THE MAGIC FERMI SEA

electronic transport
in
mesocopic conductors

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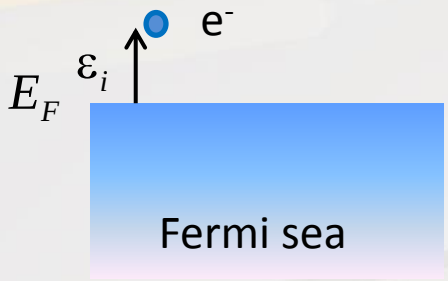
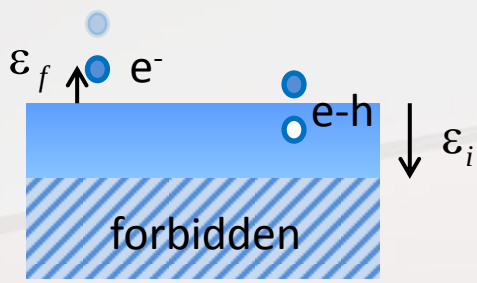
ERC Advanced Grant MeQuaNo

- >1980 quantum transport in phase coherent conductors became accessible
- new quantum effects not oberved in other systems like photons, photons+atoms, etc.
- originate from the Fermi statistics properties of electrons.

- concept of Fermi sea (Fermi 1926 : high Z atoms, Sommerfeld 1927 : metals)

- Landau quasiparticle (1957)
  - Thomas Fermi screening : $1/r \rightarrow e^{-\lambda r}/r$
  - Pauli suppression of electron-electron scattering channels





$$\tau_{qp} \sim (E_F / \varepsilon)^2$$

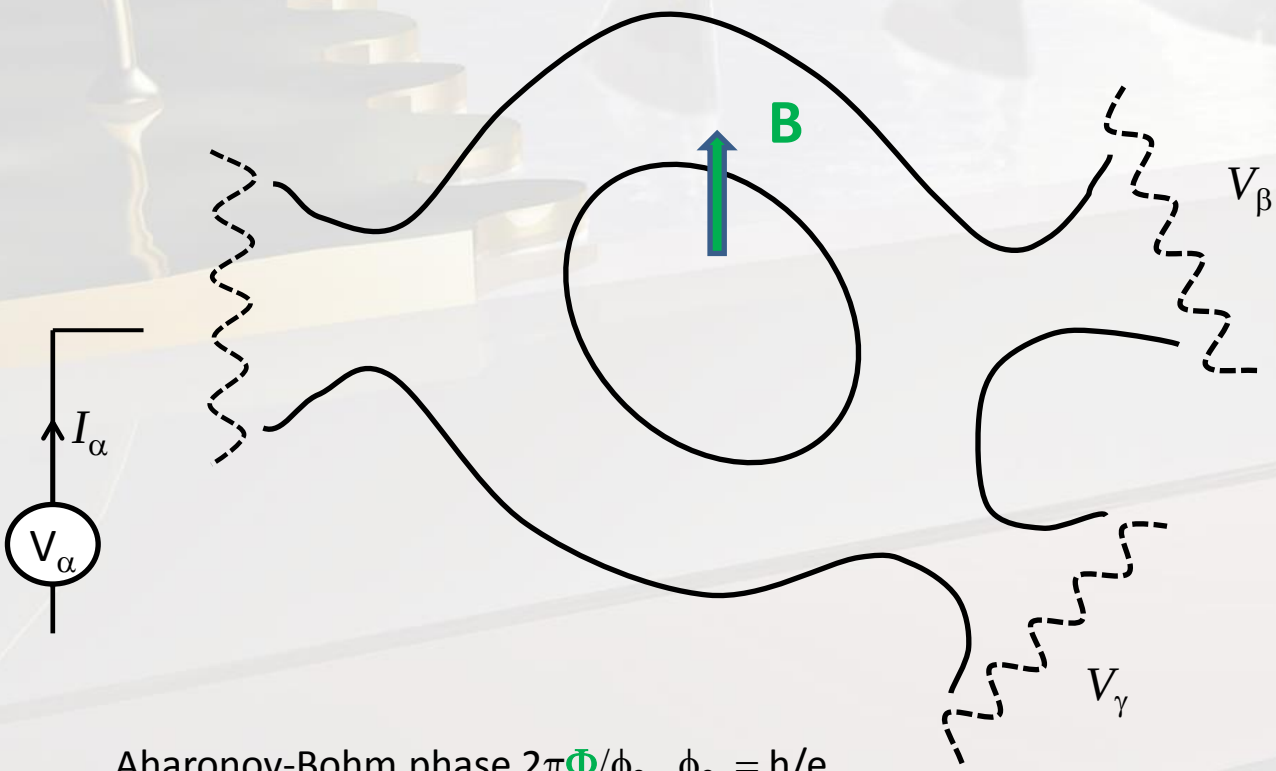
~10^4 room tpre metals

mesoscopic quantum transport:

conductor size < coherence length $l_\varphi = v_F\,\tau_\varphi$ ballistic (> 10 μm @ 1°K)

$L < l_\varphi$ $\tau_\varphi \leq \tau_{qp}$ $l_\varphi = \sqrt{v_F\, l_e\, \tau_\varphi}$ diffusive (> μm @ 1°K)

- pionneered by R. Landauer (1957) (≠ from Kubo approach 1957)
- 1980-1985 full quantum description of quantum transport (Imry, Büttiker, Landauer, …)
- elastic scattering of electron waves



$$S = \begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{pmatrix}$$

$$I_\alpha = \frac{e^2}{h} \sum_{\beta \neq \alpha} \mathrm{tr}(S^+_{\alpha\beta}\, S_{\alpha\beta})(V_\alpha - V_\beta)$$

transmission from β to α

Aharonov-Bohm phase $2\pi\Phi/\phi_0$, $\phi_0 = h/e$

(M. Büttiker, PRL 1986)
(Büttiker, Imry, Landauer, Pinhas, PRB 1985)

some magic properties of the Fermi sea:

- quantized conductance in absence of backscattering : $G = e^2/h$
- continuous single electron source for free : $I = e\,(eV/h)$
- noiseless electron (fermion) transport : $\langle \Delta I^2 \rangle = 0$
- perfect electron anti-bunching
- generation of entangled electron-hole pairs for free

All this points towards the realization of electron quantum optics experiments

emphazing or exploiting difference between bosons (photons) and fermions (electrons)

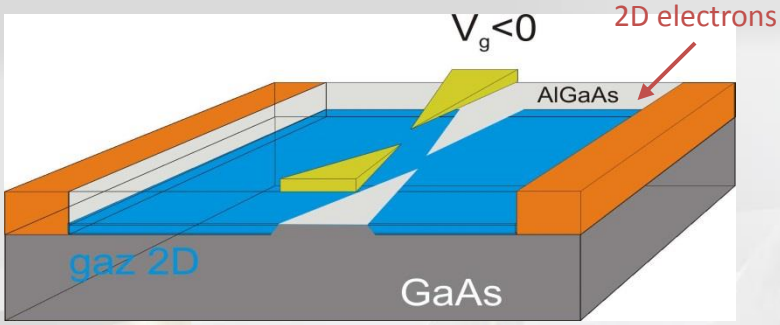
- single photons sources (photons are launched in the vacuum)
- single electron sources (electrons are launched on top of a Fermi sea)
  - in general, generates many extra el-hole excitations
  - but: a magic property of the Fermi sea allows for quiet injection of electrons in a form of a leviton.

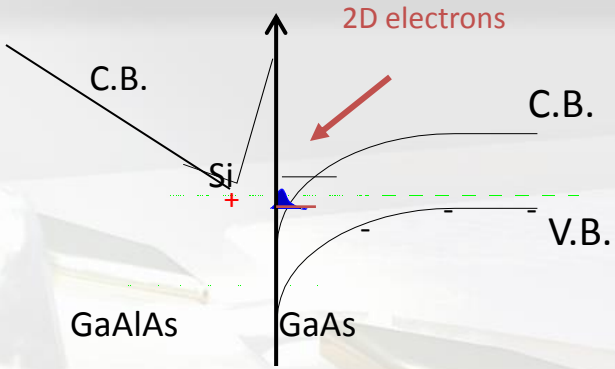
OUTLINE

- ideal conductors to explore the Fermi Sea
- quantized conductance and noiseless electron flow
- electron-hole entanglement in the Fermi-sea
- single electron sources for electron quantum optics
  - minimal excitations states of a Fermi sea: the levitons
  - experimental realization of levitons
- perspective and applications of levitons

ideal conductors to explore the Fermi sea

2D-electrons confined as a perfect 2D metal in semiconductors





III-V semi-conductor heterojunction GaAs/GaAlAs

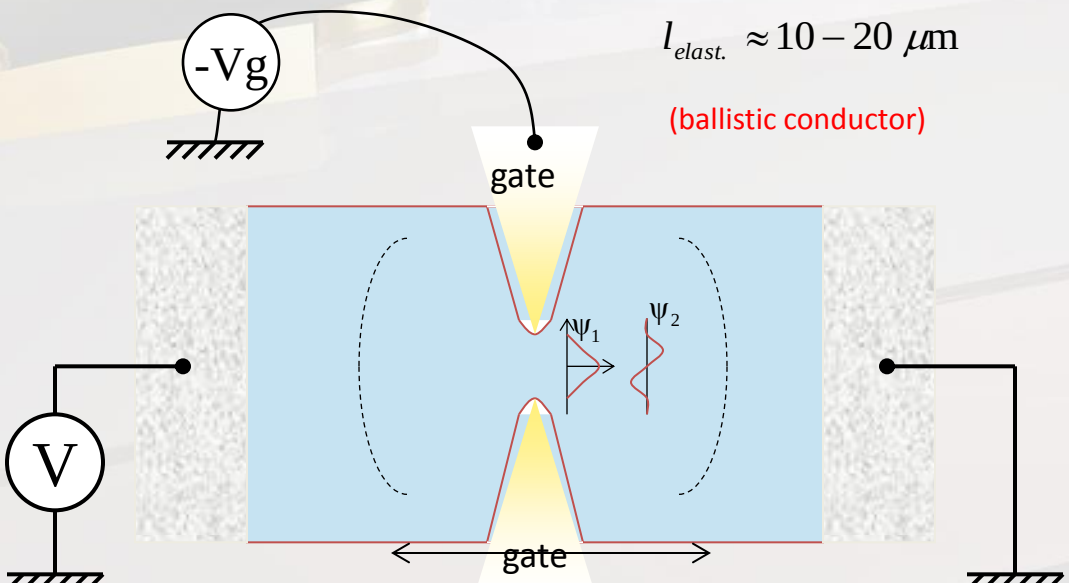
~1980 : very high quality heterojunctions

quantum Point Contact

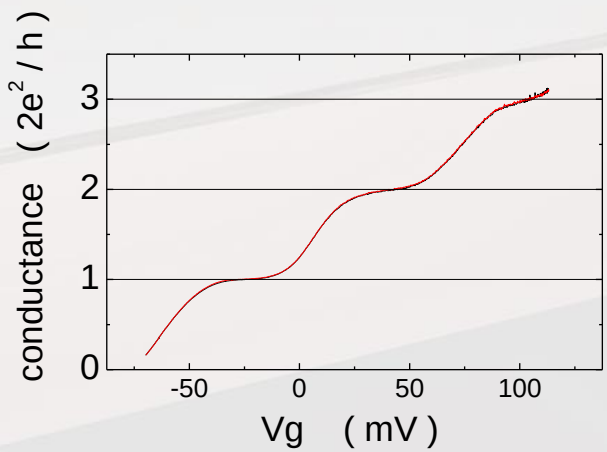
$$\lambda_F \approx 70 \text{ nm}$$

$$l_{elast.} \approx 10 - 20\ \mu\text{m}$$

(ballistic conductor)



coherence length : >10 µm (T <<1°K)



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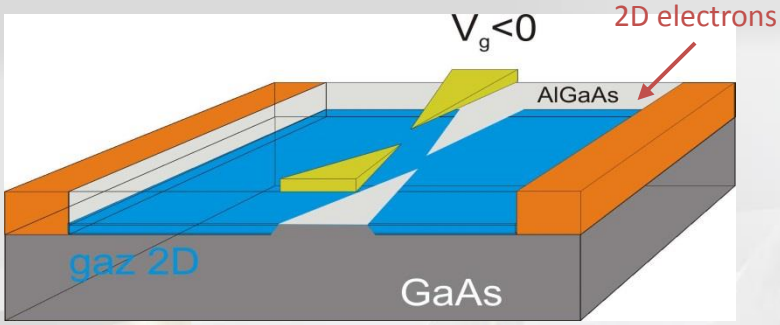

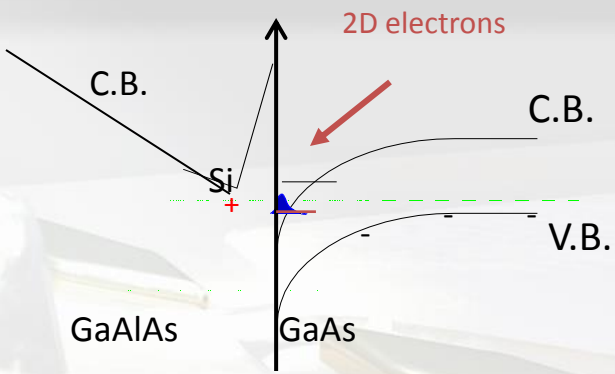

III-V semi-conductor heterojunction GaAs/GaAlAs

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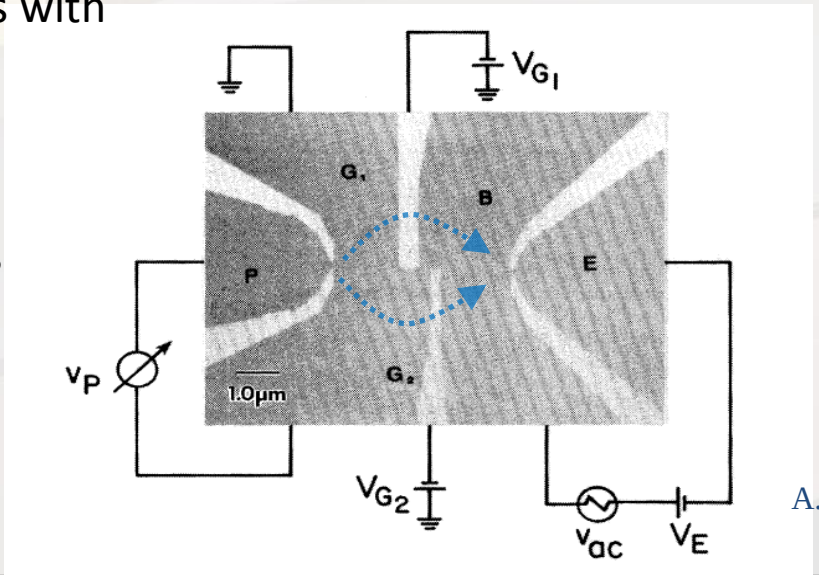
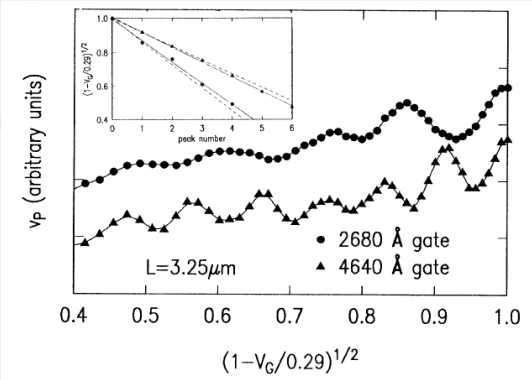
quantum optics with electron waves

electron interferences

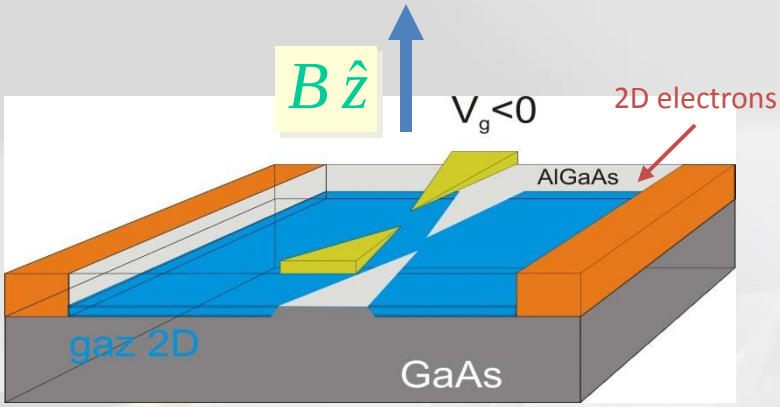
(in 2D)





A. Yacoby et al. Phys. Rev. Lett. 66, 1938 (1991)

ideal conductors to explore the Fermi sea



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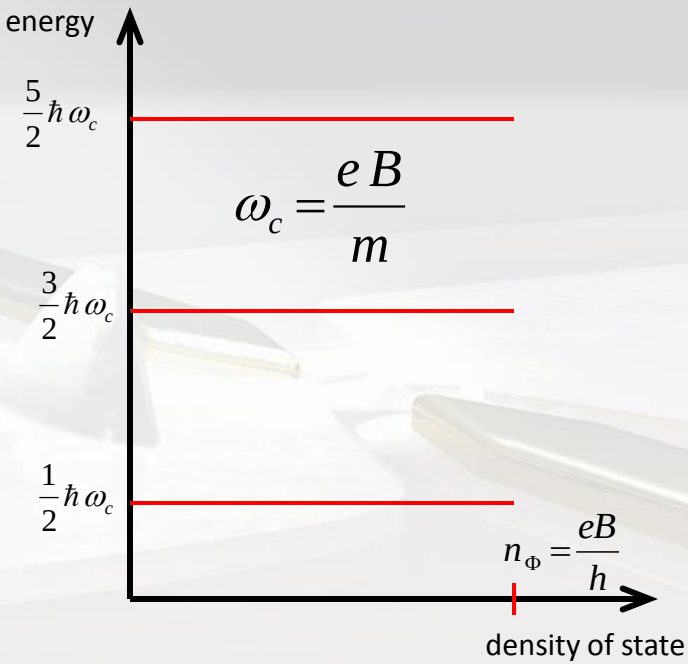
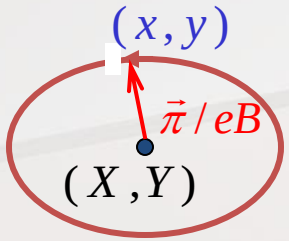


$$H = \frac{1}{2m}\left(\vec{p} + e\vec{A}\right)^2 = \frac{\vec{\pi}^2}{2m}$$

$$[\pi_x, \pi_y] = -i\hbar eB \longrightarrow E_n = \hbar\omega_c\left(n + \frac{1}{2}\right)$$

$$X = x - \frac{\pi_y}{eB}$$

$$Y = y + \frac{\pi_x}{eB}$$

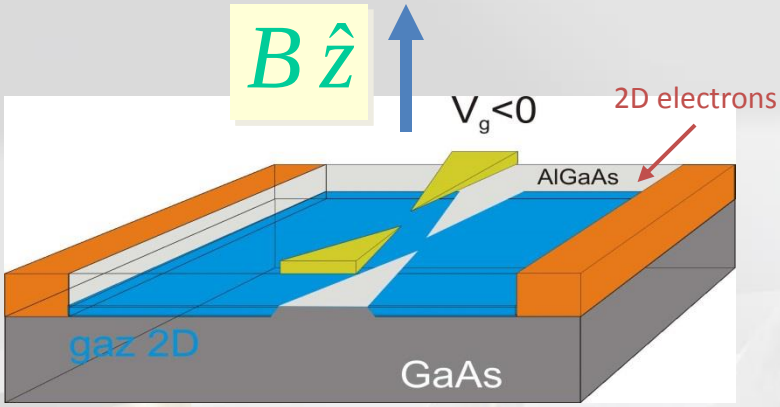


cyclotron motion

$$[X, Y] = -i\frac{\hbar}{eB} \longrightarrow B\Delta X.\Delta Y = \frac{h}{e}$$

cyclotron motion is frozen $\rightarrow$ 1D dynamics

ideal conductors to explore the Fermi sea



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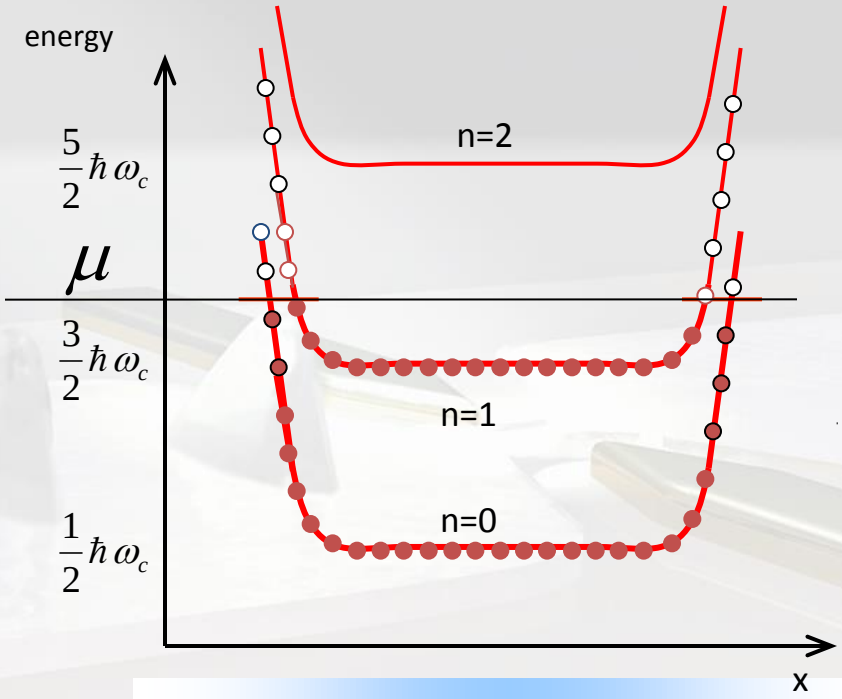
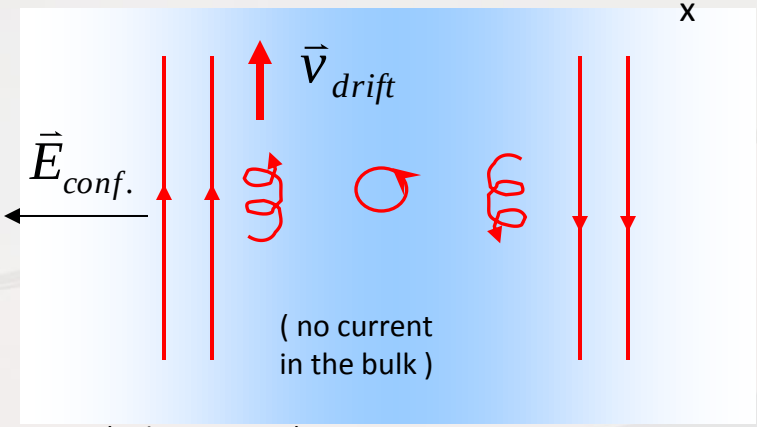


$$\vec{v}_{drift} = \frac{\vec{E}_{conf.}}{B} \times \hat{z}$$



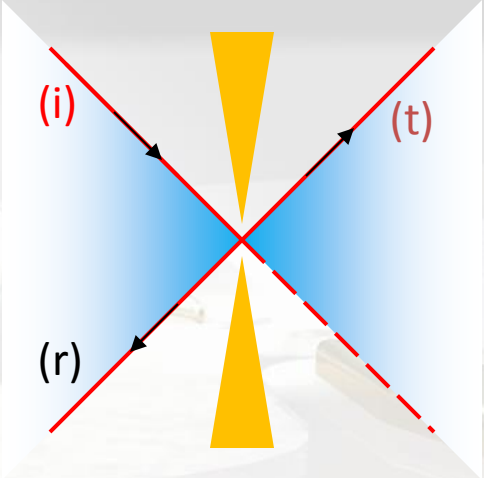
(edge current)

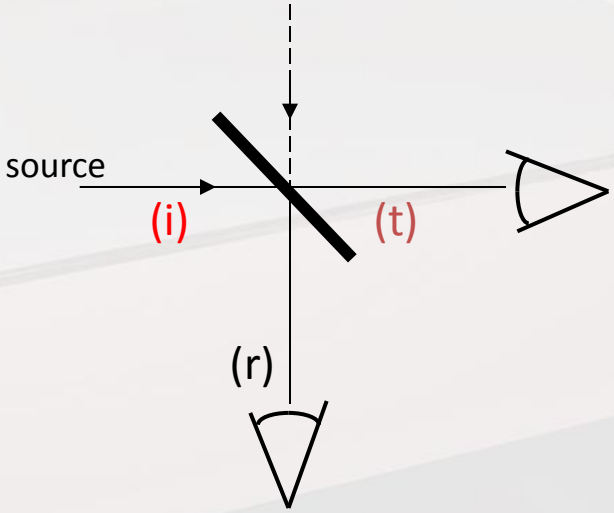
cyclotron motion drift → chiral 1D dynamics

ideal conductors to explore the Fermi sea: electron quantum optics

elementary quantum gates are realizable:

- beam splitter
- fabry-Pérot interferometer
- Mach-Zehnder interferometer





ideal conductors to explore the Fermi sea: electron quantum optics

elementary quantum gates are realizable:

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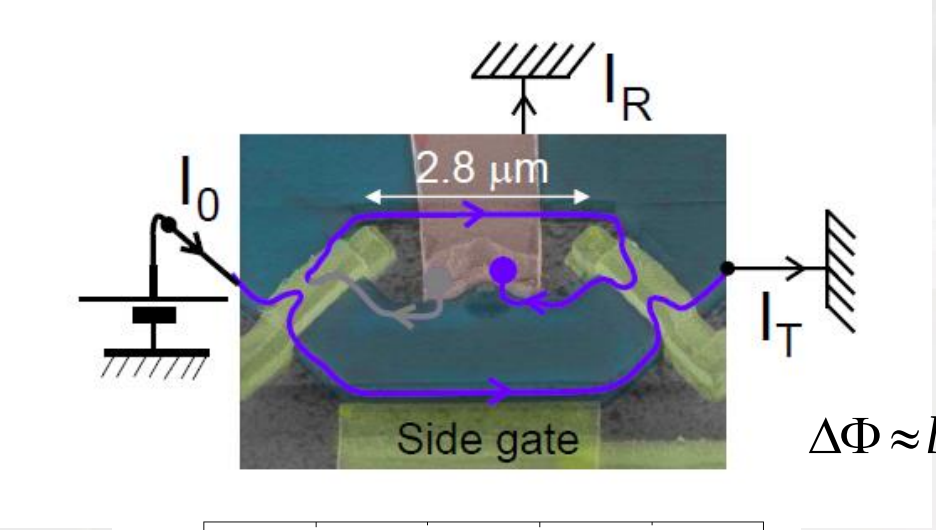
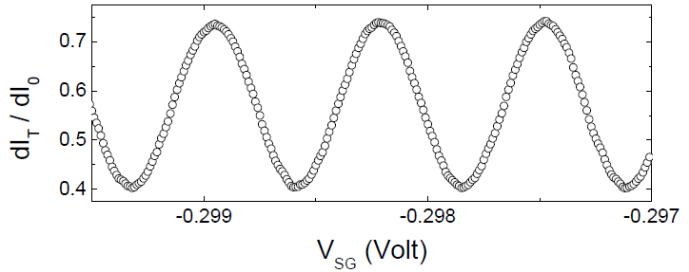
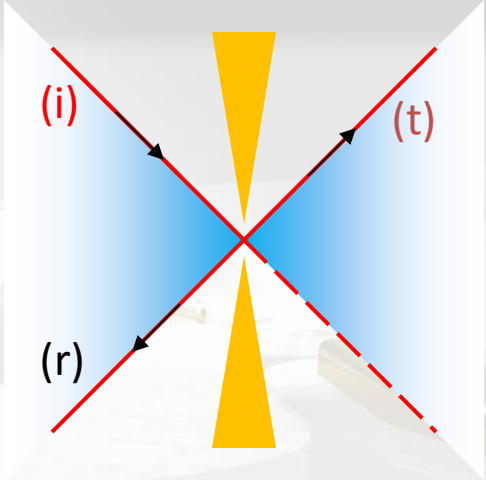
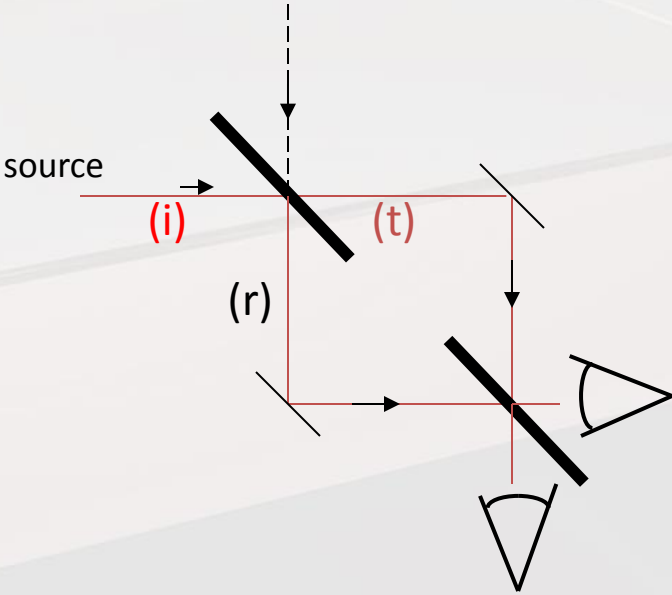


$$\Delta\Phi \approx l_1 - l_2$$



(adapted from: P. Roche, P. Roulleau,F. Portier G. Faini D. Mailly)





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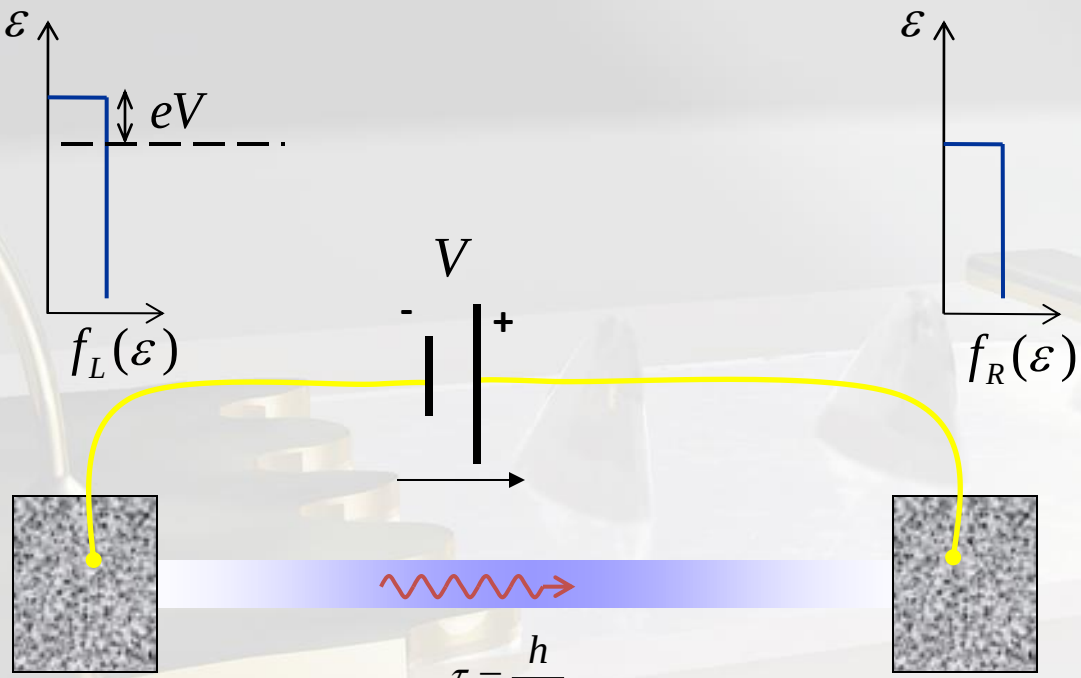
quantized conductance of a perfect conductor

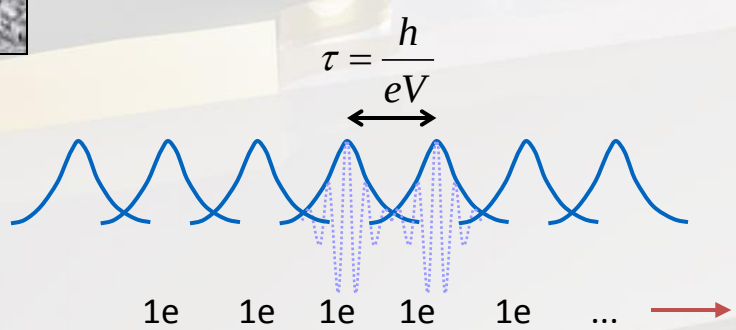


exemple: single mode (1D)

$$G = \frac{e^2}{h}$$

$$R = \frac{h}{e^2}$$

25.812 k Ω



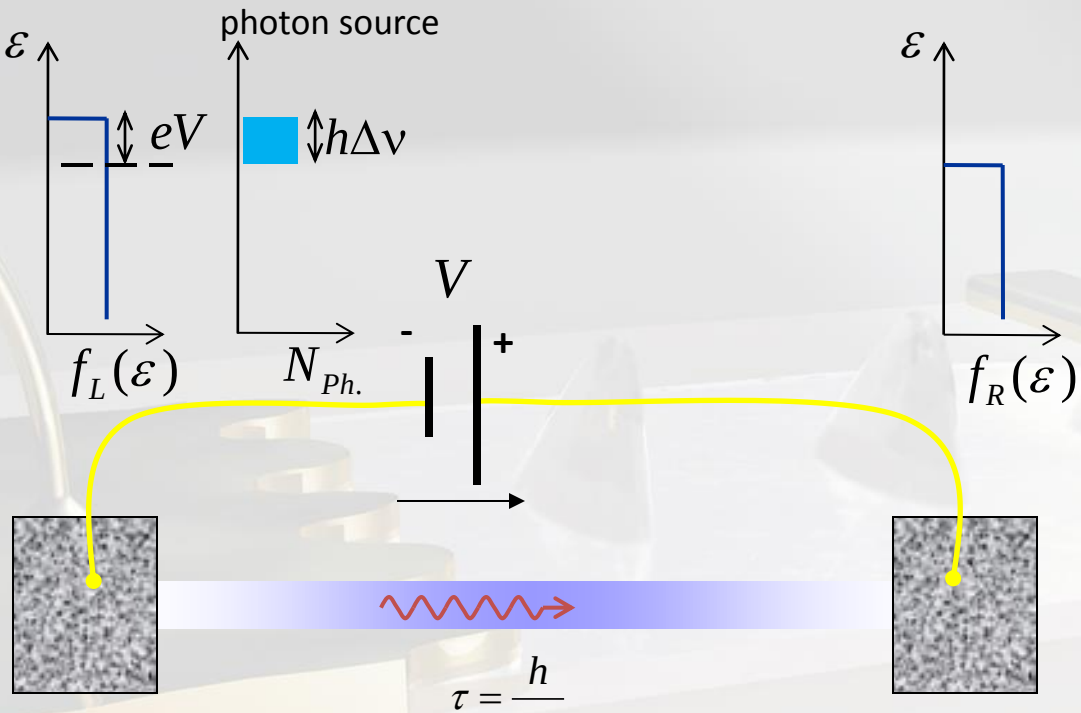
$$I = e.\frac{eV}{h}$$

Pauli

Heisenberg: $eV.\tau \sim h$

- quantization is robust to temperature
- probed in the Quantum Hall regime 10^{-9} accuracy (25 812.807 4434 Ohms)

quantized conductance of a perfect conductor



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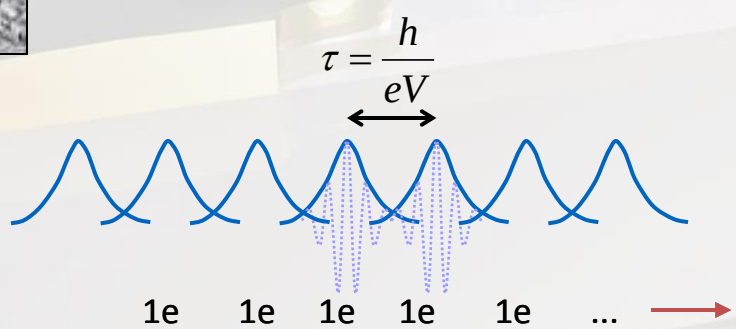


$$R = \frac{h}{e^2}$$

25.812 k Ω

$$I = e.\frac{eV}{h}$$

Pauli

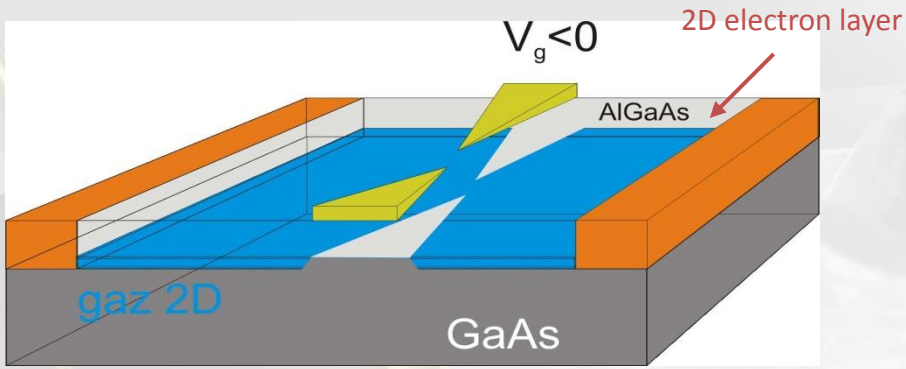
Heisenberg: $eV.\tau \sim h$

$$\dot{N}_{Ph.} = N_{Ph}.\frac{\Delta(h\nu)}{h} = N_{Ph.}\Delta\nu$$

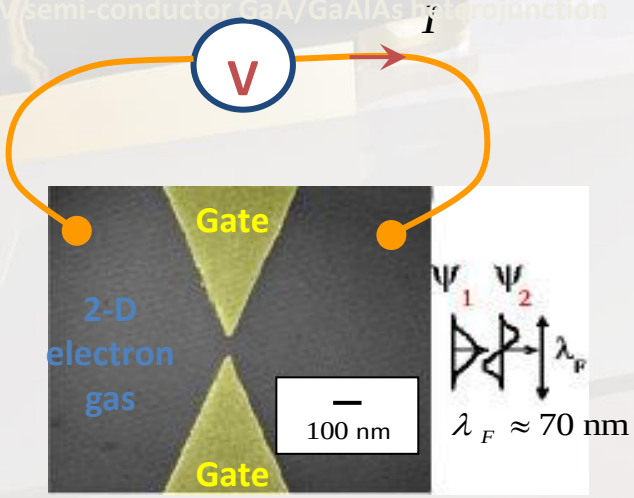
a thermodynamic reservoir of photons cannot provide photon flux quantization

⇒ conductance quantization

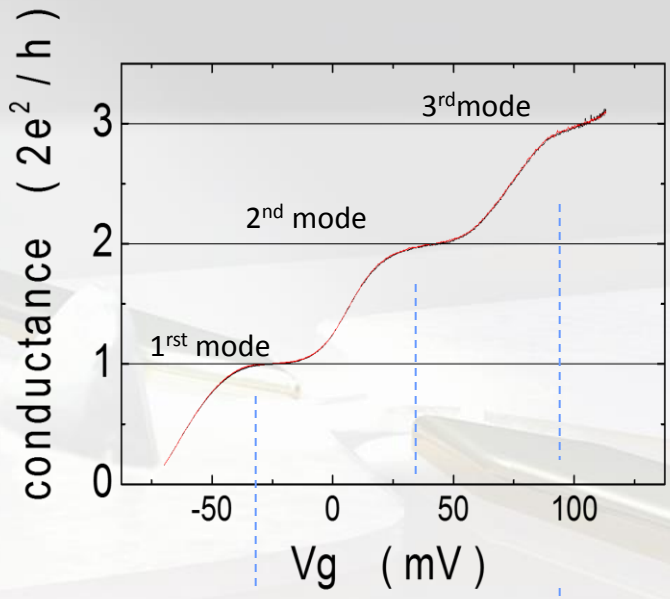
(D. Warrham ; B.J. van Wees 1988)



III-V semi-conductor GaA/GaAlAs heterojunction



Ballistic Point Contact QW



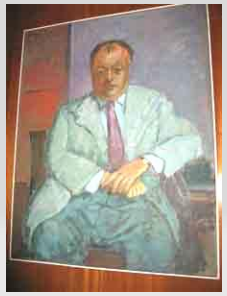
$D_1 = 0 \rightarrow 1$

$D_2 = 0 \rightarrow 1$

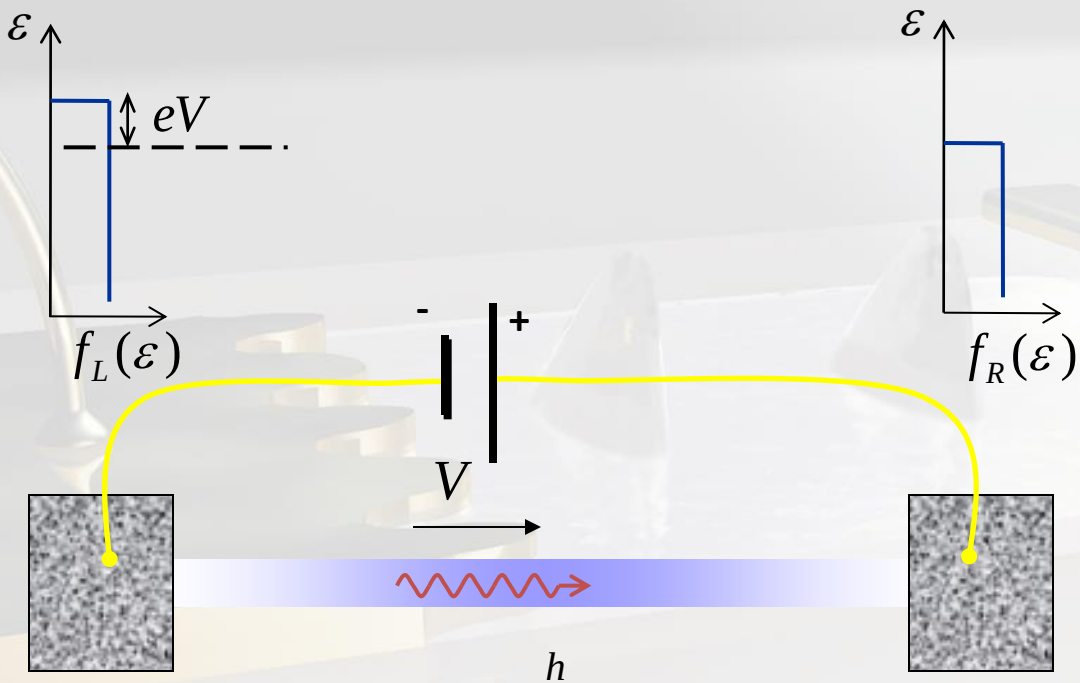
$D_3 = 0 \rightarrow 1$

$$G = \frac{2e^2}{h} \cdot \sum_n D_n$$

perfect conductors are noiseless

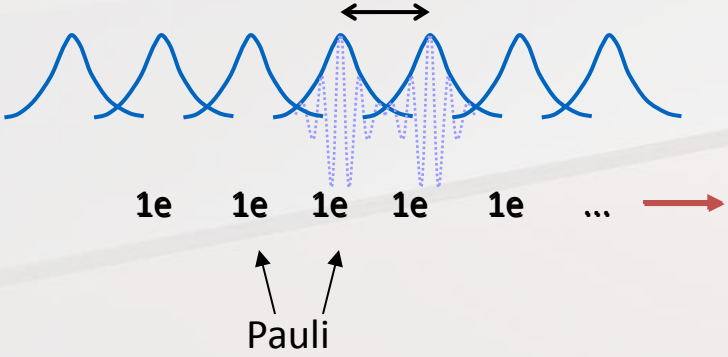


Wolfgang Pauli



$$\langle \Delta I^2 \rangle = 0$$

$$\tau = \frac{h}{eV}$$



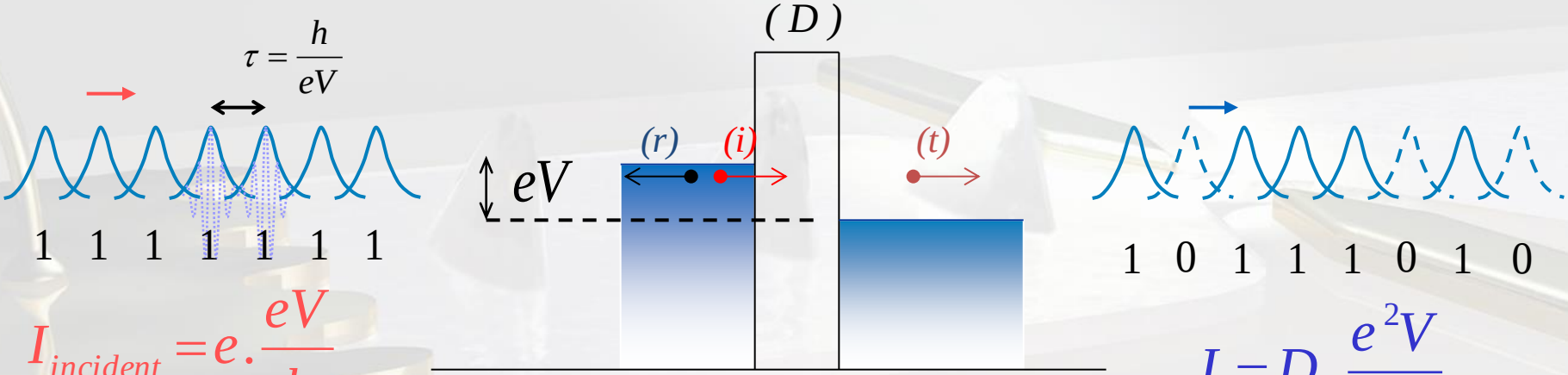
$$I = e.\frac{eV}{h}$$

Pauli

Heisenberg: $eV.\tau \sim h$

$$\langle \Delta \dot{N}_{Ph.}{}^2 \rangle \propto N_{Ph}(1 + N_{Ph.})$$

Shot noise = perfect quantum partition noise



binomial statistics

Shot Noise < Schottky (Poisson)

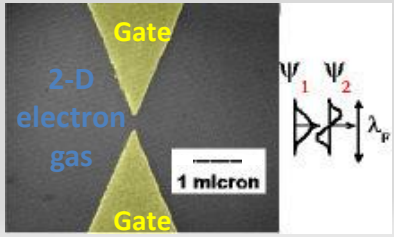
$\propto D(1-D)$

no noise for D = 1 !

$$\langle \Delta I^2 \rangle = 2eI(1-D)\Delta f$$

G. Lesovik 89, M. Büttiker 91
Th. Martin, R. Landauer 92
Khlus (1987)

Quantum Point Contact



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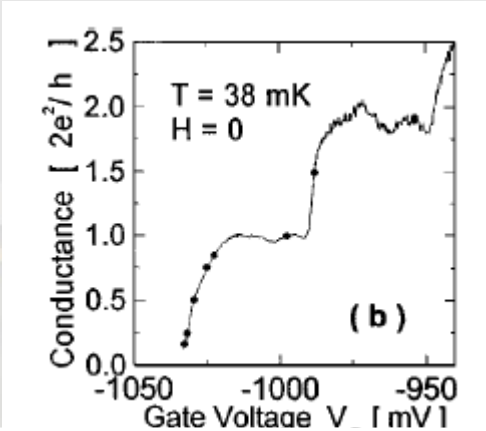
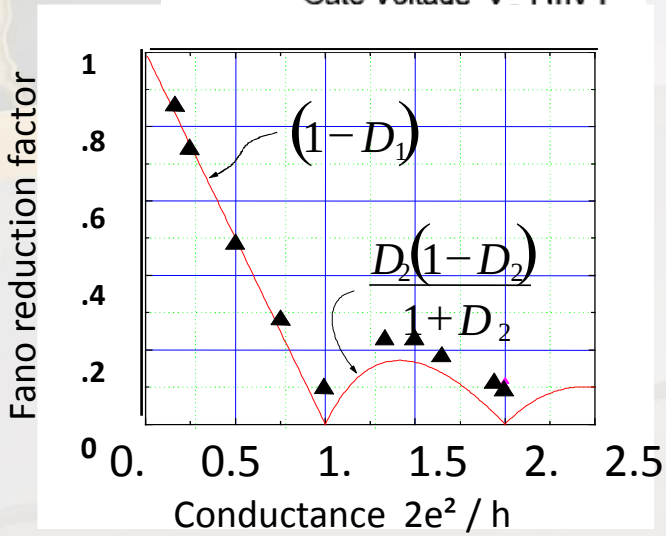
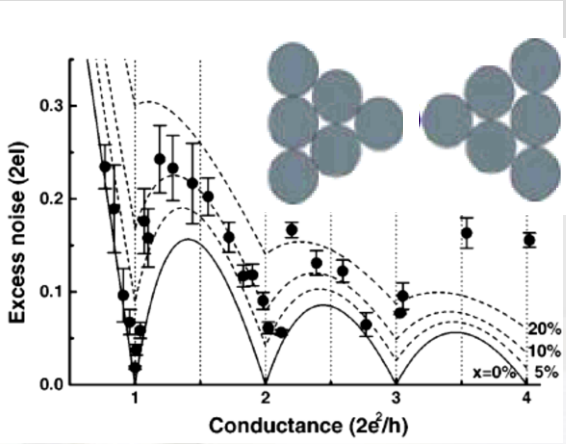




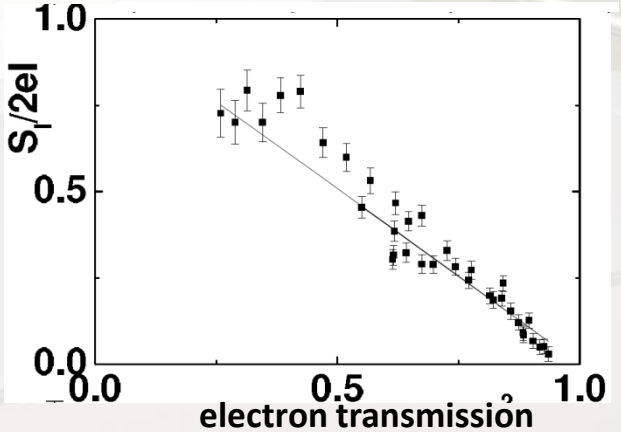
*A. Kumar et al. Phys. Rev. Lett. **76** (1996) 2778..*
*M. I. Reznikov et al., Phys. Rev. Lett. **75** (1995) 3340.*

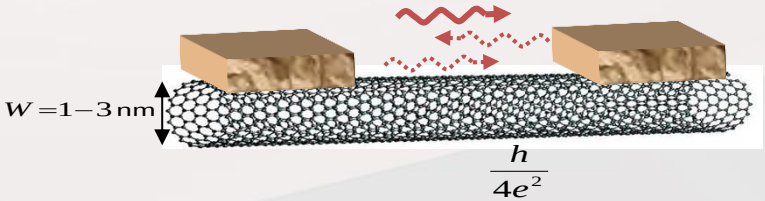
Atomic point contact



van Ruitenbeck, Phys. Rev. Lett. (1999)

Carbon Nanotube



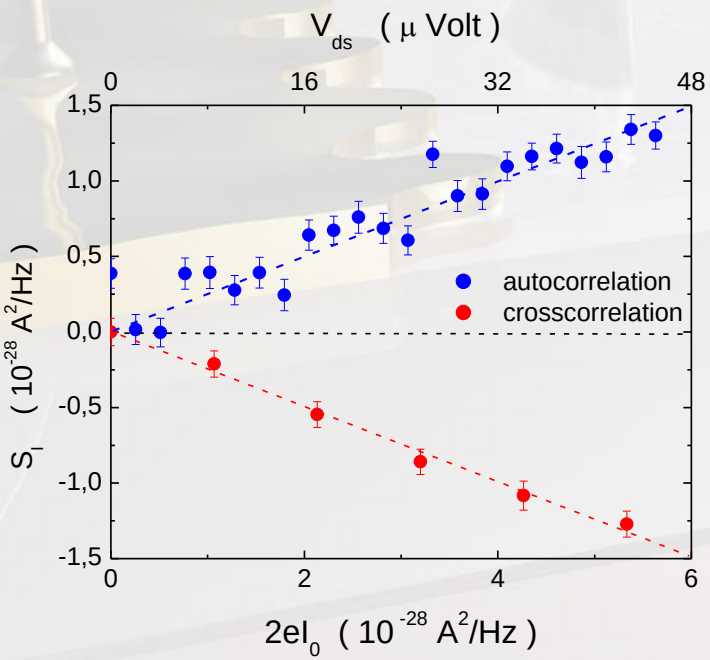


L. G. Herrmann, T. Delattre, P. Morfin, J.-M. Berroir, B. Plaçais, D. C. Glattli, and T. Kontos Phys. Rev. Lett. 99, 156804 (2007)

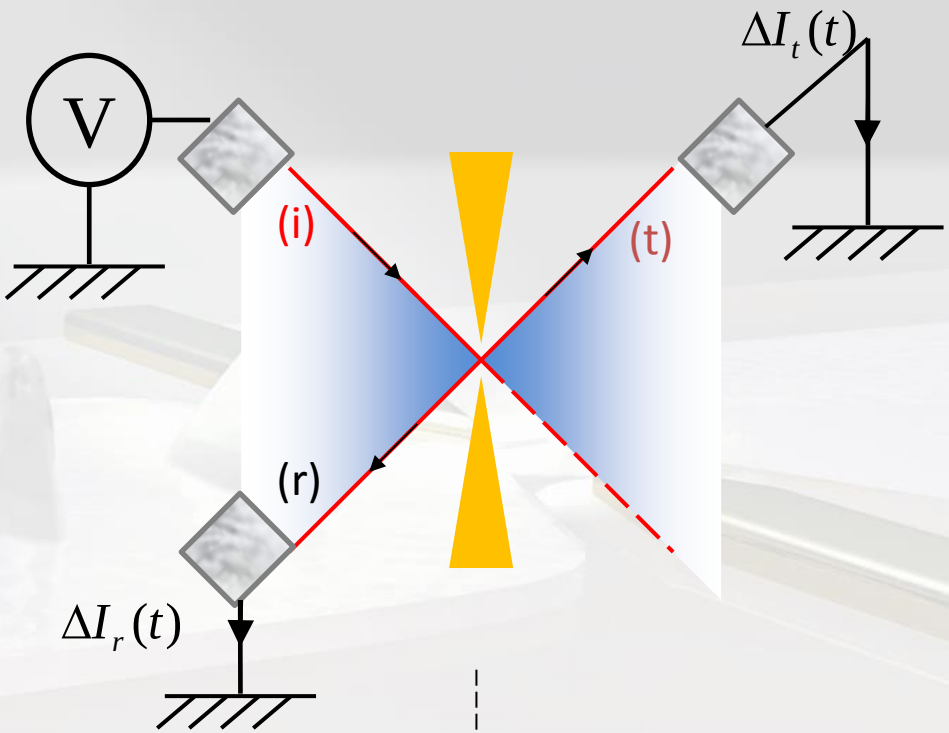
Shot noise = perfect quantum partition noise

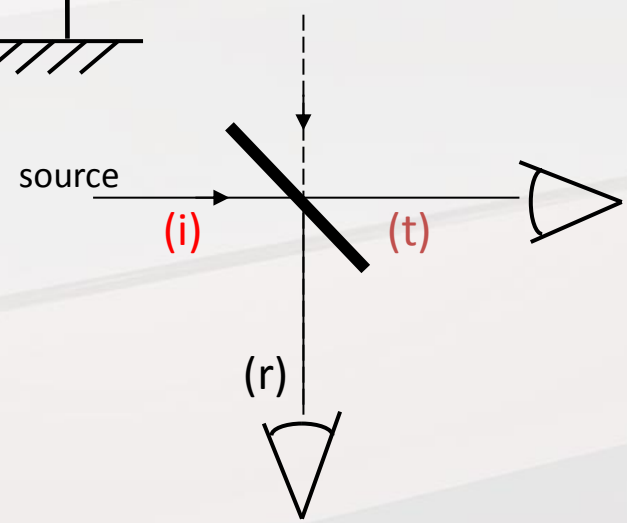
elementary quantum gates are realizable:

- beam splitter
- fabry-Pérot interferometer
- Mach-Zehnder interferometer



$$\boxed{\langle \Delta I_t . \Delta I_r \rangle = -\langle (\Delta I_t)^2 \rangle = -S_I \, \Delta\nu}$$





perfect negative correlation for electrons

VOLUME 65, NUMBER 23 PHYSICAL REVIEW LETTERS 3 DECEMBER 1990

Scattering Theory of Thermal and Excess Noise in Open Conductors

M. Büttiker

PHYSICAL REVIEW B VOLUME 46, NUMBER 19 15 NOVEMBER 1992-I

Scattering theory of current and intensity noise correlations in conductors and wave guides

M. Büttiker
IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598
(Received 16 June 1992)

$$\langle \Delta I_\alpha \Delta I_\beta \rangle_{\mathrm{tr}} = \mp 2\Delta\nu (e^2/h) \int dE \sum_{\gamma,\delta} f_\gamma f_\delta \mathrm{Tr}(\mathbf{s}^\dagger_{\alpha\gamma}\mathbf{s}_{\alpha\delta}\mathbf{s}^\dagger_{\beta\delta}\mathbf{s}_{\beta\gamma})$$

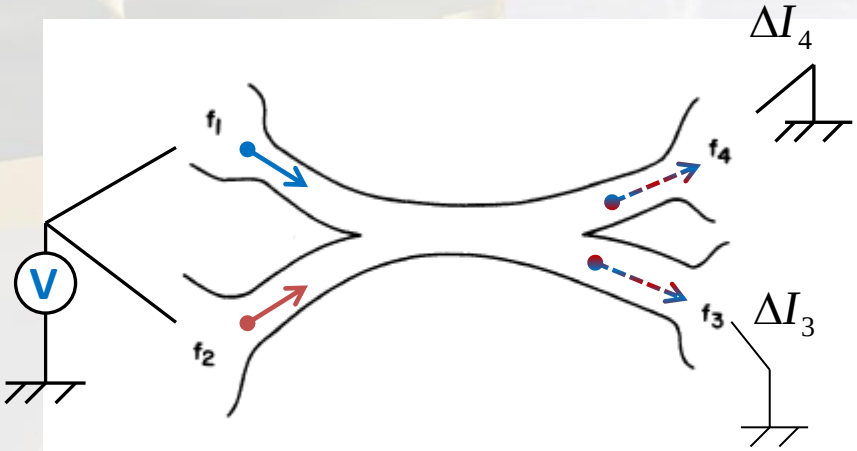


FIG. 1. Conductor (waveguide) with contacts connected to electron reservoirs (blackbody radiation sources).

exp. (A) $\langle \Delta I_4 . \Delta I_4 \rangle = -2\frac{e^2}{h} eV (T_{41} . T_{31})$

exp. (B) $\langle \Delta I_4 . \Delta I_4 \rangle = -2\frac{e^2}{h} eV (T_{42} . T_{32})$

exp. (C) $\langle \Delta I_4 . \Delta I_4 \rangle_{(C)} - \langle \Delta I_4 . \Delta I_4 \rangle_{(A)} - \langle \Delta I_4 . \Delta I_4 \rangle_{(B)}$

$$= -2\frac{e^2}{h} eV \left(s^+_{41} s_{42} s^+_{32} s_{31} \right)$$

(exchange term)

quantum statistics !

related experimental observations:
M. Henny et al., Science 284, 296 (1999)., W. D. Oliver, J. Kim, R. C. Liu, Y. Yamamoto, Science 284, 299 (1999)
Liu, R. C., Odom, B., Yamamoto, Y. & Tarucha, S., Nature 391, 263–265 (1998)

OUTLINE

- ideal conductors to explore the Fermi Sea
- quantized conductance and noiseless electron flow
- electron-hole entanglement in the Fermi-sea
- single electron sources for electron quantum optics
  - minimal excitations states of a Fermi sea: the levitons
  - experimental realization of levitons
- perspective and applications of levitons

Entangled electron-hole pairs by mixing two Fermi sea

VOLUME 91, NUMBER 14 PHYSICAL REVIEW LETTERS week ending 3 OCTOBER 2003

Proposal for Production and Detection of Entangled Electron-Hole Pairs in a Degenerate Electron Gas

C.W. J. Beenakker, C. Emary, M. Kindermann, and J. L. van Velsen
Instituut-Lorentz, Universiteit Leiden, P.O. Box 9506, 2300 RA Leiden, The Netherlands

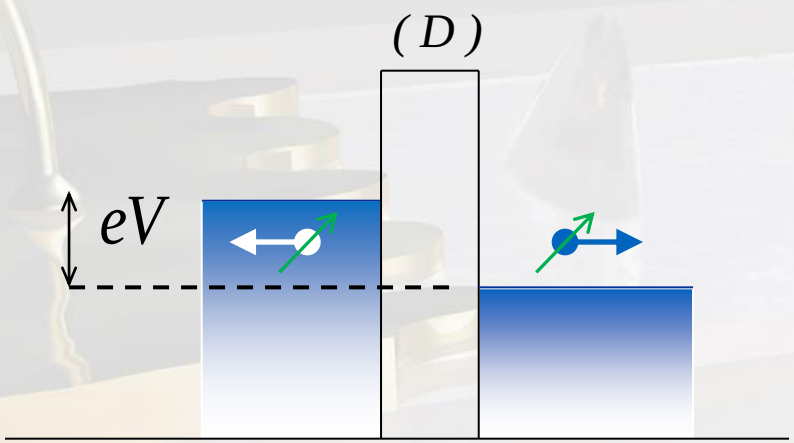


FIG. 1 Creation of an entangled electron-hole pair by application of a voltage difference V between two metals separated by a tunnel barrier. The Fermi sea consists of the filled states in the conduction band. The spins of the electron (e) that has tunneled and the hole (h) it leaves behind are entangled in the state $2^{-1/2}(|\uparrow_h\uparrow_e\rangle + |\downarrow_h\downarrow_e\rangle)$.

ground state:

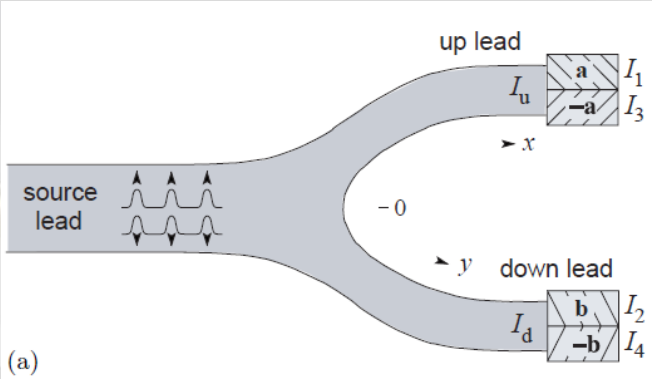
$|0\rangle \equiv$ (eV-shifted left Fermi Sea)$\otimes$ (right Fermi sea)

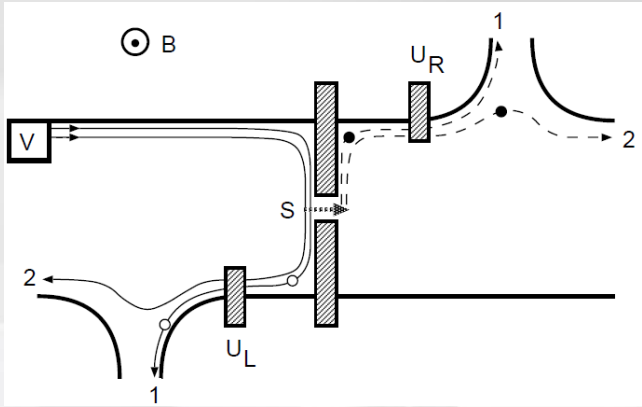
$$|0\rangle \rightarrow (1-D)|0\rangle - e^{2i\varphi} D |\uparrow\downarrow\rangle_h |\uparrow\downarrow\rangle_e$$
$$- e^{i\varphi}\sqrt{D(1-D)}\,\frac{1}{\sqrt{2}}\underbrace{\left(|\uparrow\rangle_h|\uparrow\rangle_e + |\downarrow\rangle_h|\downarrow\rangle_e\right)}_{\text{Bell pair}}$$

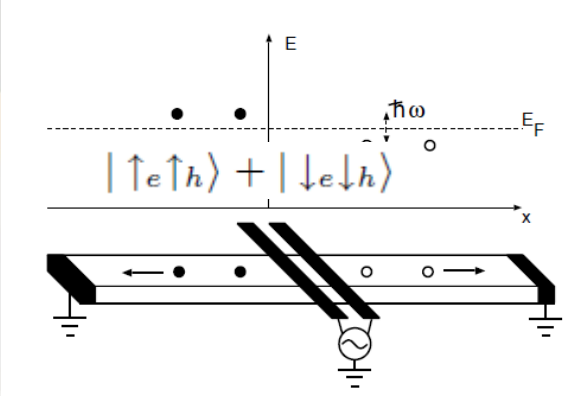
entanglement production rate: $\frac{2eV}{h} D(1-D)$

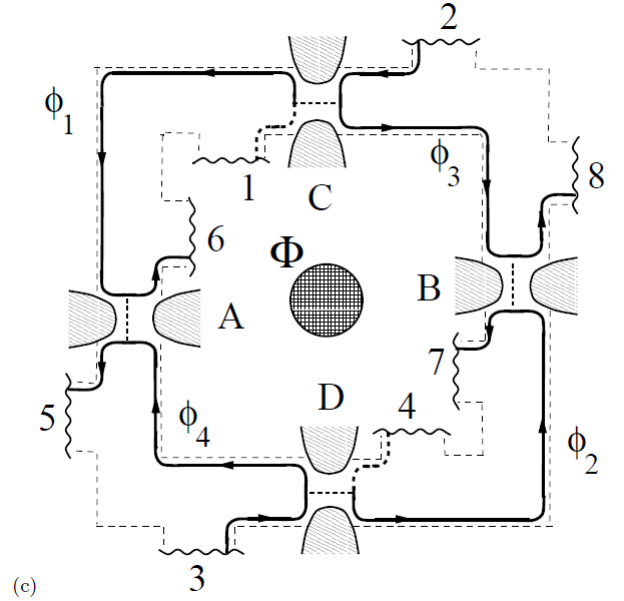
finite temperature $T \le T_C = \frac{eV}{k_B \ln(1/D)}$ $D \ll 1$

Entangled electron-hole pairs by mixing two Fermi sea

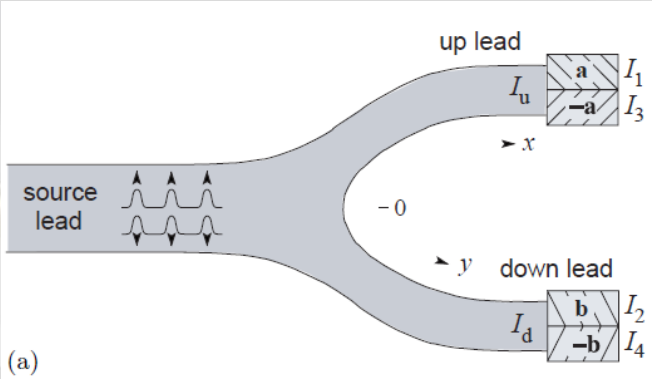


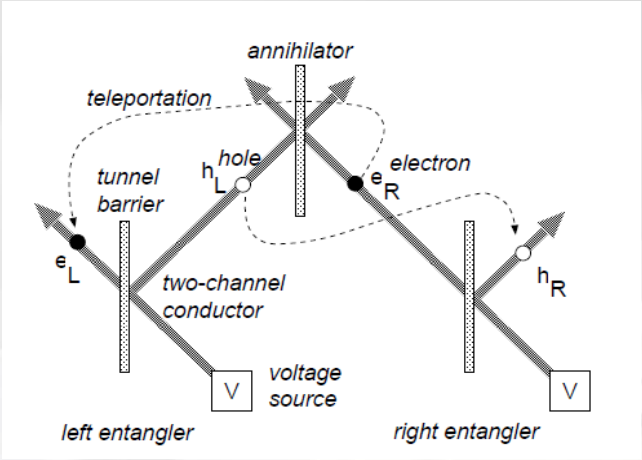


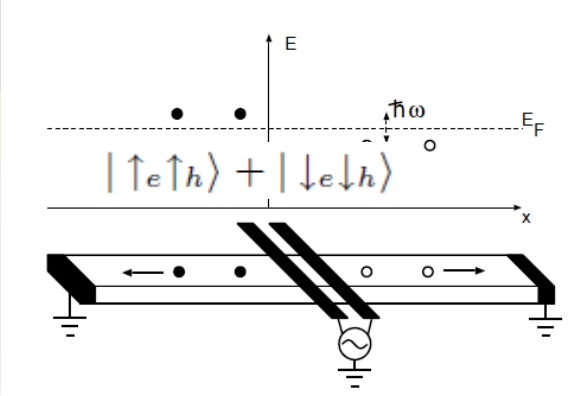


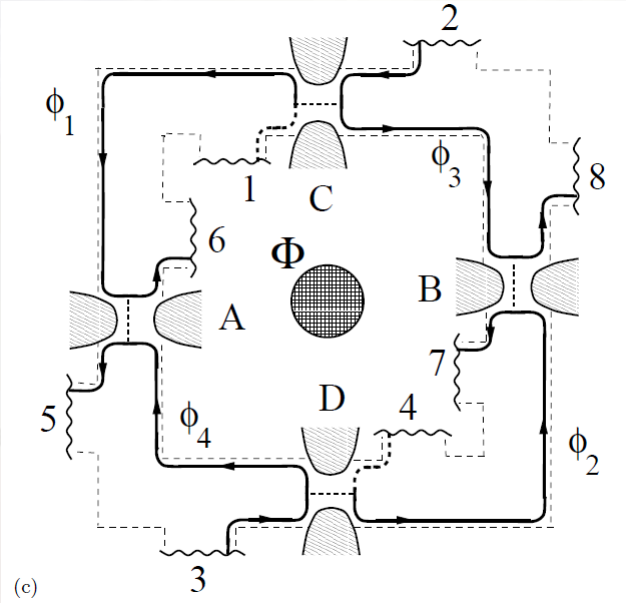


Entangled electron-hole pairs by mixing two Fermi sea

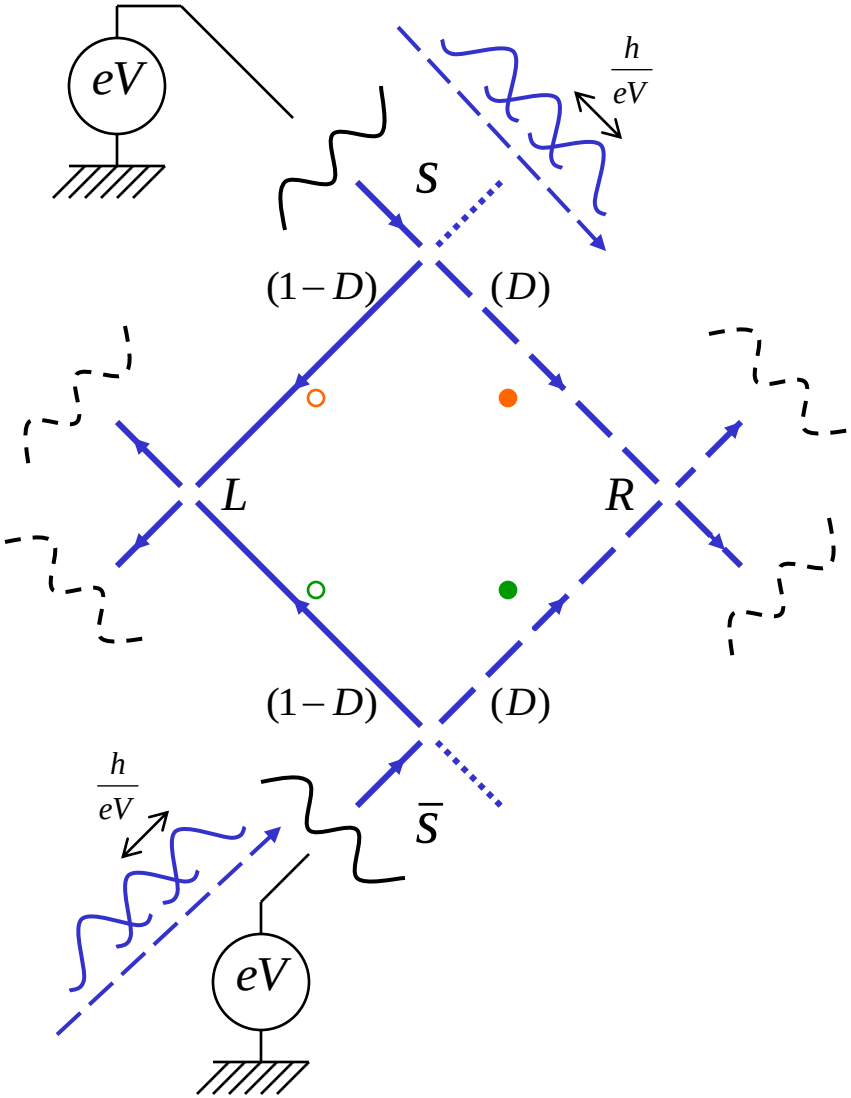








entangling Fermions emitted by thermodynamic reservoirs



- reservoirs regularly inject electrons at frequency eV/h

$$s \leftrightarrow \uparrow$$
$$\bar{s} \leftrightarrow \downarrow$$

(pseudo-spin representation)

$$|\psi\rangle_{in} = c_{\uparrow}^{+} c_{\downarrow}^{+} |0\rangle \quad \left(\equiv \prod_{0<\varepsilon\le eV} c_{\uparrow}^{+}(\varepsilon) c_{\uparrow}^{+}(\varepsilon) |0\rangle \right)$$

$$|\psi\rangle_{out} = (\sqrt{1-D}\, c_{\uparrow,L}^{+} + \sqrt{D}\, c_{\uparrow,R}^{+})(\sqrt{1-D}\, c_{\downarrow,L}^{+} + \sqrt{D}\, c_{\downarrow,R}^{+})|0\rangle$$

$$= \begin{bmatrix} (1-D) c_{\uparrow,L}^{+} c_{\downarrow,L}^{+} + D c_{\uparrow,R}^{+} c_{\downarrow,R}^{+} + \ldots \\ \ldots \sqrt{D(1-D)} \left(c_{\uparrow,L}^{+} c_{\downarrow,R}^{+} + c_{\uparrow,R}^{+} c_{\downarrow,L}^{+} \right) \end{bmatrix} |0\rangle$$

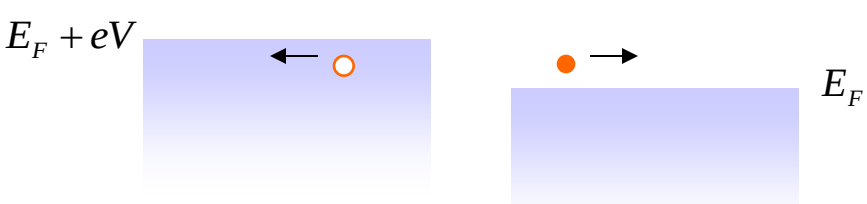
for D << 1, left states are filled up to eV

$$|\tilde{0}\rangle = c_{\uparrow,L}^{+} c_{\downarrow,L}^{+} |0\rangle \qquad \text{(new vacuum)}$$

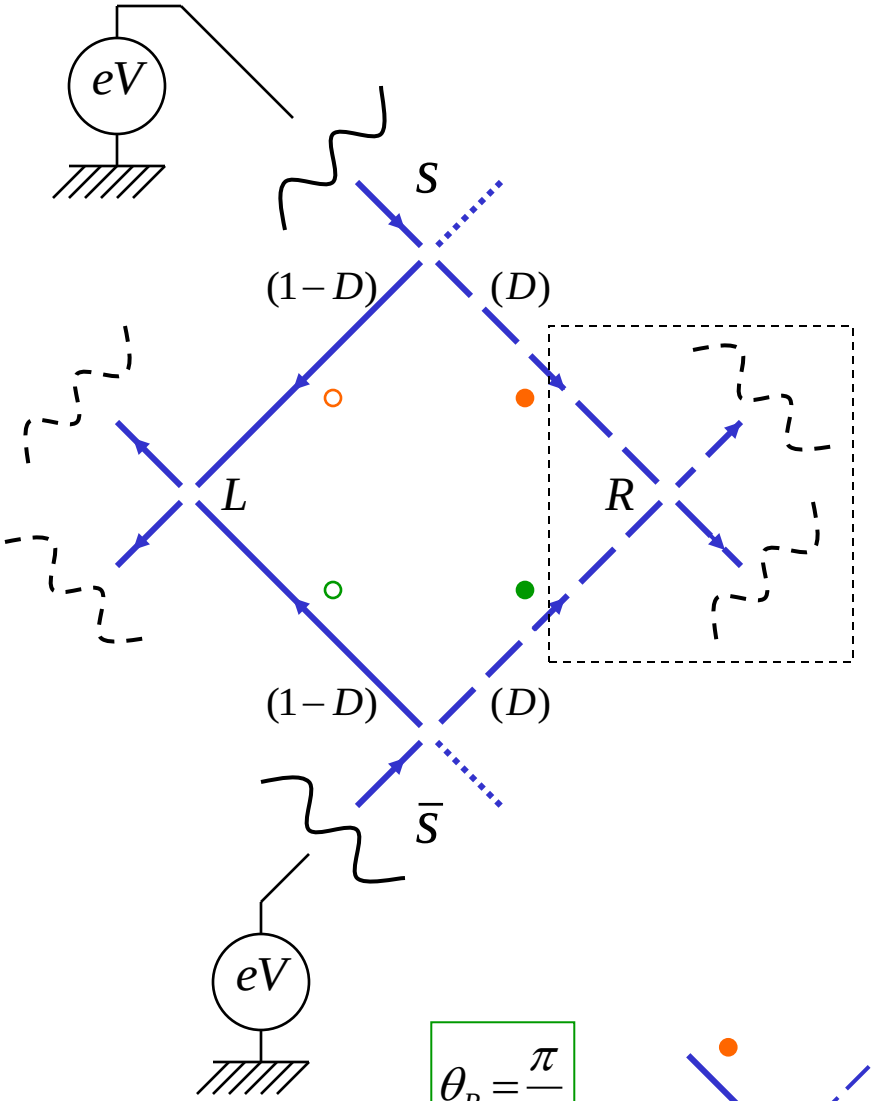
$$|\psi\rangle_{out} \approx |\tilde{0}\rangle + \sqrt{D} \left(c_{\downarrow,R}^{+} c_{\downarrow,L} + c_{\uparrow,R}^{+} c_{\uparrow,L} \right) |\tilde{0}\rangle + \vartheta(D^2)$$

$$|\psi\rangle_{out} = |\tilde{0}\rangle_{out} + \sqrt{D} \left(b_{\downarrow R}^{+} h_{\downarrow L}^{+} + b_{\uparrow R}^{+} h_{\uparrow L}^{+} \right) |\tilde{0}\rangle_{out}$$

entangled electron-hole pairs



analyzing the outputs



$$s \leftrightarrow \uparrow$$
$$\bar{s} \leftrightarrow \downarrow$$

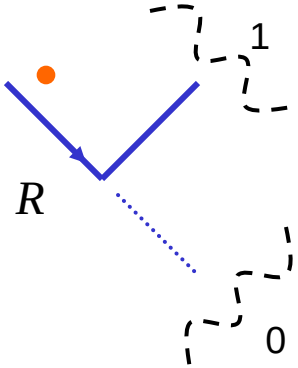
(pseudo-spin representation)

$$S_R = \begin{pmatrix} \cos\frac{\theta_R}{2} & \sin\frac{\theta_R}{2} \\ -\sin\frac{\theta_R}{2} & \cos\frac{\theta_R}{2} \end{pmatrix}$$

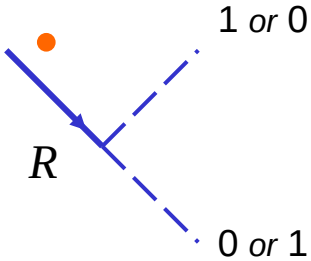
$$D_R = \sin^2\frac{\theta_R}{2}$$

$$1 - D_R = \cos^2\frac{\theta_R}{2}$$

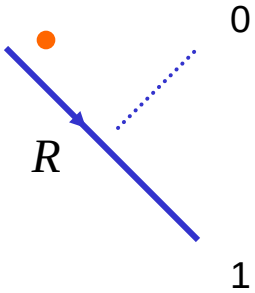
$\theta_R = 0$
$\uparrow$



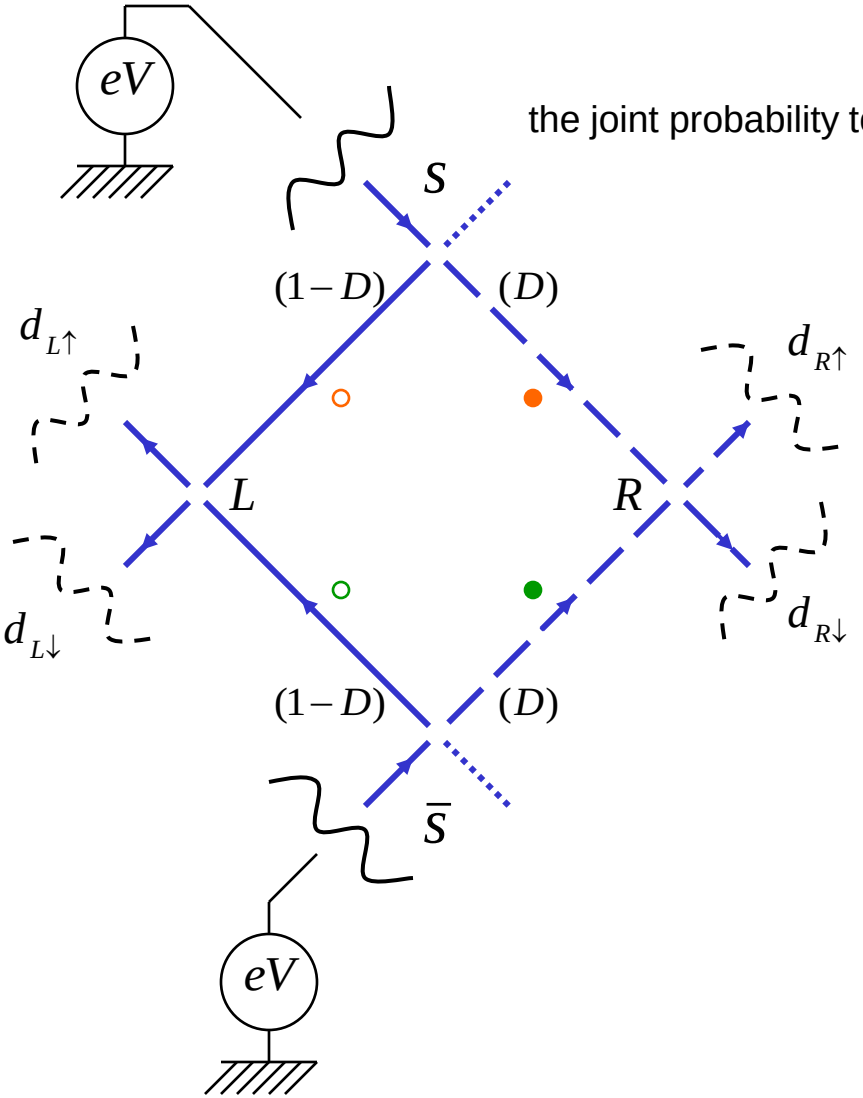
$\theta_R = \frac{\pi}{2}$
$\rightarrow$



$\theta_R = \pi$
$\downarrow$



analyzing the outputs



the joint probability to arrive in detectors (contacts) L(R),↑(↓) is obtained from:

$$S_L = \begin{pmatrix} \cos\frac{\theta_L}{2} & \sin\frac{\theta_L}{2} \\ -\sin\frac{\theta_L}{2} & \cos\frac{\theta_L}{2} \end{pmatrix} \qquad \begin{pmatrix} d_{\uparrow L} \\ d_{\downarrow L} \end{pmatrix} = S_L \begin{pmatrix} h_{\uparrow L} \\ h_{\downarrow L} \end{pmatrix}$$

$$S_R = \begin{pmatrix} \cos\frac{\theta_R}{2} & \sin\frac{\theta_R}{2} \\ -\sin\frac{\theta_R}{2} & \cos\frac{\theta_R}{2} \end{pmatrix} \qquad \begin{pmatrix} d_{\uparrow R} \\ d_{\downarrow R} \end{pmatrix} = S_L \begin{pmatrix} c_{\uparrow R} \\ c_{\downarrow R} \end{pmatrix}$$

$$P_{L\uparrow,R\uparrow} = P_{L\downarrow,R\downarrow} = D\cos^2\left(\frac{\theta_L - \theta_R}{2}\right)$$

$$P_{L\uparrow,R\downarrow} = P_{L\downarrow,R\uparrow} = D\sin^2\left(\frac{\theta_L - \theta_R}{2}\right)$$

(for zero A-B flux through the loop)

While the case $\theta_L = \theta_R = 0$ (or π) is trivial and classically expected, other cases like $\theta_L = \theta_R = \pi/2$ is not classically expected and results from quantum interferences between two possible indistinguishable electron-hole pairs.

some theoretical work on Fermi statistics entanglement

Setup of three Mach-Zehnder interferometers for production and observation of Greenberger-Horne-Zeilinger entanglement of electrons
A. Vyshnevyy, G. Lesovik, T. Jonckheere, and T. Martin ; Phys. Rev. B 87 165417 (2013)

Leggett-Garg inequality in electron interferometers
Clive Emary, Neill Lambert, and Franco Nori ; Phys. Rev. B 86 235447 (2012)

Quantum Noise as an Entanglement Meter
Israel Klich and Leonid Levitov ; Phys. Rev. Lett. 102 100502 (2009)

Optimal Spin-Entangled Electron-Hole Pair Pump
C. Beenakker, M. Titov, and B. Trauzettel ; Phys. Rev. Lett. 94 186804 (2005)

Reduced and Projected Two-Particle Entanglement at Finite Temperatures
P. Samuelsson, I. Neder, and M. Büttiker ; Phys. Rev. Lett. 102 106804 (2009)

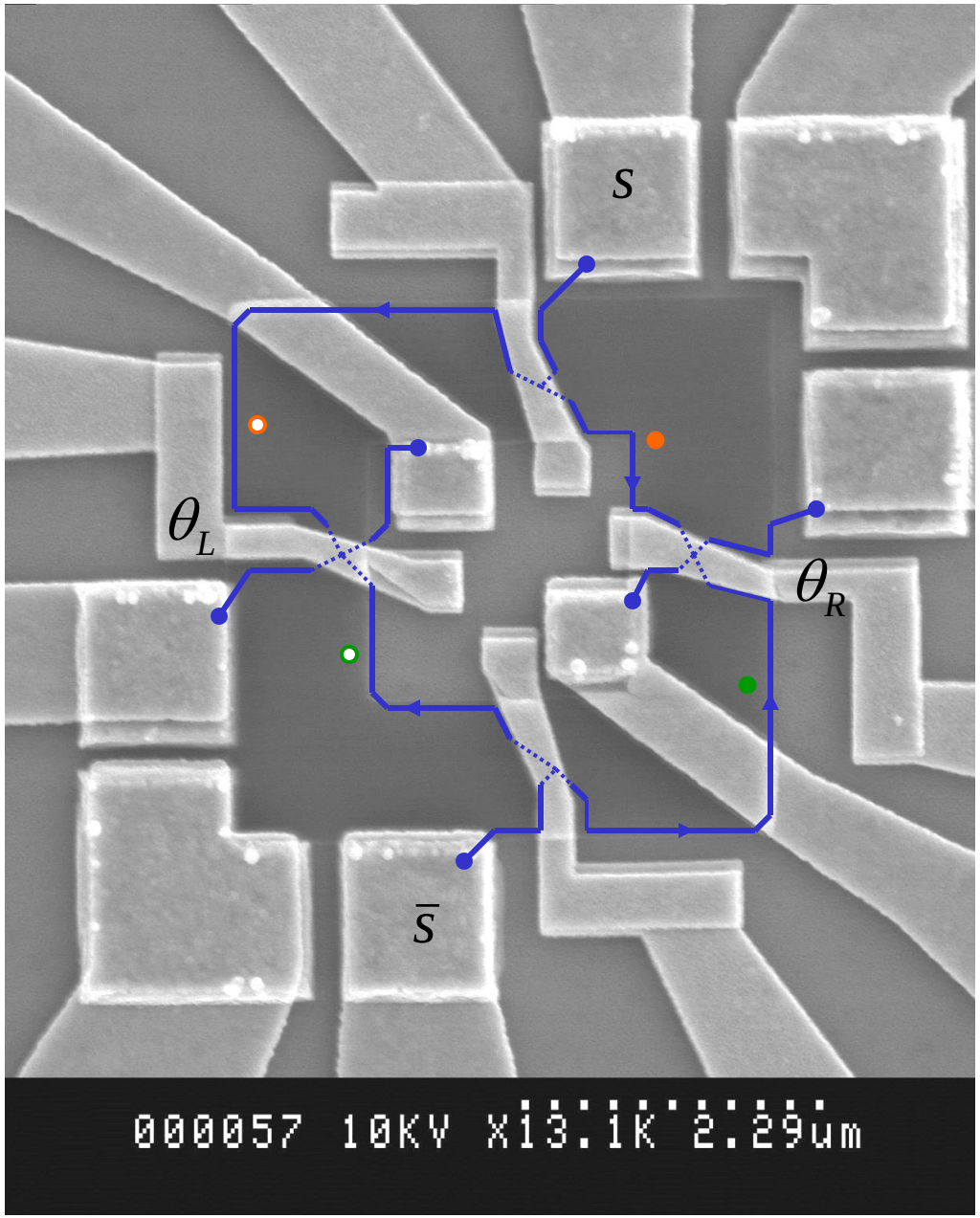
Two-Particle Aharonov-Bohm Effect and Entanglement in the Electronic Hanbury Brown–Twiss Setup
P. Samuelsson, E. Sukhorukov, and M. Büttiker ; Phys. Rev. Lett. 92 026805 (2004)

Quantum Teleportation by Particle-Hole Annihilation in the Fermi Sea
C. Beenakker and M. Kindermann Phys. Rev. Lett. 92 056801 (2004)

Entanglement in a noninteracting mesoscopic structure
A. Lebedev, G. Lesovik, and G. Blatter ; Phys. Rev. B 71 045306 (2005)

Entanglement in mesoscopic structures: Role of projection
A. Lebedev, G. Blatter, C. Beenakker, and G. Lesovik ; Phys. Rev. B 69 235312 (2004)

what could be done experimentally …



(sample: D. Mailly, P. Roche, P. Roulleau ,CEA Saclay/ LPN Marcoussis,)

Orders of magnitude :

T = 20mK

V = 25 μV (~250mK)

eV/h = 6 GHz

For transmission D = 0.1 :

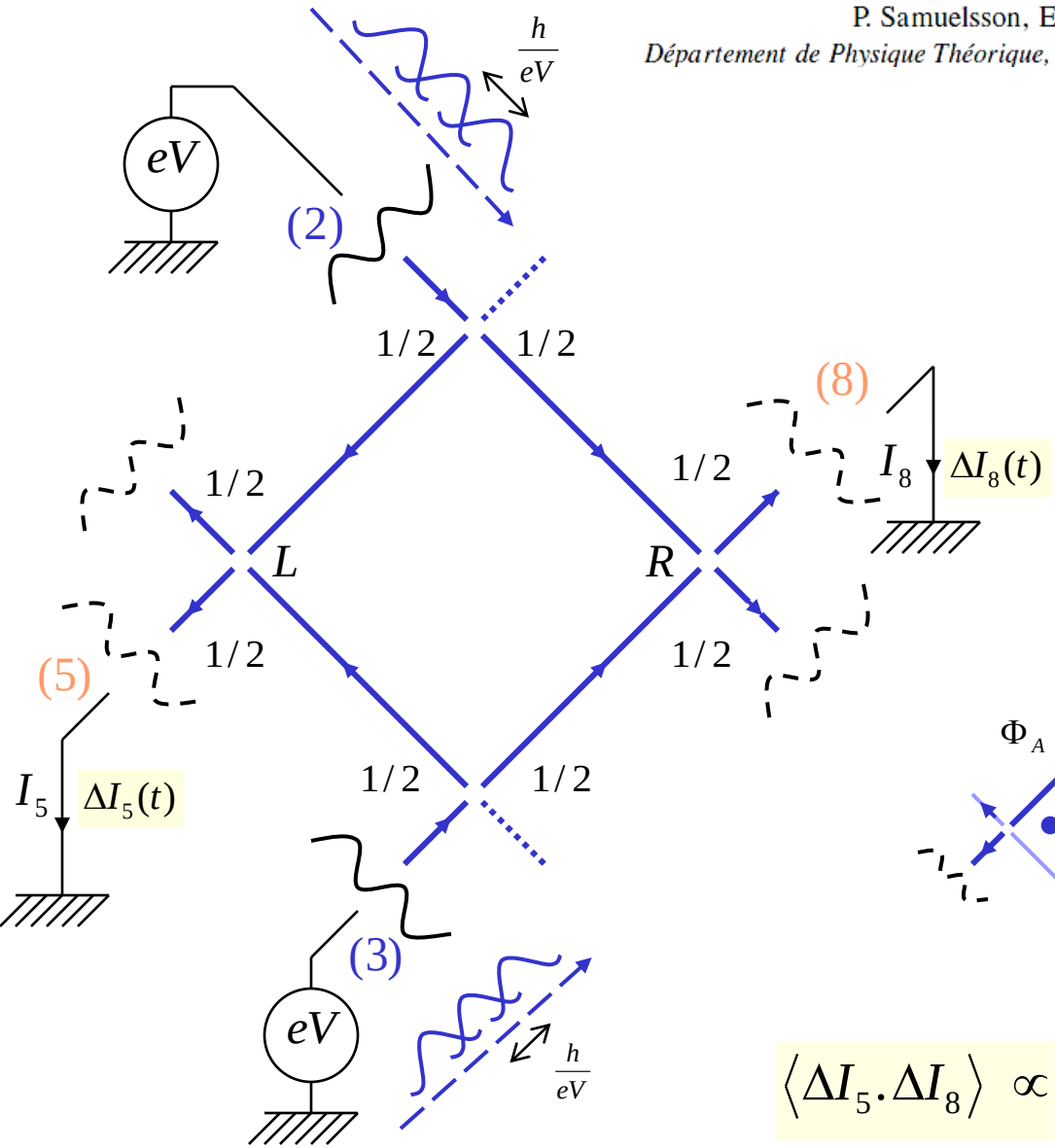
I = 100pA and τ^{-1} = I/e ~ 0.6 GHz

S_I ~ (6 fA)^2 / Hz measurable in d.c.

VOLUME 92, NUMBER 2 PHYSICAL REVIEW LETTERS week ending 16 JANUARY 2004

Two-Particle Aharonov-Bohm Effect and Entanglement in the Electronic Hanbury Brown–Twiss Setup

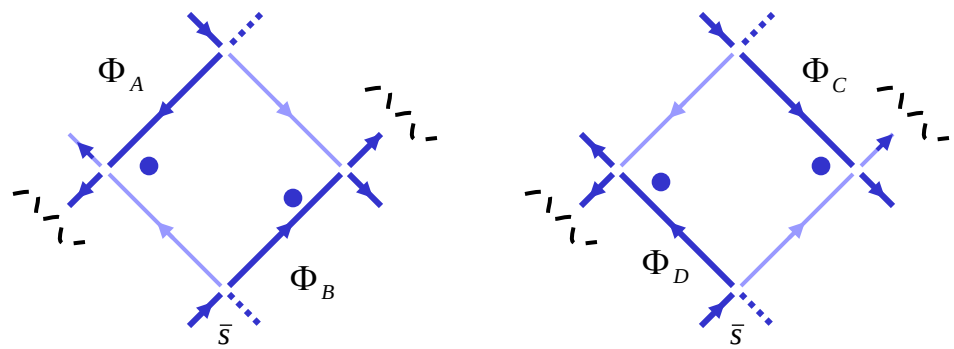
P. Samuelsson, E.V. Sukhorukov, and M. Büttiker
Département de Physique Théorique, Université de Genève, CH-1211 Genève 4, Switzerland



Electrons: injected from (2) and (3)
detected in coincidence at (5) and (8)

$$P_2 = \frac{1}{4^2}\left|e^{i\phi_A}e^{i\phi_B} + e^{i\phi_C}e^{i\phi_D}\right|^2$$

$$= \frac{1}{8}\left(1+\cos(\phi_A+\phi_B-\phi_C-\phi_D)\right)$$



$$\langle \Delta I_5 . \Delta I_8 \rangle \propto P_2$$

joint probablity to find electrons in (5) and (8) simultaneously

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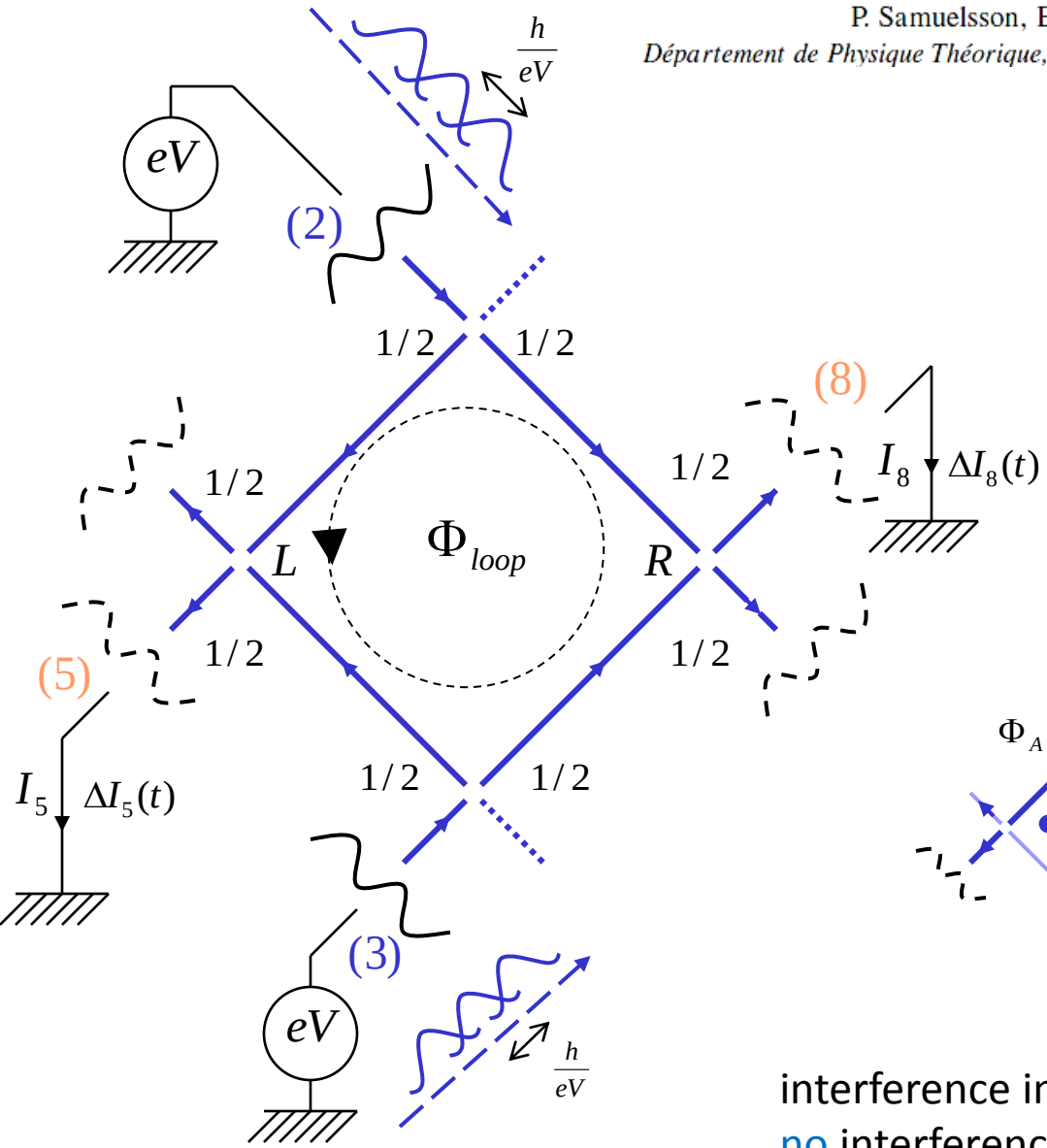

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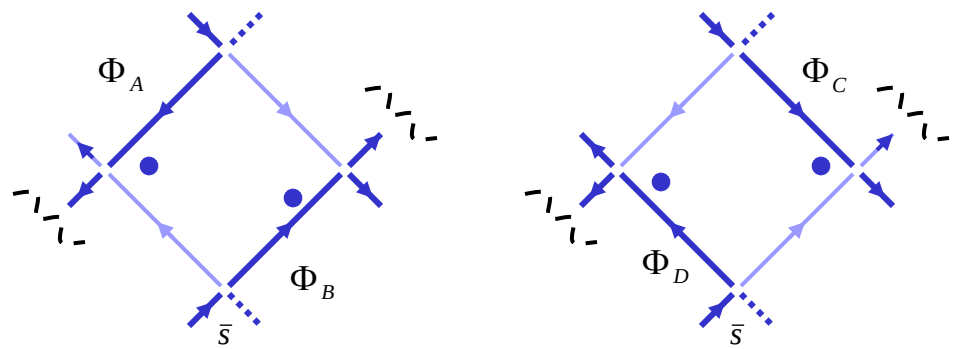
$$= \frac{1}{8}\left(1+\cos(\phi_A+\phi_B-\phi_C-\phi_D+2\pi\frac{\Phi_{loop}}{\phi_0})\right)$$



interference in the cross-correlation noise (two particle)
no interference in the current (single particle)

Two-Particle Aharonov-Bohm Effect and Entanglement in the Electronic Hanbury Brown–Twiss Setup

P. Samuelsson, E.V. Sukhorukov, and M. Büttiker
rtement de Physique Théorique, Université de Genève, CH-1211 Genève 4, Switzerland

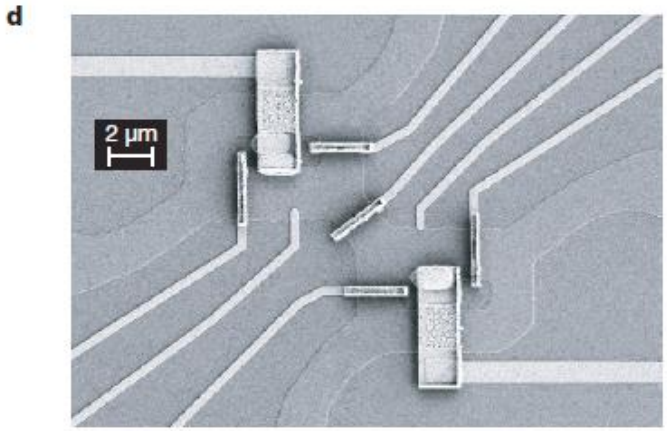
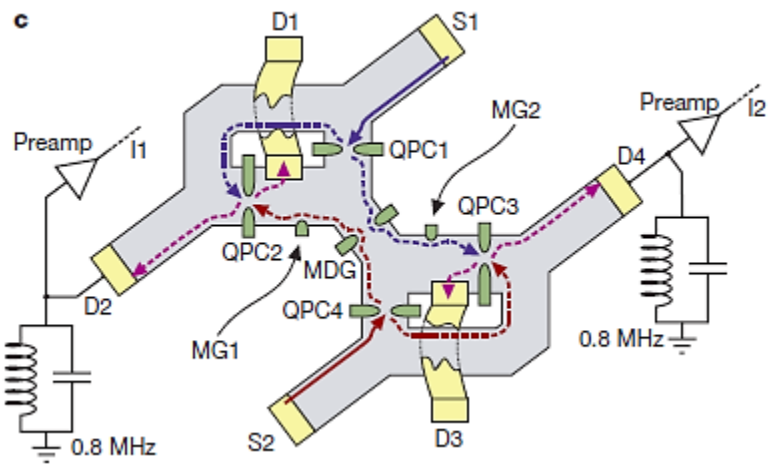
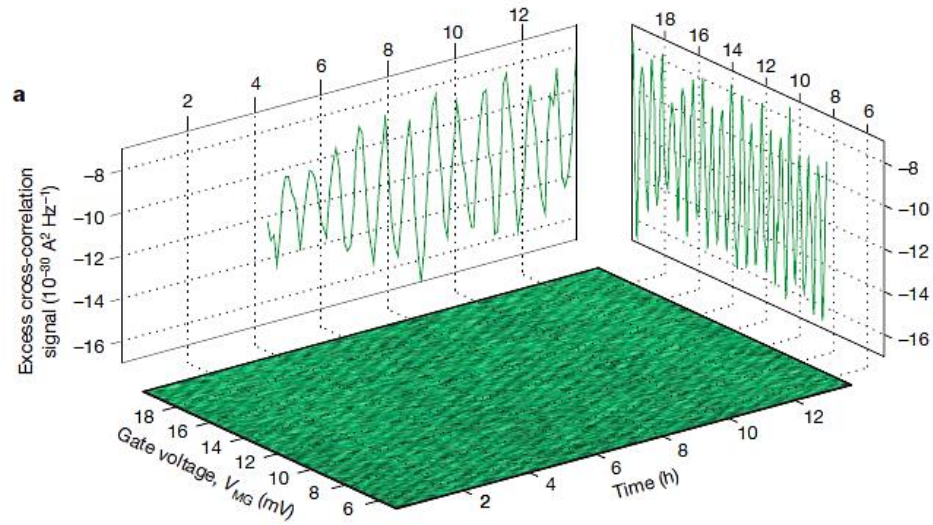


Figure 1 | The two-particle Aharonov–Bohm interferometer.





$$P_2 = \frac{1}{4^2}\left| e^{i\phi_A} e^{i\phi_B} + e^{i\phi_C} e^{i\phi_D} \right|^2$$

$$= \frac{1}{8}\left(1 + \cos\left(\phi_A + \phi_B - \phi_C - \phi_D + 2\pi \frac{\Phi_{loop}}{\phi_0}\right)\right)$$

Interference between two indistinguishable electrons from independent sources
I. Neder, N. Ofek, Y. Chung, M. Heiblum, D. Mahalu, V. Umansky
Nature 448, 333 (2007)

OUTLINE

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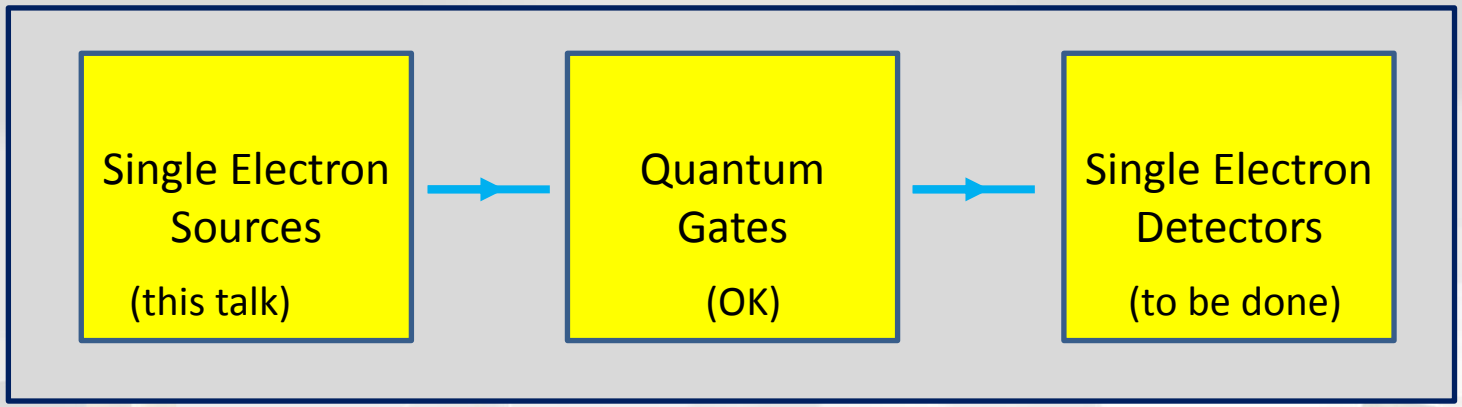
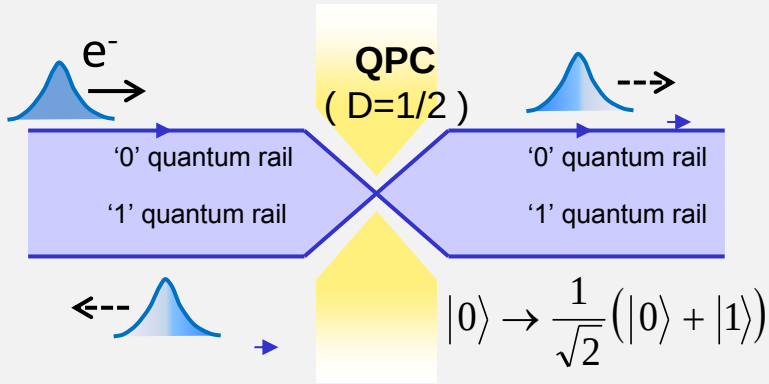
- quantized conductance and noiseless electron flow
- electron-hole entanglement in the Fermi-sea
- single electron sources for electron quantum optics
  - minimal excitations states of a Fermi sea: the levitons
  - experimental realization of levitons
- perspective and applications of levitons

electron quantum optics



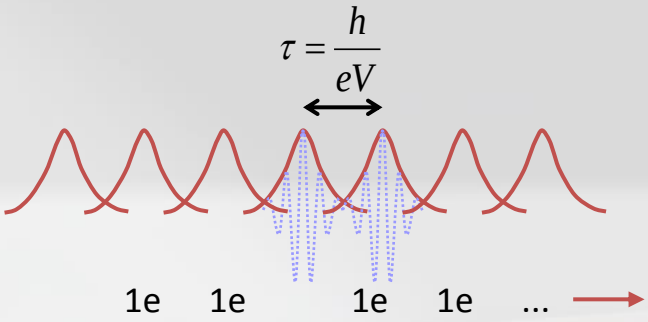


principle of a Hadamard gate with electron flying qubit in a ballistic conductor

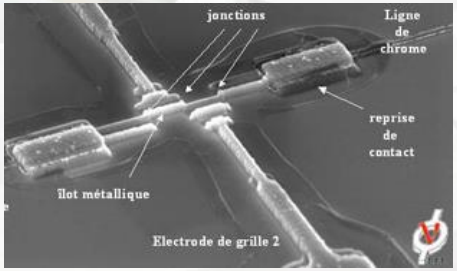
combining several similar qubits may lead to qubit operation

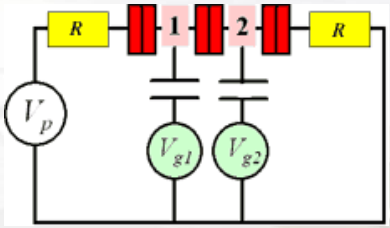
some available single electron sources

- voltage biased contact :
  continuous generation of single electrons in a form of a giant Slater determinant at eV/h pace !
  no time control

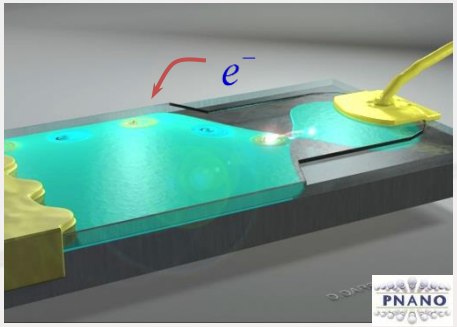


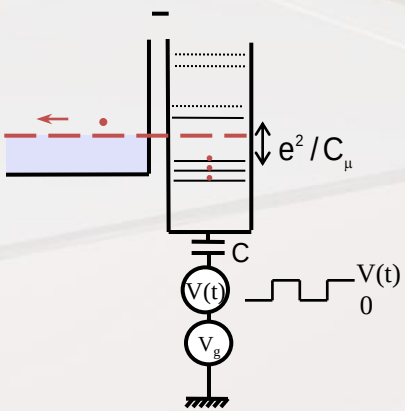
- single electron pumps :
  controlled injection of single charge
  sequential (incoherent:) electron injection
  L. J. Geerligs et al., Phys. Rev. Lett. 64, 2691 (1990).
  L. P. Kouwenhoven, et al., Phys. Rev. Lett. 67, 1626 (1991).
  Pothier et al. EPL (Europhysics Letters), Vol. 17, No. 3. (1992)





- recent on-demand single electron source:
  coherent *single* electron source
  energy resolved
  opens new field of quantum experiments
  But: do not operate for n-electrons
  Feve G., Mahe, A., Berroir J.-M., Kontos, T., Plaçais B. and Glattli D.C. Science, Vol. 316, No. 5828. (2007),

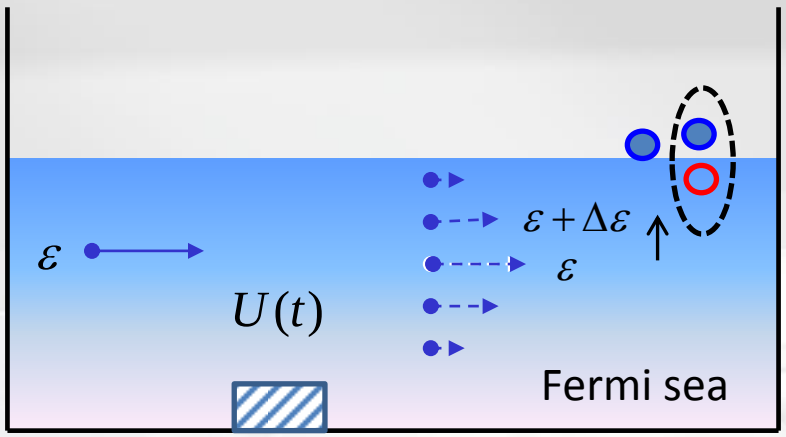




- S.A.W. based single electron sources
  itinerant quantum dots at sound speed
  Good for spin manipulation
  orbital wave-function not well defined
  Talyanskii et al, Physical Review B, Vol. 56 (Dec 1997), pp. 15180-15184
  Hermelin et al. Nature 477 (2011)

on-demand elementary excitations:

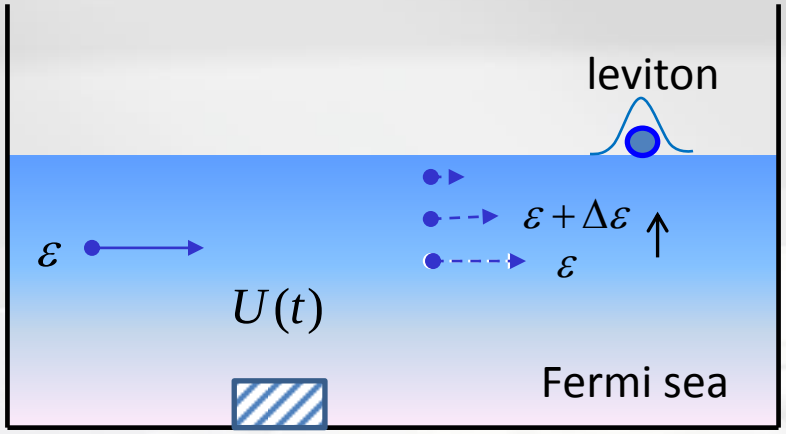
a key issue for a system made of fermions



$$e^{ikx}\, e^{-i\varepsilon t/\hbar}\, e^{-i\phi(t)}$$

$$\phi(t) = \frac{1}{\hbar}\int^{t} U(t')\,dt'$$

on-demand elementary excitations:
a key issue for a system made of fermions



$$e^{ikx}\, e^{-i\varepsilon t/\hbar}\, e^{-i\phi(t)} \qquad \phi(t) = \frac{1}{\hbar}\int^{t} U(t')\,dt'$$

$$\Delta\phi = 2\pi \quad \text{and} \quad U(t) = \frac{h}{\pi}\frac{w}{t^2 + w^2} \quad \text{lorentzian}$$

minimal fermion state: a leviton

Levitov, Lee, Ivanov,Lesovik (96,97); Keeling, Klich, Levitov (06)

PHYSICAL REVIEW B VOLUME 56, NUMBER 11 15 SEPTEMBER 1997-I

Coherent states of alternating current

Electron counting statistics and coherent states of electric current

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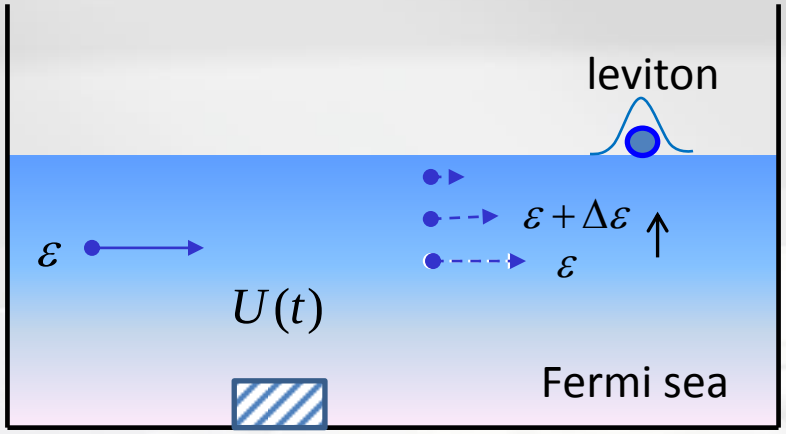
PHYSICAL REVIEW LETTERS week ending 15 SEPTEMBER 2006

Minimal Excitation States of Electrons in One-Dimensional Wires

J. Keeling,[1] I. Klich,[2] and L. S. Levitov[1]

on-demand elementary excitations:
a key issue for a system made of fermions



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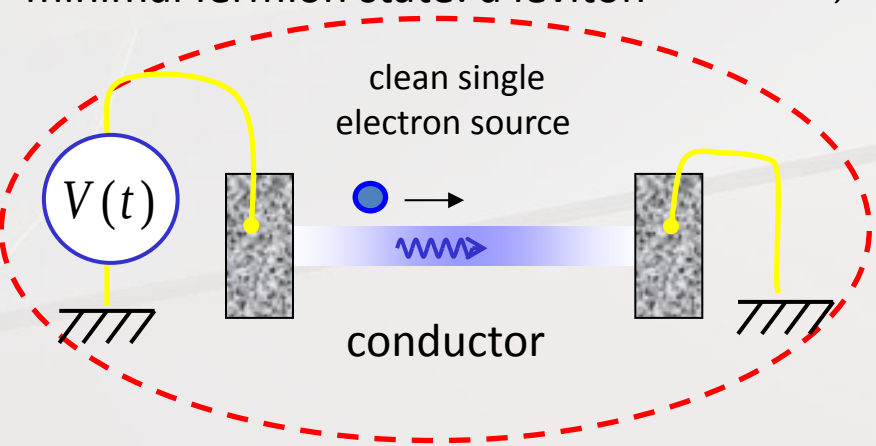
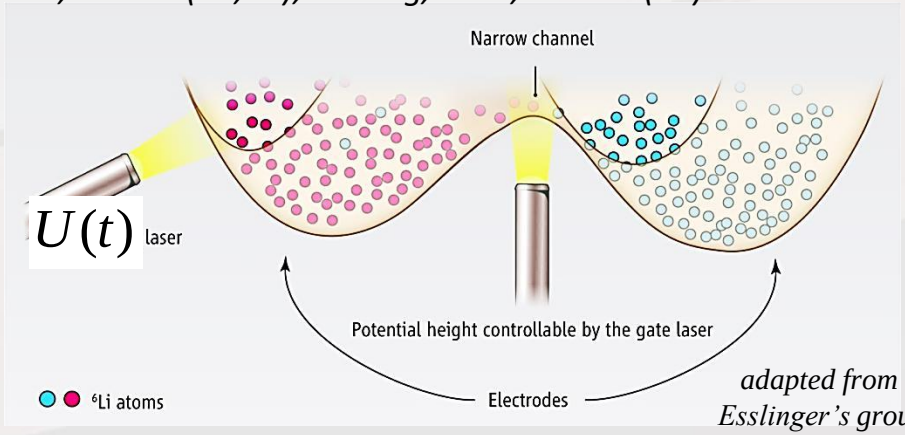
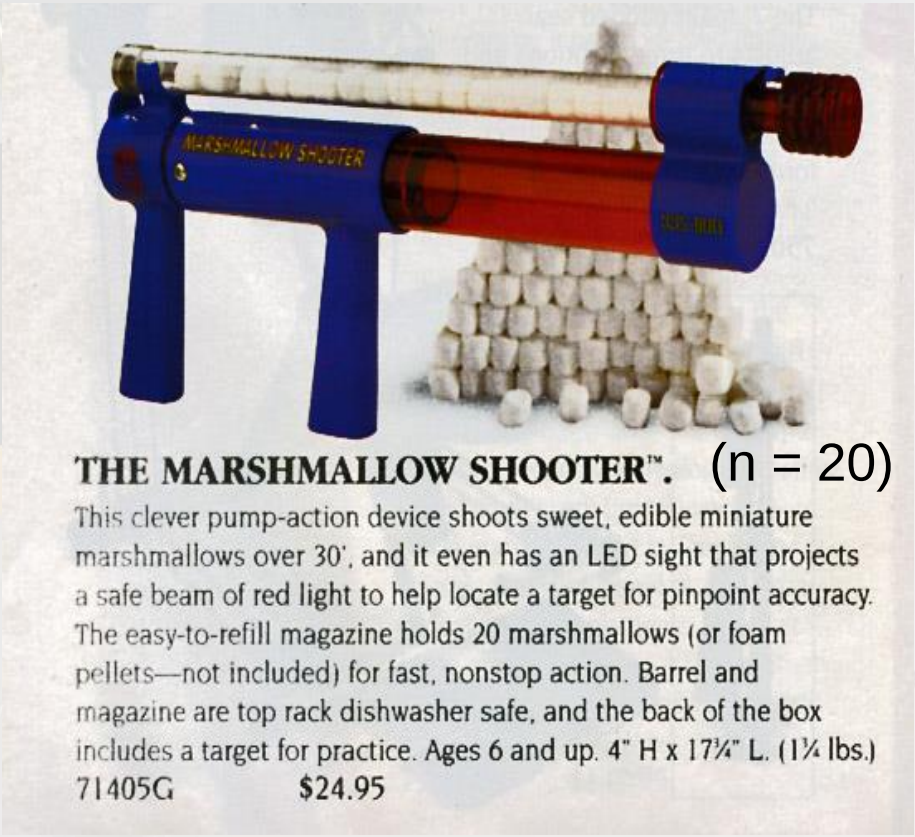




adapted from Esslinger's group

a simple on-demand single or n-electron source

requirement : as simple as possible



a simple on-demand single or n-electron source

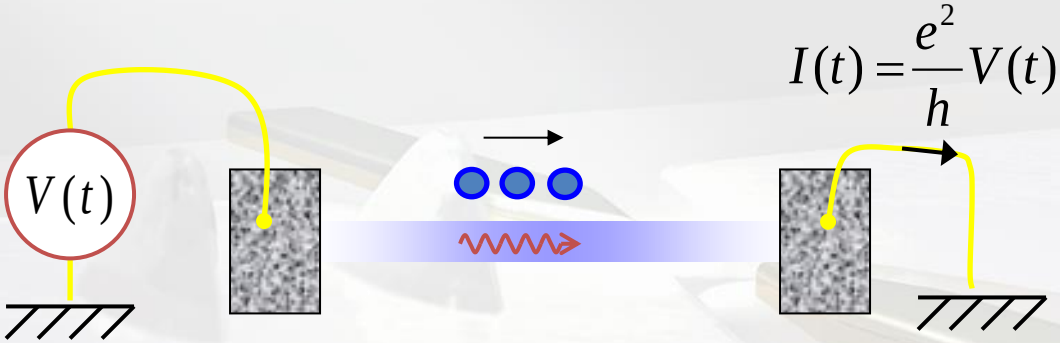
requirement : as simple as possible … and reliable

voltage pulses on a contact

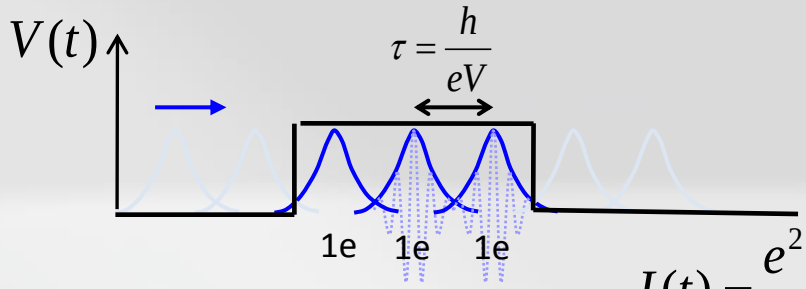
as simple as that !

$$\int I(t)dt = ne$$

or: $$\int eV(t)dt = nh$$



voltage pulses on a contact

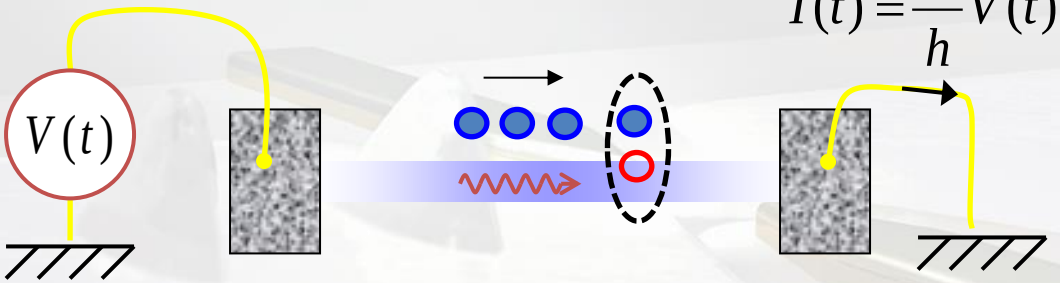


$$I(t) = \frac{e^2}{h} V(t)$$

as simple as that !

$$\int I(t)dt = ne$$

or: $$\int eV(t)dt = nh$$



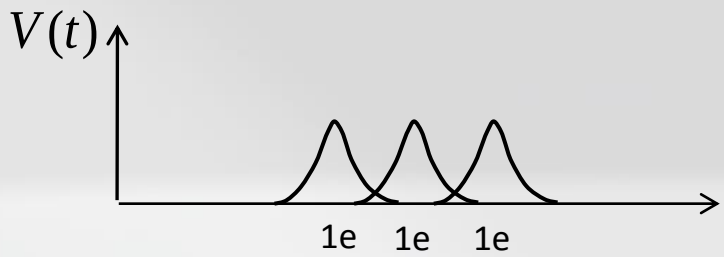
SIMPLE BUT NON TRIVIAL

$$N_e + N_h > n \quad !$$

more quasi-particles than injected charges

not suitable for quantum experiments

Lorentzian voltage pulses on a contact



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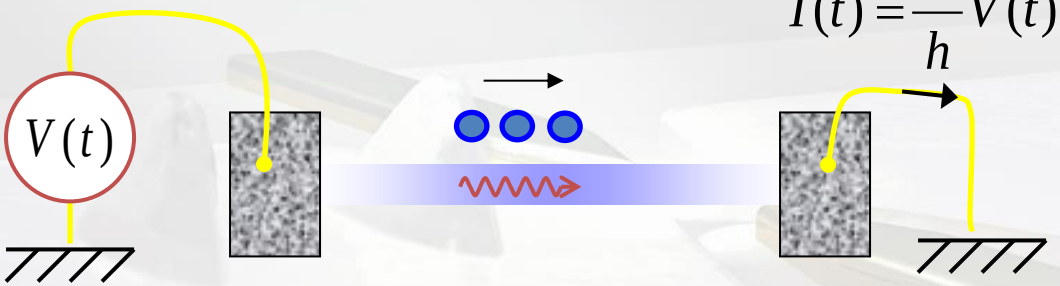
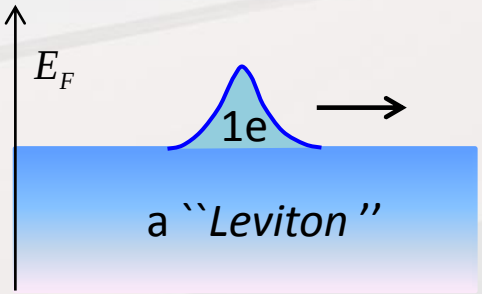


SIMPLE BUT NOT TRIVIAL :

$$N_e = n \qquad N_h = 0$$

only Lorentzian pulses
provide clean charge injection

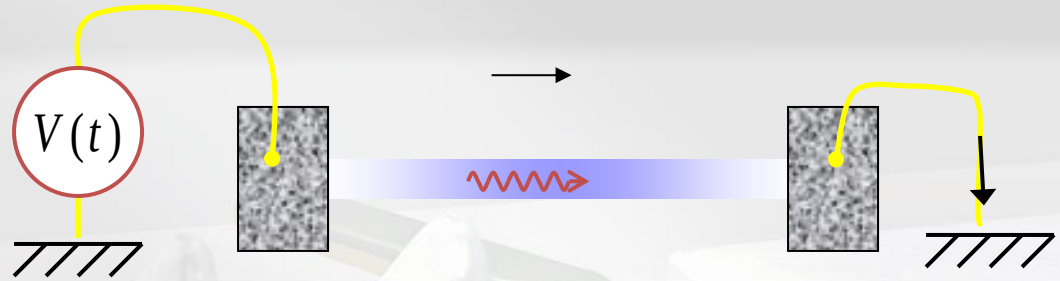
$$V(t) = \frac{h}{e\pi} \sum_{m=1}^{n} \frac{w_m}{(t - t_m)^2 + w_m^{\ 2}}$$

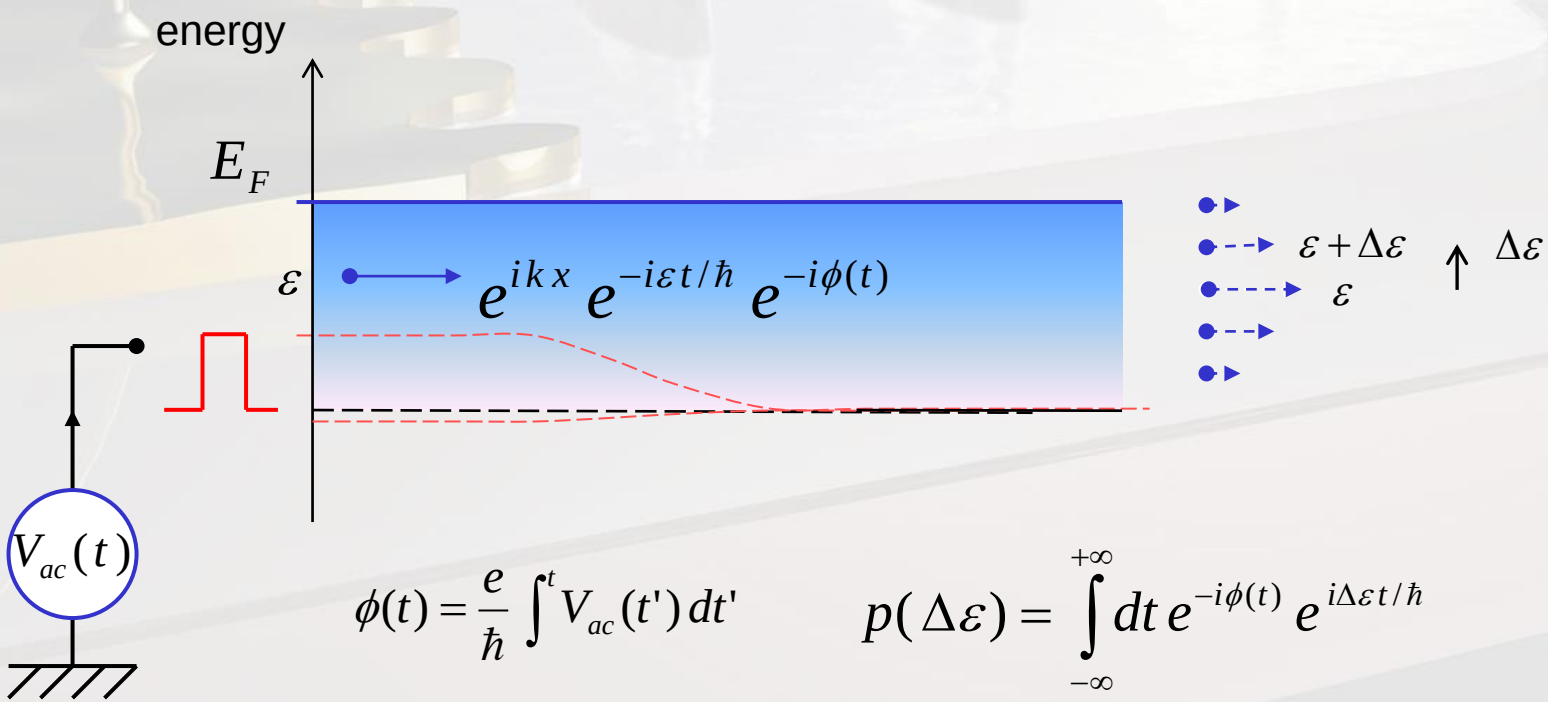


Levitov, Lee, Ivanov,Lesovik (96,97); Keeling, Klich, Levitov (06)

LEVITONS : a wave property + Fermi statistics

arbitrary pulse

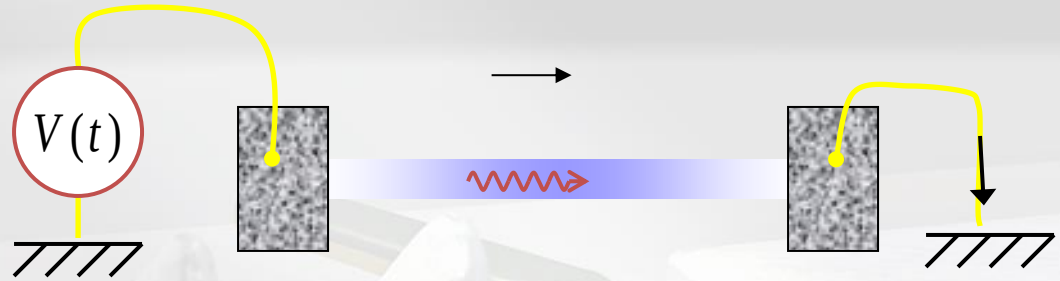


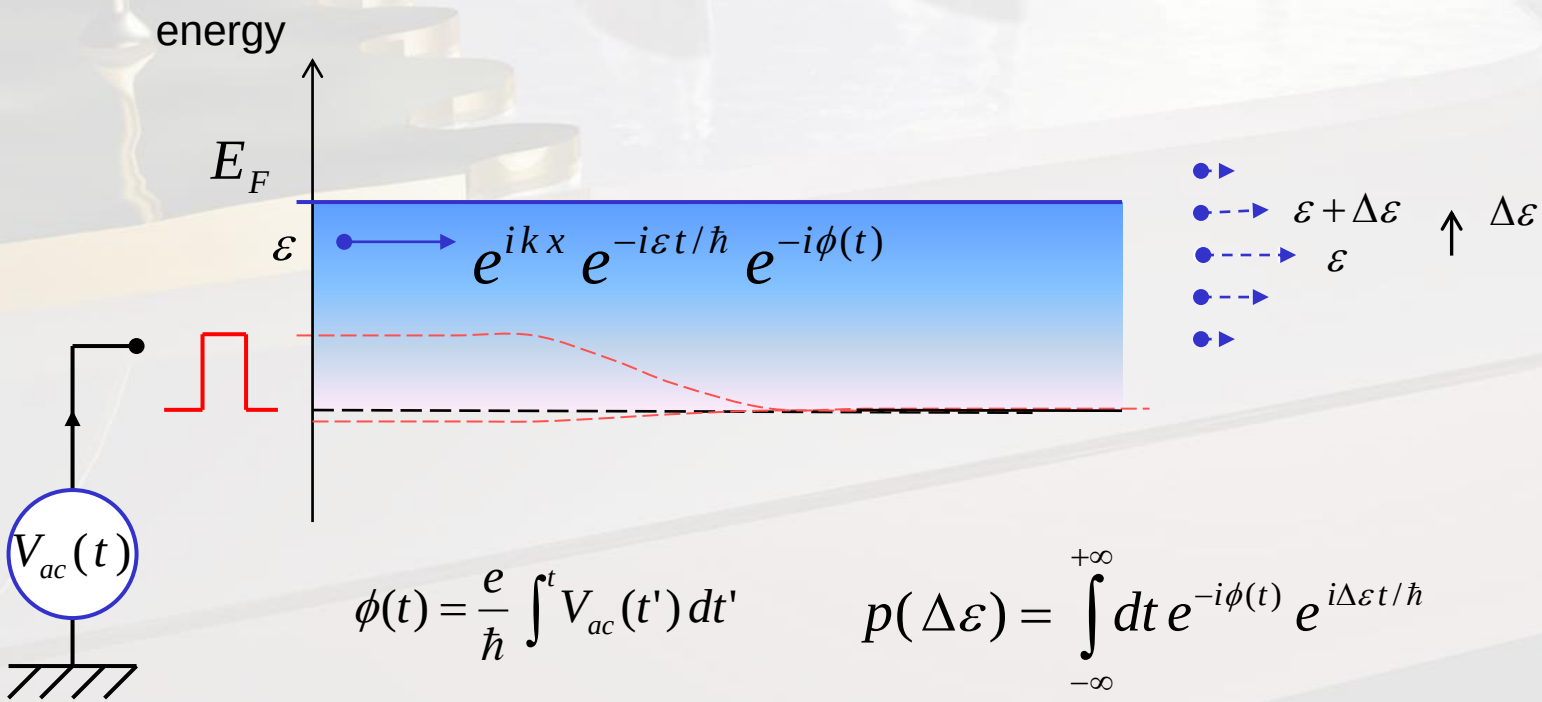


$$\phi(t) = \frac{e}{\hbar}\int^{t} V_{ac}(t')\,dt' \qquad p(\Delta\varepsilon) = \int_{-\infty}^{+\infty} dt\, e^{-i\phi(t)}\, e^{i\Delta\varepsilon t/\hbar}$$

LEVITONS : a wave property + Fermi statistics

arbitrary pulse





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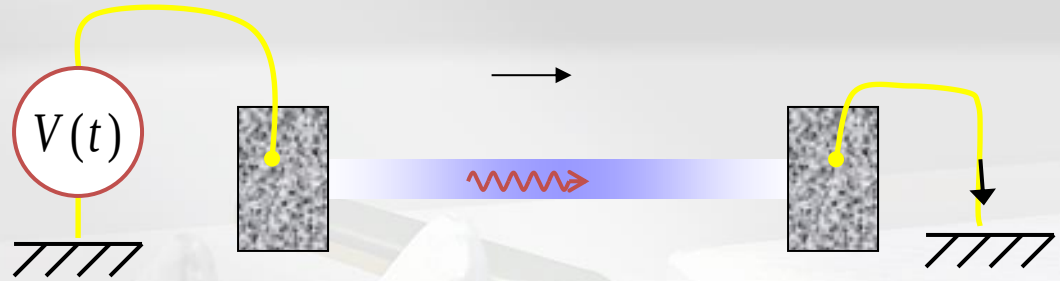

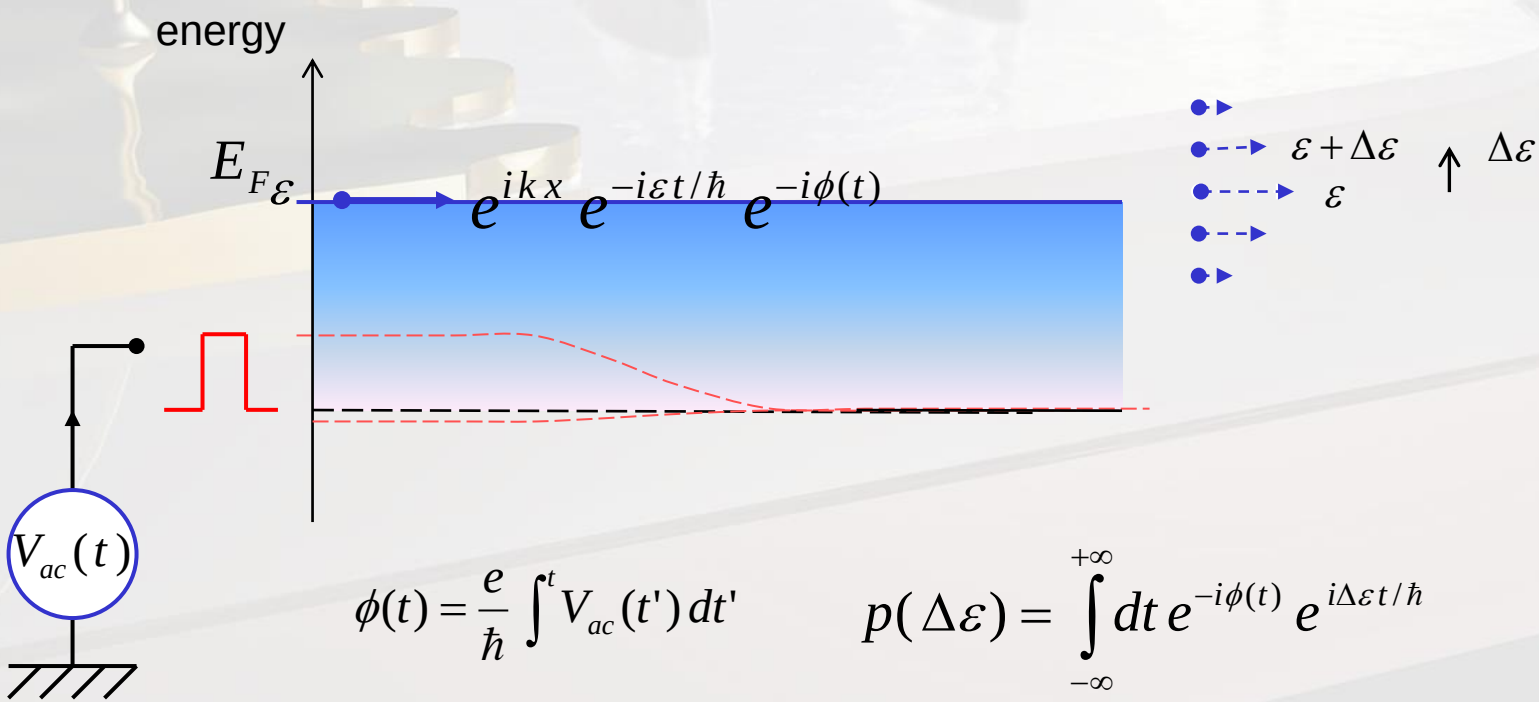

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LEVITONS : a wave property + Fermi statistics

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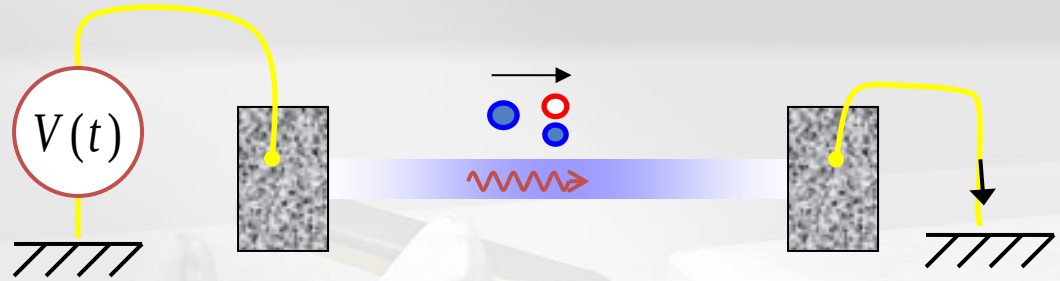

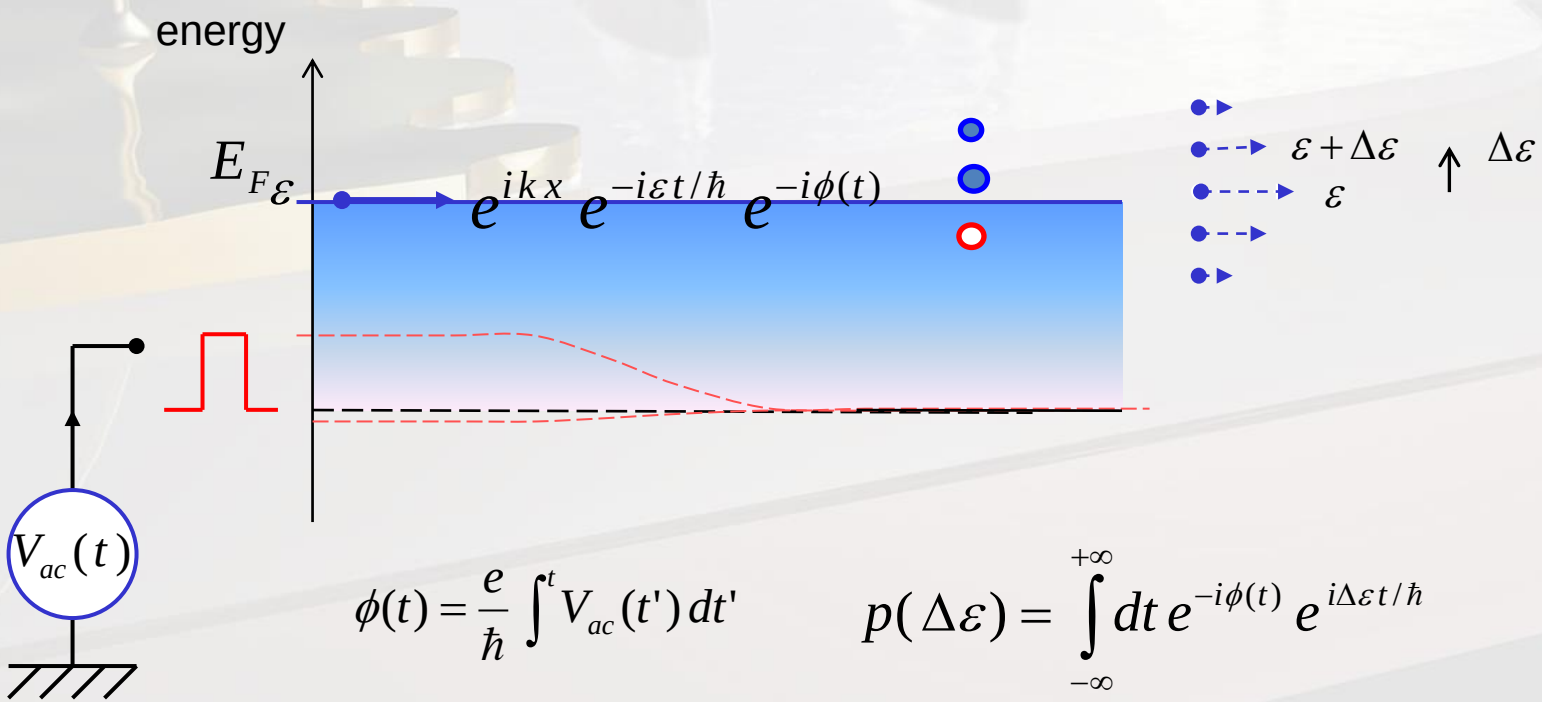

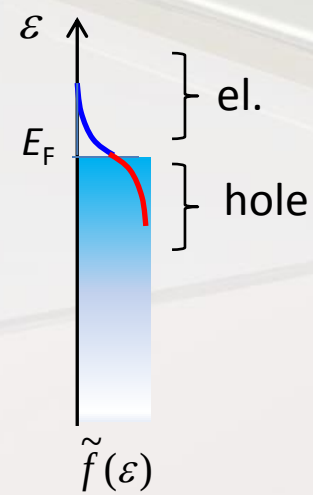

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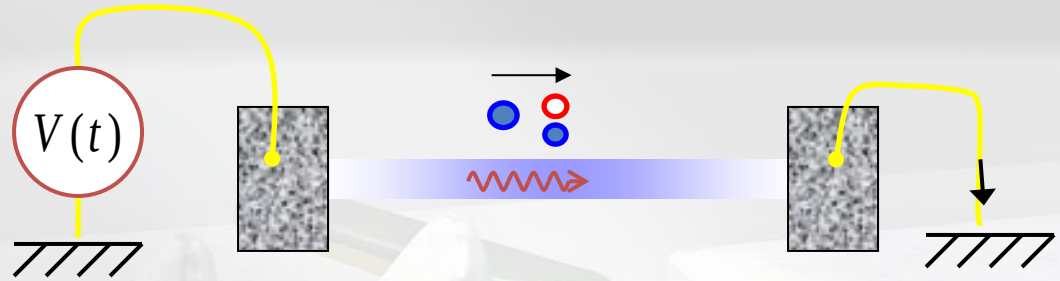
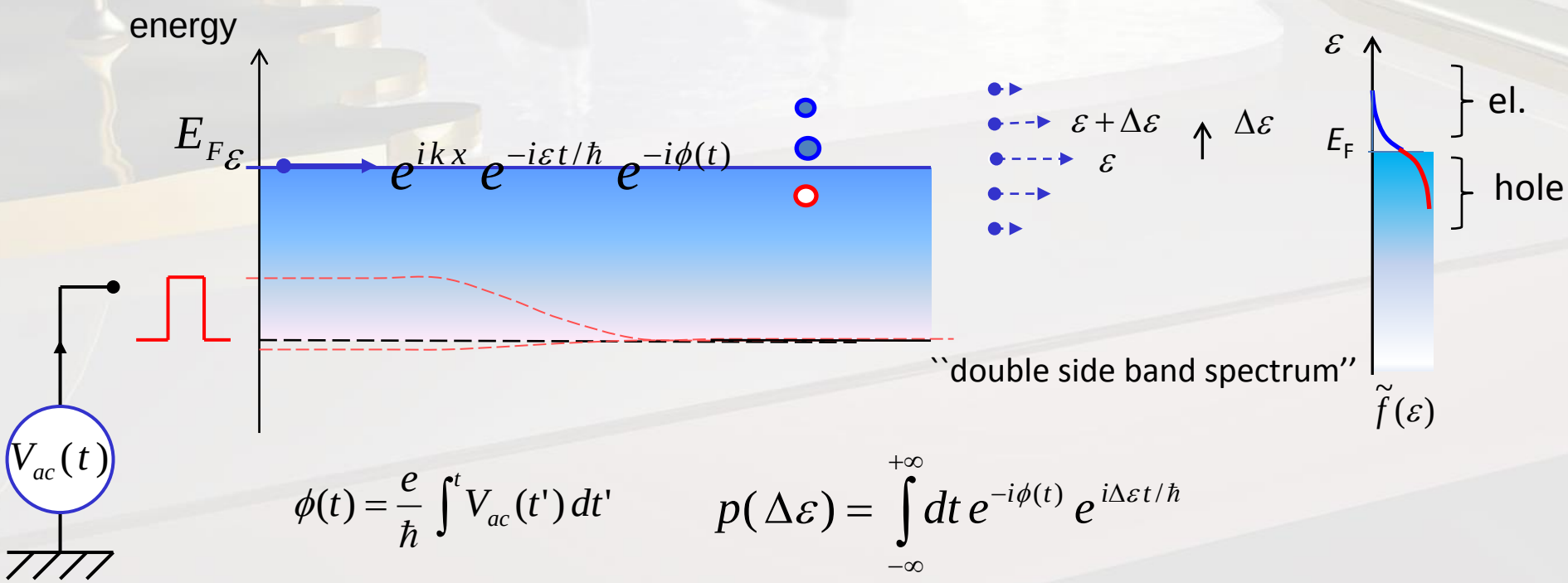
LEVITONS : a wave property + Fermi statistics

arbitrary pulse





``double side band spectrum''

$$\phi(t)=\frac{e}{\hbar}\int^{t}V_{ac}(t')\,dt'$$

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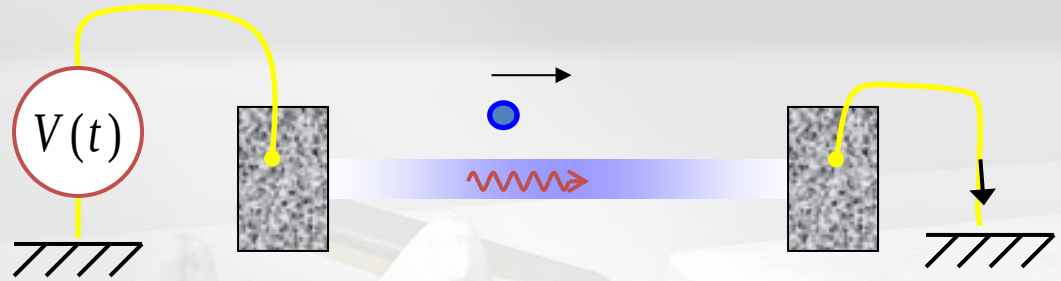
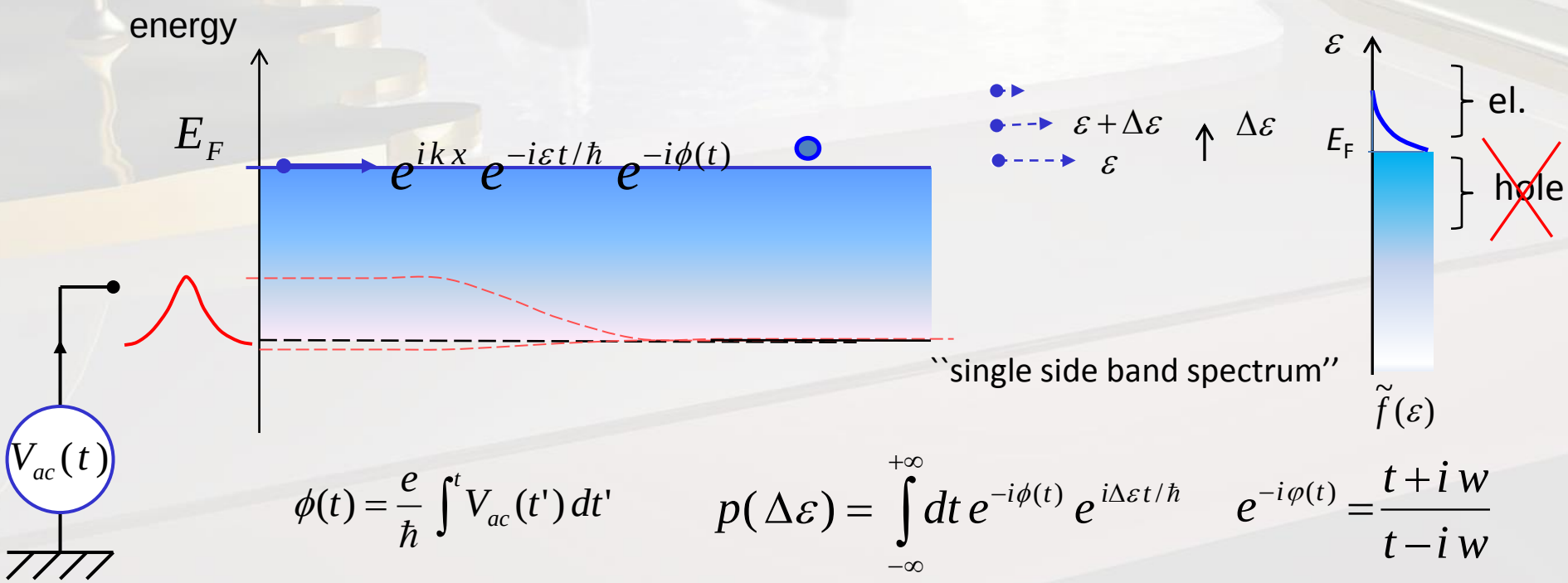
LEVITONS : a wave property + Fermi statistics

Lorentzian pulse





$$\phi(t)=\frac{e}{\hbar}\int^{t} V_{ac}(t')\,dt'$$

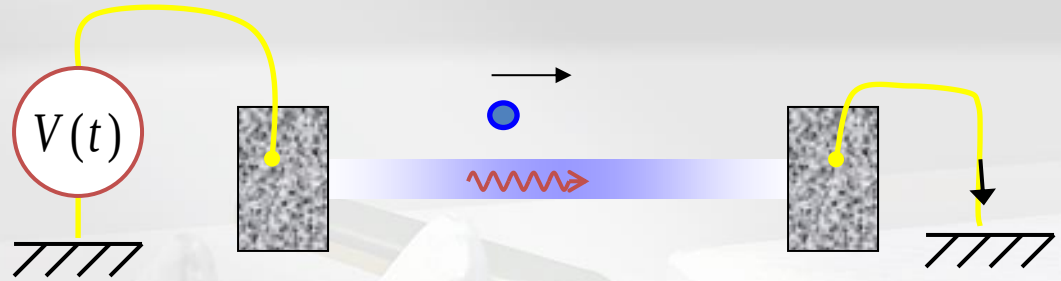
$$p(\Delta\varepsilon)=\int_{-\infty}^{+\infty} dt\, e^{-i\phi(t)}\, e^{i\Delta\varepsilon t/\hbar}$$

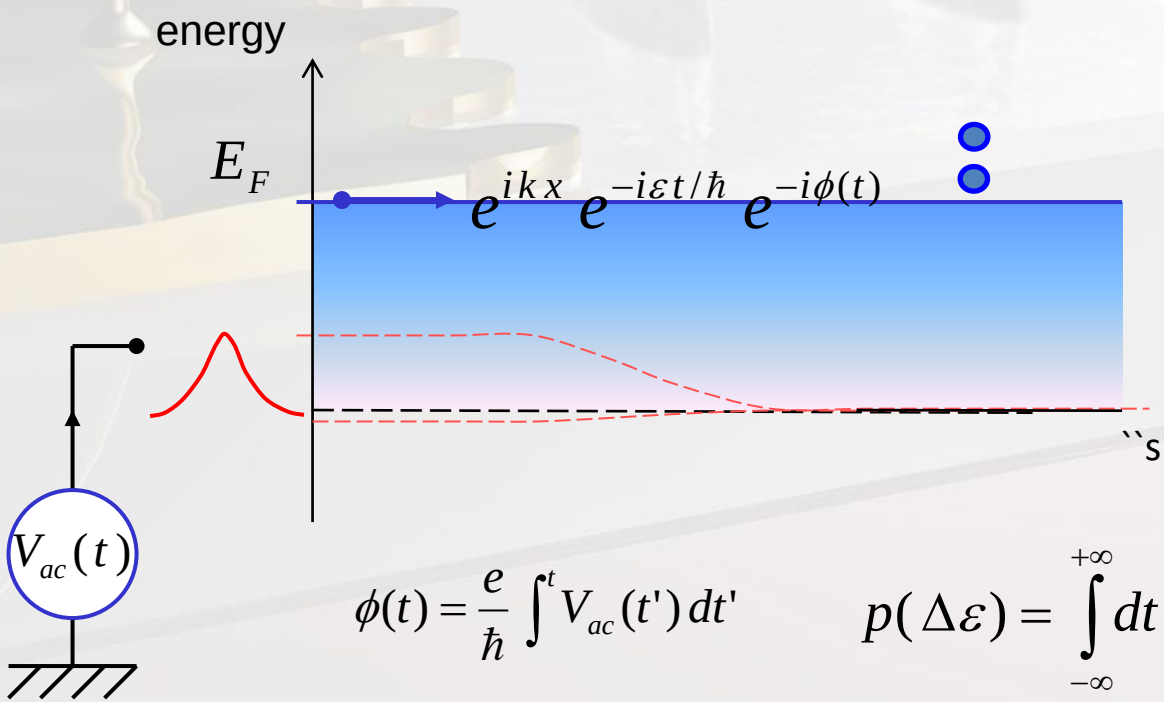
$$e^{-i\varphi(t)}=\frac{t+i\,w}{t-i\,w}$$

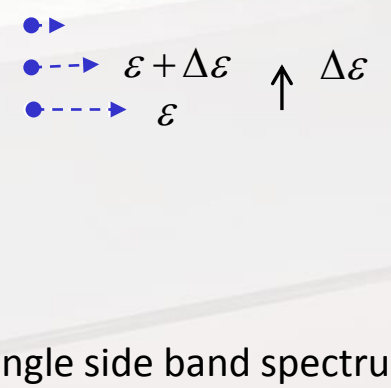
$$d\varphi/dt=2\,w/(t^2+w^2)$$

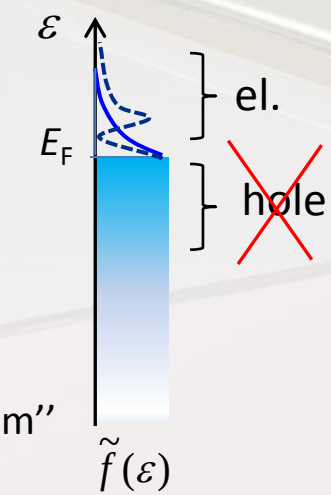
LEVITONS : a wave property + Fermi statistics

Lorentzian pulse









$$\phi(t)=\frac{e}{\hbar}\int^{t} V_{ac}(t')\,dt'$$

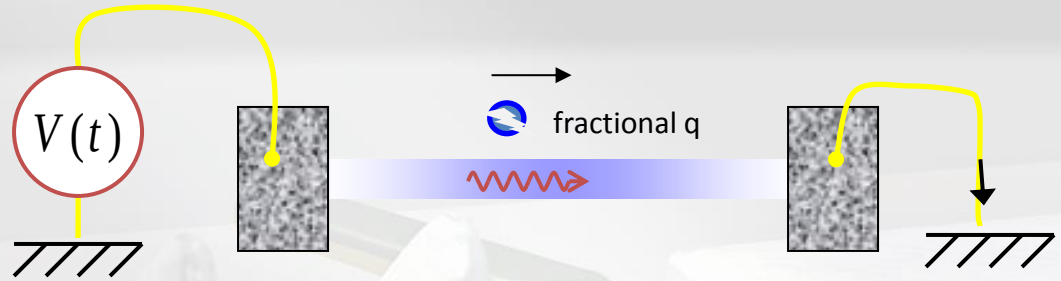
$$p(\Delta\varepsilon)=\int_{-\infty}^{+\infty} dt\, e^{-i\phi(t)}\, e^{i\Delta\varepsilon t/\hbar}$$

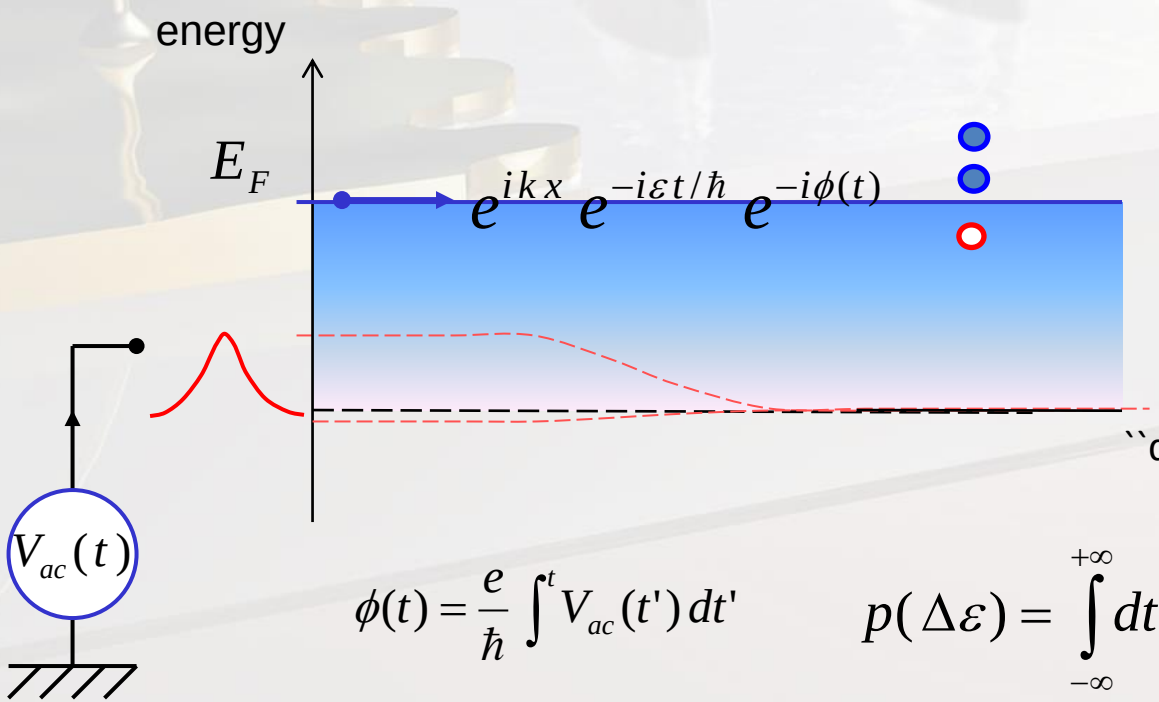
$$e^{-i\varphi(t)}=\left(\frac{t+iw}{t-iw}\right)^{2}$$

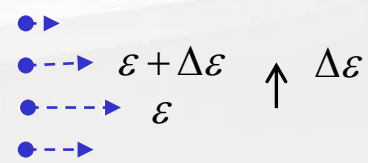
$$d\varphi/dt=4w/(t^{2}+w^{2})$$

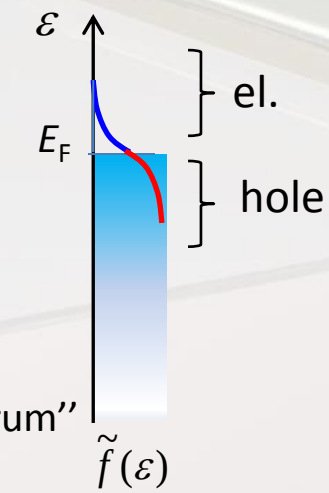
LEVITONS : a wave property + Fermi statistics

Lorentzian pulse









``double side band spectrum''
for non-integer charge

$$\phi(t)=\frac{e}{\hbar}\int^{t}V_{ac}(t')\,dt'$$

$$p(\Delta\varepsilon)=\int_{-\infty}^{+\infty}dt\,e^{-i\phi(t)}\,e^{i\Delta\varepsilon t/\hbar}$$

$$e^{-i\varphi(t)}=\left(\frac{t+iw}{t-iw}\right)^{q/e}$$

$$d\varphi/dt=\frac{q}{e}2w/(t^2+w^2)$$

arbitrary quantized Lorentzian pulses

PHYSICAL REVIEW B VOLUME 56, NUMBER 11 15 SEPTEMBER 1997-I

Coherent states of alternating current

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H. W. Lee
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(Received 2 December 1996)

PRL 97, 116403 (2006) PHYSICAL REVIEW LETTERS week ending 15 SEPTEMBER 200

Minimal Excitation States of Electrons in One-Dimensional Wires

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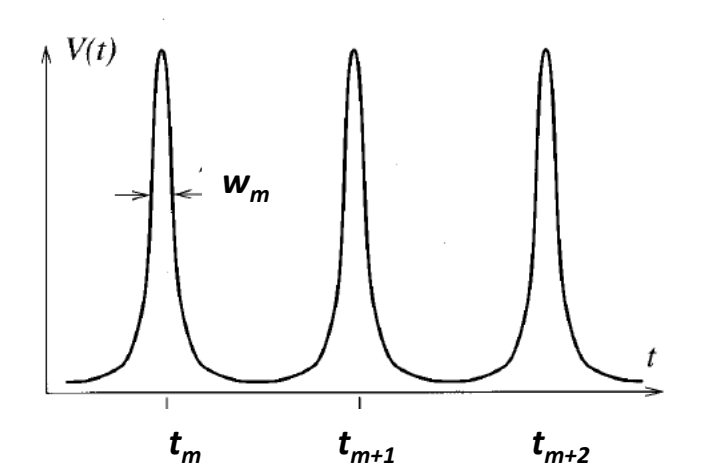
[2]*Department of Physics, California Institute of Technology, Pasadena, California 91125, USA*
(Received 1 April 2006; published 14 September 2006)

$$V(t)=\frac{h}{e\pi}\sum_{m=1}^{n}\frac{w_m}{(t-t_m)^2+w_m^{\ 2}}$$

$$\phi(t)=2\pi\frac{e}{h}\int^{t}dt'V(t')$$

$$\exp(i\phi(t))=\prod_{m=1}^{n}\frac{t-t_m+i\,w_m}{t-t_m-i\,w_m}$$

arbitrary sum of Lorentzian each Lorentzian carrying a single electron ($\Delta\Phi=\Phi_0$)



only poles in the upper plane ➡ upward energy shift of electrons of the Fermi sea = *no hole creation*

➡ generate pure n-electron excitation , whatever t_m and w_m are

$$\prod_{m=1}^{n}U_m|0\rangle=\prod_{m>m'}\frac{\xi^*_{m'}+\xi_m}{\xi_{m'}-\xi_m}A_n^\dagger A_{n-1}^\dagger\cdots A_1^\dagger|0\rangle$$

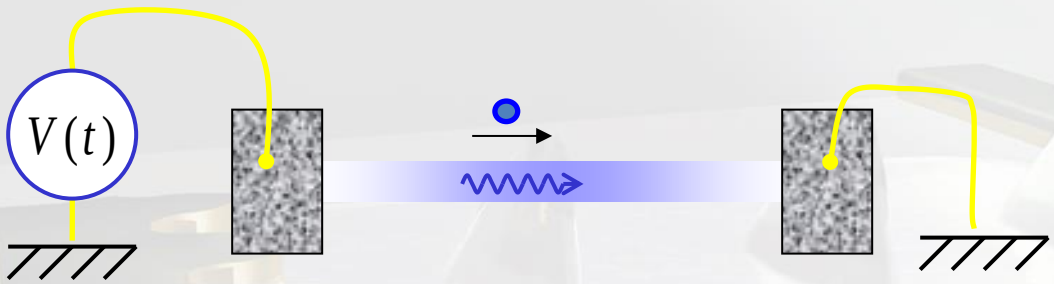
$$\xi_m=t_m+i\,w_m \qquad A_m^{\ +}=\sqrt{2w_m}\sum_{\varepsilon>E_F}\exp(-\xi_m\,\varepsilon/h)\,a_\varepsilon^{\ +}$$

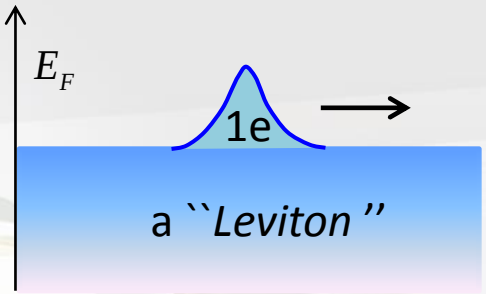
single electron wavefunction:

$$\psi(x,t)=\sqrt{\frac{v_F}{2\pi}}\frac{i\sqrt{2\tau_1}}{x-v_F(t-t_1)+iv_F\tau_1}$$

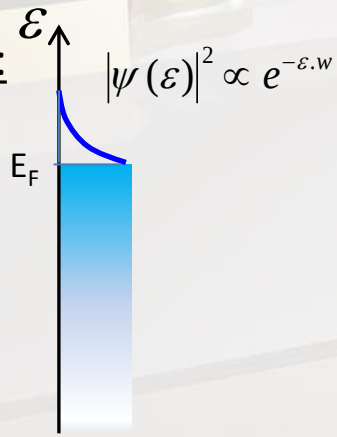
the leviton in brief

a new quasi-particle with minimal excitation



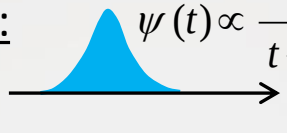


energy-domain:



$$|\psi(\varepsilon)|^2 \propto e^{-\varepsilon.w}$$

time-domain:



$$\psi(t) \propto \frac{1}{t+iw}$$

Levitov, Lee, Ivanov,Lesovik (96,97); Keeling, Klich, Levitov (06)

OUTLINE

- ideal conductors to explore the Fermi Sea
- quantized conductance and noiseless electron flow
- electron-hole entanglement in the Fermi-sea
- single electron sources for electron quantum optics
  - minimal excitations states of a Fermi sea: the levitons
  - experimental realization of levitons
- perspective and applications of levitons

physical implementation of Levitons

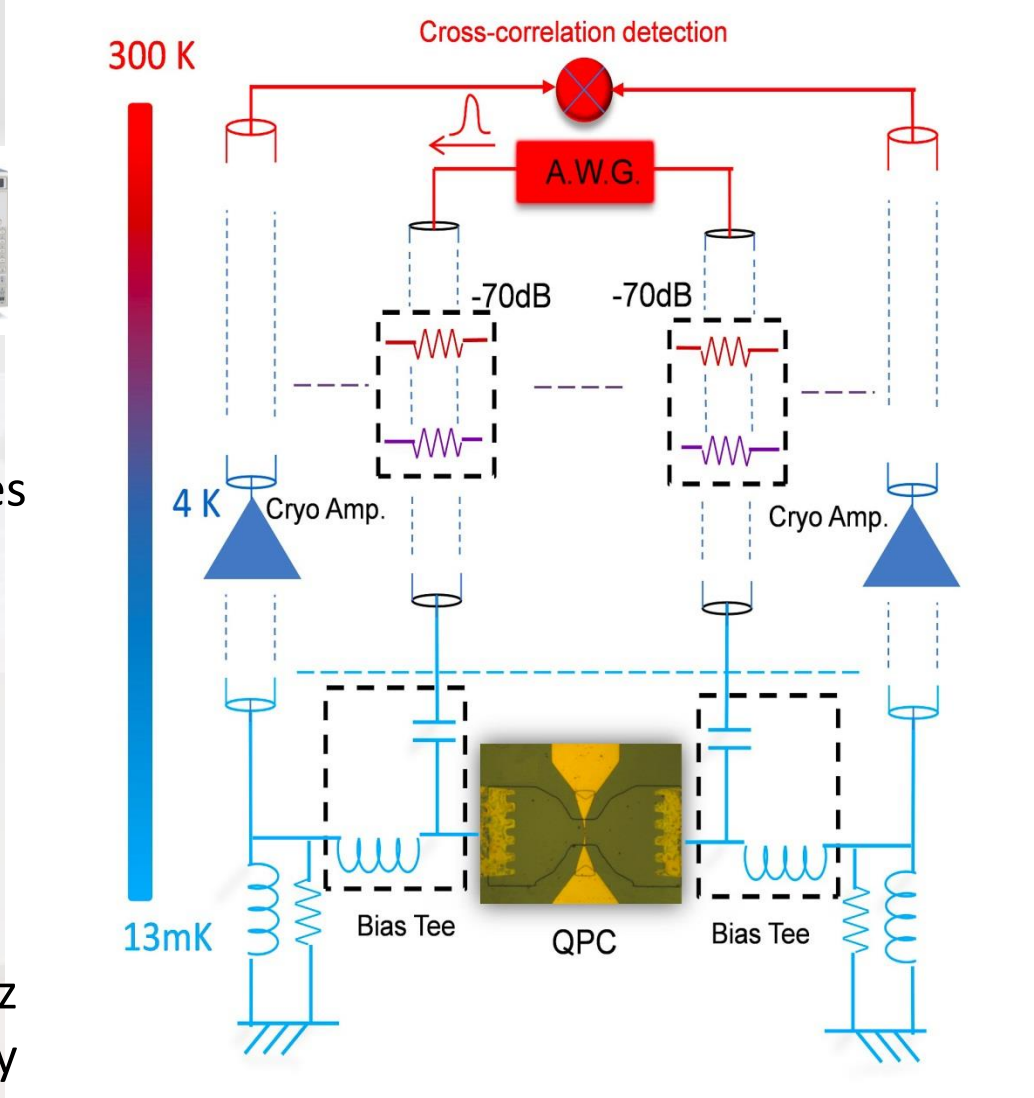
- AWG : 24 Gs/s 10bits
or four harmonic generation

- 40 GHz low loss short coaxial cables

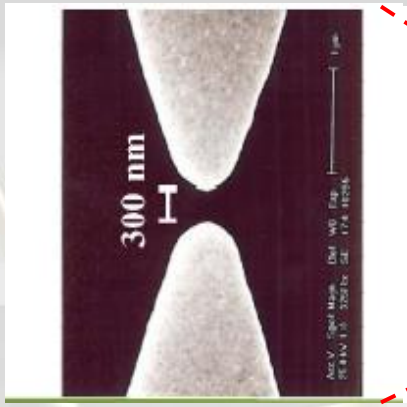
- 13 mK Cryoconcept pulse-tube dilution fridge

- ~ 35 mK electron temperature

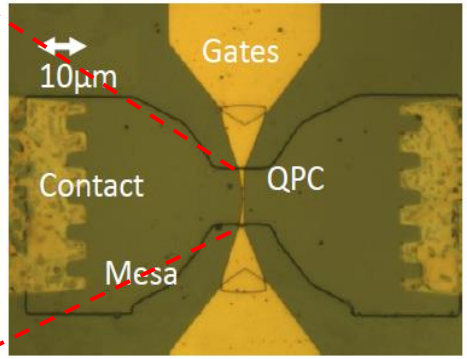
- world state of the art < 10^{-30} A^2/Hz current noise sensitivity



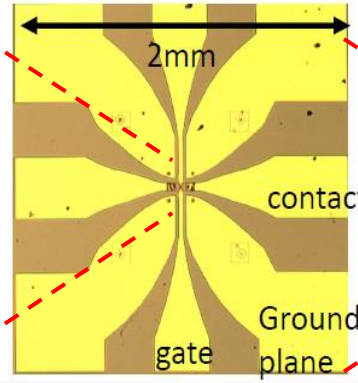
physical implementation of Levitons



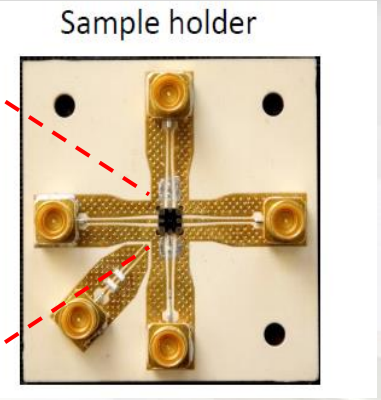
Quantum Point Contact



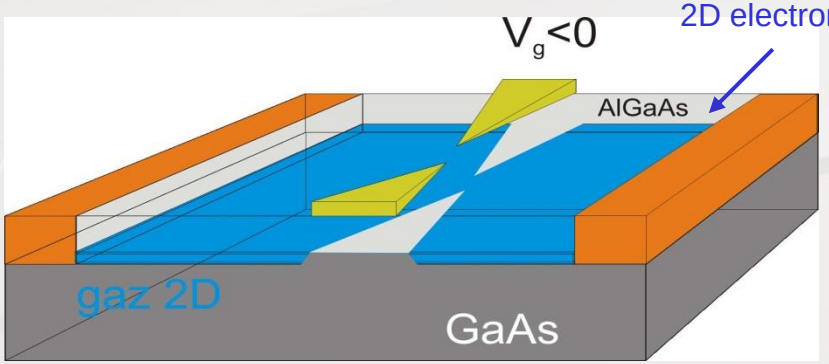
quantum conductor
(2DEG in GaAs)



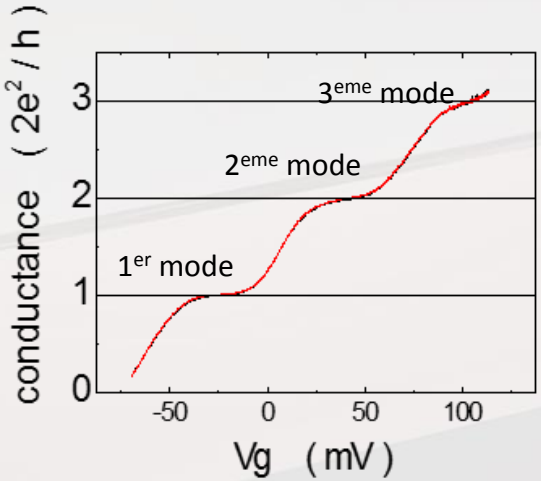
GaAs/Ga(Al)As chip
40GHz coplanar lines



High frequency (40 GHz)
Printed Circuit Board

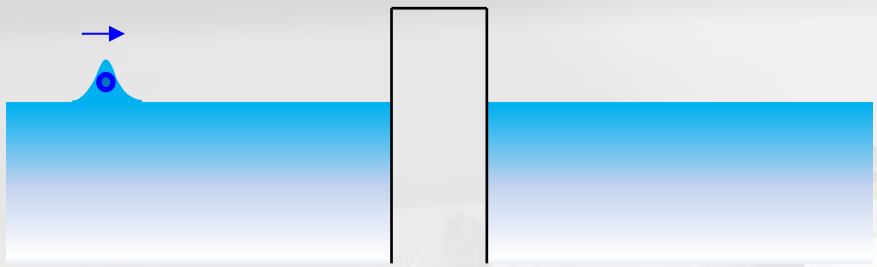


III-V semi-conductor heterojunction GaAs/GaAlAs



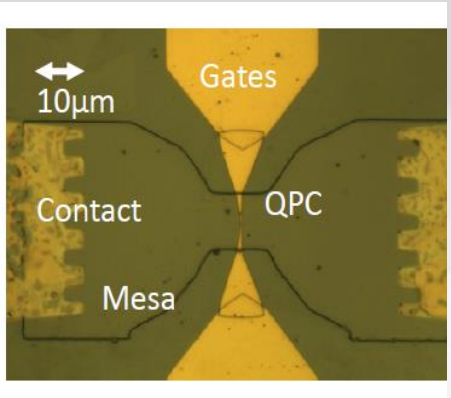
counting excitations with shot noise

- on-demand injection of n-electron(s)

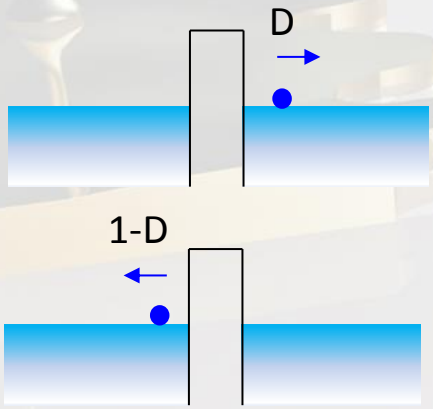


$n = N_e$

$n = 1$



- NOISE as a tool to characterize the on-demand electron injection



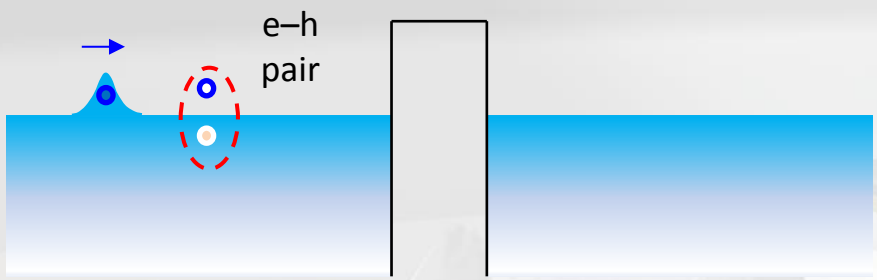
$$S_I = 2e^2\nu\, D(1-D)\, n$$

(ν : repetition frequency)

G.B. Lesovik JETP Lett.49, 592 (1989)
M. Büttiker, PRL 65, 2901 (1990)
Th. Martin and R. Landauer, PRB 46,1889 (1992)

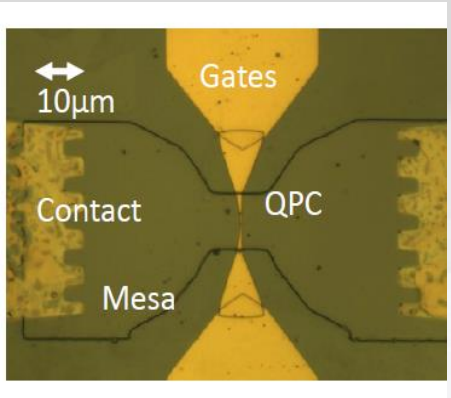
counting excitations with shot noise

- NOISELESS on-demand injection of n-electron(s)

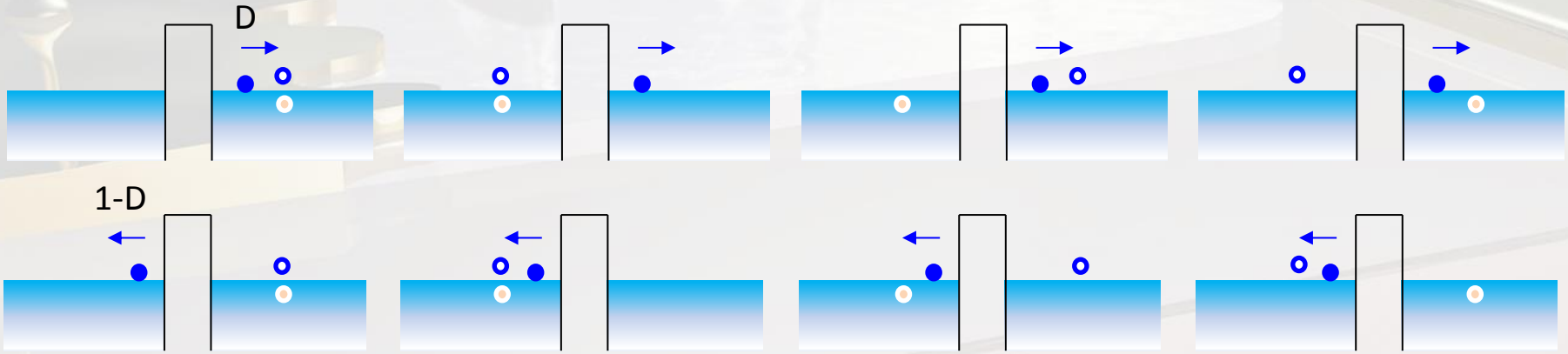


$n = N_e - N_h$

$n < N_e + N_h$



- NOISE as a tool to characterize the on-demand electron injection



$S_I = 2e^2\nu\, D(1-D)\, n$

(ν : repetition frequency)

$S_I = 2e^2\nu\, D(1-D)\, (N_e + N_h)$

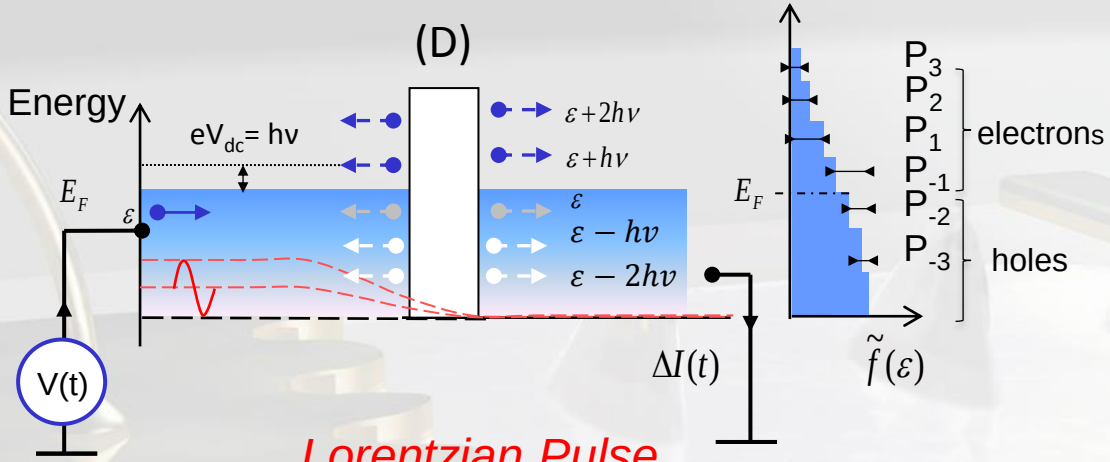
(current: $I = e\,\nu\,(N_e - N_h) = e\,\nu\, n$)

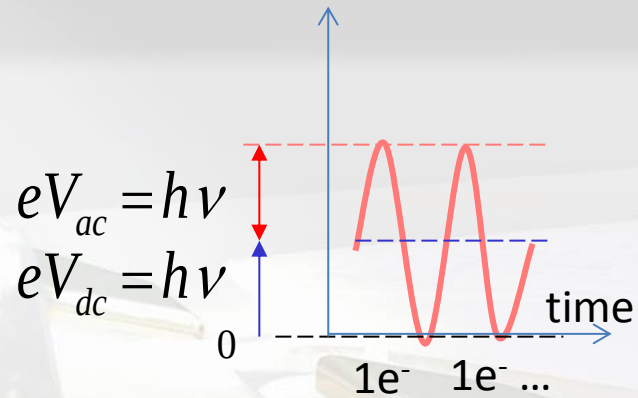
exact relation @ $k_B T \ll h\nu$

periodic voltage pulses applied on a contact

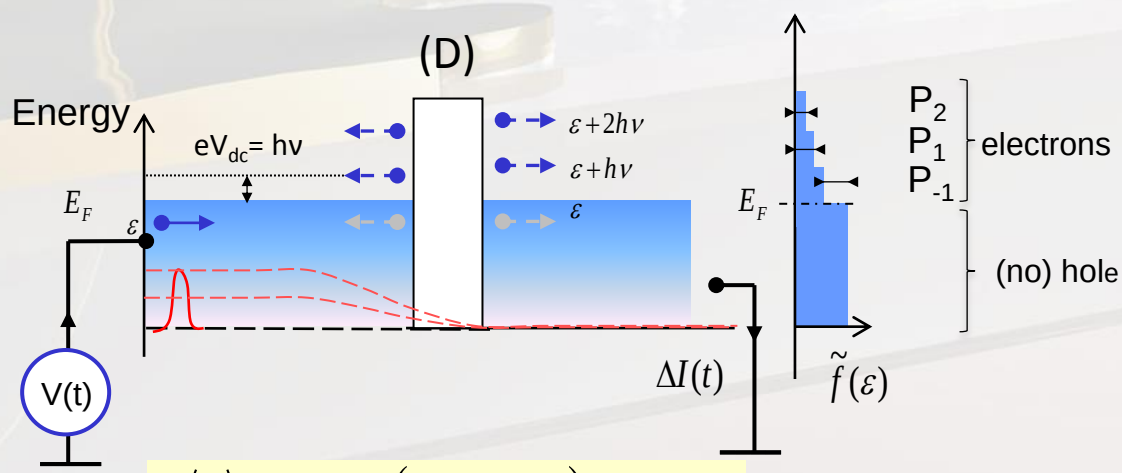
$$V(t) = V_{dc} + V_{ac}(t) \qquad I_{dc} = e\nu$$

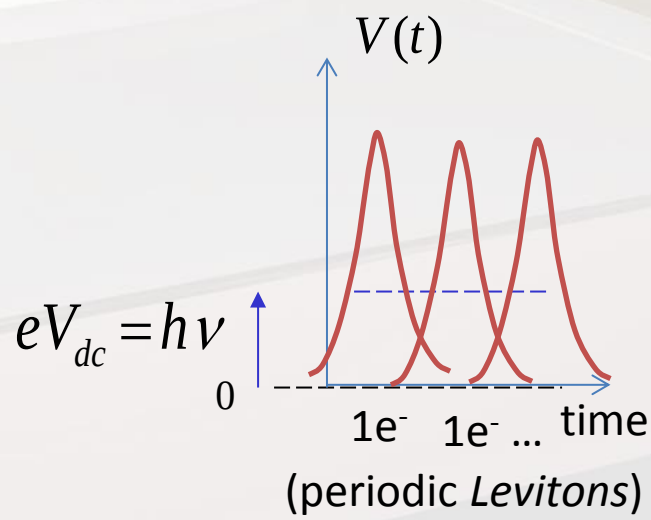
Sine Pulse





Lorentzian Pulse





$$\langle I \rangle = e\nu\left(N_e - N_h\right) = ne\nu$$

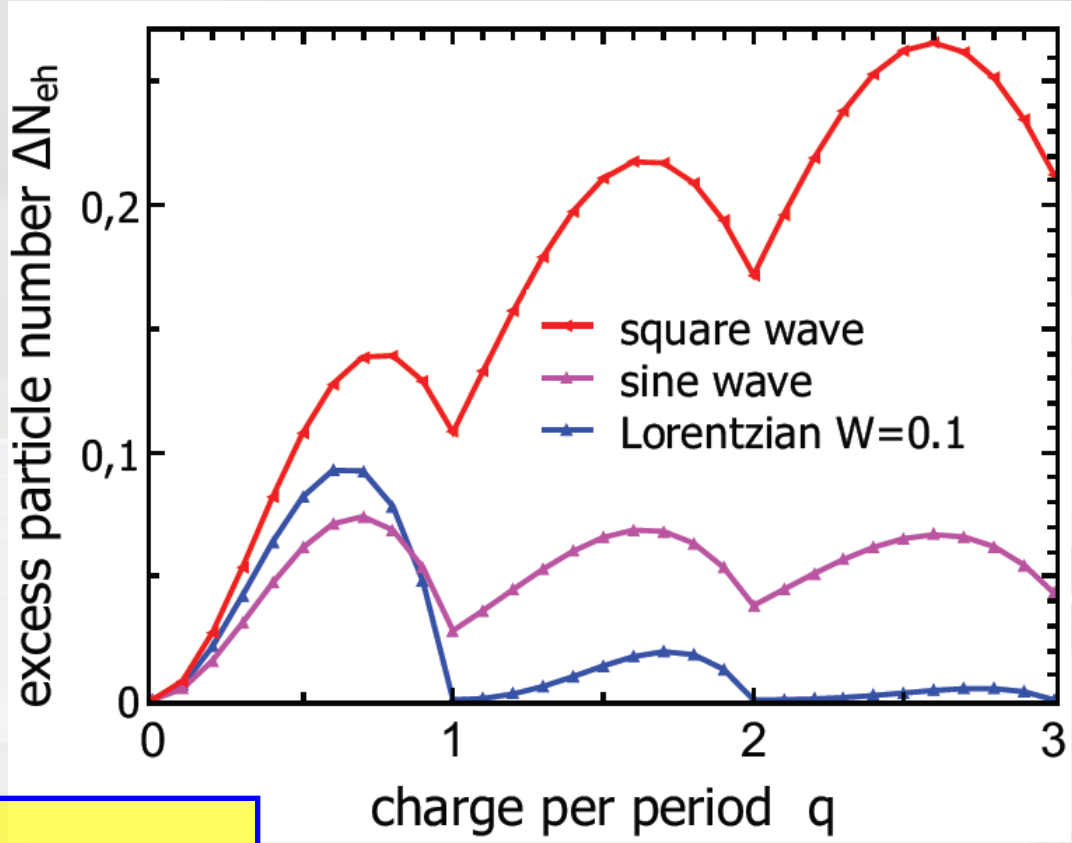
$$\langle \Delta I^2 \rangle \propto e^2\nu\left(N_e + N_h\right)$$

periodic voltage pulses applied on a contact

separate V_{dc} and V_{ac}

$$V(t) = V_{dc} + V_{ac}(t)$$

experimentally:
noise rf On - noise rf Off)



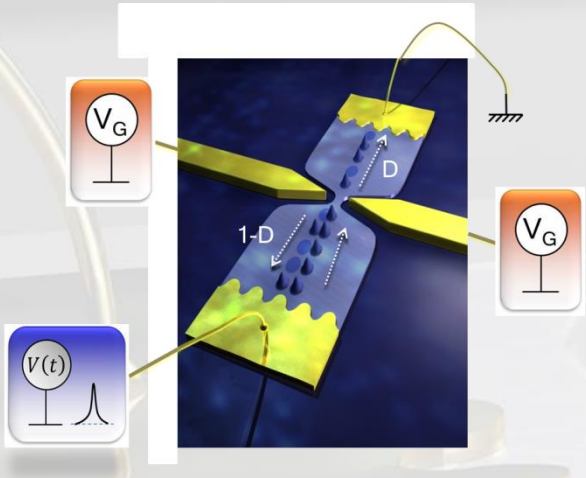
$$\Delta S_I = S_I(V_{DC} + V_{ac}) - S_I(V_{DC}) = 2e\frac{e^2}{h}D(1-D)h\nu.\ \Delta N_{eh}$$

$$\Delta N_{eh} = N_e + N_h - n$$

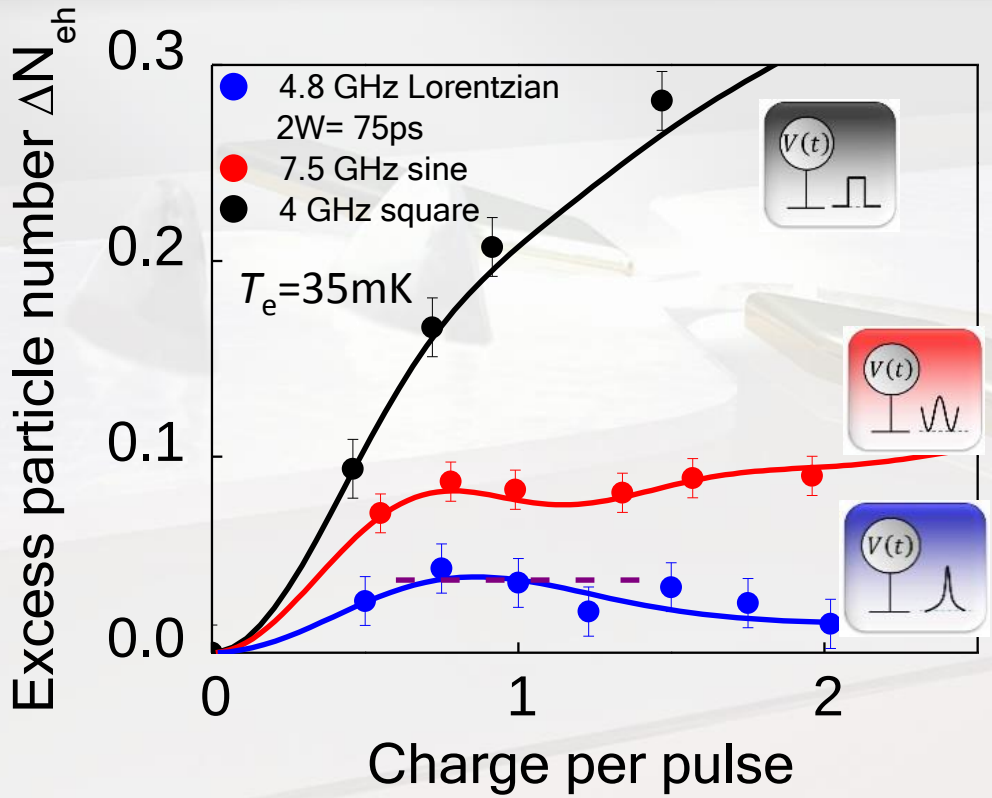
- *minimum* excess e-h particle for *integer* charge : general for *any* pulse shape
- remarkable *quantization of the charge* in a system made of many *delocalized* electrons

M. Vanevic, Y. V. Nazarov, and W. Belzig, Phys. Rev. Lett. 99, 076601 (2007)
Lee and Levitov, arxiv., cond-mat, 9312013v1 (1993)

EXPERIMENTAL NOISE for VARIOUS PULSE SHAPES



electron-hole pair number per pulse



square pulses : 20% e-h pairs per electron

sine wave pulse: 4.6 % e-h pairs per electron

Lorentzian : <1% e-h pairs per electron

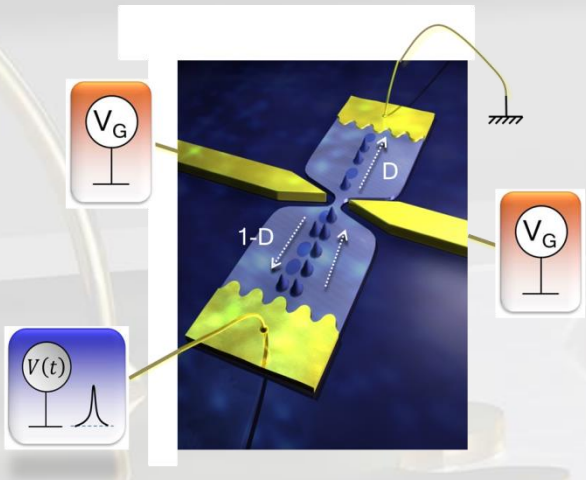
thermal excitations : ~ 3.6 % @ q=1

$\sim 2\theta \exp(-4\pi W/T)$ leviton

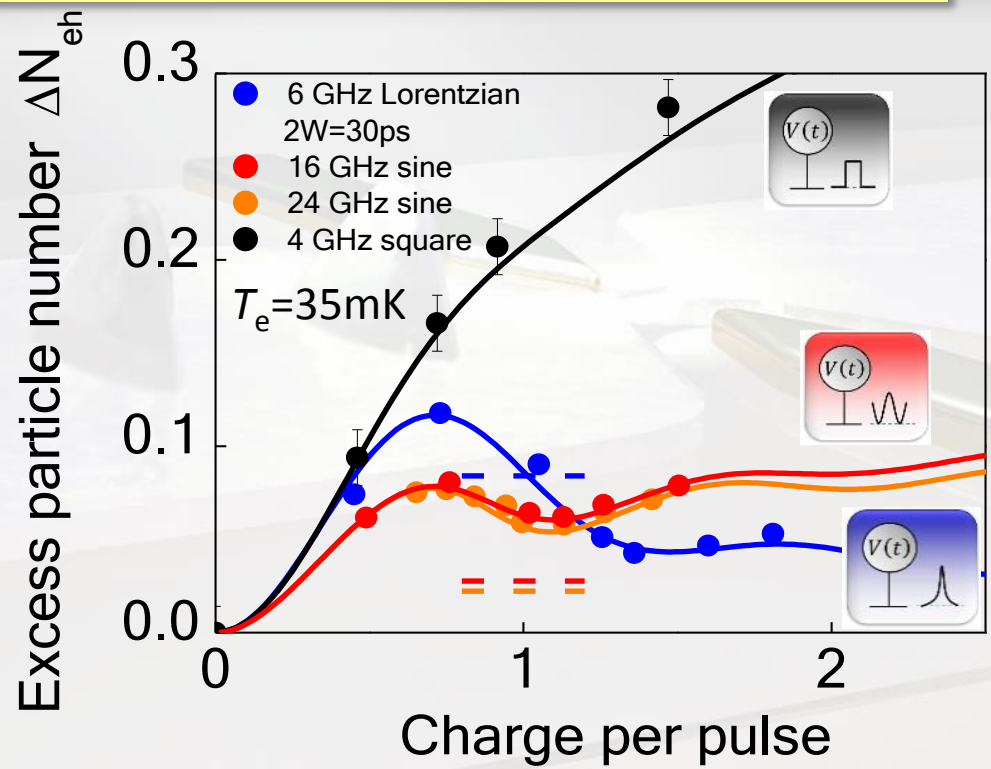
$\sim 2\theta J_1(q)^2$ sine

J. Dubois et al, Nature 502, 659 (2013)

EXPERIMENTAL NOISE for VARIOUS PULSE SHAPES



electron-hole pair number per pulse



SINE-WAVE : noise minimum at integer charge

LEVITON: noise minimum shifted by temperature

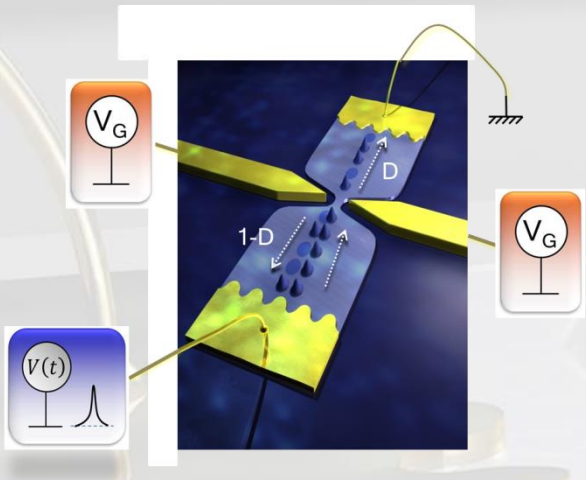
thermal excitations : ~ 8.6 % @ q=1

$\sim 2\theta \exp(-4\pi W/T)$ leviton

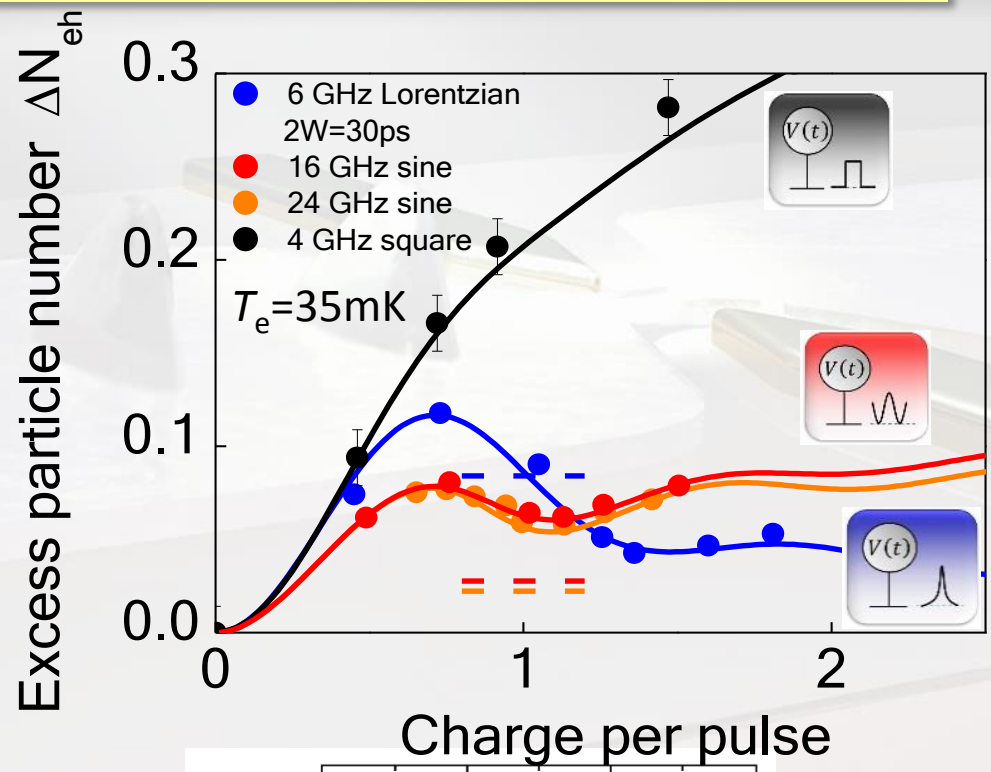
$\sim 2\theta J_1(q)^2$ sine

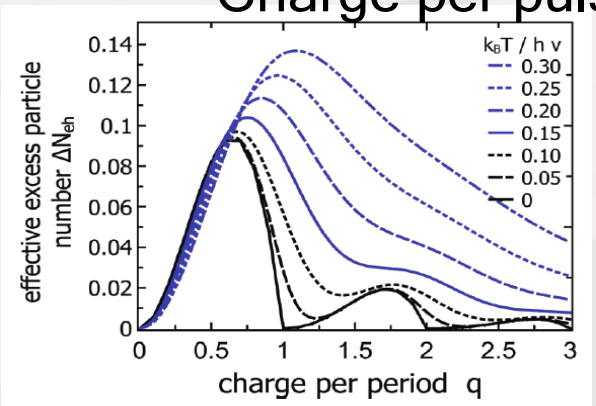
J. Dubois et al, Nature 502, 659 (2013)

EXPERIMENTAL NOISE for VARIOUS PULSE SHAPES



electron-hole pair number per pulse

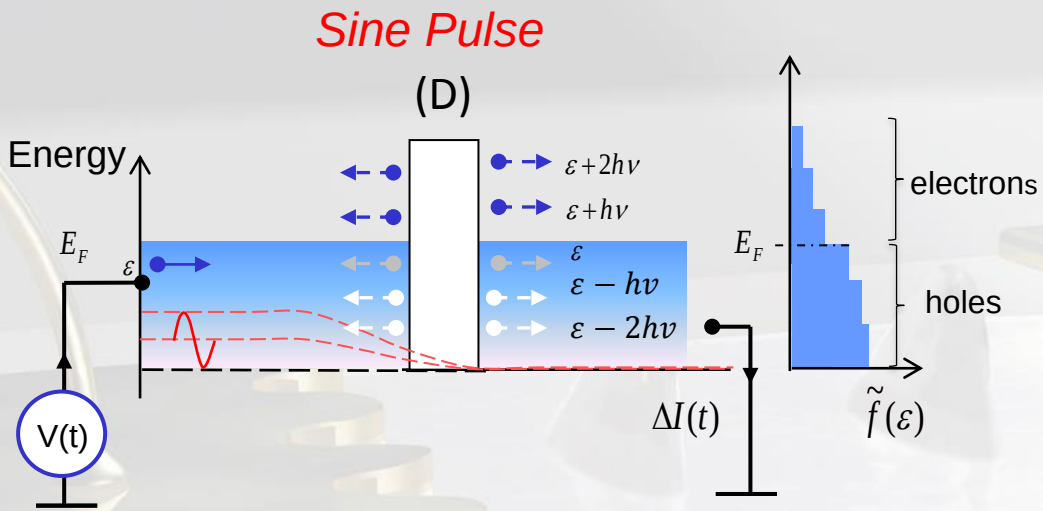


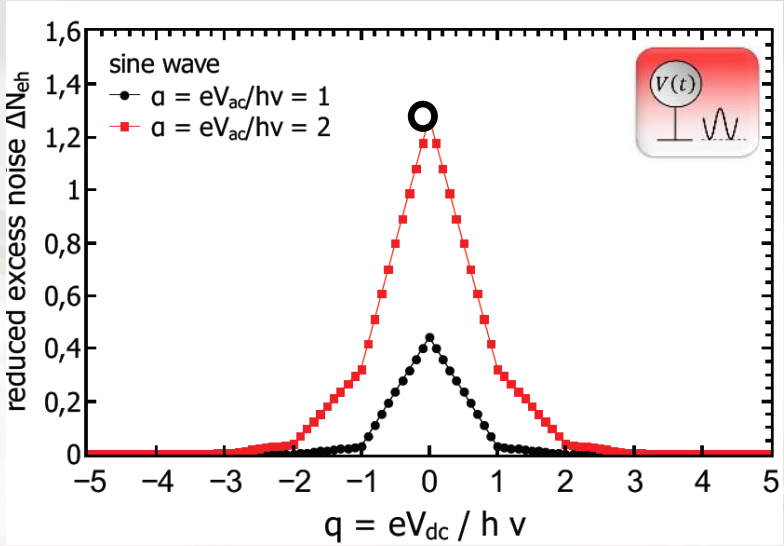


SINE-WAVE : noise minimum at integer charge

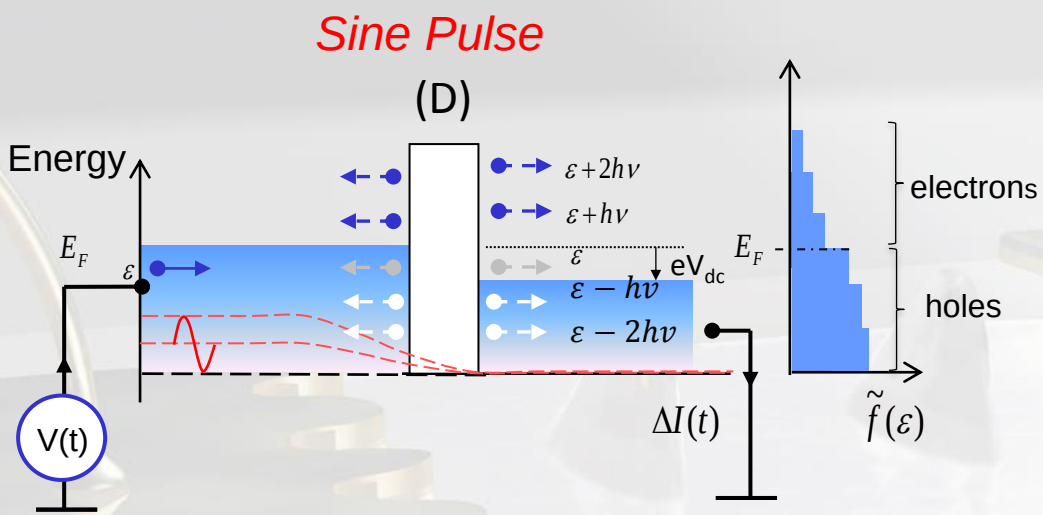
LEVITON: noise minimum shifted by temperature

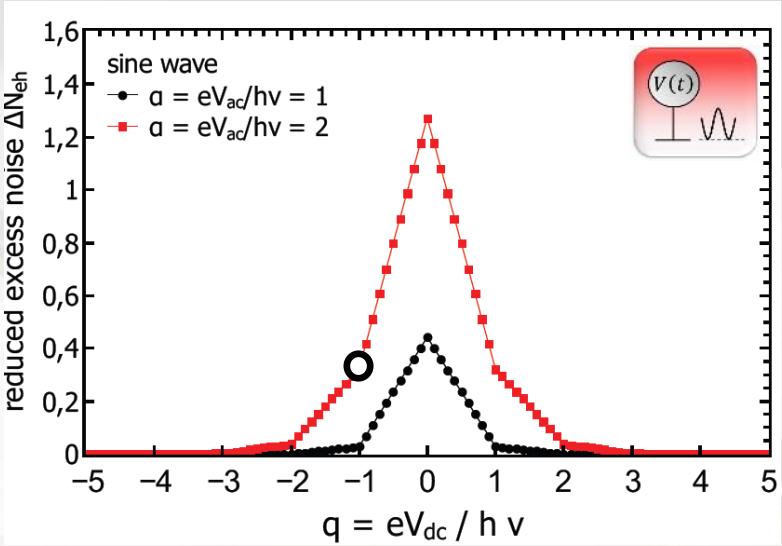
Energy domain: shot noise spectroscopy



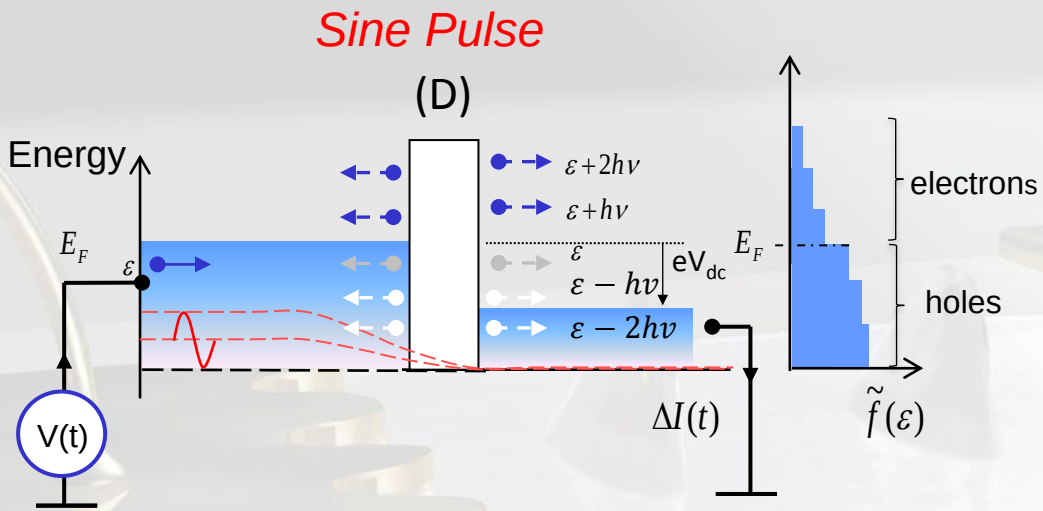


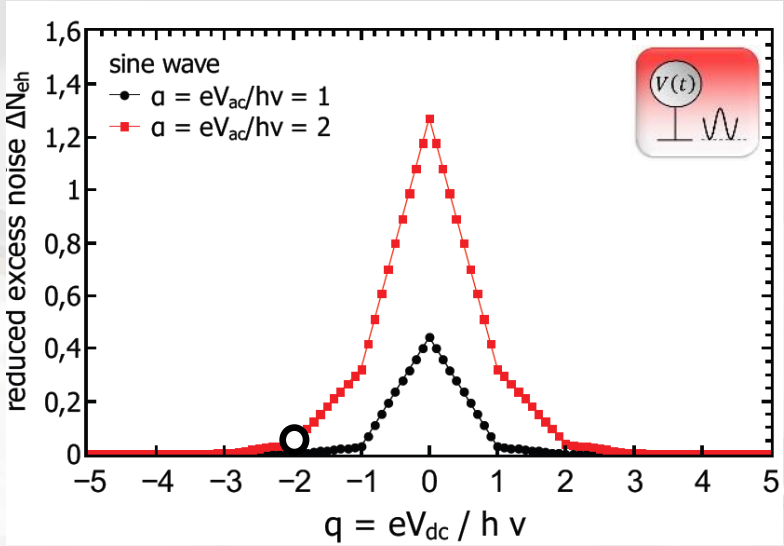
Energy domain: shot noise spectroscopy



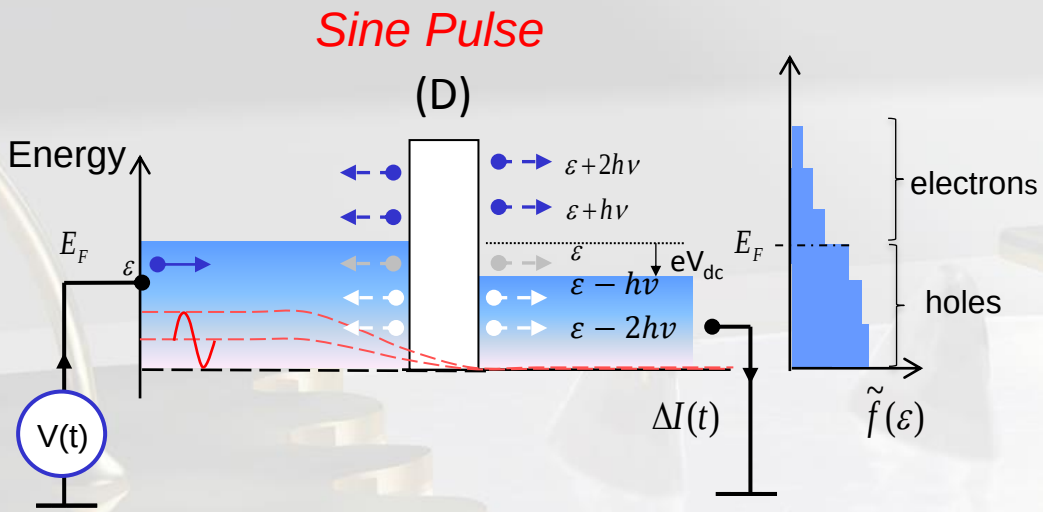


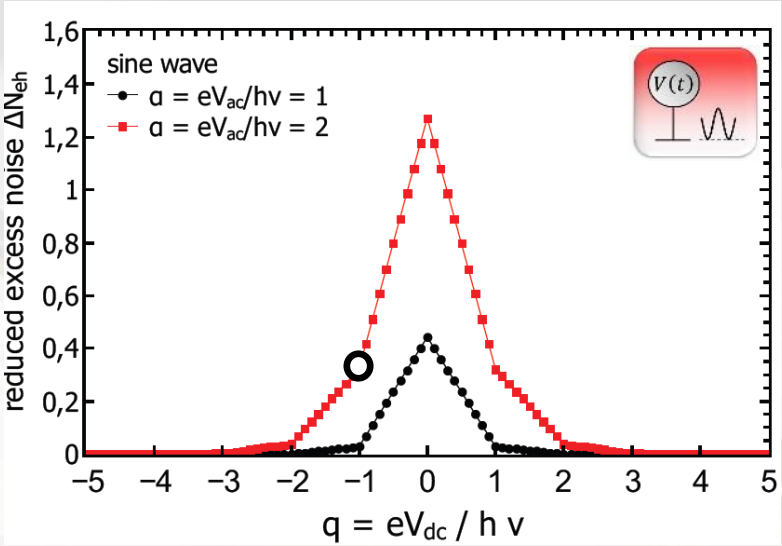
Energy domain: shot noise spectroscopy



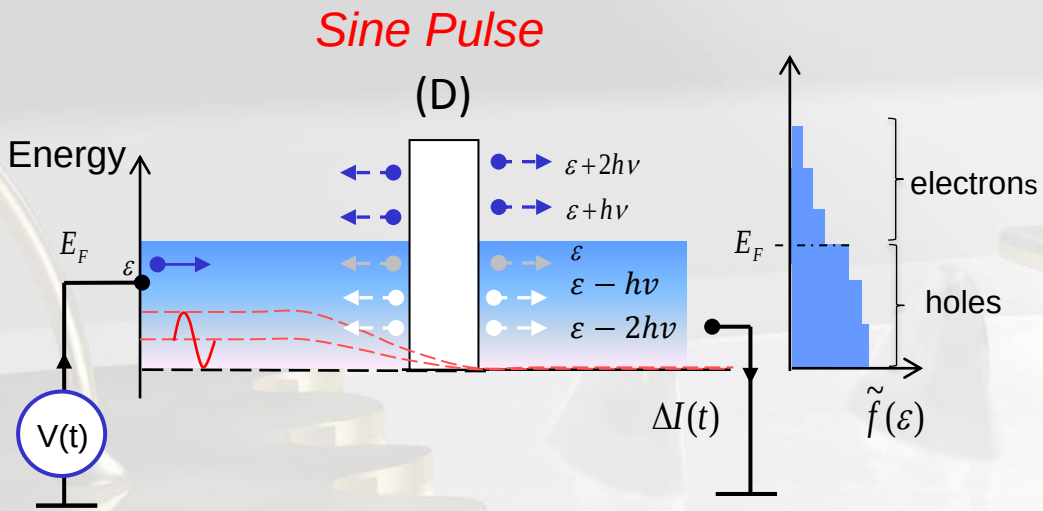


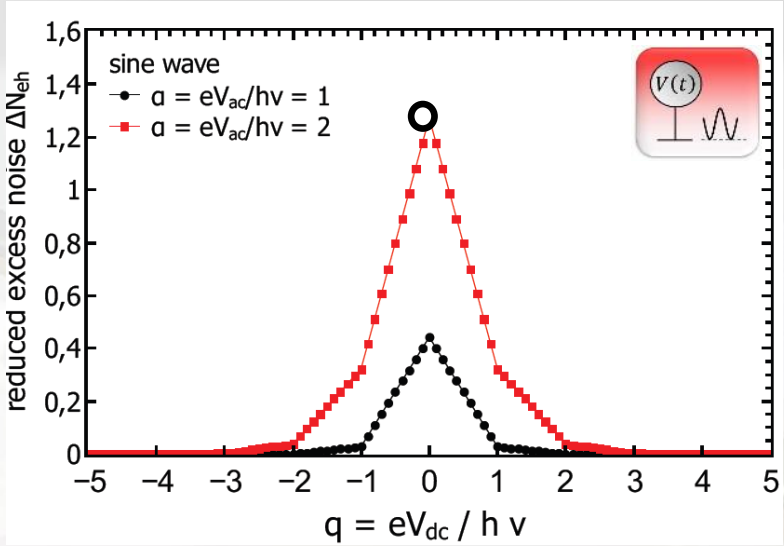
Energy domain: shot noise spectroscopy



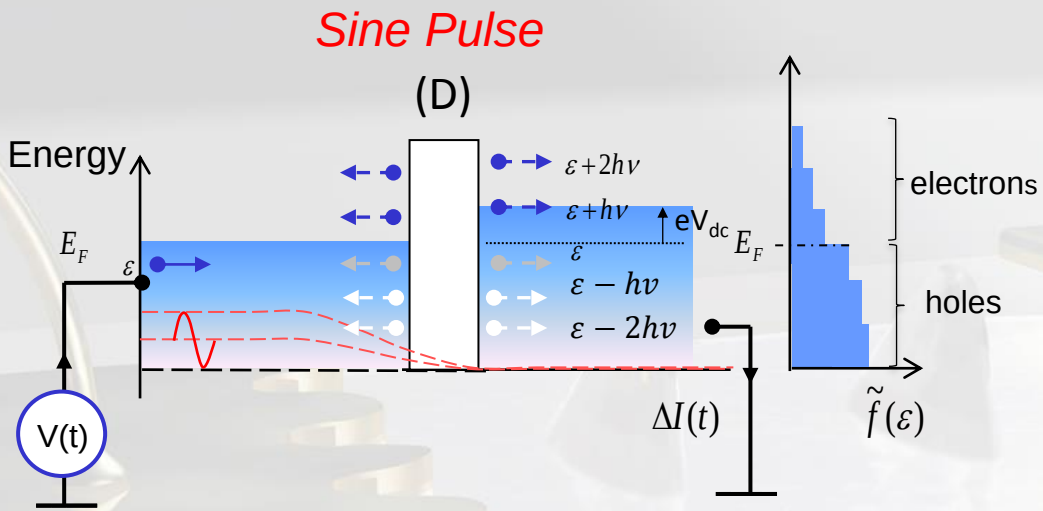


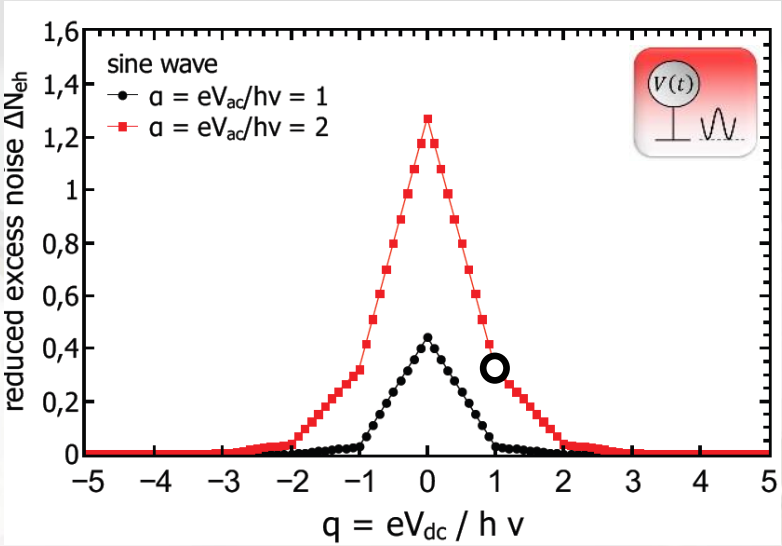
Energy domain: shot noise spectroscopy



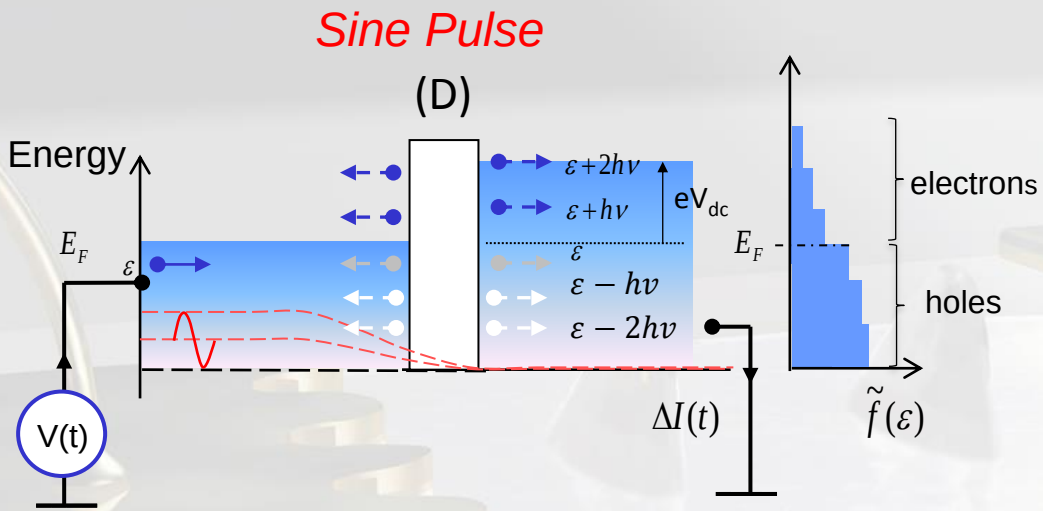


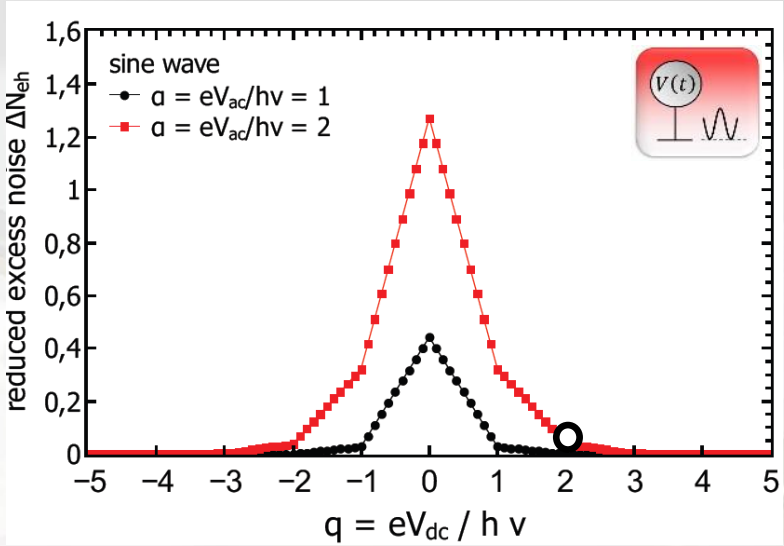
Energy domain: shot noise spectroscopy



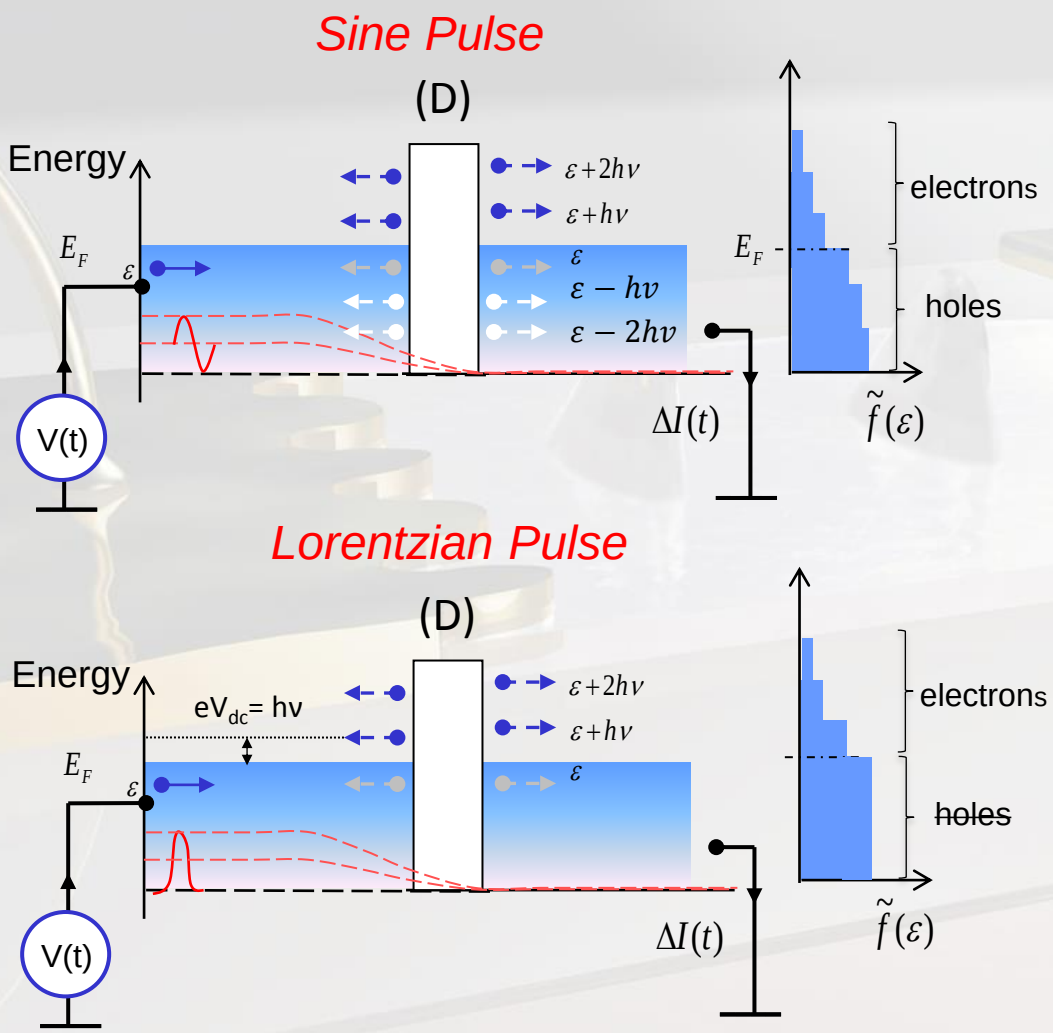


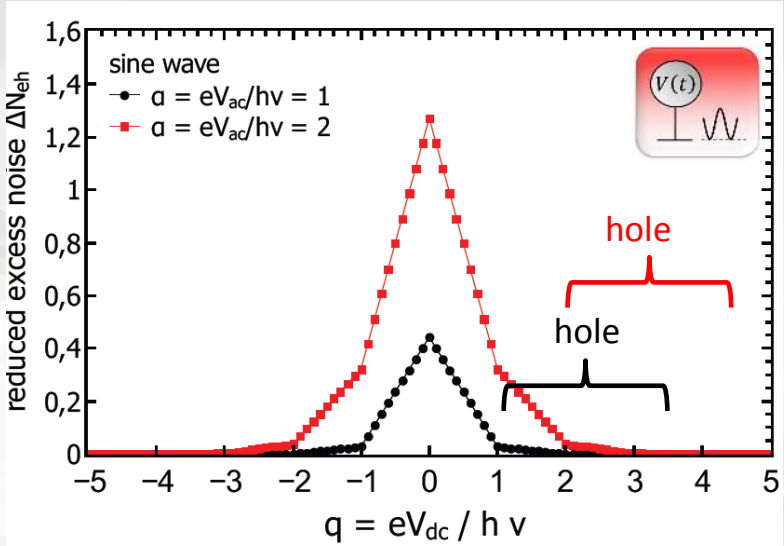
Energy domain: shot noise spectroscopy

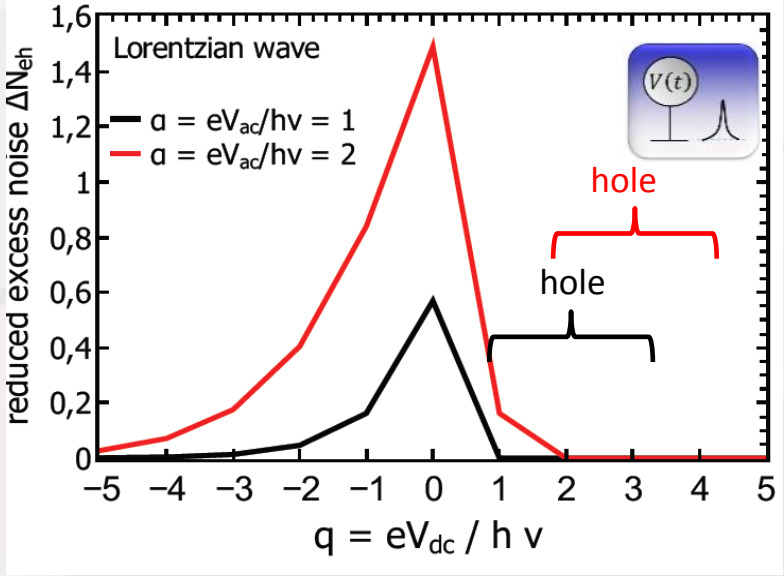




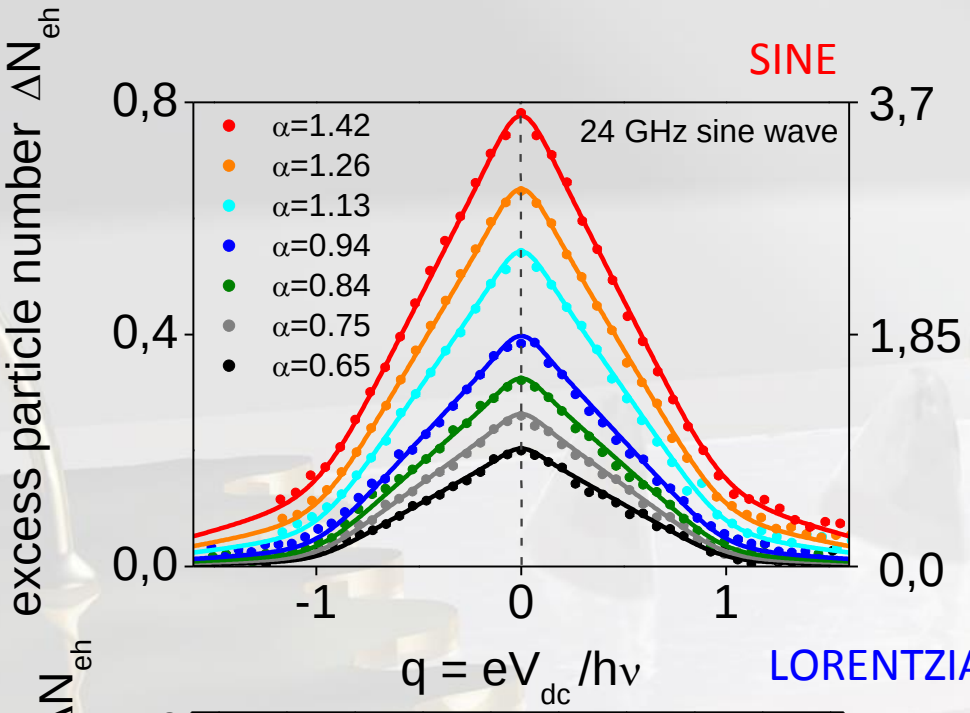
Energy domain: shot noise spectroscopy

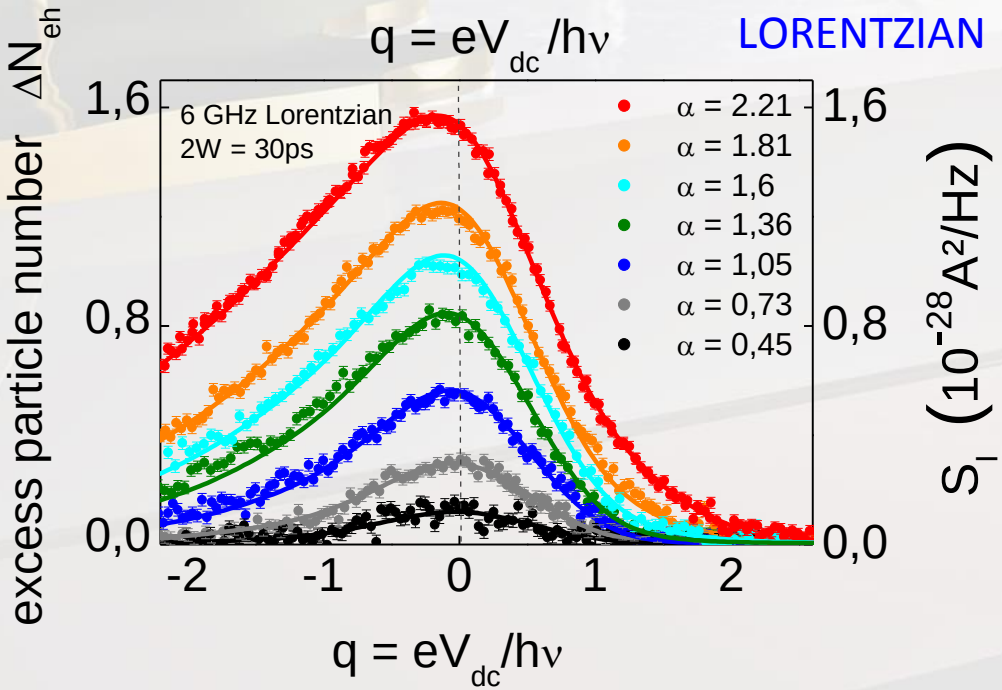


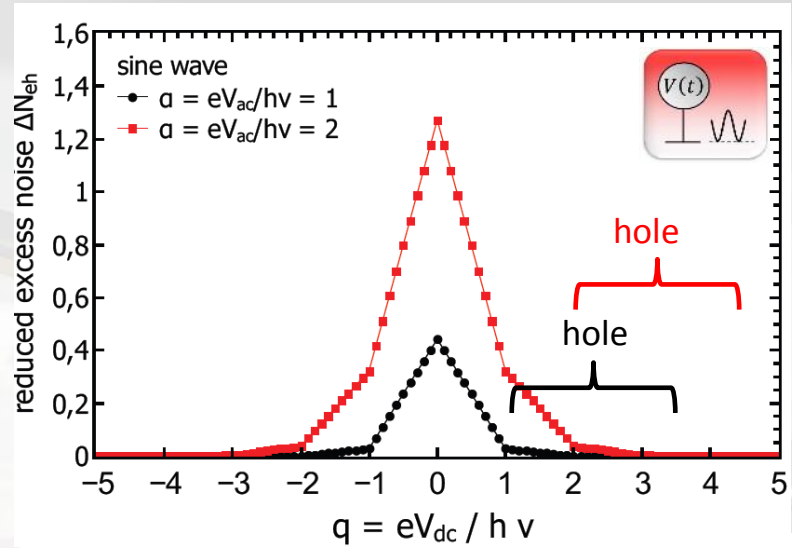


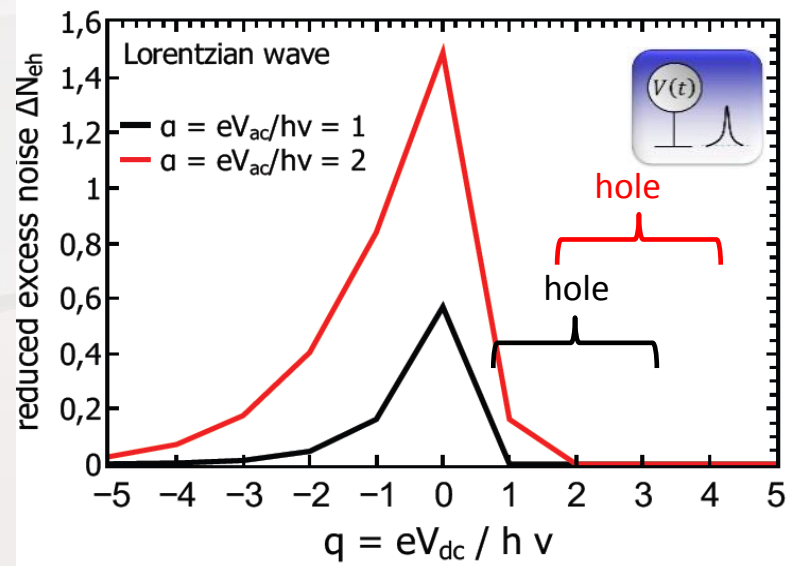


Energy domain: shot noise spectroscopy





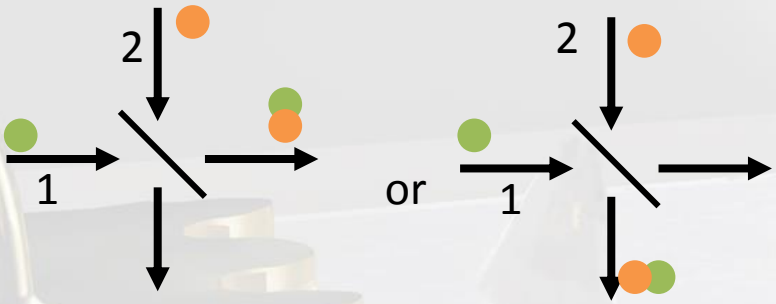




J. Dubois et al, Nature 502, 659 (2013)

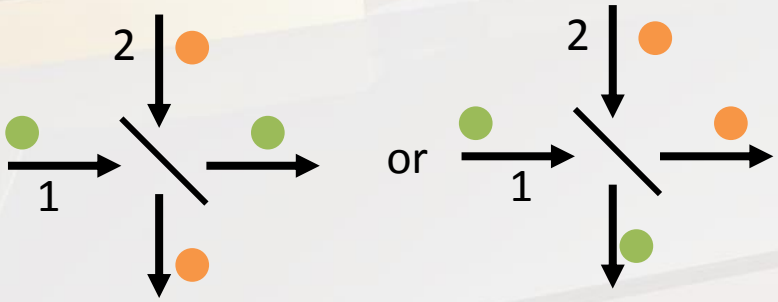
time domain: electron pulse collisions

bosons



$$\frac{S_I^{HOM}}{S_I^{HBT}} = 2$$

fermions



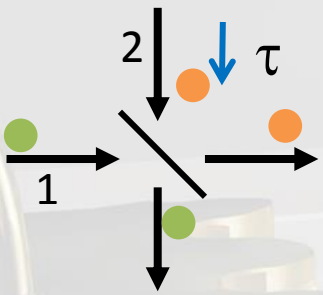
$$\frac{S_I^{HOM}}{S_I^{HBT}} = 0$$

electrons: dc voltage
W. Oliver, J. Kim, R. Liu, and Y. Yamamoto, Science 284, 299 (1999).
M. Henny et al., Science 284, 296 (1999)

C. K. Hong, Z. Y. Ou, and L. Mandel. PRL, 59(18), (Nov 1987) pp 2044-2046

time domain: electron pulse collisions

bosons

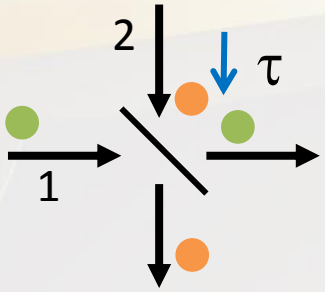


$$\frac{S_I^{HOM}}{S_I^{HBT}} = 1 + \left|\langle\psi(0)|\psi(\tau)\rangle\right|^2$$

noise measures the overlap of wavefunctions

→ time-domain information on Levitons

fermions



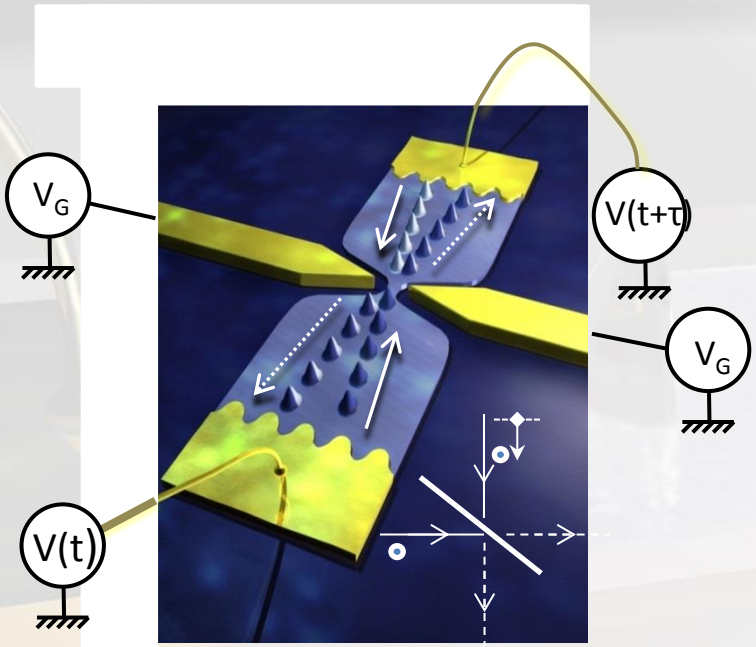
$$\frac{S_I^{HOM}}{S_I^{HBT}} = 1 - \left|\langle\psi(0)|\psi(\tau)\rangle\right|^2$$

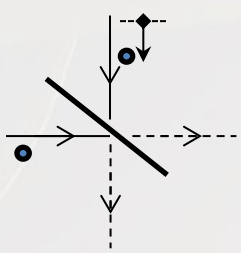
electrons: dc voltage
W. Oliver, J. Kim, R. Liu, and Y. Yamamoto, Science 284, 299 (1999).
M. Henny et al., Science 284, 296 (1999)
on-demand electron sources
E. Bocquillon et al., Science 339,1054 (2013).
J. Dubois et al., Nature 502, 659 (2013)

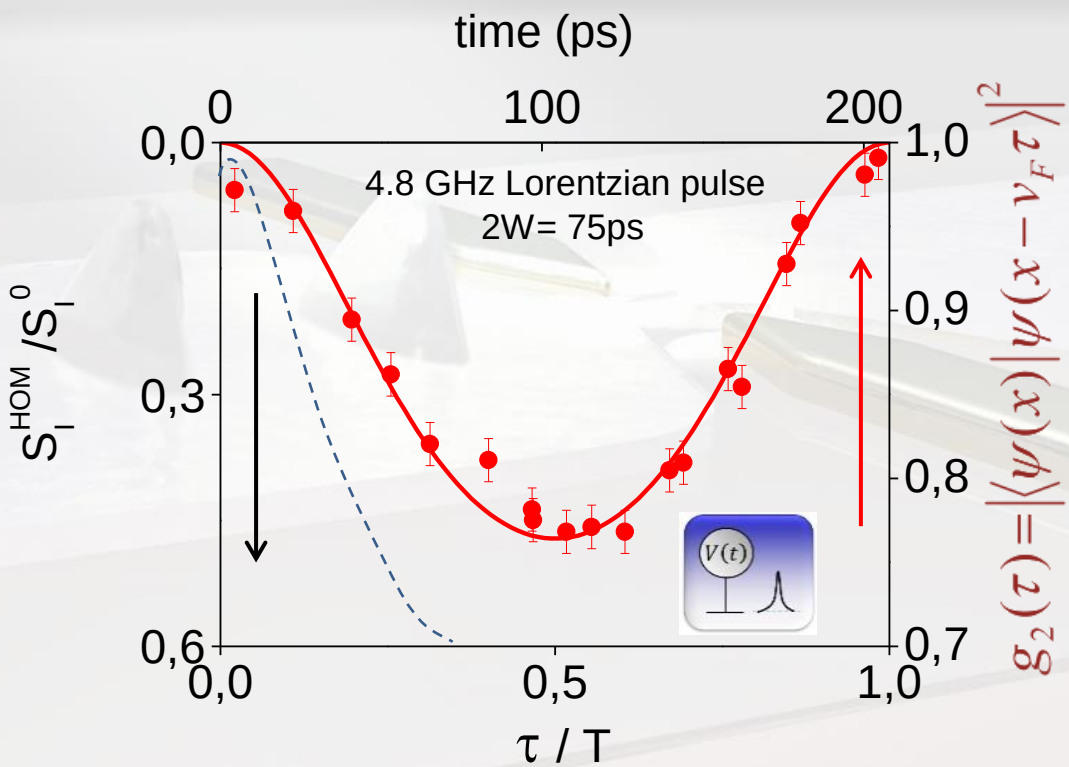
C. K. Hong, Z. Y. Ou, and L. Mandel. PRL, 59(18), (Nov 1987) pp 2044-2046

experimental Hong Ou Mandel correlations with Levitons

SHOT NOISE + FERMI Statistics probes the OVERLAP of ELECTRON WAVE-FUNCTIONS







$$g_2(\tau)=\left|\langle\psi(0)|\psi(\tau)\rangle\right|^2$$

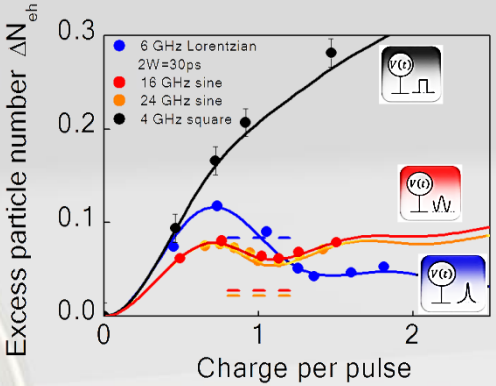
- excellent agreement with theory
- time-domain shape is Lorentzian (1e$^-$)

$$g_2(\tau)=1-\frac{S_I^{HOM}}{2S_I^0}$$

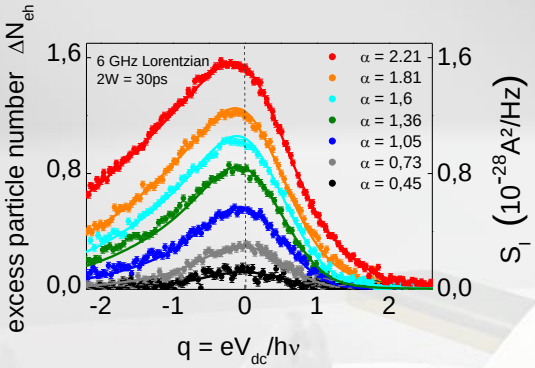
$$S_I/S_I^0=2(1-g_2(\tau))=\frac{8\beta^2\sin(\pi\nu\tau)^2}{1-2\beta^2\cos(2\pi\nu\tau)+\beta^2}$$

J. Dubois et al, Nature 502, 659 (2013)

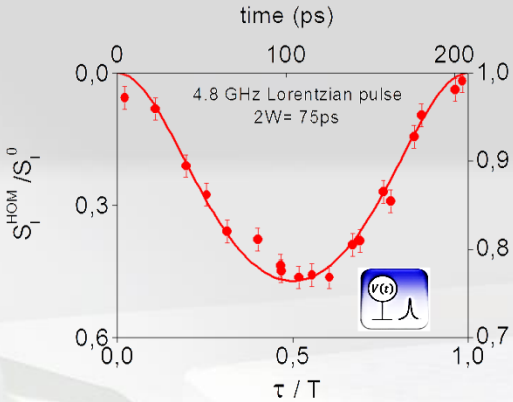
shot noise:



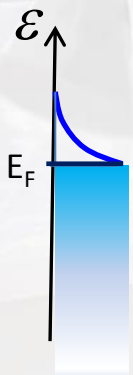
energy-domain:



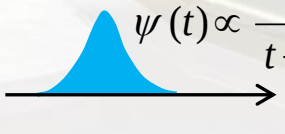
time-domain:



J. Dubois, T. Jullien,P. Roulleau, F. Portier,P. Roche, Y. Jin, W. Wegscheider, and D.C. Glattli , NATURE 502, 659 (2013)



$$|\psi(\varepsilon)|^2 \propto e^{-\varepsilon . w}$$

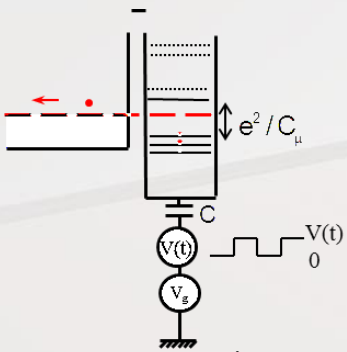


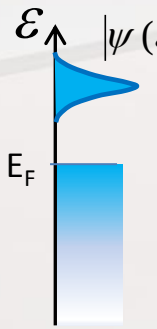
$$\psi(t) \propto \frac{1}{t+iw}$$

Levitons :

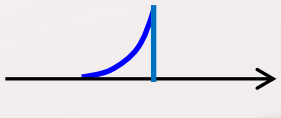
- minimal excitation states
- time resolved electrons

dual of the energy resolved electron source





$$|\psi(\varepsilon)|^2 \propto \frac{1}{(\varepsilon-\varepsilon_0)^2+\Gamma^2}$$



$$\psi(t) \propto e^{-\Gamma t}$$

G. Fève, A. Mahé, J.-M. Berroir, T. Kontos, B. Plaçais, D. C. Glattli, B. Etienne, Y. Jin, , Science 316, 1169 (2007)

some recent published work related to levitons (… not exhaustive list!)

Mach-Zehnder interferometry with periodic voltage pulses
Hofer, Patrick P.; Flindt, Christian PHYS REV B 90 Issue: 23 235416 2014

Distributions of electron waiting times in quantum-coherent conductors
Haack, Geraldine; Albert, Mathias; Flindt, Christian PHYS REV B 90 205429 2014

Fermi-sea correlations and a single-electron time-bin qubit
Moskalets, Michael PHYS REV B 90 Issue: 15 155453 2014

Projective versus weak measurement of charge in a mesoscopic conductor
Oehri, D.; Lebedev, A. V.; Lesovik, G. B.; et al., PHYS REV B 90 075312 2014

Dynamical control of interference using voltage pulses in the quantum regime
Benoit Gaury, Xavier Waintal Nature Communications, 10.1038/ncomms4844

Wigner function approach to single electron coherence in quantum Hall edge channels
D. Ferraro,et al. Physical Review B, 10.1103/PhysRevB.88.205303

Two-electron state from the Floquet scattering matrix perspective
Michael Moskalets, Physical Review B, 10.1103/PhysRevB.89.045402

Annihilation of Colliding Bogoliubov Quasiparticles Reveals their Majorana Nature
C. W. J. Beenakker, Physical Review Letters, 10.1103/PhysRevLett.112.070604

Electronic Hong-Ou-Mandel interferometry in two-dimensional topological insulators
D. Ferraro, et al. Physical Review B, 10.1103/PhysRevB.89.075407

Floquet Scattering Matrix Theory of Heat Fluctuations in Dynamical Quantum Conductors
Michael Moskalets, Physical Review Letters, 10.1103/PhysRevLett.112.206801

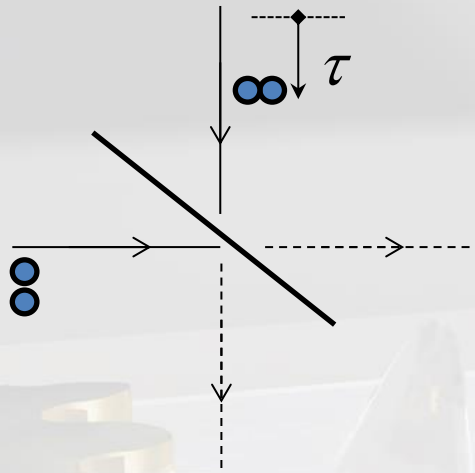
Floquet Theory of Electron Waiting Times in Quantum-Coherent Conductor
David Dasenbrook, Christian Flindt, Markus Büttiker, Physical Review Letters, 10.1103/PhysRevLett.112.146801

Emission of time-bin entangled particles into helical edge states
Patrick P. Hofer, Markus Büttiker, Physical Review B, 10.1103/PhysRevB.88.241308

OUTLINE

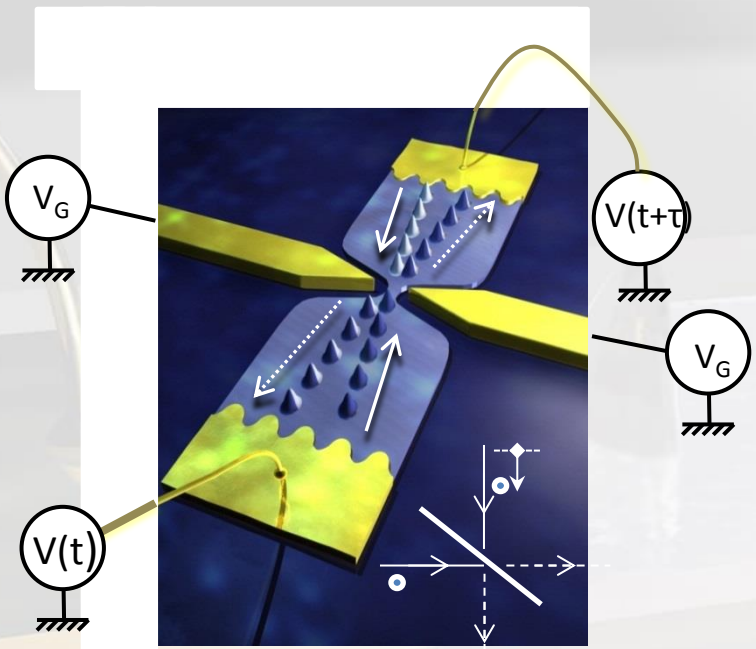
- ideal conductors to explore the Fermi Sea
- quantized conductance and noiseless electron flow
- electron-hole entanglement in the Fermi-sea
- single electron sources for electron quantum optics
  - minimal excitations states of a Fermi sea: the levitons
  - experimental realization of levitons
- perspective and applications of levitons

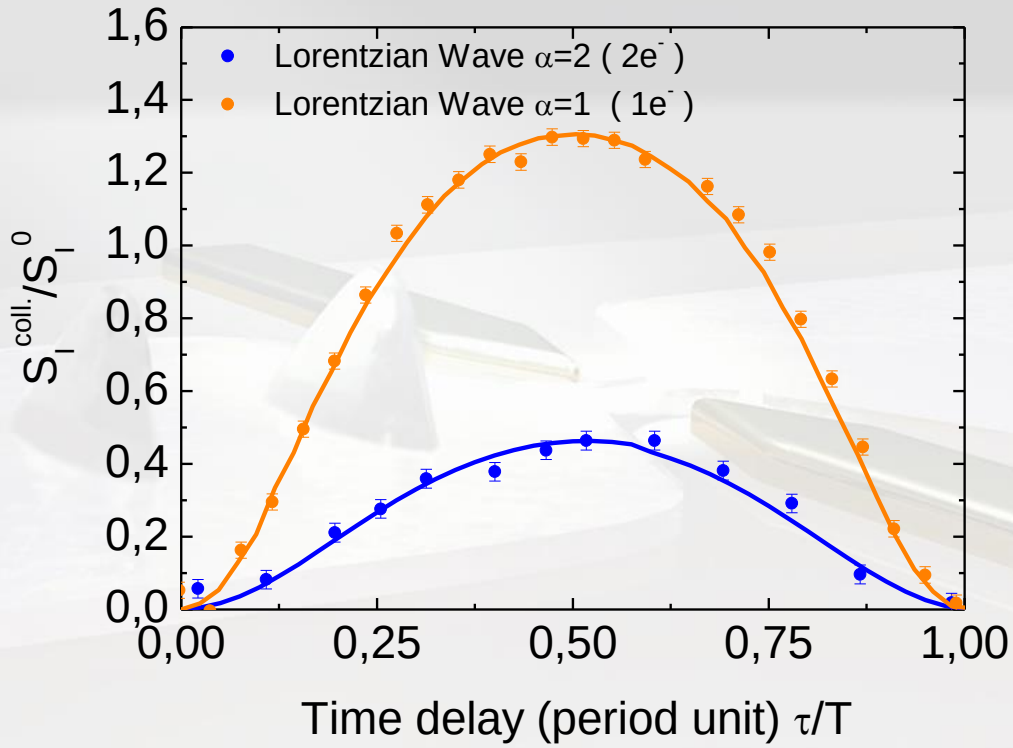
multiple Fermion Hong Ou Mandel

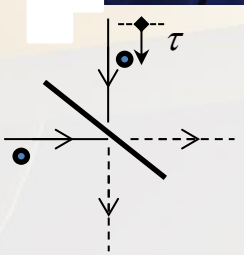


Hong Ou Mandel correlations with 2e levitons

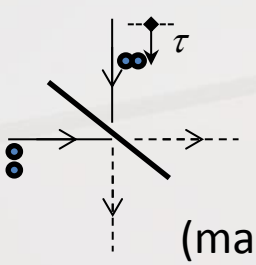
2e levitons versus 1e levitons







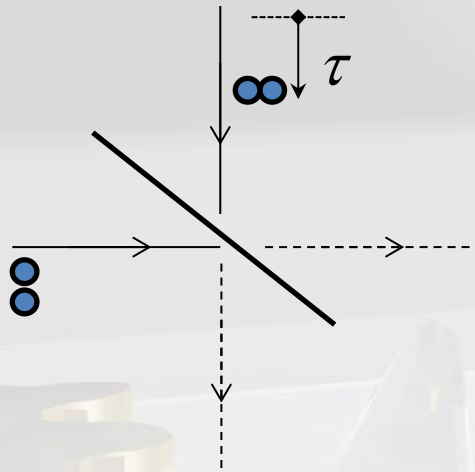
$$S_I^{HOM}(1e) \propto 1 - \left|\langle \psi_1(0) | \psi_1(\tau) \rangle\right|^2$$



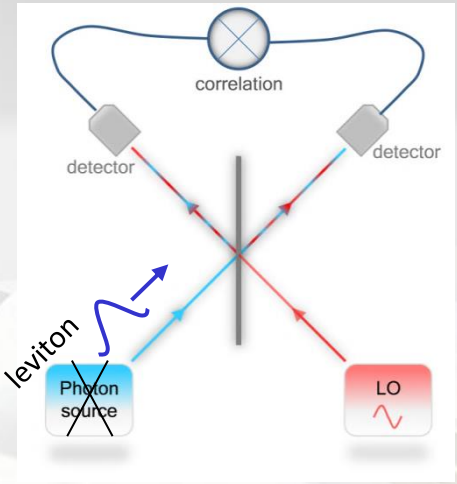
$$S_I^{HOM}(2e) \propto 2 - \left|\langle \psi_1(0) | \psi_1(\tau) \rangle\right|^2 - \left|\langle \psi_2(0) | \psi_2(\tau) \rangle\right|^2$$

(many particle Leviton HOM experiments open new field of quantum investigations)

multiple Fermion Hong Ou Mandel



`homodyne' tomography of a leviton



quantum state tomography of a leviton

Full knowledge of a quantum state: $\langle \psi^+(t') | \psi(t) \rangle$ or $\langle \psi^+(\varepsilon') | \psi(\varepsilon) \rangle$

For **fermions** (electrons) $\psi(t - x/v_F) = \varphi(t - x/v_F) | f \rangle$

wavefunction

Fock state

For **bosons** (photons) $E_0\, u(t - x/c) | b \rangle$

E.M. mode
(~ classical)

HERE : quantum **wavefunction** tomography

for a leviton: $| f \rangle = | 1 \rangle$ and focus on: $\langle \psi^+(\varepsilon') | \psi(\varepsilon) \rangle \equiv \varphi^*(\varepsilon')\, \varphi\ (\varepsilon)$

quantum state tomography of a leviton

Full knowledge of a quantum state: $\langle \psi^+(t') | \psi(t) \rangle$ or $\langle \psi^+(\varepsilon') | \psi(\varepsilon) \rangle$

For **fermions** (electrons) $\psi(t - x/v_F) = \varphi(t - x/v_F) | f \rangle$

wavefunction

Fock state

For **bosons** (photons) $E_0\, u(t - x/c) | b \rangle$

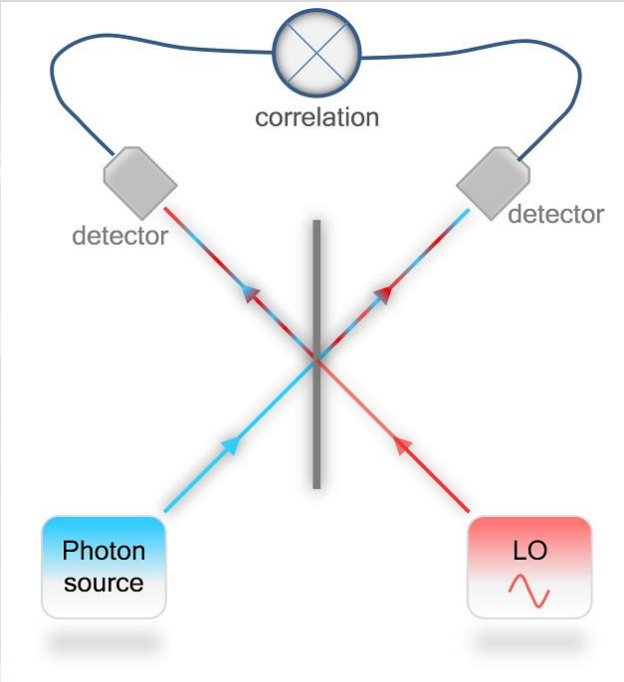
E.M. mode
(~ classical)

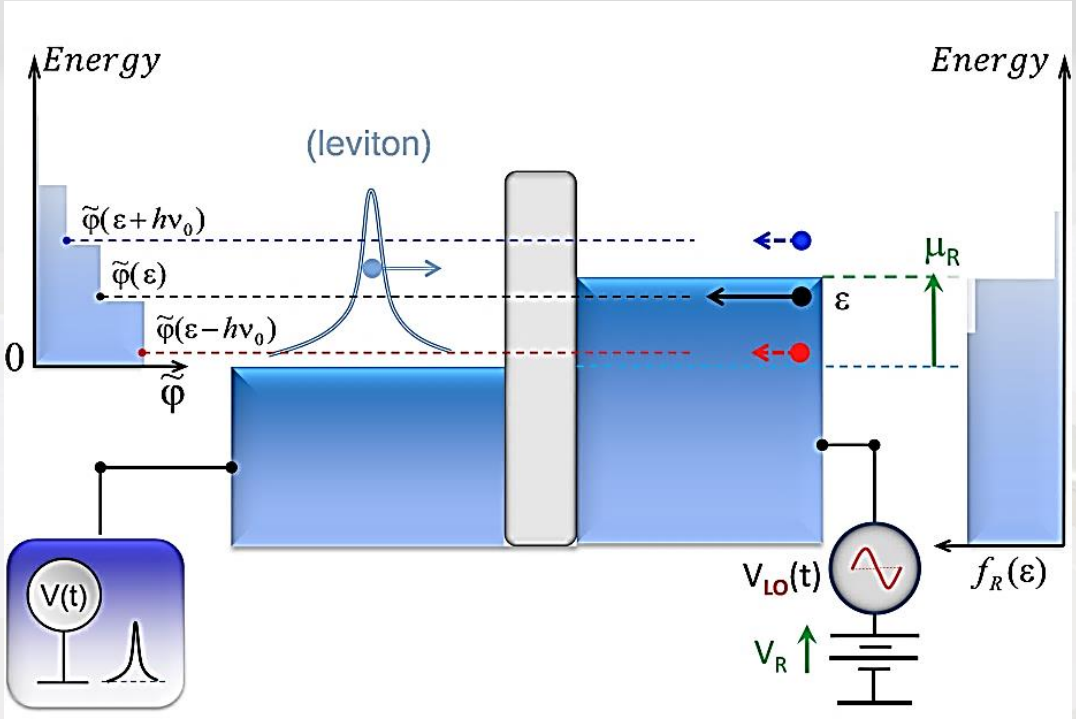
for a **periodic** train of **levitons**

$$\langle \psi^+(\varepsilon')\psi(\varepsilon) \rangle = \sum_k \delta(\varepsilon' - \varepsilon - kh\nu_0)\, \tilde{\varphi}^*(\varepsilon + kh\nu_0)\tilde{\varphi}(\varepsilon)$$

from which one gets de **WIGNER** fct.: $W(\bar{t}, \varepsilon) = \int_{-\infty}^{+\infty} \langle \psi^+(\varepsilon + \delta/2)\psi\ (\varepsilon - \delta/2) \rangle e^{-i\delta\bar{t}/\hbar}\, d\delta$

homodyne quantum state tomography





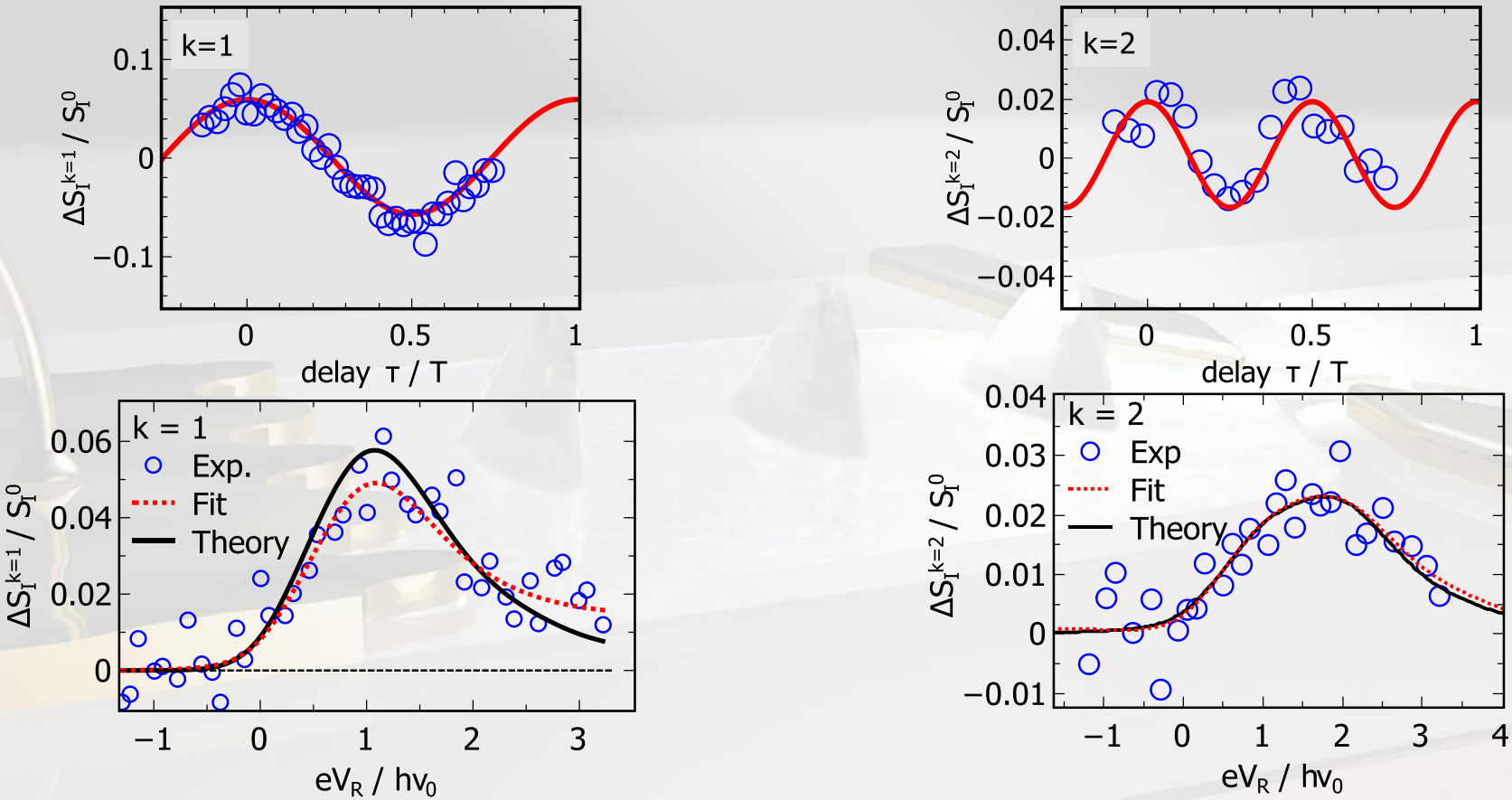
$V(t)$: periodic (ν_0) Lorentzian

$$V_{LO}(t) = eV_{ac}\cos(2\pi k\,\nu_0\,(t-\tau))$$

$$\Delta S_I^{\,k}(eV_R) = S_I^{\,0}\,\frac{eV_{ac}}{2h\nu_0}\cos(2\pi k\,\nu_0\tau)\left(\int_0^{eV_R}\left(\varphi(\varepsilon)\,\varphi(\varepsilon+kh\nu_0)-\varphi(\varepsilon)\,\varphi(\varepsilon-kh\nu_0)\right)d\varepsilon\right)/\,h\nu_0$$

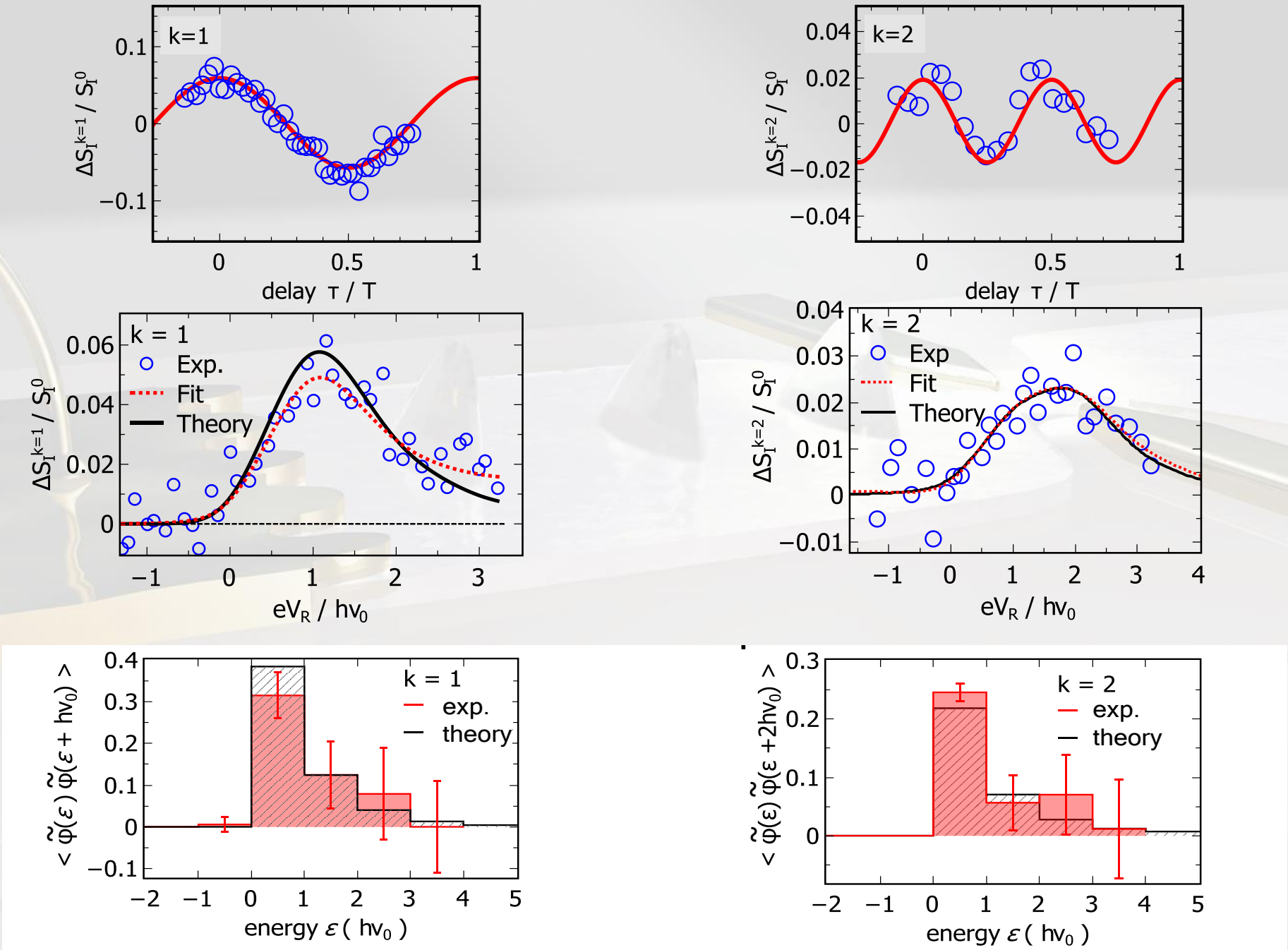
theor. proposal by C. Grenier et al. *New Journal of Physics* **13**, 093007 (2011)

homodyne quantum state tomography



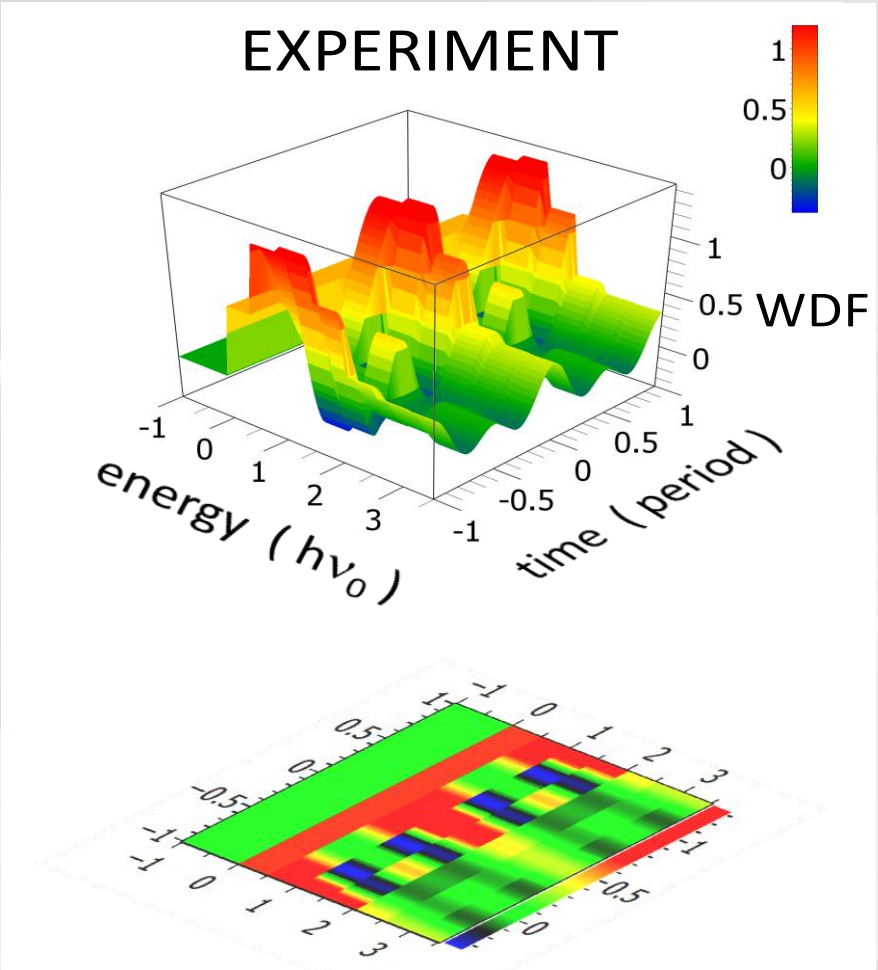
$$\Delta S_I^{\,k}(eV_R) = S_I^{\,0}\,\frac{eV_{ac}}{2h\nu_0}\cos(2\pi k\,\nu_0\tau)\left(\int_0^{eV_R}\left(\varphi(\varepsilon)\,\varphi(\varepsilon+kh\nu_0)-\varphi(\varepsilon)\,\varphi(\varepsilon-kh\nu_0)\right)d\varepsilon\right)/\,h\nu_0$$

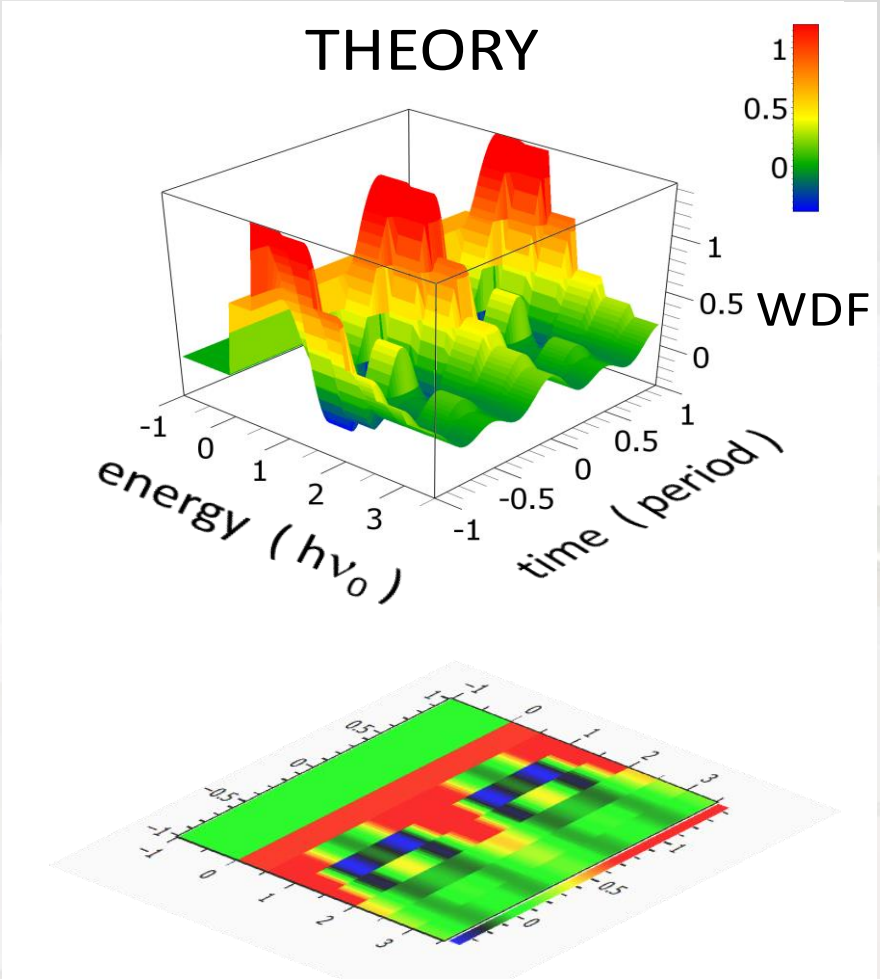
T. Jullien, P. Roulleau, B. Roche and D. C. Glattli **Nature**, 514 603-607 (2014)



T. Jullien, P. Roulleau, B. Roche and D. C. Glattli **Nature**, 514 603-607 (2014)

homodyne quantum state tomography





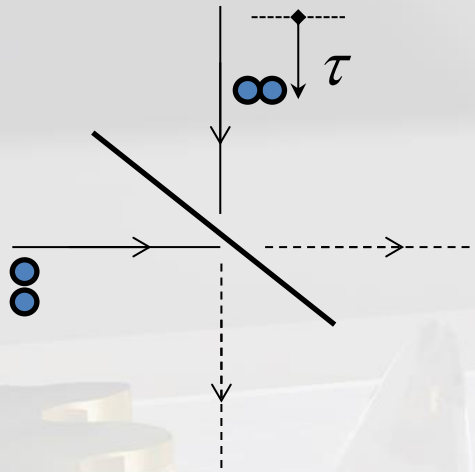
Wigner function
(quasi-probability)

$$W(\bar{t},\varepsilon)=\int_{-\infty}^{+\infty}\left\langle \psi^{+}(\varepsilon+\delta/2)\psi\ (\varepsilon-\delta/2)\right\rangle e^{-i\delta\bar{t}/\hbar}\,d\delta$$

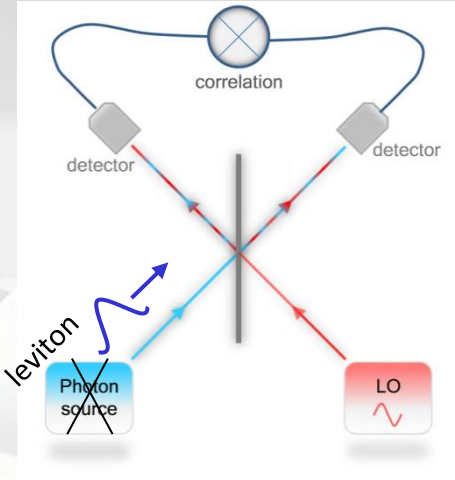
NEGATIVE PARTS (blue) reflect Quantum Coherence)

T. Jullien, P. Roulleau, B. Roche and D. C. Glattli **Nature**, 514 603-607 (2014)

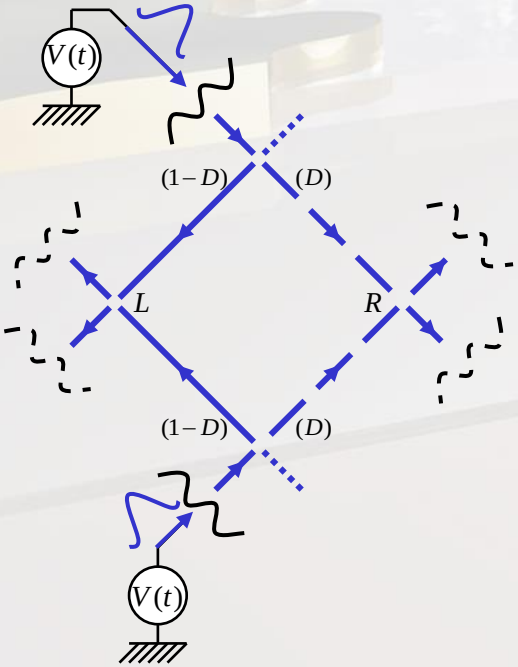
multiple Fermion Hong Ou Mandel



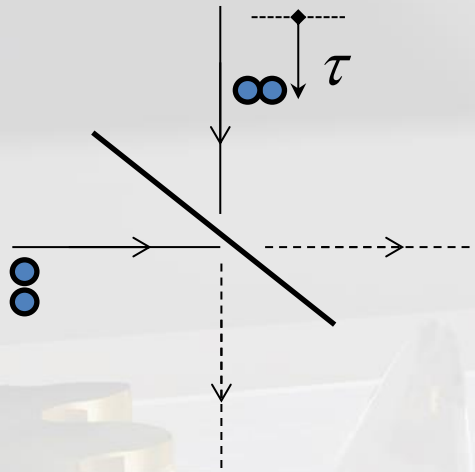
`homodyne' tomography of a leviton



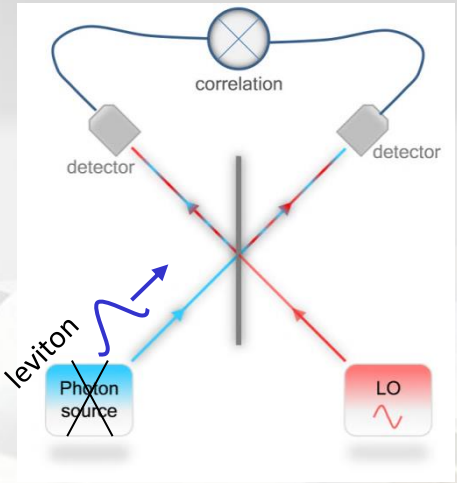
2- leviton Hanbury Brown Twiss



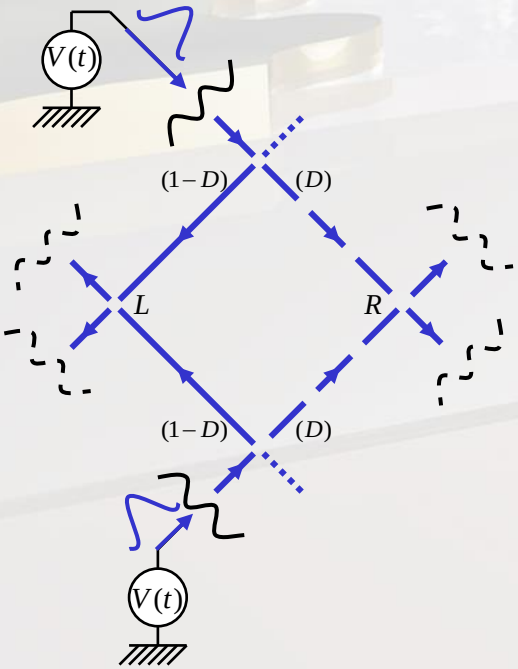
multiple Fermion Hong Ou Mandel



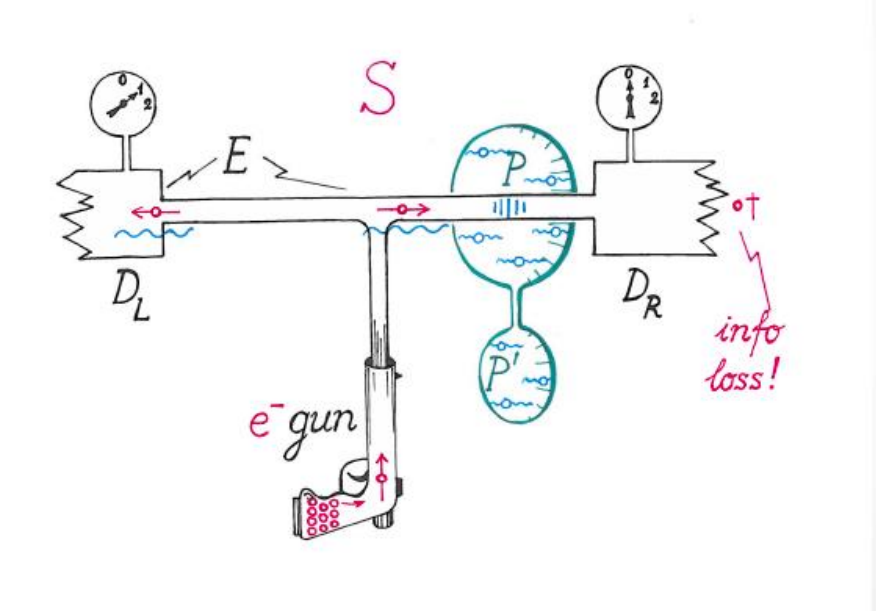
`homodyne' tomography of a leviton



2- leviton Hanbury Brown Twiss



… and much more !



magic properties of the Fermi sea:

- quantized conductance in absence of backscattering : $G = e^2/h$
- continuous single electron source for free : $I = e\,(eV/h)$
- noiseless electron (fermion) transport : $\langle \Delta I^2 \rangle = 0$
- perfect electron anti-bunching
- generation of entangled electron-hole pairs for free
- minimal excitations states (levitons) for single electron injection
- ….
- … it’s worth looking deeper in the Fermi sea.

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Preden Roulleau
Matthieu Santin
Fabien Portier
Patrice Roche
Patrice Jacques
Christian Glattli

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Y. Jin, A. Cavanna
LPN Marcoussis, Fr.

NTT Bas. Res. Lab Japan

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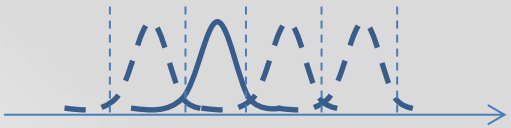
F. Hassler,
ETHZ theory group
L. Levitov (MIT USA)

LPA ENS Paris

Levitons :
J. Dubois et al, Nature 502, 659 (2013)
T. Jullien et al., Nature 514, 603 (2014)

HOM with multiple electrons

single pulse



single pulse n-flux leviton wavefunction

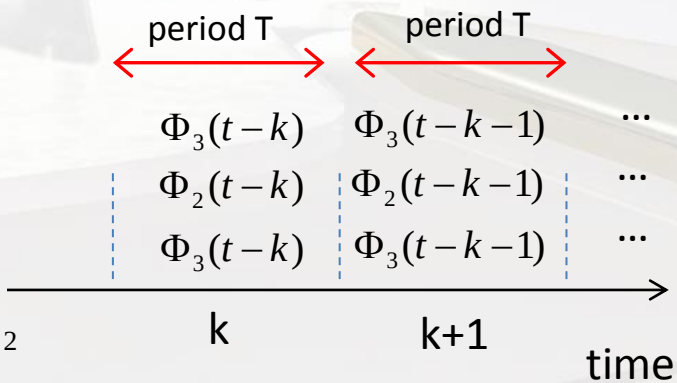
$$\Phi_n(t) = \frac{1}{t-iw}\left(\frac{t+iw}{t-iw}\right)^{n-1}$$

example : 2-electron leviton $\Psi_{2e}(1,2) = \begin{vmatrix} \Phi_1(1) & \Phi_2(1) \\ \Phi_1(2) & \Phi_2(2) \end{vmatrix}$

periodic pulses

periodic n-flux leviton wavefunction

$$\Phi_n(t) = \frac{1}{\sin(\pi(t-iw))}\left(\frac{\sin(\pi(t+iw))}{\sin(\pi(t-iw)}\right)^{n-1}$$



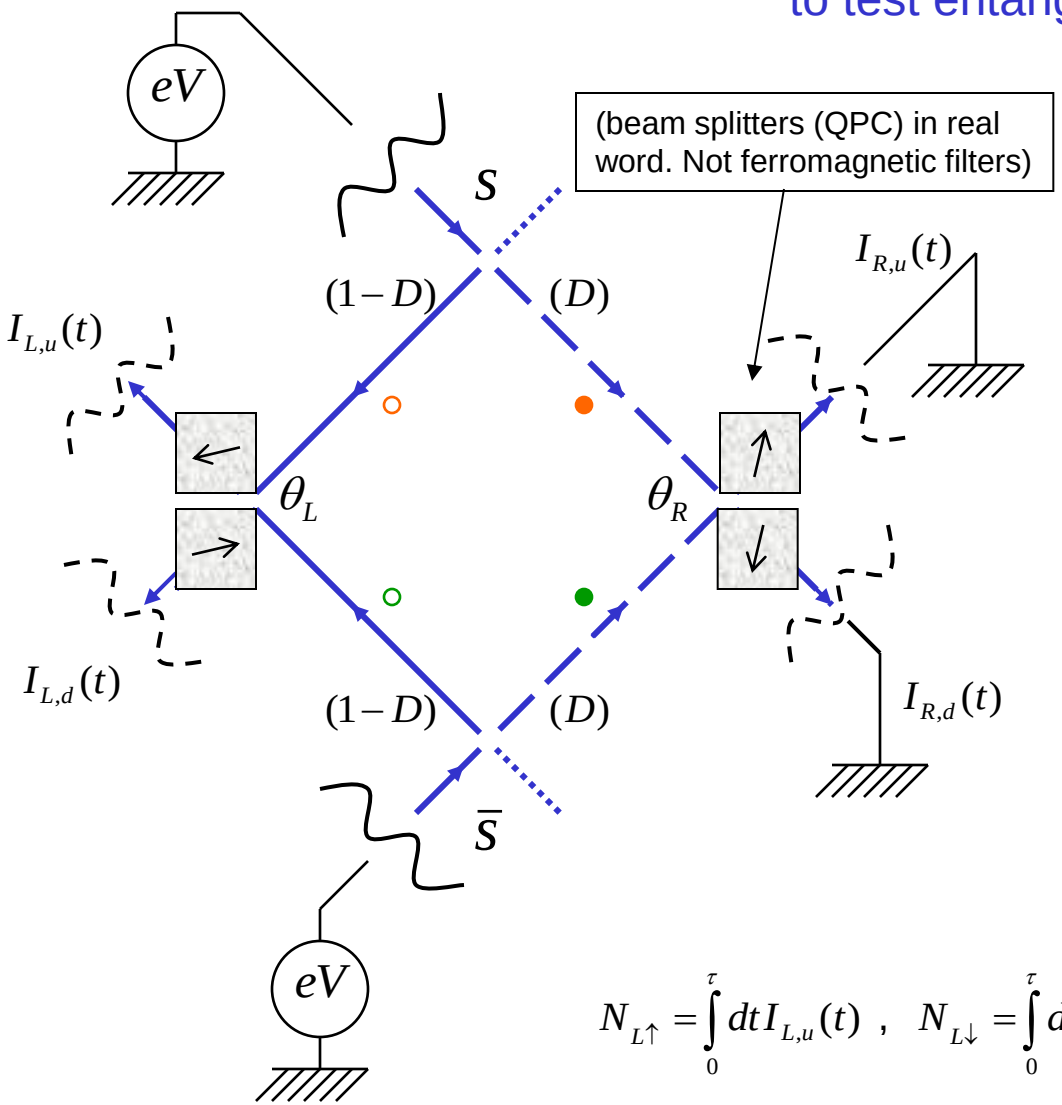
$$S_I^{HOM} / S_I^{0} = 2(1 - \left|\langle \Phi_1(t) | \Phi_1(t+\tau)\rangle\right|^2) = \frac{8\beta^2 \sin(\pi\nu\tau)^2}{1 - 2\beta^2\cos(2\pi\nu\tau) + \beta^2}$$

identifies to conventional
Floquet Shot Noise calculation.

Same for 2-electron HOM noise:

$$S_I^{HOM}(2e) \propto 2 - \left|\langle \Phi_1(0) | \Phi_1(\tau)\rangle\right|^2 - \left|\langle \Phi_2(0) | \Phi_2(\tau)\rangle\right|^2$$

current shot noise cross-correlations to test entanglement



$$s \leftrightarrow \uparrow$$
$$\bar{s} \leftrightarrow \downarrow$$

(pseudo-spin representation)

$$C_{\vec{a}\vec{b}} = \left\langle (\vec{a}.\vec{\sigma})_L \otimes (\vec{b}.\vec{\sigma})_R \right\rangle$$
$$= \frac{\left\langle \left(N_{L\uparrow}(\vec{a}) - N_{L\downarrow}(\vec{a})\right)\left(N_{R\uparrow}(\vec{b}) - N_{R\downarrow}(\vec{b})\right)\right\rangle}{\left\langle \left(N_{L\uparrow}(\vec{a}) + N_{L\downarrow}(\vec{a})\right)\left(N_{R\uparrow}(\vec{b}) + N_{R\downarrow}(\vec{b})\right)\right\rangle}$$

$\vec{a}$ and $\vec{b}$ pseudo-spin polarizer direction

Clauser-Horne-Shimony-Holt form of Bell inequality:

$$\mathrm{B} = \left| C_{\vec{a}\vec{b}} + C_{\vec{a}'\vec{b}} + C_{\vec{a}\vec{b}'} - C_{\vec{a}'\vec{b}'} \right| \leq 2$$

$$N_{L\uparrow} = \int_0^\tau dt I_{L,u}(t) \ , \quad N_{L\downarrow} = \int_0^\tau dt I_{L,d}(t) \ , \quad N_{R\uparrow} = \int_0^\tau dt I_{L,u}(t) \ , \text{and } N_{R\downarrow} = \int_0^\tau dt I_{L,d}(t)$$

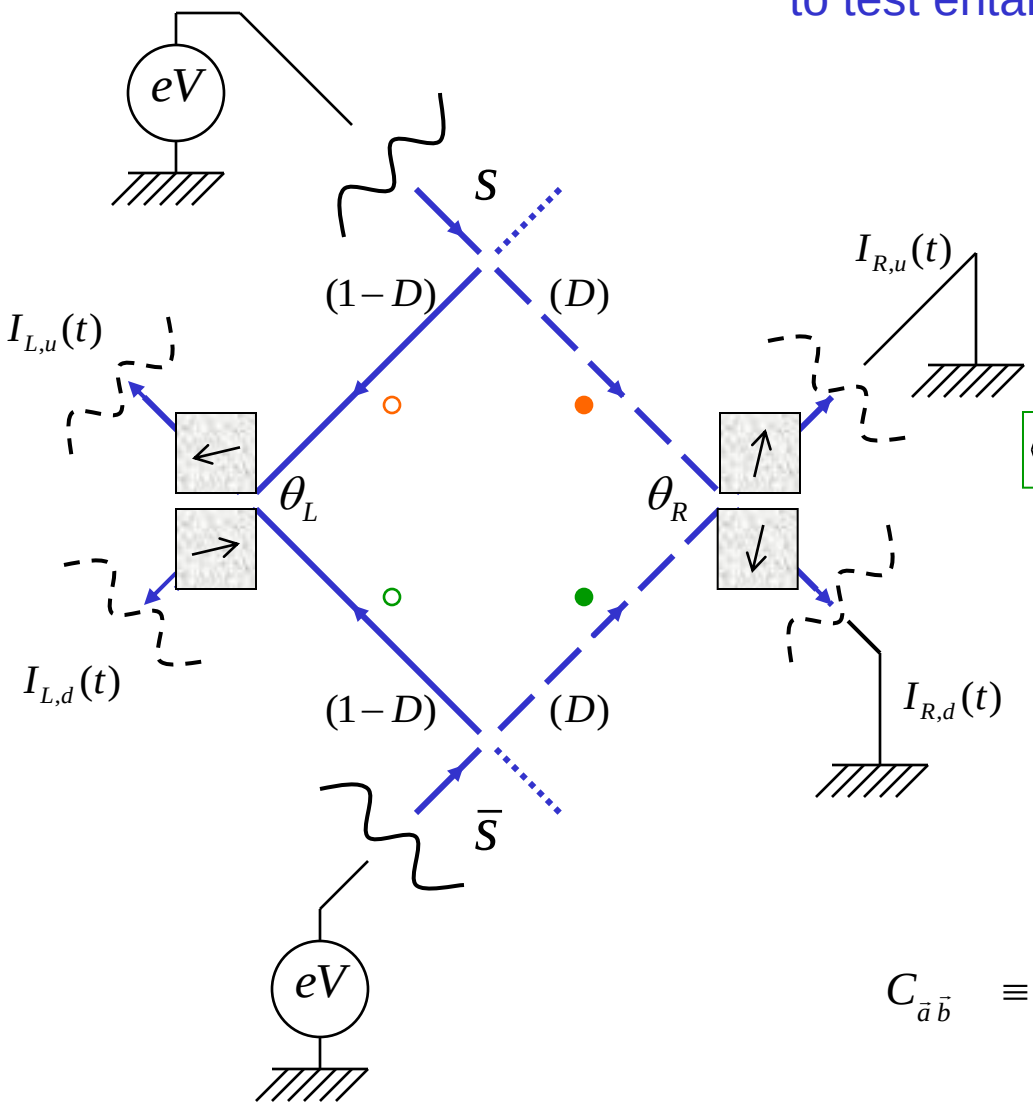
$$\boxed{\left\langle N_{Li} N_{Rj} \right\rangle = S_{Li,Rj}(\omega \approx 0)\ \tau + \left\langle I_{Li} \right\rangle \left\langle I_{Rj} \right\rangle \tau^2 \quad i, j = \uparrow, \downarrow}$$

current noise correlation

average currents

measurement time

current shot noise cross-correlations to test entanglement



$$s \leftrightarrow \uparrow$$
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$$\boxed{\langle N_{Li}\, N_{Rj}\rangle = S_{Li,Rj}(\omega \approx 0)\ \tau + \langle I_{Li}\rangle\langle I_{Rj}\rangle \tau^2 \quad i, j = \uparrow, \downarrow}$$

for $\dfrac{e}{\max(I_\alpha^{e(h)})} > \tau >> \dfrac{h}{eV}$

$$C_{\vec{a}\,\vec{b}} \equiv \hat{E}(\theta_R, \theta_L) = \frac{S_{R\downarrow,L\uparrow} + S_{R\uparrow,L\downarrow} - S_{R\downarrow,L\downarrow} - S_{R\uparrow,L\uparrow}}{\sum_{i,j=\uparrow,\downarrow} S_{Ri,Lj}}$$

$$\boxed{\left|\tilde{E}(\theta_R,\theta_L) - \tilde{E}(\theta_R',\theta_L) + \tilde{E}(\theta_R',\theta_L) + \tilde{E}(\theta_R',\theta_L')\right| \le 2 \quad \text{can be violated : } 2\sqrt{2}}$$