

# Class forcing and topos theory

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It is well-known that forcing over a model of material set theory corresponds to taking sheaves over a small site (a poset, a complete Boolean algebra, and so on). One phenomenon that occurs is that given a small site, all *new* subsets created are smaller than a fixed bound depending on the size of the site. There is a more general notion of forcing invented by Easton to create new subsets of arbitrarily large sets, namely *class forcing*, where one starts with a partially ordered class. The existing theory of class forcing is entirely classical, with no corresponding intuitionist theory as in ordinary forcing.

Our understanding of its relation to topos theory is in its infancy, but it is clear that class forcing is about taking small sheaves on a large site. That these do not automatically form a topos means that the theory has interesting twists and turns. This talk will outline the theory of class forcing from a category/topos point of view, give examples and constructions, and finally a list of open questions – not least being whether an intuitionistic version of Easton’s theorem on the continuum function holds.