

# There are categories of ‘spaces’ that are not categories of locales

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## **Abstract**

We described a short list of categorical axioms that make a category behave like the category of locales. In summary the axioms assert that the category has an object that behaves like the Sierpiński space and this object is double exponentiable. A number of the usual results of locale theory can be derived using the axioms: the (weakly) closed subgroup theorem proved, closed and proper surjection are of effective descent, parallel theories of discrete and compact Hausdorff spaces emerge. An example is given of a category that satisfies the axioms but which is not the category of locales for any topos. We show how to embed the category of elementary toposes into the category whose objects are categories that satisfy the axioms.