Zoran Škoda: Localizations and semiquantals

In the context of noncommutative algebraic geometry, and analogously to their role in topos theory, flat localization functors on abelian categories are sometimes viewed as analogues of Zariski open sets. However in this context they suffer a failure of mutual compatibility: the composition of the corresponding endofunctors is not commutative. Wuth this in mind, the first guess is that a locale of open sets is replaced by a quantale, as the latter comprises a noncommutative product. It is known that only weaker axioms can be satisfied in general. In the case of categories of modules over rings, Simmons exhibited what he calls a *semiquantale*. I extended some of his basic results to a wider class of abelian categories as well as to the *equivariant* setup, partly motivated by unpublished results on equivariant localization and gluing obtained with G. Böhm, some of which will be sketched in the lecture. In a collaboration with P. Resende (in April 2014), I have tried to find how the notions of sheaves on (good) quantales can be extended to the contexts of semiquantales in a way which would fit with the motivating case of localizations; a satisfactory solution seems elusive so far.