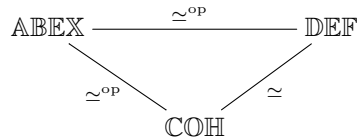# Model-theoretic imaginaries and localisation for additive categories

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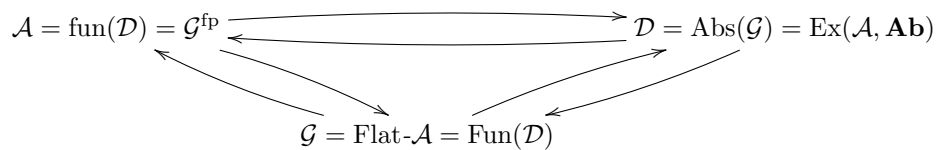Manchester M13 9PL
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October 14, 2015

I will talk about and around the following diagram of additive 2-categories (which is 1.1 of M. Prest, Abelian categories and definable additive categories, arXiv:1202.0426). It shows the setting for the model theory of modules (and other additive structures) and is an additive version of some aspects of the topos-theoretic world.

Theorem 0.1. *There is a diagram of equivalences and anti-equivalences between* $\mathbb{ABEX}$, $\mathbb{COH}$ *and* $\mathbb{DEF}$ *as follows.*

$$
\begin{array}{ccc}
\mathbb{ABEX} & \xrightarrow{\simeq^{\mathrm{op}}} & \mathbb{DEF} \\
& \searrow^{\simeq^{\mathrm{op}}} \quad \swarrow^{\simeq} & \\
& \mathbb{COH} &
\end{array}
$$

Explicitly:

$$
\mathcal{A} = \mathrm{fun}(\mathcal{D}) = \mathcal{G}^{\mathrm{fp}} \rightleftarrows \mathcal{D} = \mathrm{Abs}(\mathcal{G}) = \mathrm{Ex}(\mathcal{A}, \mathbf{Ab})
$$

$$
\mathcal{G} = \mathrm{Flat}\text{-}\mathcal{A} = \mathrm{Fun}(\mathcal{D})
$$

Here $\mathbb{ABEX}$ is the 2-category which has for its objects the skeletally small abelian categories and for its morphisms the exact functors, $\mathbb{DEF}$ is the category of definable additive categories and interpretation functors, and $\mathbb{COH}$ is the category of locally coherent Grothendieck abelian categories and coherent morphisms. In each case the 2-arrows are just the natural transformations.