Soutenance d’Habilitation à Diriger des Recherches

Topological defects and other properties of multicomponent superconductors

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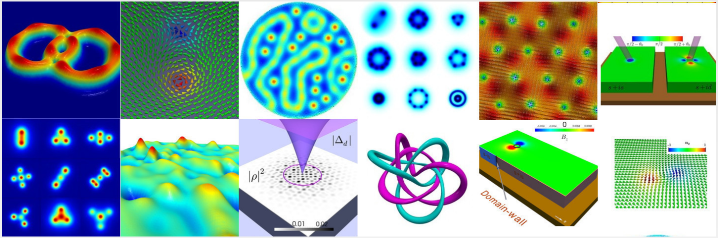


Outline

Based on works from 2011 – 2021 with

E. Babaev, J. Carlström, M. Speight, M. Silaev, D. Agterberg, J. Jäykka, K. Sellin, A. Zyuzin, A. Corticelli, F. Rybakov

Outline

Topological defects and vortices

Topological defects are ubiquitous in modern physics

superfluid, superconductors, cold atoms BEC, (chiral) magnets, ferroelectric, (liquid) crystals, spin ices,. . .
also in models of early universe cosmology, high-energy, . . .

e.g. dislocations, monopoles, domain-walls, skyrmions, . . .

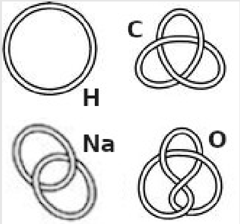
(quantum) vortices (superfluid, superconductors)

- [Onsager 1949; Feynman 1955]: circulation of the superflow is quantized; [Abrikosov 1957]: magnetic vortices in SC
- [Onsager 1949; Peskin 1978; Dasgupta, Halperin 1981]: phase transitions: thermal proliferation of vortex loops
- [Berezinskiǐ 1971; Kosterlitz, Thouless 1972]: in 2d (V/AV)
- Overall, vortices control the thermal, rotational/magnetic responses of superfluids and superconductors

[Kelvin 1867]: atoms consist in topologically inequivalent linked and knotted loops of vortices in the luminferous aether

Vortex [Helmholtz 1858]

Vortex atom



Superconductivity – Generalities

Conventional mechanism [Bardeen, Cooper, Schrieffer 1957]

- in a metal Fermi sphere of occupied states
- states near Fermi surface can interact via phonons

electron-phonon interaction scattering mediates attraction

- e^- moves in a potential and excites a phonon
- later absorbed by another e^-
- small attraction between electrons causes (bound) paired state (Cooper pair) with opposite momenta

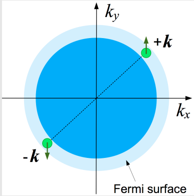
Cooper pairs are bosons

- they can undergo Bose-Einstein condensation
- macroscopic occupation of the zero momenta states

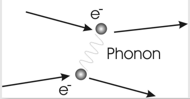
At mean field, one single macroscopic wave function

Ginzburg-Landau: effective classical mean field theory near T_c

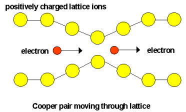
Fermi sphere



Electron-phonon



Cooper pairs



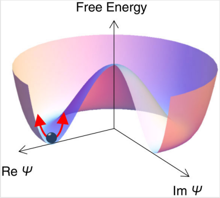
Superconductivity – Properties & Ginzburg-Landau

$$E = \int_{\mathbb{R}^3} |\nabla \times \boldsymbol{A}|^2 + D_\mu \Psi^* D^\mu \Psi + \kappa(|\Psi|^2 - 1)^2\,, \text{ with } D_\mu = \nabla_\mu - iA_\mu$$

Spontaneous symmetry breaking

- at the mean field level, one macroscopic wave function (density of Cooper pairs), the gap function $\Psi = |\Psi| e^{i\varphi}$
- Ψ: charged bosonic scalar field; $\boldsymbol{A}$ gauge field (photon)
- longitudinal component of the photon becomes massive
- Anderson-Higgs mechanism [Anderson 1962; Higgs 1964]

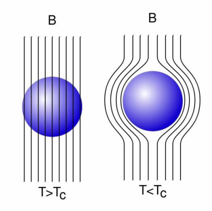
Broken $U(1)$



Properties of superconductors

- dissipationless current (no viscosity for BEC)
- perfect diamagnetism (Meissner effect): $\boldsymbol{B}$ is screened by the superflow of Cooper pairs $\boldsymbol{J} = 2e|\Psi|^2(\nabla\varphi + \boldsymbol{A})$
- Massive photon $\Rightarrow$ London eq.: $\lambda \nabla \times \nabla \times \boldsymbol{B} = \boldsymbol{B}$ (Proca)
- Quantized flux $\Phi = \frac{\Phi_0}{2\pi} \oint \nabla\varphi \, \boldsymbol{d}\ell = n\Phi_0$ and $n \in \pi_1(S^1) = \mathbb{Z}$
- $\Rightarrow$ topo. defects (vortices) [Onsager 1949; Abrikosov 1957]

Meissner effect



Properties of superconducting vortices and loops

Classical field theory: Ginzburg-Landau/Abelian-Higgs

$$E = \int_{\mathbb{R}^3} |\nabla\times\boldsymbol{A}|^2 + D_\mu\Psi^* D^\mu\Psi + \kappa(|\Psi|^2-1)^2\,,\ \text{with } D_\mu = \nabla_\mu - iA_\mu$$

- Ψ: complex (bosonic) scalar field, $\boldsymbol{A}$: gauge field
- $U(1)$ symmetry spontaneously broken by vacuum $|\Psi|=1$
- vacuum manifold is $\sim S^1$ $\qquad \Psi \propto e^{in\varphi}$

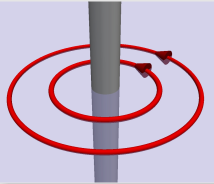
Straight infinite vortex solution — in 2d, $\Psi : S^1 \to S^1$

$E<\infty$ field configurations are classified by $n\in\pi_1(S^1)=\mathbb{Z}$

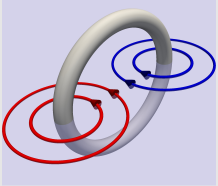
Instability of vortex loops — in 3d, $\Psi : S^2 \to S^1$

- topologically trivial ($\pi_2(S^1)=1$)
- dynamical scaling instability (Derrick) (loops collapse)

Straight ∞ vortex



Vortex loop



What about more general models of superconductivity?

Multicomponent superconductors

Many recently discovered materials are multi-band

- pairing on several sheets of a Fermi Surface; Cooper pairs can tunnel btw FS [Suhl, Matthias, Walker 1959; Moskalenko 1959]
- the gap function cannot be reduced to a single complex field thus $\Psi \to \Psi = (\Psi_1, \Psi_2, \cdots) \in \mathbb{C}^N$

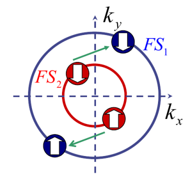
intercomponent interactions open many new physical phenomena

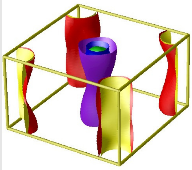
Multicomponent Ginzburg-Landau $\Psi = (\Psi_1, \Psi_2, \cdots) \in \mathbb{C}^N$

$$E = \int_{\mathbb{R}^3} |\boldsymbol{\nabla} \times \boldsymbol{A}|^2 + \sum_a |\boldsymbol{D}\Psi_a|^2 + V\left[\Psi^\dagger, \Psi\right], \text{ with } \boldsymbol{D} = \boldsymbol{\nabla} + ie\boldsymbol{A}$$

- potential $V\left[\Psi^\dagger, \Psi\right]$, such that the ground state is $\Psi^\dagger\Psi \neq 0$
- in SC, V typically contains explicitly breaks symmetries terms

Fermi surfaces





Multicomponent theories are relevant to many systems beyond the solid state

e.g. mixtures of charged condensates (LMH, LMD); metallic superfluids; nuclear SC in neutron stars; superfluid He; spinor condensates; Weinberg-Salam theory; ...

Outline

Ginzburg-Landau theory for multicomponent superconductors

Ginzburg-Landau free energy

$\Psi = (\psi_1, \psi_2, \cdots) \in \mathbb{C}^N$ and $\psi_a = |\psi_a| e^{i\varphi_a}$

$$\mathcal{F}/\mathcal{F}_0 = \int \frac{1}{2}|\nabla\times\boldsymbol{A}|^2 + \sum_a \frac{1}{2}|\boldsymbol{D}\psi_a|^2 + V(\Psi,\Psi^\dagger)\,, \quad \text{and} \quad \boldsymbol{D} \equiv \nabla + ie\boldsymbol{A}$$

$$\text{and} \quad V(\Psi,\Psi^\dagger) = \alpha_{ab}\psi_a^*\psi_b + \beta_{abcd}\psi_a^*\psi_b^*\psi_c\psi_d \quad \in \mathbb{R}$$

Ginzburg-Landau and Ampère-Maxwell equations

$\boldsymbol{B} = \nabla\times\boldsymbol{A}$

$$\boldsymbol{D}\boldsymbol{D}\psi_a = 2\delta V/\delta\psi_a^* \quad \text{and} \quad \nabla\times\boldsymbol{B} + \boldsymbol{J} = 0\,, \quad \text{with} \quad \boldsymbol{J} = e\,\mathrm{Im}(\Psi^\dagger\boldsymbol{D}\Psi)$$

Meissner supercurrent

$\boldsymbol{J} := \delta\mathcal{F}/\delta\boldsymbol{A} = e\,\mathrm{Im}(\Psi^\dagger\boldsymbol{D}\Psi)$

$$\boldsymbol{J} = e^2\varrho^2\boldsymbol{A} + e\sum_a |\psi_a|^2\nabla\varphi_a\,, \quad \text{with} \quad \varrho^2 = \sum_a |\psi_a|^2$$

Separation in charged and neutral modes

$\varphi_{ab} = \varphi_b - \varphi_a$

$$\mathcal{F}/\mathcal{F}_0 = \int \frac{\boldsymbol{B}^2}{2} + \sum_a \frac{1}{2}(\nabla|\psi_a|)^2 + \frac{\boldsymbol{J}^2}{2e^2\varrho^2} + \sum_{a,b>a} \frac{|\psi_a|^2|\psi_b|^2}{2\varrho^2}(\nabla\varphi_{ab})^2 + V(\Psi,\Psi^\dagger)\,.$$

Flux quantization and fractional vorticity

Flux quantization – Fractional vortices (at ∞, $J \to 0$)

$$\Phi = \int_{\mathbb{R}^2} \boldsymbol{B} d\boldsymbol{S} = \oint_\infty \boldsymbol{A} d\boldsymbol{\ell} = \oint_\infty \frac{\boldsymbol{J}}{e^2\varrho^2} - \sum_a \frac{|\psi_a|^2}{e\varrho^2}\boldsymbol{\nabla}\varphi_a$$

$$= \frac{-\Phi_0 \sum_a |\Psi_a|^2}{2\pi\varrho^2} \oint_\infty \boldsymbol{\nabla}\varphi_a \cdot d\boldsymbol{\ell} \quad \text{with} \quad \oint_\infty \boldsymbol{\nabla}\varphi_a = 2\pi n_a \quad \text{and} \quad \Phi_0 = \frac{2\pi}{e}$$

- thus *e.g.* $(n_1, n_2) = (1, 0)$ carries a fraction $|\psi_1|^2/(|\psi_1|^2 + |\psi_2|^2)$ of Φ_0

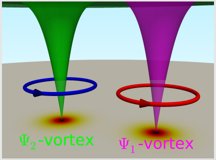
$\Rightarrow$ elementary topological excitations are fractional vortices in different components

if only $n_1 \neq 0$, the neutral sector has a contribution

$$\mathcal{F} = \int \cdots + \frac{\sum_{b\neq 1} |\psi_1|^2|\psi_b|^2}{e^2\varrho^2}(\boldsymbol{\nabla}\varphi_{1b})^2 + \cdots$$

- but $\int^R rdr \left(\frac{2\pi}{r}\right)^2 \to$ divergent energy $\sim \log R$
- only the configurations with all $n_b = n_1$ have finite energy

Composite vorticity

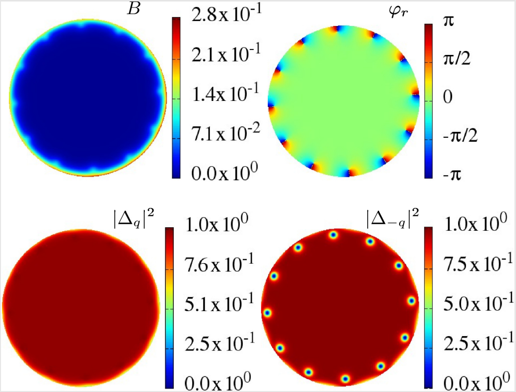


Composite vortex: either singular (singularities overlap) or coreless (they do not)

Fractional vortices

Bulk fractional vortices have divergent energy per unit length

Yet they can form due to finite-size effects, in mesoscopic samples



see *e.g.* [Chibotaru, Dao 2007; Silaev 2011; Agterberg, Babaev, **JG** 2014;. . .]

Extra topological properties in multicomponent superconductors

Fractional vortices are characterized by with the winding $\oint \nabla\varphi_a = 2\pi n_a$

elems. of the 1st homotopy of the circle: $n \in \pi_1(\mathbb{S}^1) = \mathbb{Z}$ [maps $\mathbb{S}^1 \to \mathbb{S}^1$]

Multicomponent SC are characterized by additional topological invariants

In 2d: $\mathbb{CP}^{N-1}$ topological invariants **[JG, Carlstrom, Babaev, Speight 2013]**

Whenever $\Psi \neq 0$, *i.e.* if singularities do not overlap, the index $\mathcal{Q}(\Psi) \in \mathbb{Z}$

$$\mathcal{Q}(\Psi) = \int_{\mathbb{R}^2} \frac{i\varepsilon_{ji}}{2\pi|\Psi|^4}\left[|\Psi|^2\partial_i\Psi^\dagger\partial_j\Psi + \Psi^\dagger\partial_i\Psi\partial_j\Psi^\dagger\Psi\right]dxdy \quad \text{and} \quad \int \boldsymbol{B} = \Phi_0\mathcal{Q}(\Psi)$$

In 3d, for two-components: the Hopf invariant **$\Psi \neq 0 \Leftrightarrow$ Coreless**

- SC *d.o.f.* are cast in a 4d unit vector $\boldsymbol{\zeta} = (\mathrm{Re}\,\psi_1, \mathrm{Im}\,\psi_1, \mathrm{Re}\,\psi_2, \mathrm{Im}\,\psi_2)/\sqrt{\Psi^\dagger\Psi}$
- maps from the compactified space to the target $\mathbb{S}^3$, $\boldsymbol{\zeta} : \mathbb{S}^3\,[\cong \mathbb{R}^3 \cup \{\infty\}] \mapsto \mathbb{S}^3_\Psi$
- since $\pi_3(\mathbb{S}^3_\Psi) = \mathbb{Z}$, these maps are characterized the degree of the map

$$\mathcal{I} := \deg\boldsymbol{\zeta} = \frac{-1}{12\pi^2}\int_{\mathbb{R}^3} \varepsilon_{ijk}\varepsilon_{abcd}\,\zeta_a\,\partial_i\zeta_b\,\partial_j\zeta_c\,\partial_k\zeta_d\,d\boldsymbol{r} \quad \in \mathbb{Z}$$

$\mathcal{I}$ = Hopf charge of the combined Hopf map $h \circ \boldsymbol{\zeta} : \mathbb{S}^3 \to \mathbb{S}^2$ as long as $\Psi \neq 0$

Mapping to the Faddeev-Skyrme model (for two-components)

A pseudo-spin unit vector $\boldsymbol{n} \in \mathbb{S}^2$ can be defined as the projection $\boldsymbol{n} = \frac{\Psi^\dagger \boldsymbol{\sigma} \Psi}{\Psi^\dagger \Psi}$

And the free energy can be mapped onto a Faddeev-Skyrme model coupled to

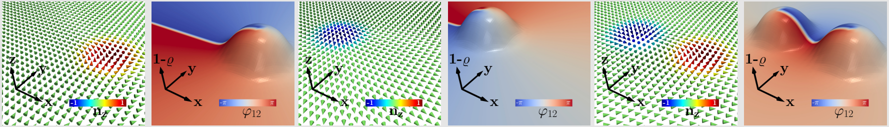
$$\mathcal{F} = \int \frac{\boldsymbol{B}^2}{2} + \frac{1}{2}(\boldsymbol{\nabla}\varrho)^2 + \frac{\varrho^2}{8}\partial_k n_a \partial_k n_a + \frac{\boldsymbol{J}^2}{2e^2\varrho^2} + V(\varrho, \boldsymbol{n})$$

where the magnetic field is $B_k = \varepsilon_{ijk}\left(\partial_i\left(\frac{J_j}{e^2\varrho^2}\right) - \frac{1}{4e}\varepsilon_{abc} n_a \partial_i n_b \partial_j n_c\right)$

Since $\boldsymbol{n} : \mathbb{S}^2\, [\cong \mathbb{R}^3 \cup \{\infty\}] \mapsto \mathbb{S}^2_{\boldsymbol{n}}$ it is classified by $\pi_2(\mathbb{S}^2_{\boldsymbol{n}})$

$$\mathcal{Q}(\boldsymbol{n}) = \frac{1}{4\pi}\int_{\mathbb{R}^2} \boldsymbol{n} \cdot \partial_x \boldsymbol{n} \times \partial_y \boldsymbol{n}\, dxdy \in \mathbb{Z} \quad \text{as long as } \Psi \neq 0$$

There, fractional vortices $\mapsto$ merons, and composite vortices $\mapsto$ skyrmions



Interaction between fractional vortices

There are new topological properties when fractional vortices do not overlap

In the London limit ($|\psi_a|$=const); R is a cut-off; λ is the London penetration depth

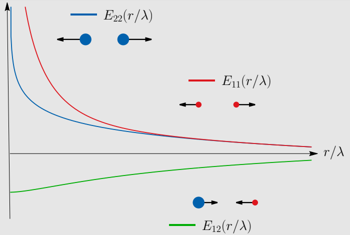
The interaction between fractional vortices is

$$\frac{E^{(int)}_{ab}}{2\pi} = \frac{|\psi_a|^2|\psi_b|^2}{\varrho^2}\left(\ln\frac{r}{R} + K_0\left(\frac{r}{\lambda}\right)\right)$$

$$\frac{E^{(int)}_{aa}}{2\pi} = -\frac{|\psi_a|^2\sum_{c\neq a}|\psi_c|^2}{\varrho^2}\ln\frac{r}{R} + \frac{|\psi_a|^4}{\varrho^2}K_0\left(\frac{r}{\lambda}\right)$$

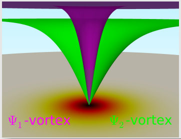
- charged sector mediates repulsion
- neutral mode is attractive if $a \neq b$

⇒ Favours co-centred vortices



London limit: bulk fractional vortices are log/linearly confined

- Is it possible to have skyrmions or coreless vortices? ⇒ Yes
- beyond the London limit because of some potential terms
- or modify the simplest multicomponent Ginzburg-Landau



Scenarii that promote coreless (composite) vortices

Why should we bother about skyrmions? [it's nice :)]

- they are observable and behave differently than usual vortices
- they can reveal the presence of fractional vortices

Skyrmions can be stabilized by various mechanisms

- mixtures of commensurately charged condensates (*e.g.* LMD) [**JG**, Babaev 2017]
- dissipationless drag: Andreev-Bashkin effect [**JG**, *et al.* 2014; 2019] (see next)
- interaction with domain-walls $\Rightarrow$ chiral $\mathbb{C}\mathrm{P}^2$ skyrmions [**JG**, *et al.* 2013] (see next)

Also by condensate repulsion (*i.e.* beyond the London limit)

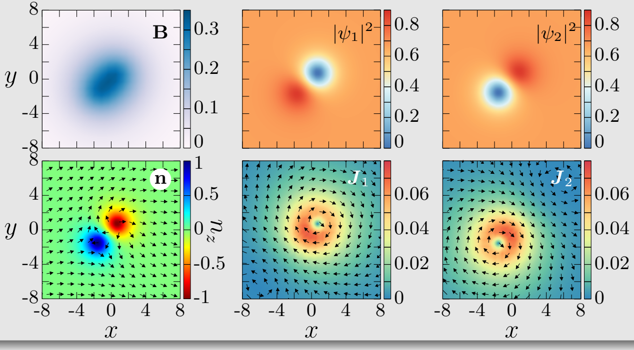
- interface superconductors [Agterberg, Babaev, **JG** 2014]
- immiscible mixtures of condensates [**JG**, Babaev 2014; 2015]
- nematic superconductors with odd-parity pairing [Zyuzin, Babaev, **JG** 2017]
- chiral $p+ip$ superconductors [**JG**, Babaev 2012; 2015] also [**JG**, *et al.* 2016; 2021]

Skyrmions can be stabilized by condensate repulsion

Numerical construction: Finite Elements + Non Linear Conjugate Gradients

- choose an initial configuration with desired topological property
- minimize – topological barriers prevent the change of topological sectors

Core-splitting by condensate repulsion: $\mathcal{F} = \int \cdots + \sum_{a,b} \beta_{ab}|\psi_a|^2|\psi_b|^2 + \cdots$



Skyrmions and hopfions stabilized by dissipationless drag (1/2)

[Andreev, Bashkin 1975] effect in multicomponent superfluids

- originates from the interactions between two superfluids, (forms quasi-particles with nonzero content of either ψ_a)
- The flow of one component is accompanied by mass transport of the other component: $\boldsymbol{j}_1 = \rho_{11}\boldsymbol{v}_1 + \rho_{12}\boldsymbol{v}_2$
  $\Rightarrow$ superflow $\boldsymbol{j}_1$ is partially induced by superfluid velocity $\boldsymbol{v}_2$

Ginzburg-Landau theory with Andreev-Bashkin term

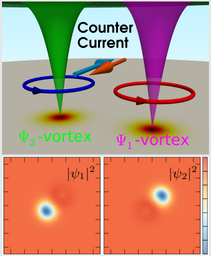
$$\mathcal{F} = \int \cdots + \sum_{a,b} \mu_{ab} \boldsymbol{J}_a \cdot \boldsymbol{J}_b + \cdots$$

$\mu_{ab} > 0$ favours counterflow and penalizes co-flow

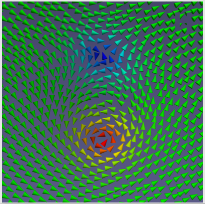
AB-term favours conterflows thus induces core-"splitting"

- the London limit is modified [JG, Sellin, Jäykkä, Babaev 2013]
- skyrmions exist and form unusual lattices that differ from usual vortices $\Psi \neq 0 \Rightarrow \mathcal{Q}(\Psi) \in \mathbb{Z}$

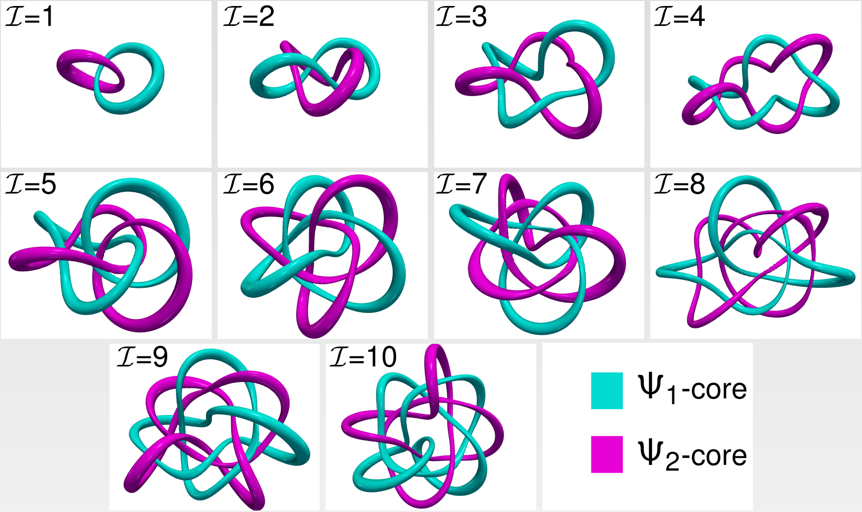
Core-splitting



Texture $\boldsymbol{n} = \Psi^\dagger \boldsymbol{\sigma} \Psi$



Skyrmions and hopfions stabilized by dissipationless drag (2/2)



Outline

States that Break the Time-Reversal Symmetry (BTRS)

Several materials break the time-reversal symmetry $p+ip$, $s+is$, $s+id$, ...

Observation of spontaneous bulk magnetic field below T_c, via muon spin relaxation (μSR) or/and the Kerr effect Sr_2RuO_4, UPt_3, $Ba_{1-x}K_xFe_2As_2$, ...

The time-reversal symmetry is broken if $\mathcal{T}(\Psi_0) \neq \Psi_0 e^{i\chi}\ \forall\chi$ $\mathcal{T}(\Psi_0) \equiv \Psi_0^*$

BTRS due to competing of phase-locking terms ($\Psi_0 = \operatorname{argmin} V$)

e.g. if $V(\Psi) = \cdots + \sum_{b>a}^{3} \eta_{ab}|\psi_a||\psi_b|\cos\varphi_{ab} + \ldots$ and $\eta_{ab} > 0$

- each Josephson term anti-locks the phases ($\varphi_{ab} = \pi$)
- they cannot be simultaneously satisfied $\Rightarrow$ frustration.

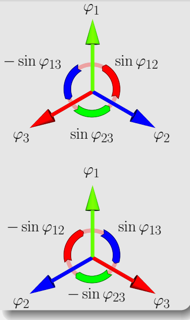
A simple example if $\eta_{ab} = 1$ φ_{ab} is neither 0 nor π

$(\varphi_{12}, \varphi_{13})$ is either $(2\pi/3, -2\pi/3)$ or $(-2\pi/3, 2\pi/3)$

The potential $V(\Psi, \Psi^\dagger)$ is invariant under $\varphi_{ab} \to -\varphi_{ab}$

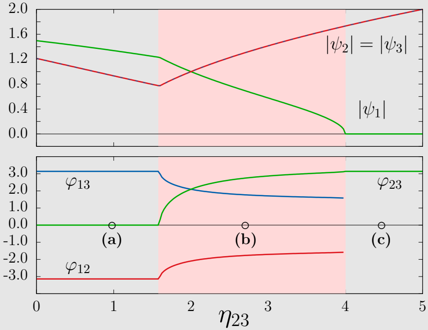
there is an additional discrete $\mathbb{Z}_2$ degeneracy of the ground state

Phase-lockings

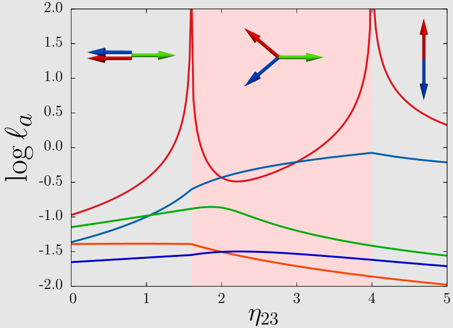


Phase diagram for the $s+is$ state [JG, *et al.* 2011; 2014; 2017; 2018]

Ground-state densities and phases



length-scales (mass spectrum)



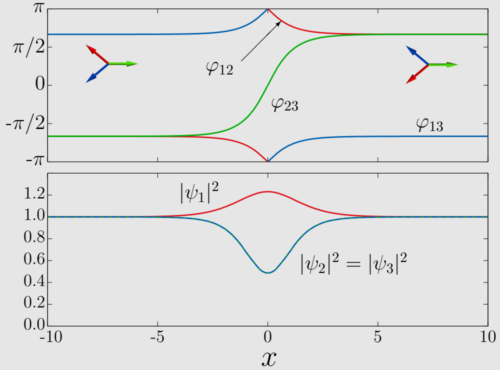
the time-reversal symmetry breaking transition usually occurs at $T_c^{\mathbb{Z}_2} < T_c$

- typically a dome of BTRS state inside the superconducting state
- the BTRS transition is associated with a divergent length-scale (2nd order)

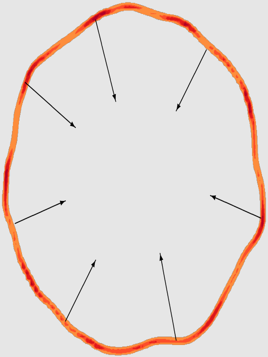
Domain-walls in the BTRS $s+is$

[JG, Babaev 2014]

in 1d domain-wall is topologically stable



in 2d, a closed DW collapses

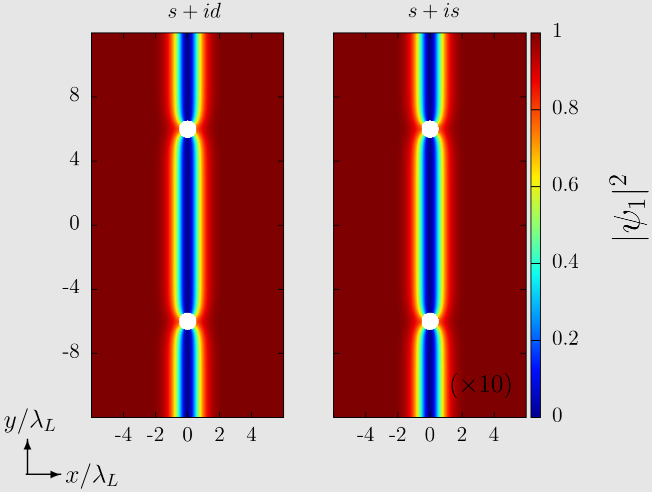


Domain-walls are produced by Kibble-Zurek mechanism during the transition

- Domain-walls can be pinned by impurities or non-convex geometries
- Domain-walls react to thermal gradients

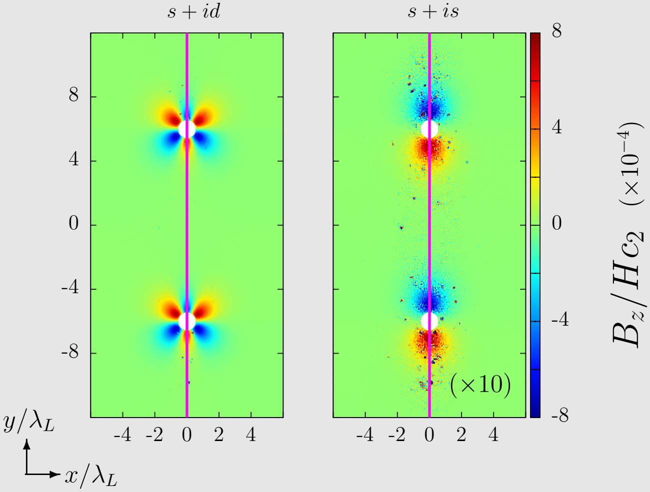
Domain-walls can be pinned *e.g.* by impurities [JG, Babaev 2014]

they exhibit magnetic fields because of the modified electrodynamics



Domain-walls can be pinned *e.g.* by impurities [JG, Babaev 2014]

they exhibit magnetic fields because of the modified electrodynamics



Chiral $\mathbb{C}P^2$ skyrmions: Domain-walls and vortices [JG, *et al.* 2011; 2013]

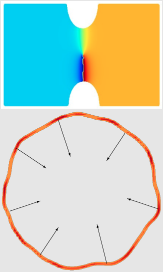
Bulk fractional vortices are linearly confined

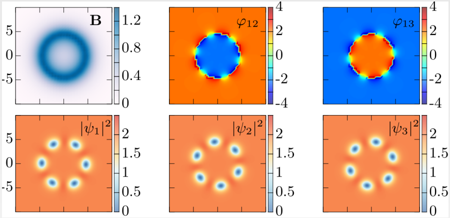
On a domain-wall fractional vortices are linearly deconfined

- field gradients suppress the superfluid density at domain-walls
  $\Rightarrow$ vortices are confined on the domain-wall
- domain-wall has energetically unfavourable relative phase φ_{ab}
  $\Rightarrow$ integer vortices are split and repel on the domain-wall
- closed domain-walls shrink due to the line tension

Composite fractional vortices and closed domain-walls are stable

DW trap FV

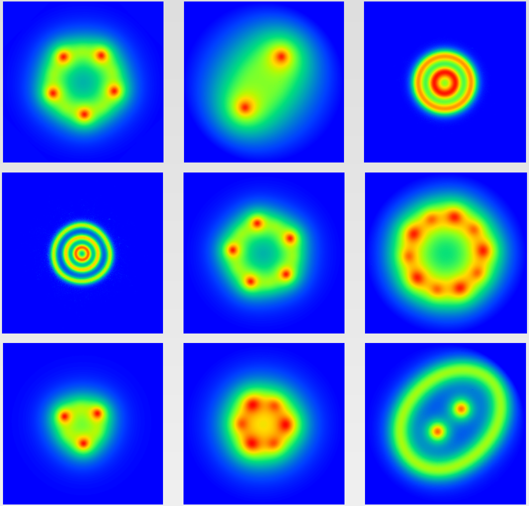




Fractional vortices do not overlap

- since $\Psi \neq 0$ then $\mathcal{Q}(\Psi) \in \mathbb{Z}$
- ***B*** is distributed along the domain-wall
- chiral $\mathbb{C}P^2$ skyrmions carry quantized flux, and can be discriminated from singular vortices

Chiral $\mathbb{CP}^2$ skyrmions have very exotic profile of *B* [JG, *et al.* 2011; 2013]



Electrodynamics of multicomponent superconductors

Current in multicomponent systems reads as

$$\boldsymbol{J} = e\varrho^2 \boldsymbol{Q}_\Sigma + \frac{e}{N} \sum_{a,\, b>a} \left(|\psi_b|^2 - |\psi_a|^2\right) \boldsymbol{\nabla}\varphi_{ab}$$

$\varphi_{ab} = \varphi_b - \varphi_a$: inter-components relative phase

Condensate's *com* momentum

$$\boldsymbol{Q}_\Sigma = \boldsymbol{\nabla}\varphi_\Sigma - e\boldsymbol{A};\ \varphi_\Sigma = \sum_a \varphi_a / N$$

Gauge invariant potential Φ

$$\Phi = A_0 - \partial_t \varphi_\Sigma$$

Extra contribution to London's electrodynamics

$$\boldsymbol{B} = \boldsymbol{\nabla}\times\left(\frac{\boldsymbol{J}}{e^2\varrho^2}\right) - \boldsymbol{\nabla}\times\boldsymbol{\nabla}\varphi_\Sigma + \sum_{a,\, b>a} \boldsymbol{\nabla}\times\left(\frac{|\psi_a|^2 - |\psi_b|^2}{Ne\varrho^2}\boldsymbol{\nabla}\varphi_{ab}\right)$$

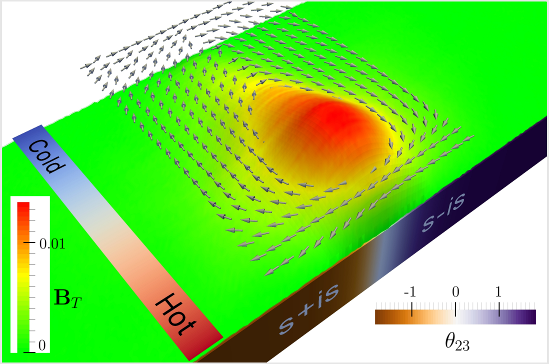
$$\boldsymbol{E} = \partial_t\left(\frac{\boldsymbol{J}}{e^2\varrho^2}\right) - \boldsymbol{\nabla}\Phi + \sum_{a,\, b>a} \partial_t\left(\frac{|\psi_b|^2 - |\psi_a|^2}{Ne\varrho^2}\boldsymbol{\nabla}\varphi_{ab}\right)$$

The extra contribution to London's magnetostatics is nonzero

especially when relative density gradients: $\boldsymbol{\nabla}\times\left(\frac{|\psi_a|^2-|\psi_b|^2}{Ne\varrho^2}\right)$ are non-collinear with the relative phase gradients $\boldsymbol{\nabla}\varphi_{ab}$ $\Rightarrow$ possible non-screened contributions

Ex.: domain-walls react to thermal gradients [Silaev, JG, Babaev 2015]

Magnetic response of a domain-wall to thermal gradients

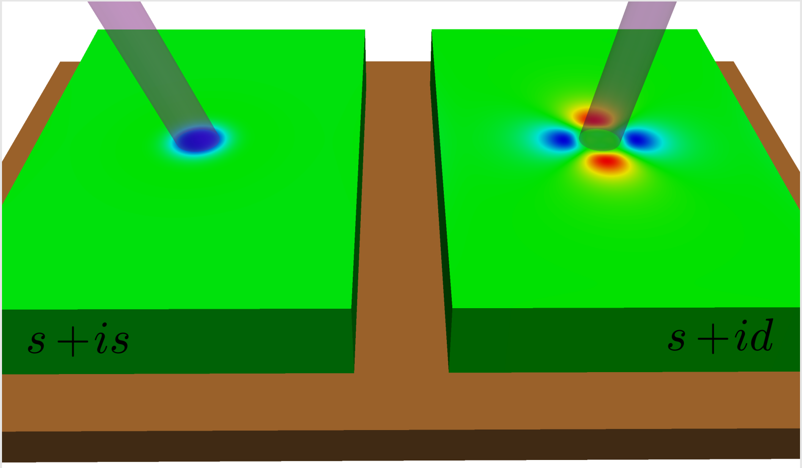


Because of the modified electrodynamics and thermoelectric properties

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \left(\frac{\boldsymbol{J}}{e^2 \varrho^2} \right) + \sum_{a,b>a} \boldsymbol{\nabla} \left(\frac{|\psi_a|^2 - |\psi_b|^2}{Ne\varrho^2} \right) \times \boldsymbol{\nabla} \varphi_{ab}$$

Ex.: a hotspot can induce magnetic fields [JG, Silaev, Babaev 2016]

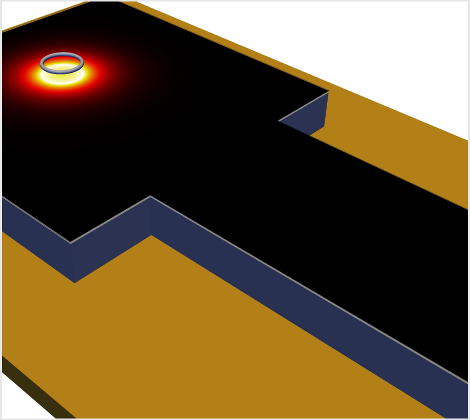
Excite relative phase between Cooper pairs on different Fermi surface



Ex.: a hotspot can induce magnetic fields [JG, Silaev, Babaev 2016]

Inducing variations of the contributions to the screening by local heating

- additional contributions to the electrodynamics due to relative phases
$$\propto \nabla \times \left(\frac{|\psi_a|^2 - |\psi_b|^2}{Ne\varrho^2} \nabla \varphi_{ab} \right) \neq 0$$
- these can be excited when the contribution to screening varies in space [$\psi_a(\boldsymbol{x})$], then :
$$\nabla \left(\frac{|\psi_a|^2 - |\psi_b|^2}{Ne\varrho^2} \right) \not\parallel \nabla \varphi_{ab}$$
- varying the contribution to the screening can be done by locally heating the sample



$\Rightarrow$ in other words deplete density by locally heating the system

Conclusion

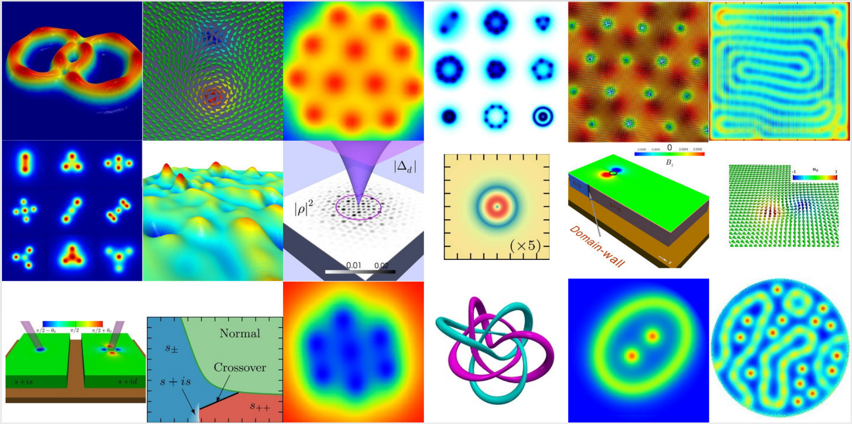
Multicomponent superconductors allow for a rich physics

- broad variety of topological defects: fractional vortices, singular vortices, domain-walls, skyrmions (coreless vortices), hopfions (knots), ...
- many mechanisms allow to stabilize the coreless vortices (skyrmions)
- modified electrodynamics and thermoelectric properties

Many aspects of multicomponent superconductivity where not discussed here

- only isolated topological defects but what is the response in external fields?
  $\Rightarrow$ magnetization processes, field cooled experiments, ...
- microscopic theories that yield the multicomponent Ginzburg-Landau theories
  $\Rightarrow$ can be consistently derived from BCS, Eilenberger (clean), Usadel (dirty)
- singular vortices can interact non-trivially due to multiple length-scales
  $\Rightarrow$ semi-Meissner/type-1.5 superconductivity, vortex aggregates
- other pairing symmetries $s+id$, $d+d$, $p+ip$, $d+id$, $d+ig$, ...
- numerical methods

Thank you for your attention!



Based on works from 2011 – 2021 with

E. Babaev, J. Carlström, M. Speight, M. Silaev, D. Agterberg, J. Jäykka, K. Sellin, A. Zyuzin, A. Corticelli, F. Rybakov