

# MORITA-EQUIVALENCES FOR MV-ALGEBRAS

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We shall make a survey of the most recent results obtained in connection with the programme of investigating notable categorical equivalences for MV-algebras from a topos-theoretic perspective commenced in [3].

In [3] and [2] we generalize to a topos-theoretic setting two classical equivalences arising in the context of MV-algebras: Mundici's equivalence [4] between the category of MV-algebras and the category of  $\ell$ -u groups (i.e., lattice-ordered abelian groups with strong unit) and Di Nola-Lettieri's equivalence [5] between the category of perfect MV-algebras and the category of  $\ell$ -groups (i.e., lattice-ordered abelian groups, not necessarily with strong unit). These generalizations yield respectively a Morita-equivalence between the theory  $\text{MV}$  of MV-algebras and the theory  $\mathbb{L}_u$  of  $\ell$ -u groups and one between the theory  $\mathbb{P}$  of perfect MV-algebras and the theory  $\mathbb{L}$  of  $\ell$ -groups. These Morita-equivalences allow us to apply the 'bridge technique' of [1] to transfer properties and results from one theory to the other, obtaining new insights on the theories which are not visible by using classical techniques. Among these results, we mention a bijective correspondence between the geometric theory extensions of the theory  $\text{MV}$  and those of the theory  $\mathbb{L}_u$ , a form of completeness and compactness for the infinitary theory  $\mathbb{L}_u$ , the identification of three different levels of bi-interpretability between the theory  $\mathbb{P}$  and the theory  $\mathbb{L}$  and a representation theorem for the finitely presentable objects of Chang's variety as finite products of perfect MV-algebras.

Given the fact that perfect MV-algebras are exactly the local MV-algebras in the variety generated by Chang's algebra, it is natural to wonder whether analogues of Di Nola-Lettieri's equivalence exist for local MV-algebras in a given proper subvariety of MV-algebras. In a forthcoming paper, we prove that the theory of local MV-algebras in any subvariety  $V$  of MV-algebras is of presheaf type (i.e., classified by a presheaf topos) and establish a Morita-equivalence with a theory that extends that of  $\ell$ -groups. Furthermore, we generalize to this setting the representation results obtained in [2].

## REFERENCES

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