

## **Olivia Caramello**

“Introduction to categorical logic, classifying toposes and the 'bridge' technique”

The course will begin by presenting the basic notions and results of first-order categorical logic, with the aim of reaching the theory of classifying toposes by Makkai and Reyes and illustrating the general techniques allowing to use them as unifying 'bridges' for transferring information across distinct mathematical theories. The exposition will be accompanied by several examples and applications. The lectures will require a basic familiarity with the fundamental notions of topos theory, as reviewed in André Joyal's lectures on Monday.

**Lecture 1:** First-order logic and its interpretation in categories. Geometric theories and syntactic categories. Universal models and representability.

**Lecture 2:** Construction of classifying toposes for geometric theories. Duality between the subtoposes of the classifying topos of a geometric theory and the quotients of the theory. Transfer of topos-theoretic notions across the duality and their logical interpretations.

**Lecture 3:** Theories classified by a presheaf topos and their quotients. Finite presentability, irreducible formulae and homogeneous models.

**Lecture 4:** The ‘bridge-building’ technique: Morita-equivalences as ‘decks’ and site characterizations as ‘arches’. Some examples and applications.

### **References:**

O. Caramello, Topos-theoretic background (and the papers cited therein), available at <http://preprints.ihes.fr/2014/M/M-14-27.pdf> (2014), to appear in a forthcoming book for Oxford University Press.

P. T. Johnstone, Sketches of an Elephant: a topos theory compendium, Vols. I and 2, Oxford University Press (2002).

S. Mac Lane and I. Moerdijk, Sheaves in Geometry and Logic, Springer Universitext (1992).

M. Makkai and G. Reyes, First-order categorical logic, Lecture Notes in Mathematics, Vol. 611, Springer-Verlag (1977).