**André Joyal** :

"A crash course in topos theory: the big picture"

The notion of topos was introduced by Grothendieck more than 50 years ago for its applications to algebraic geometry. It has since experienced three important mutations: the first with the notion of elementary topos introduced by Lawvere and Tierney, the second with the theory of classifying toposes and geometric logic by Makkai and Reyes, and the third with the theory of higher toposes by Rezk, Lurie, Toen and Vezzosi.

**Lecture 1**: About half of the topos theory of SGA4 is devoted to categorical generalities.

They are now subsumed by the modern theory of (locally) presentable categories. I will sketch this theory, stressing the results that are important for topos theory. The category of complete lattices and sup-preserving maps is a toy example of this theory.

**Lecture 2**: I will sketch an overall picture of topos theory and of the theory of locales. It includes the notion of sheaf on a site, the notion of forcing topology, of geometric morphism and Giraud's theorem.

A useful principle is that a topos is a commutative ring-like object.

Every topos is a quotient of a free topos, like every commutative ring is a quotient of a polynomial ring.

**Lectures 3:** I will introduce the notion of elementary topos and its logical interpretation. I will discuss the notion of frames internal to a topos. I will describe the equivalence between geometric theories and frames internal to the free topos $S[X]$. I will propose a new approach to classifying toposes.

See the course of Olivia Caramello for the theory of classifying toposes.

**Lectures 4:** This lecture will be a very brief introduction to higher category theory and to higher topos theory.

References:

M. Artin, A. Grothendieck et J.L. Verdier. <Théorie des topos> (SGA 4 Tome 1), Séminaire de Géométrie Algébrique du Bois-Marie (1963–1964).

A. Joyal and M. Tierney. <An extension of the Galois theory of Grothendieck>, AMS Memoir 309 (1984)

P. T. Johnstone, <Sketches of an Elephant: a topos theory compendium>, Vols.1and 2, Oxford University Press (2002).

J. Lurie, <Higher Topos Theory>, Princeton University Press (AM-170) (2009).

S. Mac Lane and I. Moerdijk, <Sheaves in Geometry and Logic>, Springer Universitext (1992).

M. Makkai and G. Reyes, <First-order categorical logic>, Springer Lecture Notes in Mathematics, Vol. 611, Springer-Verlag (1977).

P. Cartier: <Logique, catégories et faisceaux>. Séminaire Bourbaki, exp. 513, (1977-1978)

D.C. Cisinski: <Catégories supérieures et théorie des topos>, Séminaire Bourbaki, exp 1097, (2015)