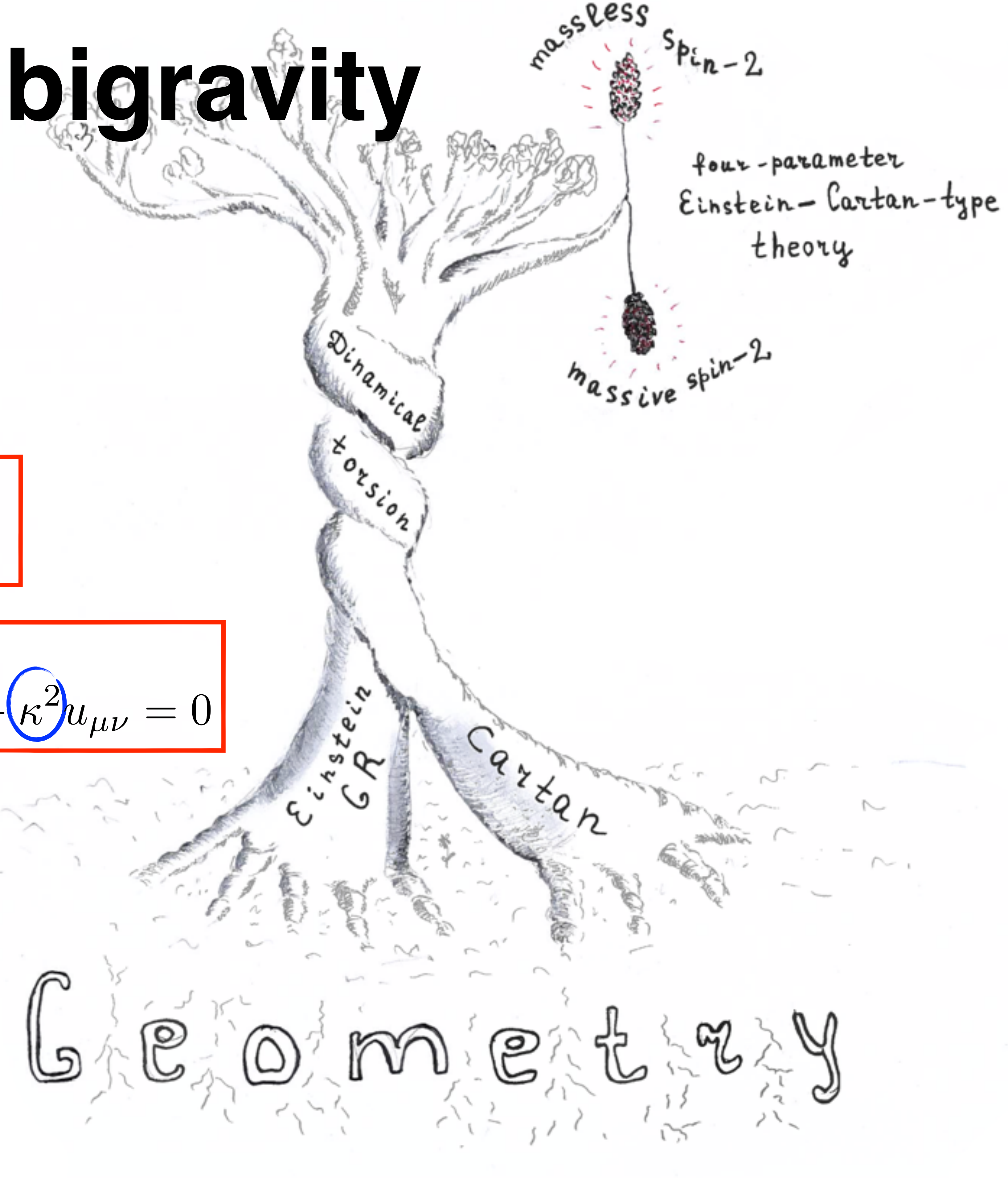
$$\Box\bar{h}_{\mu\nu} + \partial_{\mu\nu}\bar{h} - \partial_{\mu\sigma}\bar{h}^{\sigma}_{\nu} - \partial_{\nu\sigma}\bar{h}^{\sigma}_{\mu} = 0$$

massive spin-2

$$\Box u_{\mu\nu} + \partial_{\mu\nu}u - \partial_{\mu\sigma}u^{\sigma}_{\nu} - \partial_{\nu\sigma}u^{\sigma}_{\mu} - \kappa^2 u_{\mu\nu} = 0$$

$$u_{\mu\nu} \equiv \mathcal{R}_{(1)\,\mu\nu} - \frac{1}{6}g_{\mu\nu}\mathcal{R}_{(1)}$$

Hayashi-Shirafuji'80



Theories with massive spin-2:

Theoretical consistency

bimetric gravity

$$g_{\mu\nu}\ , \quad f_{\mu\nu}$$

$$L \sim \sqrt{g}R(g) + \sqrt{f}R(f) - V\left(\sqrt{g^{-1}f}\right)$$

☺ $m = 0,\ s = 2$ $\quad \alpha\, \delta g_{\mu\nu} + \beta\, \delta f_{\mu\nu}$

☺ $m \neq 0,\ s = 2$ $\quad \bar{\alpha}\, \delta g_{\mu\nu} + \bar{\beta}\, \delta f_{\mu\nu}$

ghost-free
(5 dof for the massive spin-2;
generic existence of 5
constraints)

de Rham-Gabadadze-Tolley'10,
Hassan-Rosen'11,
Volkov'14

torsion bigravity

$$g_{\mu\nu}\ , \quad T^{\lambda}{}_{\mu\nu}$$

$$L \sim \sqrt{g}R(g) + \sqrt{g}\mathcal{R}(\Gamma) + \sqrt{g}\left(\mathcal{R}_{(\mu\nu)}\mathcal{R}^{(\mu\nu)} - \frac{1}{3}\mathcal{R}^2\right)$$
$$+\sqrt{g}\mathcal{R}_{[\mu\nu]}\mathcal{R}^{[\mu\nu]}$$

☺ $m = 0,\ s = 2$ $\quad \delta g_{\mu\nu} + \frac{1}{\kappa^2}\delta\mathcal{R}_{\mu\nu}$

☺ $m \neq 0,\ s = 2$ $\quad \delta\mathcal{R}_{\mu\nu} - \frac{1}{6}g_{\mu\nu}\delta\mathcal{R}$

ghost-free
(5 dof for the massive spin-2) around flat space,
Einstein spaces $(R_{\mu\nu} = \Lambda g_{\mu\nu})$,
in nonlinear static spherically symm. solutions

Sezgin-van Nieuwenhuizen'80, Hayashi-Shirafuji'81, Nair-Randjbar-Daemi-Rubakov'09
Nikiforova-Randjbar-Daemi-Rubakov'09, Damour-Nikiforova'19, Nikiforova'20

Theories with massive spin-2:

Theoretical consistency

bimetric gravity	torsion bigravity

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ghost-free
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Einstein spaces $(R_{\mu\nu} = \Lambda g_{\mu\nu})$,
in nonlinear static spherically sy olutions

future direction

Sezgin-van Nieuwenhuizen’80, Hayashi-Shirafuji’81, Nair-Randjbar-Daemi-Rubakov’09
Nikiforova-Randjbar-Daemi-Rubakov’09, Damour-Nikiforova’19, Nikiforova'20

Phenomenology:
Torsion bigravity world

OK let us model a world
on a base of torsion bigravity.

And then will see
whether it looks like our real world or not.

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Torsion bigravity world

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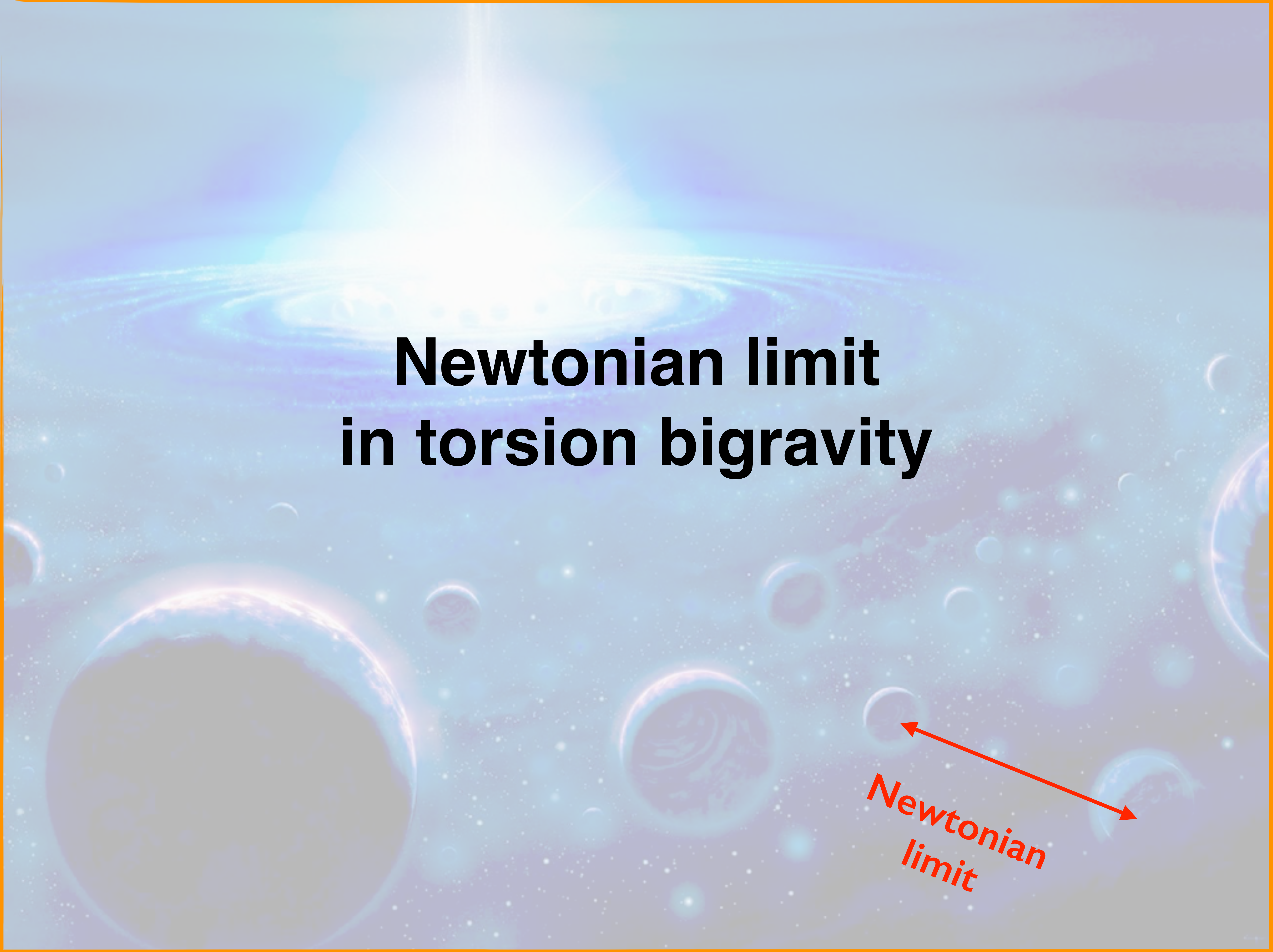
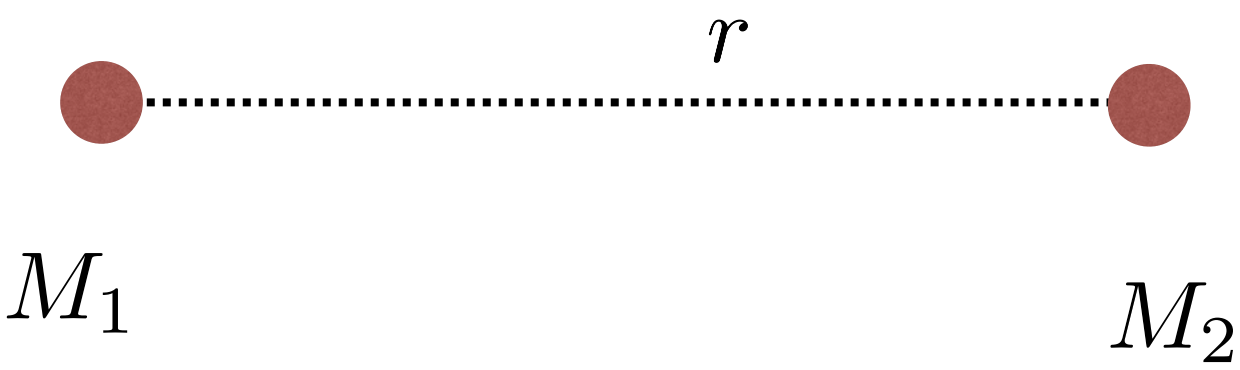
Newtonian limit

Newtonian limit in torsion bigravity

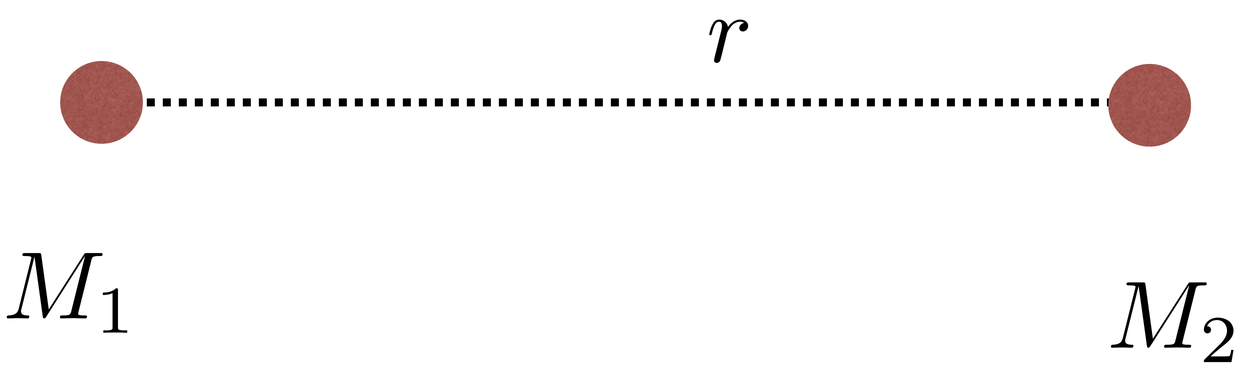


Newtonian limit



Hayashi-Shirafuji’81,
Nikiforova-Randjbar-Daemi-Rubakov’09

Newtonian limit



$$\boxed{V_{\rm int}^{\rm Newtonian} = -\underbrace{G_0 \frac{M_1 M_2}{r}}_{\text{massless spin-2}} - \underbrace{G_m \frac{M_1 M_2}{r} e^{-\kappa r}}_{\text{massive spin-2}}}$$

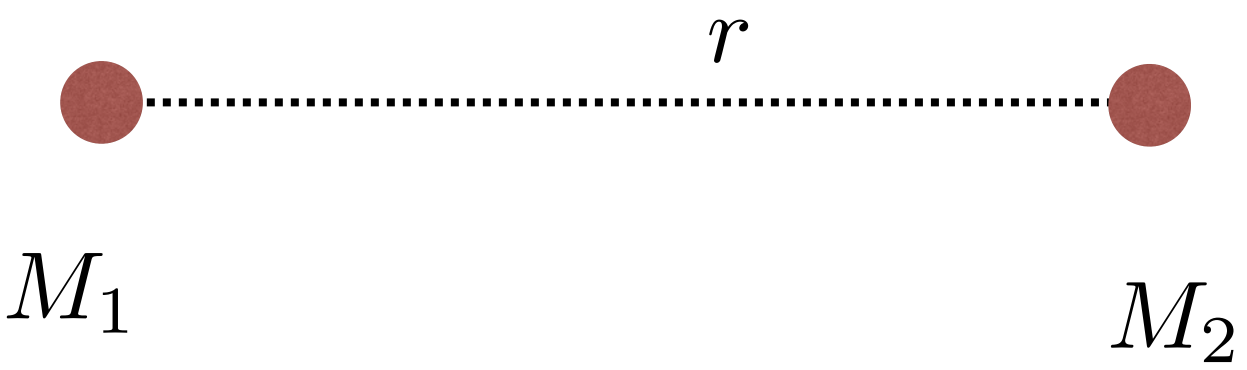
Coupling constant :

$$\boxed{\frac{G_m}{G_0} = \frac{4}{3}\eta}$$

$$L_{\rm interaction} = G_0 T_{00}\left(\frac{-4\pi}{\Delta}\right)T_{00} + G_m T_{00}\left(\frac{-4\pi}{\Delta - \kappa^2}\right)T_{00}$$

Hayashi-Shirafuji'81,
Nikiforova-Randjbar-Daemi-Rubakov'09

Newtonian limit

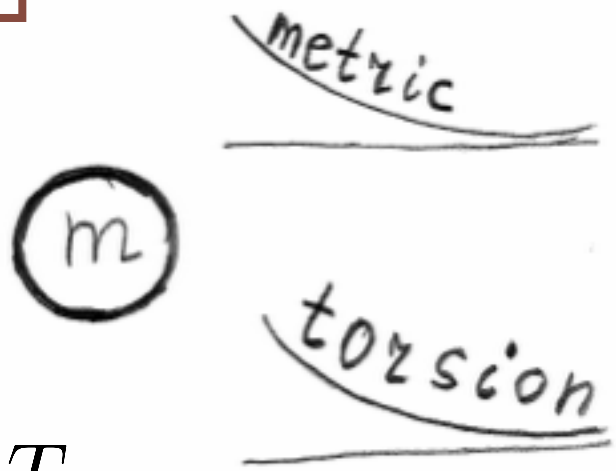


$$\boxed{V_{\text{int}}^{\text{Newtonian}} = -G_0 \frac{M_1 M_2}{r} - G_m \frac{M_1 M_2}{r} e^{-\kappa r}}$$

massless spin-2

massive spin-2

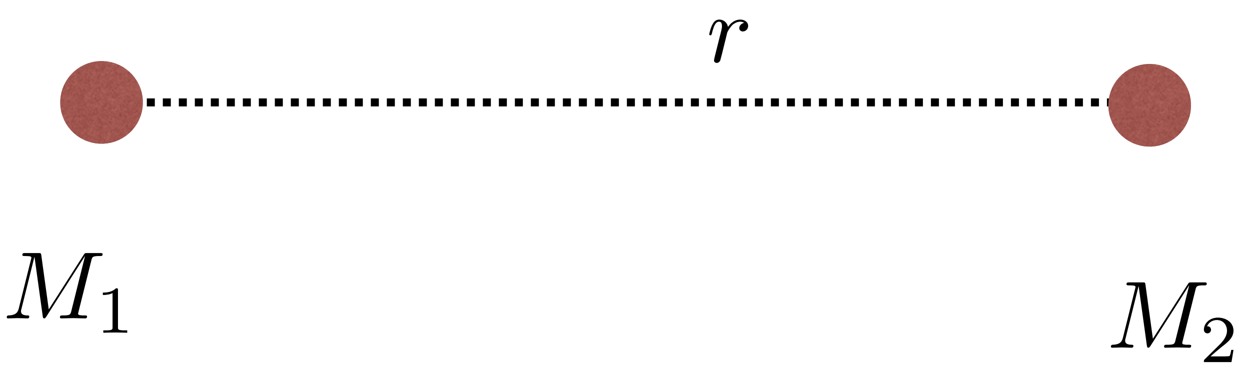
$$\boxed{\frac{G_m}{G_0} = \frac{4}{3}\eta}$$



$$L_{\text{interaction}} = G_0 T_{00} \left(\frac{-4\pi}{\Delta} \right) T_{00} + G_m T_{00} \left(\frac{-4\pi}{\Delta - \kappa^2} \right) T_{00}$$

Hayashi-Shirafuji'81,
Nikiforova-Randjbar-Daemi-Rubakov'09

Newtonian limit



$$V_{\text{int}}^{\text{Newtonian}} = -\underbrace{G_0 \frac{M_1 M_2}{r}}_{\text{massless spin-2}} - \underbrace{G_m \frac{M_1 M_2}{r} e^{-\kappa r}}_{\text{massive spin-2}}$$

$$\boxed{\frac{G_m}{G_0} = \frac{4}{3}\eta}$$

$$\boxed{\eta \lesssim 3 \times 10^{-4} \quad \text{for } \kappa^{-1} \lesssim 10 \text{ km} \quad (\kappa \gtrsim 10^{-10} \text{ eV})}$$

Fischbach-Talmadge'99

Stars in torsion bigravity

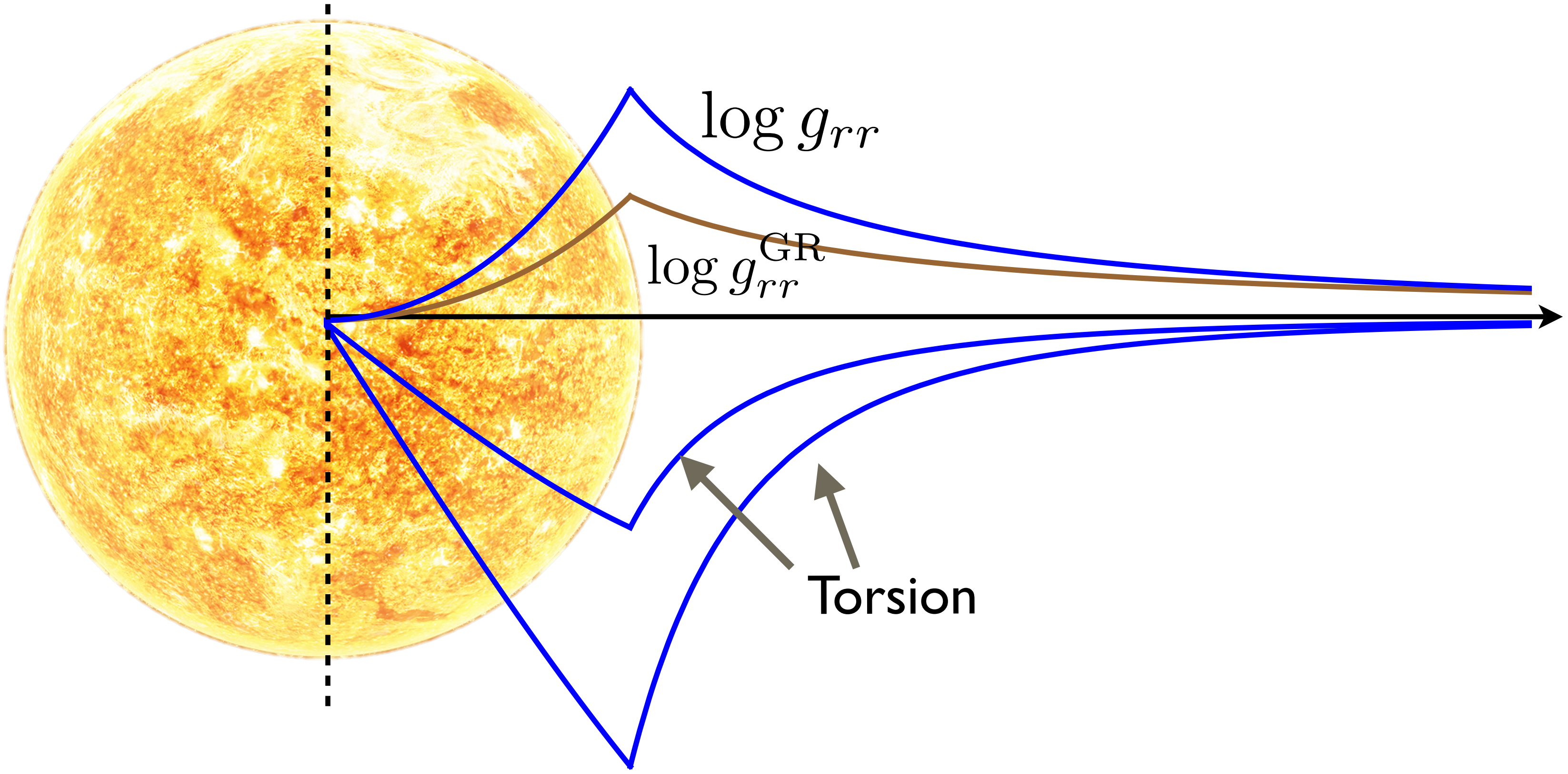
Spherically symmetric star solutions

Damour, Nikiforova’19

dynamical torsion is generated by $T_{\mu\nu}$ of matter !

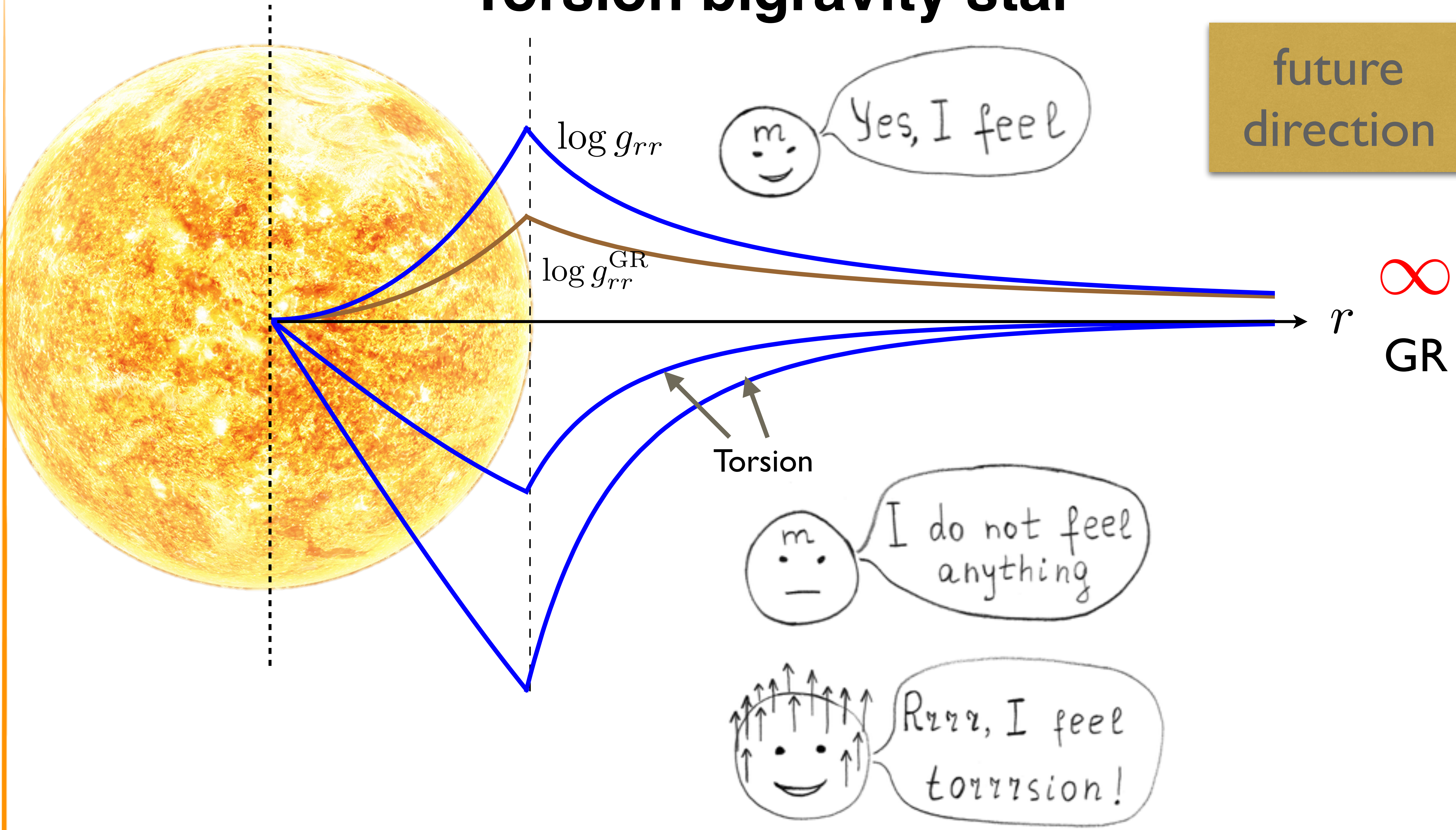
⇩

metric is different from the one predicted in GR
(compactness is different)



Torsion bigravity star

future direction



Direct source of torsion, direct torsion probe

Dirac fermions

$$\mathcal{L}_m = \frac{i\hbar c}{2}(\overline{\psi}\gamma^\mu\nabla_\mu\psi - \nabla_\mu\overline{\psi}\gamma^\mu\psi) - mc^2\overline{\psi}\psi$$

torsionfull connection

Direct source of torsion, direct torsion probe

Dirac fermions

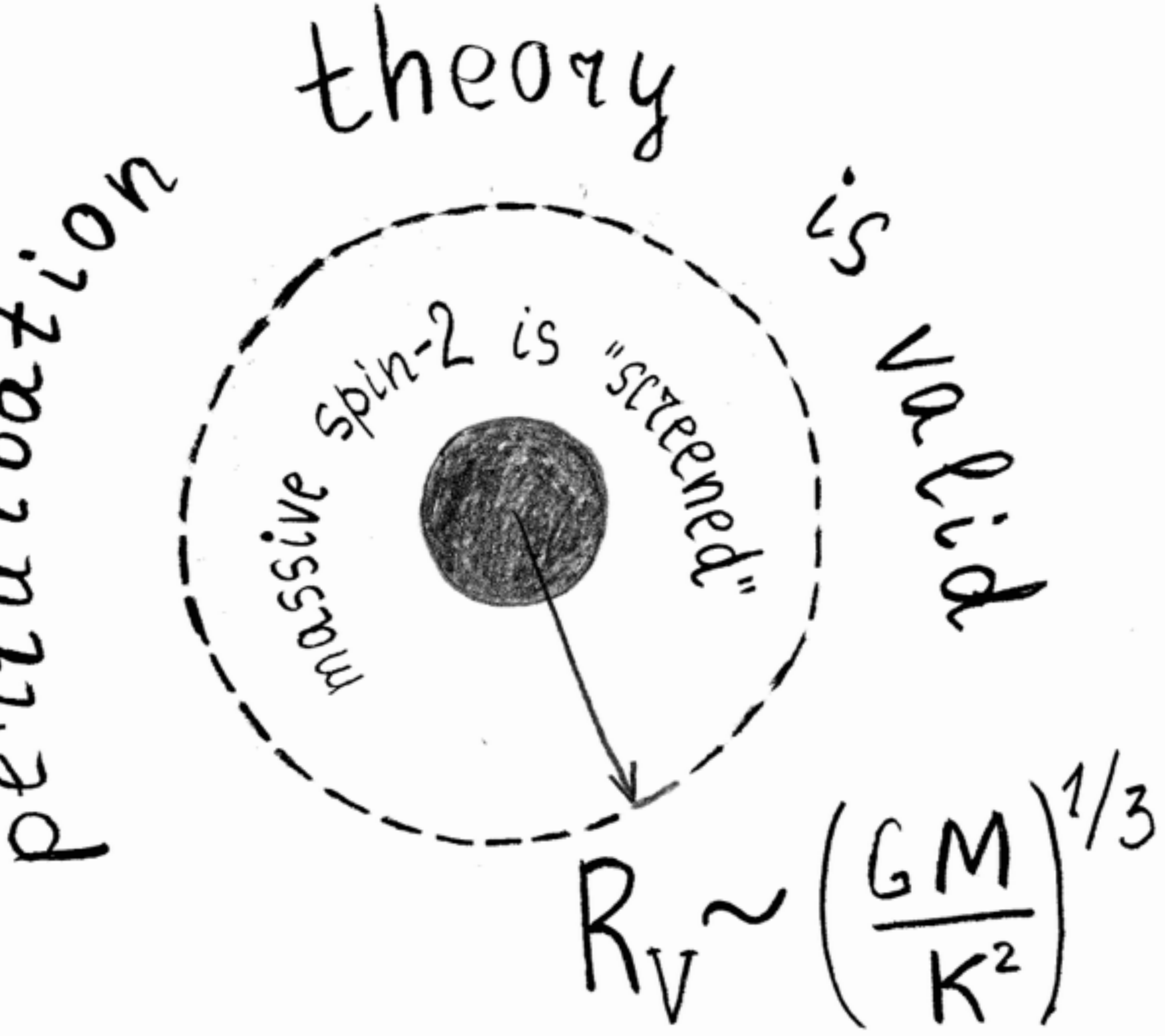
$$\mathcal{L}_m = \frac{i\hbar c}{2}(\overline{\psi}\gamma^{\mu}\nabla_{\mu}\psi - \nabla_{\mu}\overline{\psi}\gamma^{\mu}\psi) - mc^2\overline{\psi}\psi$$

torsionfull connection

$$\boxed{\bar{\psi}\gamma^{[\lambda\mu\nu]}\psi\, T_{\lambda\mu\nu}}$$

torsion is not easy to detect directly

Vainshtein mechanism

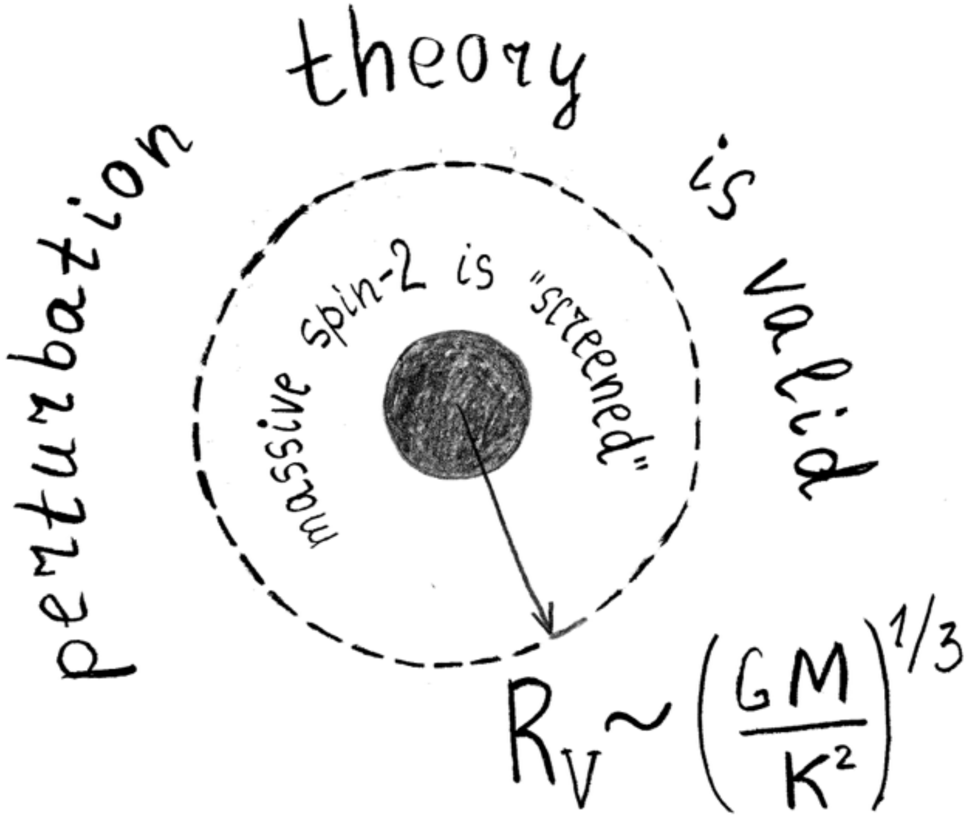


Theories with massive spin-2:

Vainshtein mechanism

bimetric gravity	torsion bigravity

Vainshtein screening



Babichev-Deffayet-Ziour'10,
Babichev-Deffayet'13 ….

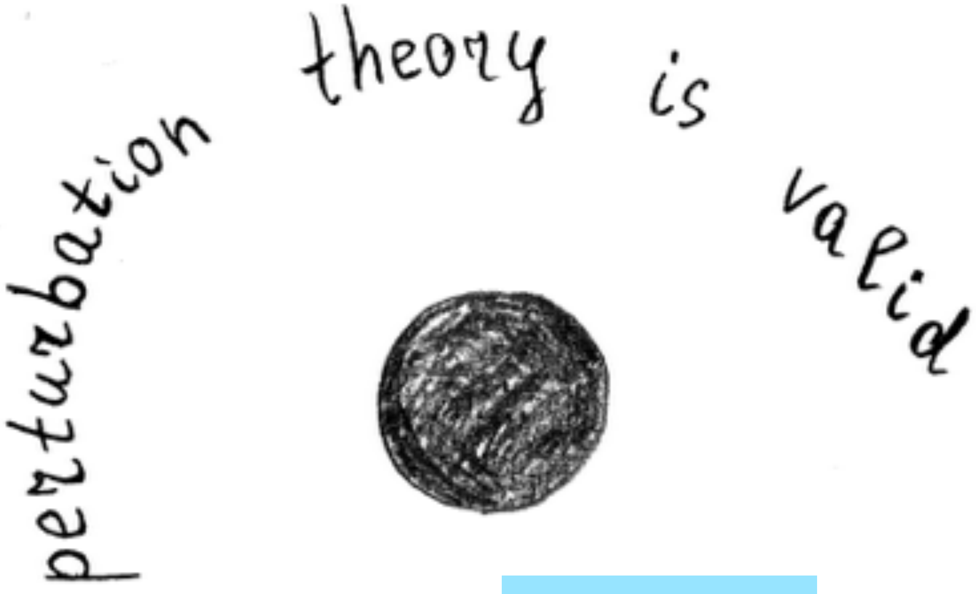
no $\frac{1}{\kappa^2}$ denominators

$\Downarrow$

absence of Vainshtein radius

$\Downarrow$

no Vainshtein screening



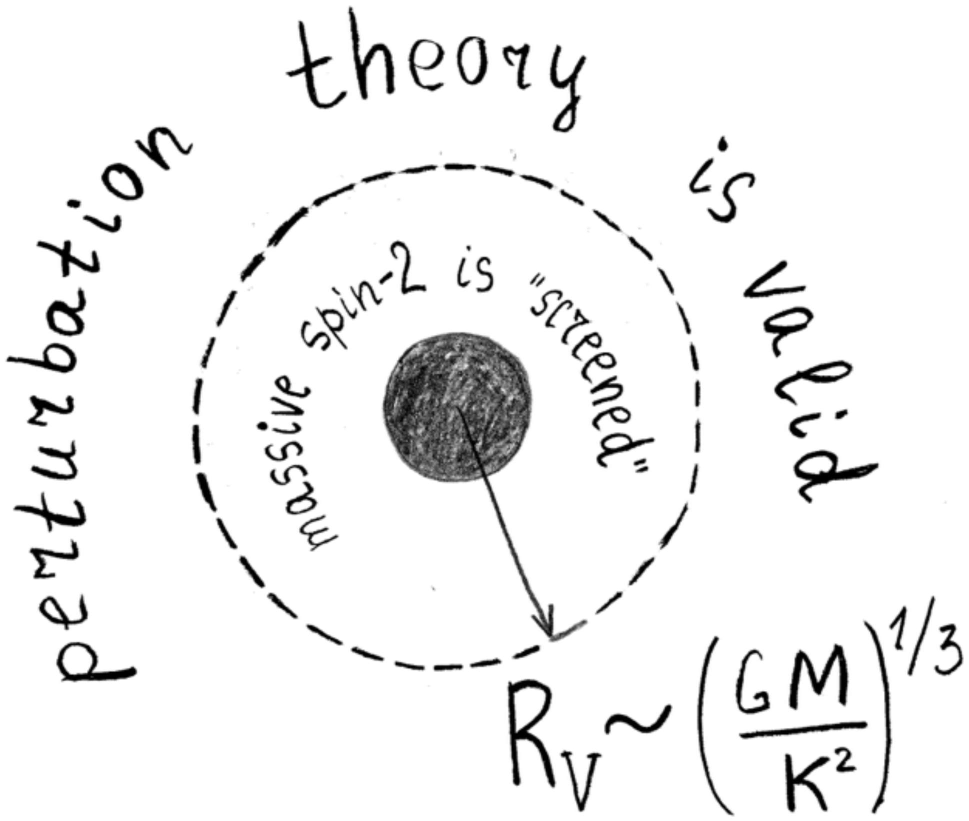
Nikiforova'20

Theories with massive spin-2:

Vainshtein mechanism

bimetric gravity	torsion bigravity

Vainshtein screening



Babichev-Deffayet'13 ….

no $\frac{1}{\kappa^2}$ denominators

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absence of Vainshtein radius

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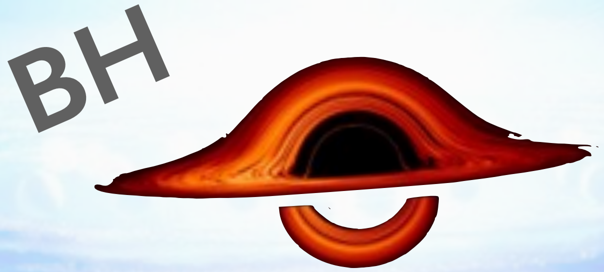
future direction

Nikiforova'20



Black Holes in torsion bigravity

Theories with massive spin-2:

(Asymptotically flat) black hole ZOO

bimetric gravity

Einstein BHs (incl. Kerr)
are exact solutions

Einstein BHs

(Volkov'12)

BHs
with massive graviton hair

(Brito-Cardoso-Pani'13,
Gervalle-Volkov'20)

Theories with massive spin-2:

(Asymptotically flat) black hole ZOO

torsion bigravity

Einstein BHs (incl. Kerr)
are exact solutions
(with zero torsion)

Einstein BHs

$\nexists$ Schwarzschild BHs
with linearized torsion hair

Nikiforova-Damour’20

in the limit of
infinite range
$\kappa \to 0$

(different
from
the Einstein
BHs)

**asymp. flat
hairy BHs**

Theories with massive spin-2:

(Asymptotically flat) black hole ZOO

torsion bigravity

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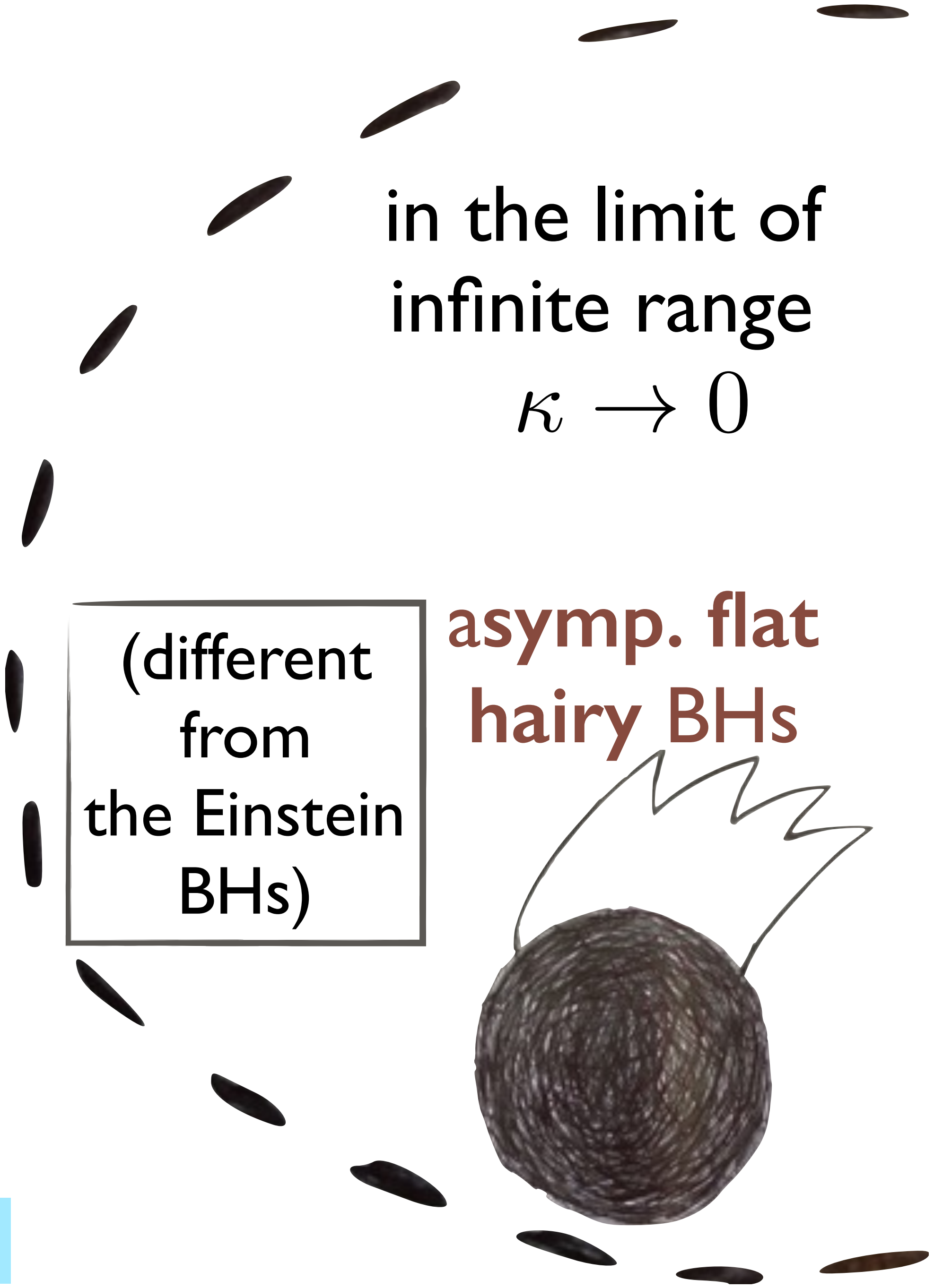

in the limit of
infinite range
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(different
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the Einstein
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**asymp. flat
hairy BHs**

Nikiforova-Damour'20

Theories with massive spin-2:

(Asymptotically flat) black hole ZOO

torsion bigravity

cut here

Einstein BHs (incl. Kerr)
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$\nexists$ Schwarzschild BHs
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(Nikiforova-Damour'20)

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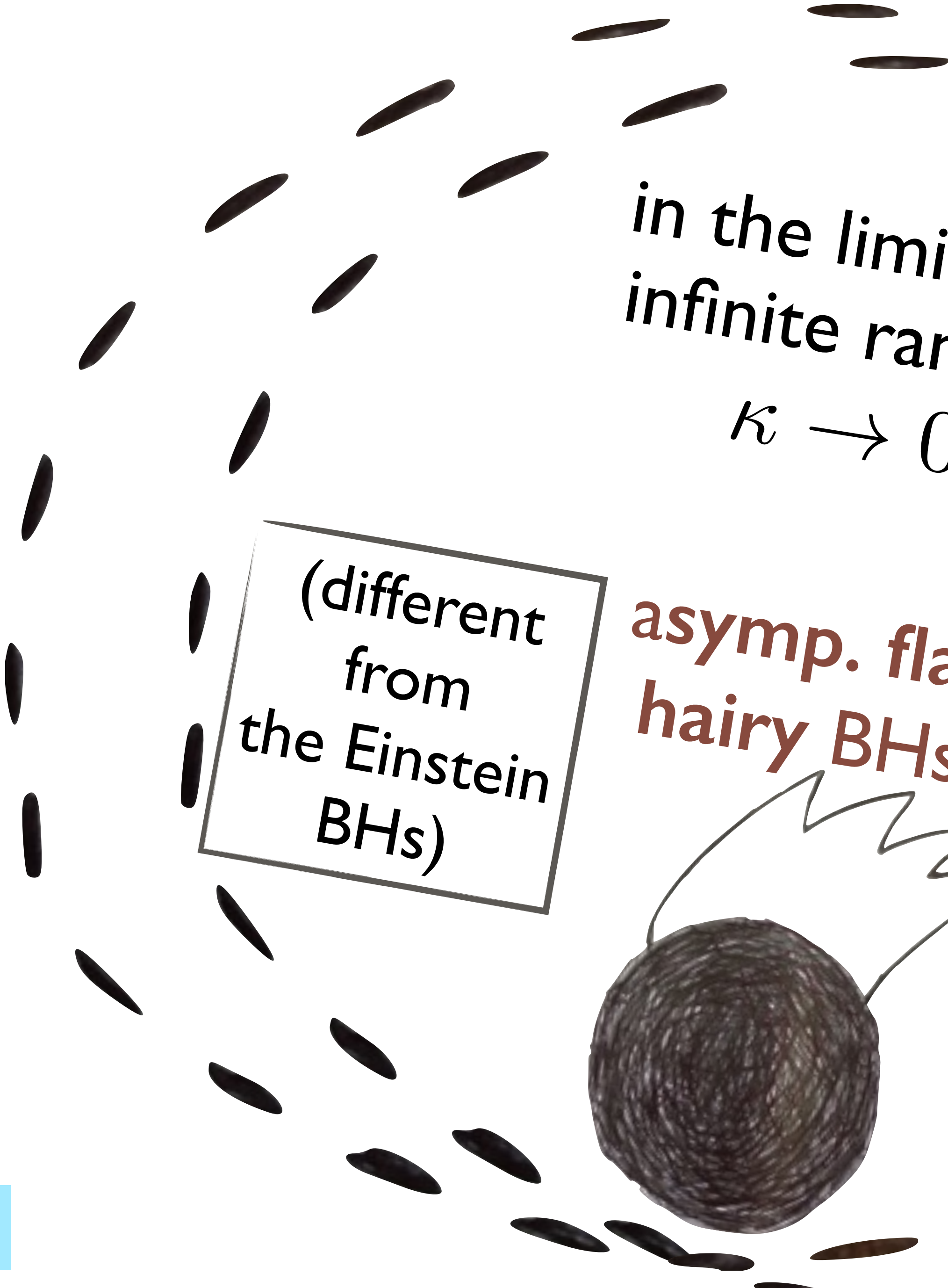


in the limi
infinite ra
$\kappa \to$

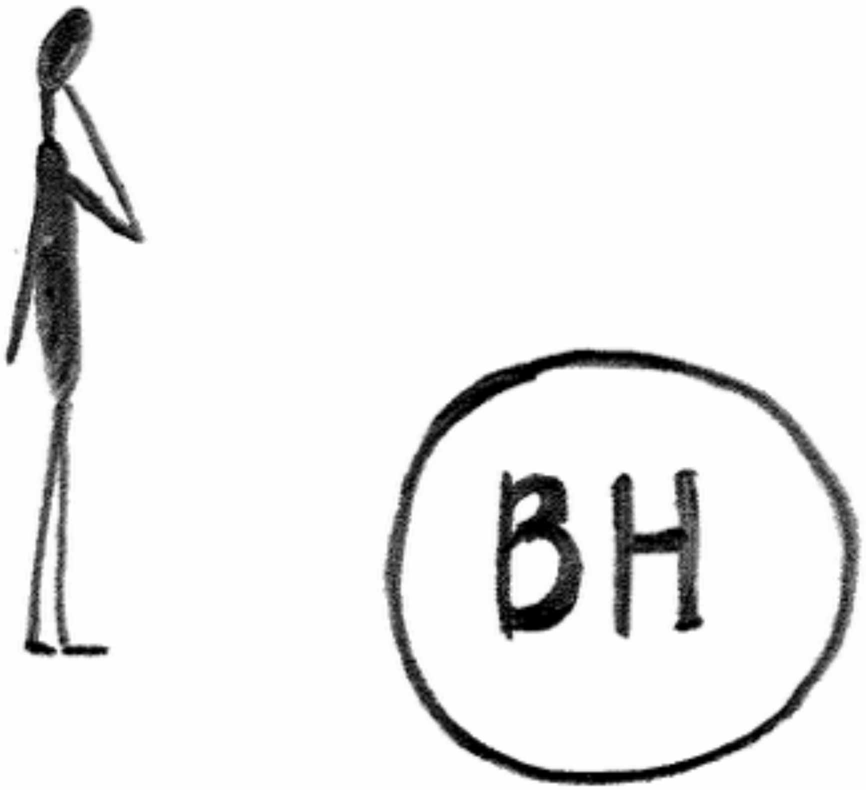
(different
from
the Einstein
BHs)

asymp. fla
hairy BH

Nikiforova-Damour'20

Black hole perturbations

Nikiforova'21 + Nikiforova'22 + future works



Black hole perturbations

Nikiforova'21 + Nikiforova'22 + future works

Black hole perturbations

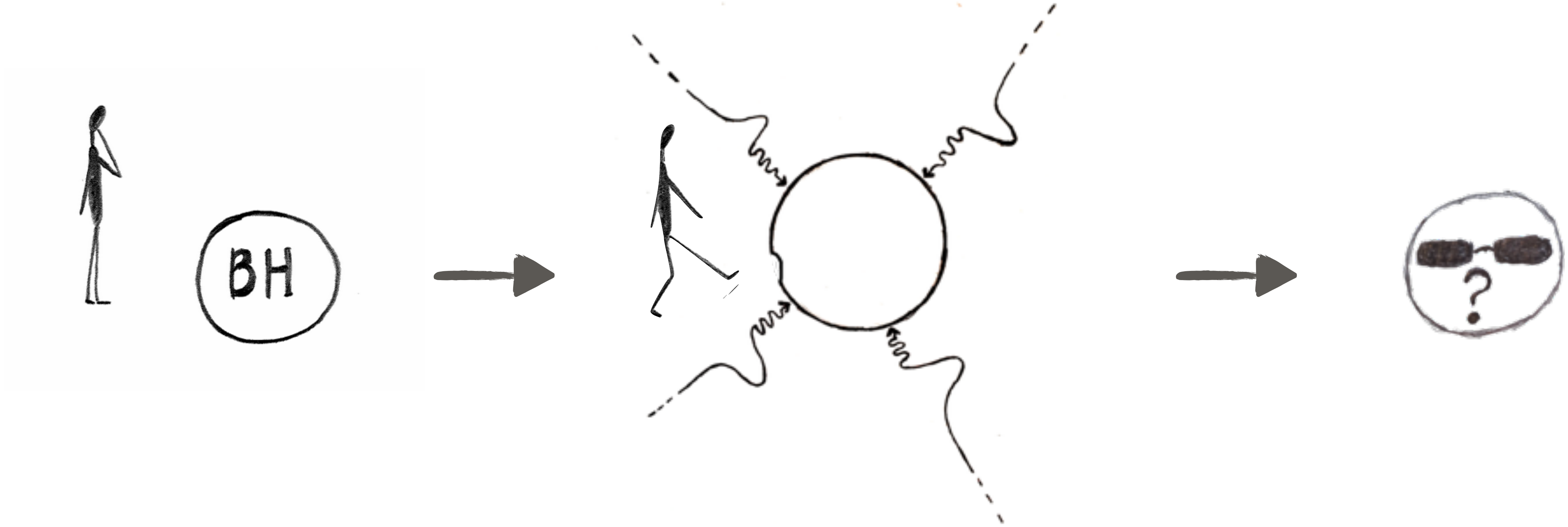
Nikiforova'21 + Nikiforova'22 + future works

- quasi-bound states and stability
- quasi-normal modes

Black hole perturbations

Nikiforova'21 + Nikiforova'22 + future works

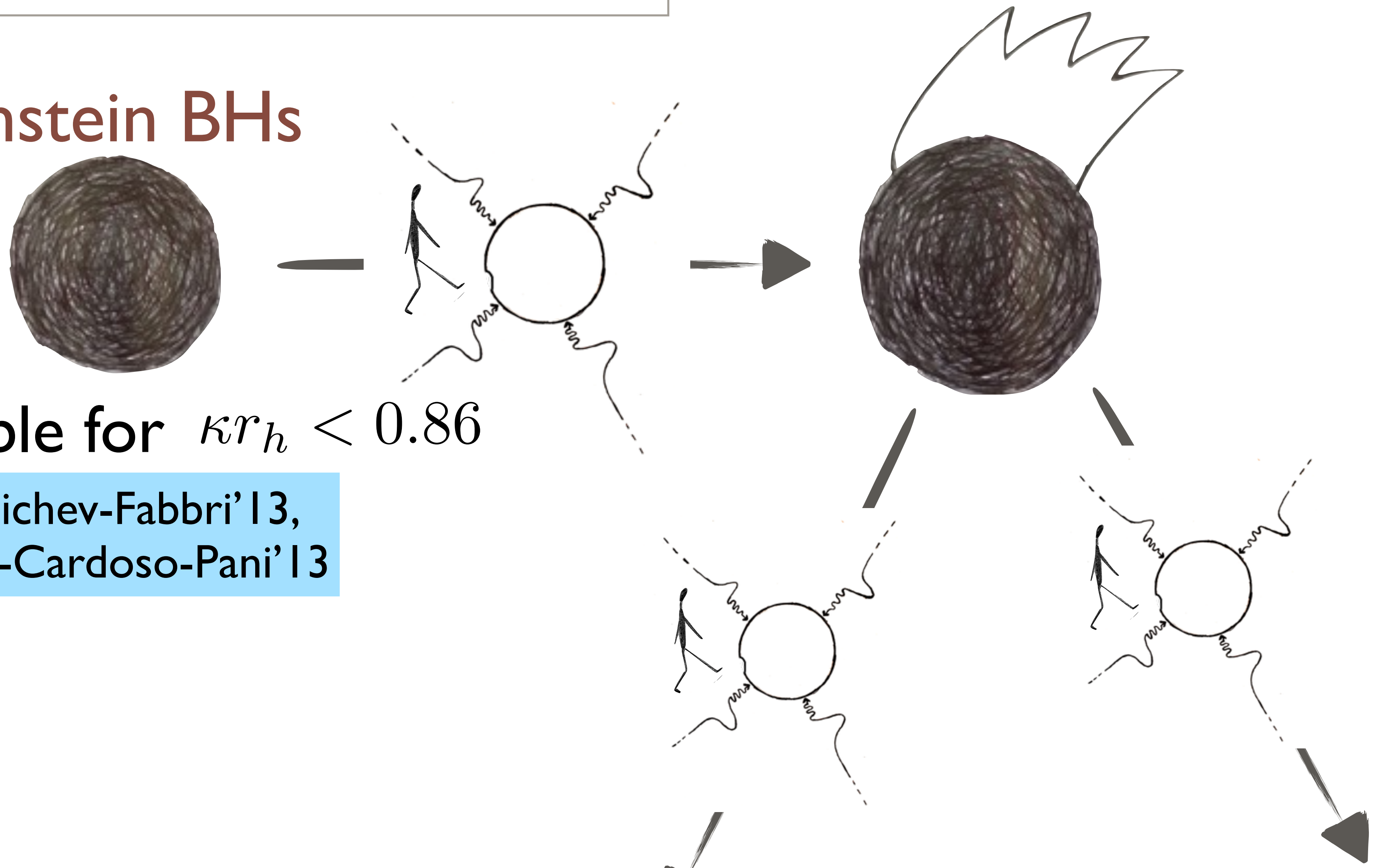
- quasi-bound states and stability
- quasi-normal modes



Theories with massive spin-2:

Stability of Schwarzschild BHs

bimetric gravity



Einstein BHs

unstable for $\kappa r_h < 0.86$

Babichev-Fabbri'13,
Brito-Cardoso-Pani'13

stable

Gervalle-Volkov'20

unstable
???

Theories with massive spin-2:

Stability of Schwarzschild BHs

torsion bigravity

Schw. BH is linearly stable

Nikiforova’21,
Nikiforova'22

Theories with massive spin-2:

Stability of Schwarzschild BHs

torsion bigravity

lower bound on the mass of the massive spin-2
to avoid singularities

$$\kappa > \frac{\sqrt{1+\eta}}{r_h}$$

Schw. BH is linearly stable

Nikiforova’21,
Nikiforova'22

Theories with massive spin-2:

Stability of Schwarzschild BHs

torsion bigravity

lower bound on the mass of the massive spin-2
to avoid singularities

$$\kappa > \frac{\sqrt{1+\eta}}{r_h}$$

for $2M_\odot$ black hole:

$\kappa \gtrsim 10^{-10}$ eV $\qquad \kappa^{-1} < 6$ km

Nikiforova'21,
Nikiforova'22

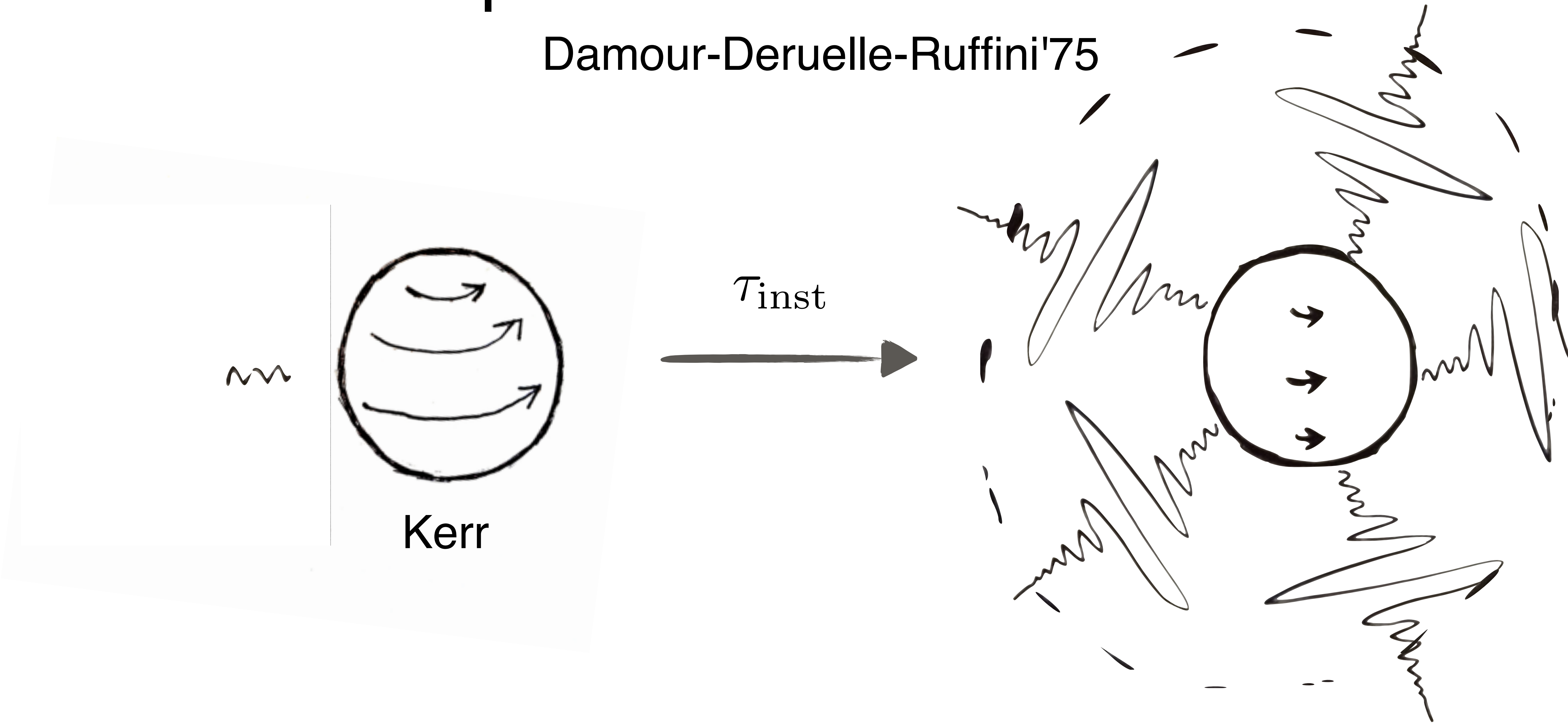
Superradiant instabilities: NO

Damour-Deruelle-Ruffini'75

Kerr

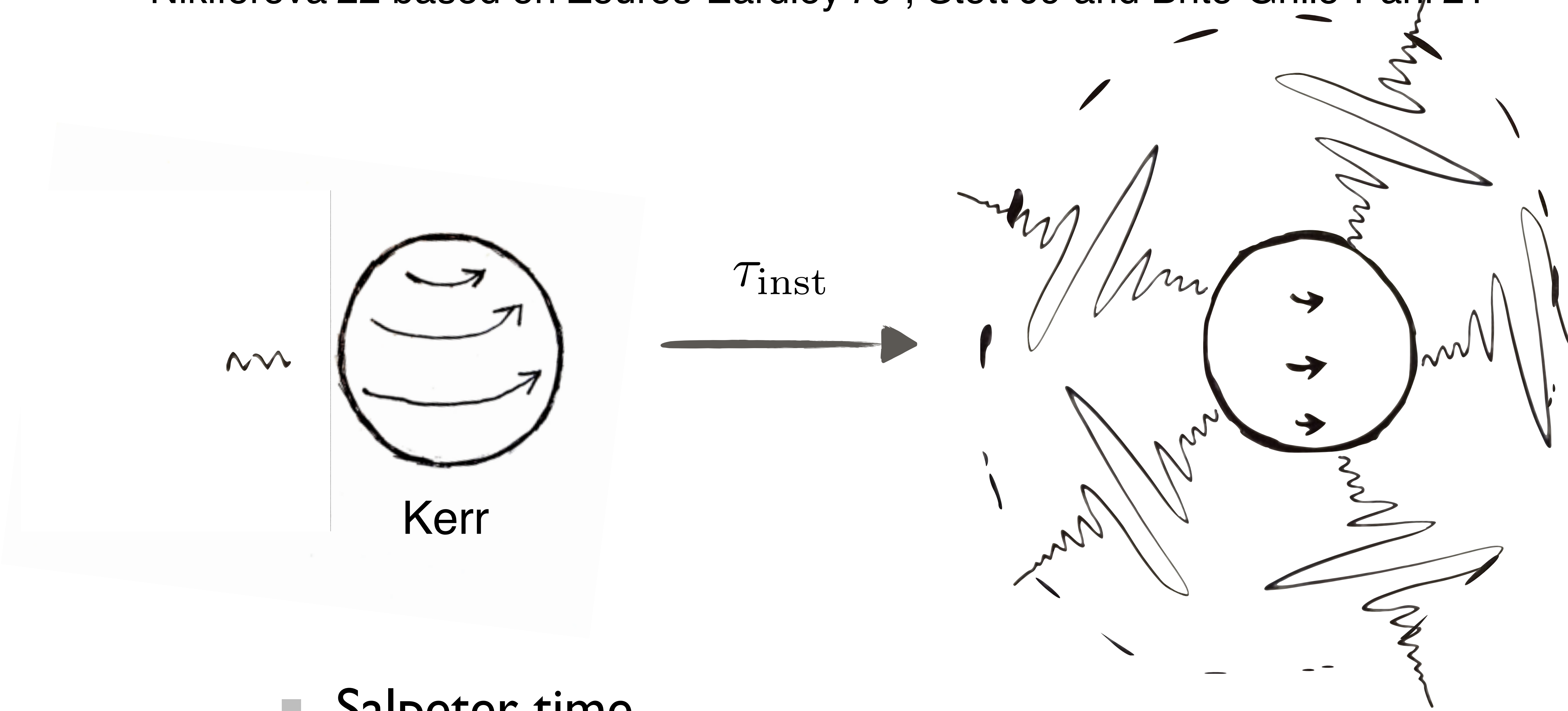
Superradiant instabilities: NO

Damour-Deruelle-Ruffini'75



Superradiant instabilities: NO

Nikiforova’22 based on Zouros-Eardley'79 , Stott’09 and Brito-Grillo-Pani’21

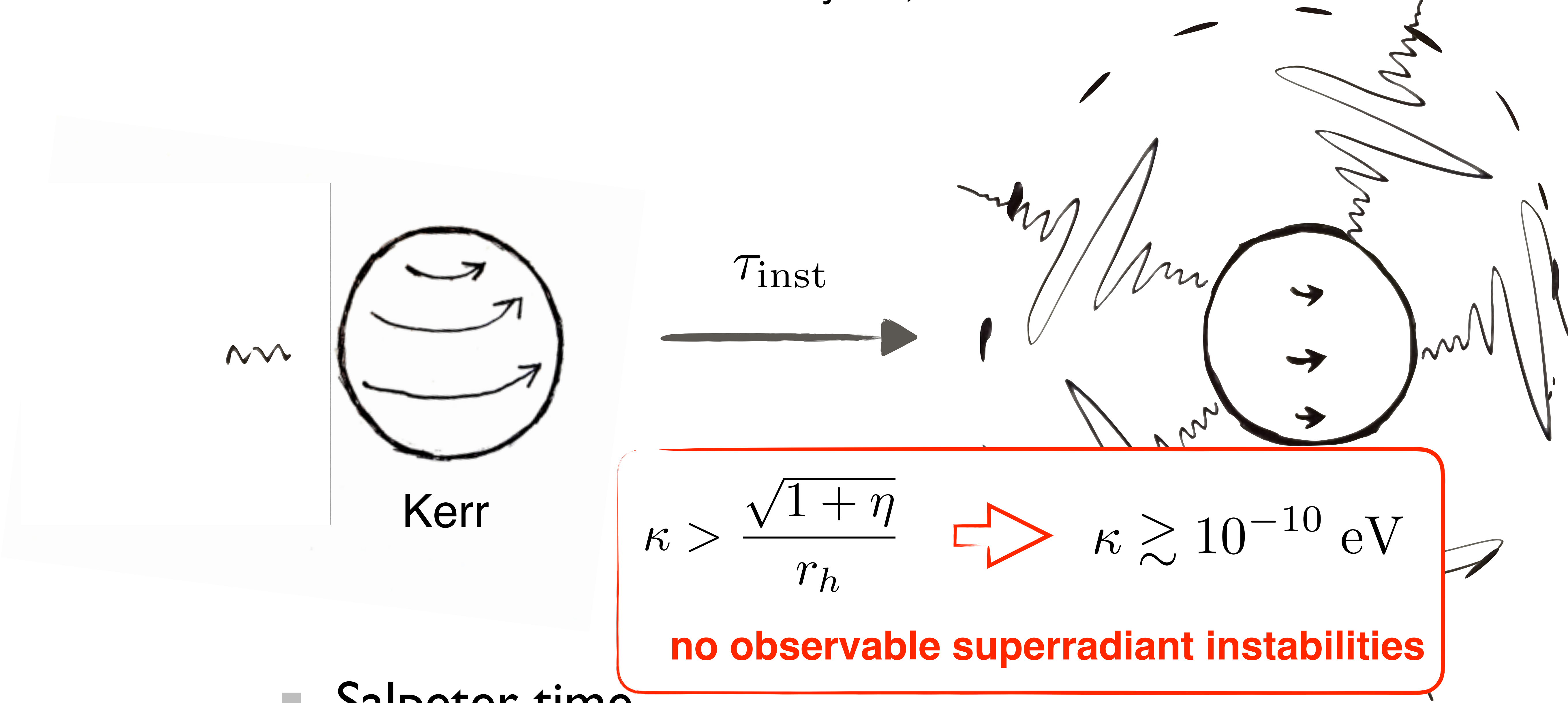


- Salpeter time
- time of observation of a spinning black hole

EXCLUDED: $10^{-13}\ \mathrm{eV} \lesssim\ \kappa\ \lesssim 10^{-11}\ \mathrm{eV}$

Superradiant instabilities: NO

Nikiforova’22 based on Zouros-Eardley'79 , Stott’09 and Brito-Grillo-Pani’21



$$\kappa > \frac{\sqrt{1+\eta}}{r_h} \Rightarrow \kappa \gtrsim 10^{-10}\ {\rm eV}$$

no observable superradiant instabilities

- Salpeter time
- time of observation of a spinning black hole

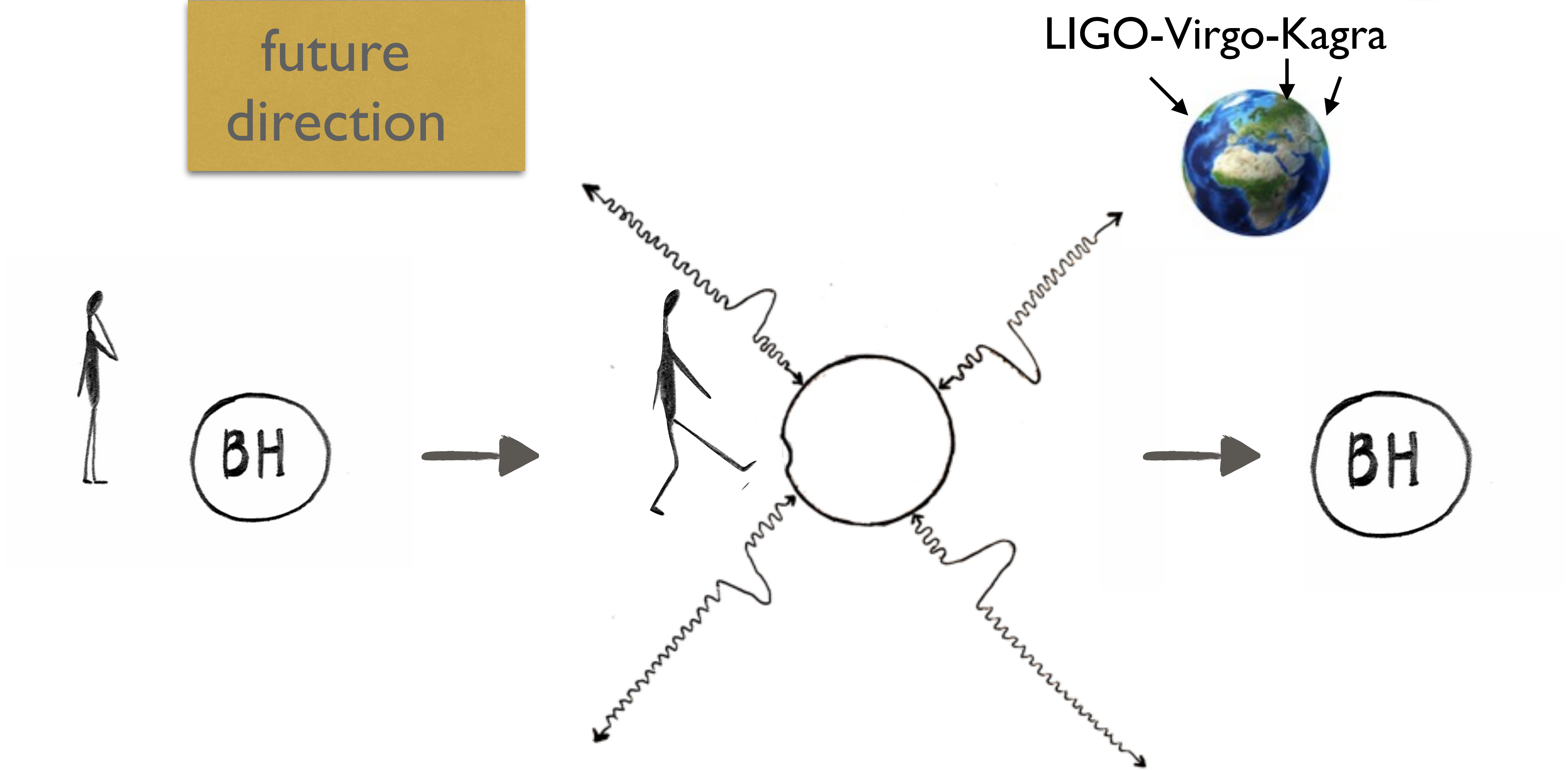
EXCLUDED: $10^{-13}\ {\rm eV} \lesssim \kappa \lesssim 10^{-11}\ {\rm eV}$

Black hole perturbations

Nikiforova'21 + work in progress

- quasi-bound states and stability
- quasi-normal modes

future direction



Take away message

- it’s a good time to do gravity. good time to formulate questions, good time to look beyond GR

- gravity with dynamical torsion is a natural extension of GR

- gravity with dynamical torsion gives
  an interesting alternative to GR,
  which can be useful both for phenomenology
  and
  for theoretical development of gravity.
  TO THIS END, I will continue study.

A 5-parameter class of dynamical torsion theories revived with cosmological motivation

Nair—Randjbar-Daemi—Rubakov (2009)
V. Nikiforova, S. Randjbar-Daemi, V. Rubakov (2009)
Deffayet—Randjbar-Daemi (2011)
V. Nikiforova, S. Randjbar-Daemi, V. Rubakov (2016)
V. Nikiforova (2017)
V. Nikiforova, T. Damour (2018)

self-accelerating solution
where torsion accelerates the Universe

but
instabilities found.