Soliton stars, boson constellations and hairy black holes

Yakov Shnir

Thanks to my collaborators:
J Kunz, C.Herdeiro,
I.Perapechka, and E Radu

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Phys.Lett.B 810 (2020) 135847
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JHEP 07 (2019) 109

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Outline

- Boson Stars & Q-balls
- U(1) gauged boson stars
- Multipolar boson stars
- Boson stars and hairy BHs
- Boson Stars and hairy BHs in the O(3) sigma-model
- Spinning black holes with skyrmionic hairs
- Einstein-Maxwell BHs with scalar hairs
- Dirac stars
- Summary

Q-balls

G. Rosen (1968),

R. Friedberg, T.D. Lee & A. Sirlin (1976)

S. Coleman (1985)

$$L = |\partial_\mu \phi|^2 - V(|\phi|)$$

$$Q = i \int d^3x \, (\phi \partial_t \phi^* - \phi^* \partial_t \phi)$$

$$\phi = f(r) e^{i\omega t}$$

Spherically symmetric Q-ball

$$Q = 8\pi\omega \int_0^\infty dr r^2 f^2$$

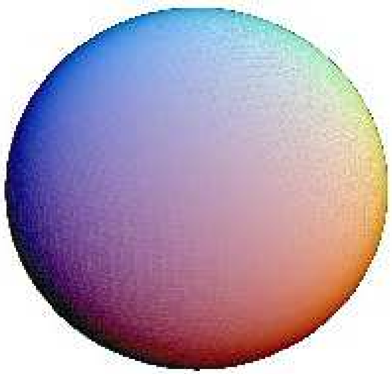
Field equation:

$$\frac{d^2 f}{dx^2} + \frac{2}{r}\frac{df}{dr} + \omega^2 f = \frac{1}{2}\frac{dV}{df}$$

$$f \sim \frac{1}{r} e^{-\sqrt{m^2-\omega^2}\, r}$$

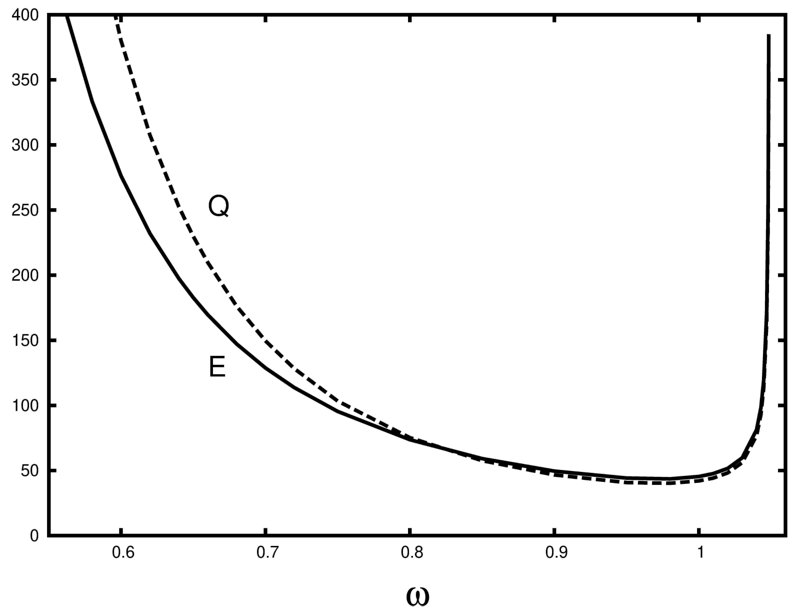
Potential:

$$V = a|\phi|^2 - b|\phi|^4 + |\phi|^6$$



Angular frequency is restricted:

$$\omega_{min} \leq \omega \leq \omega_{max}$$



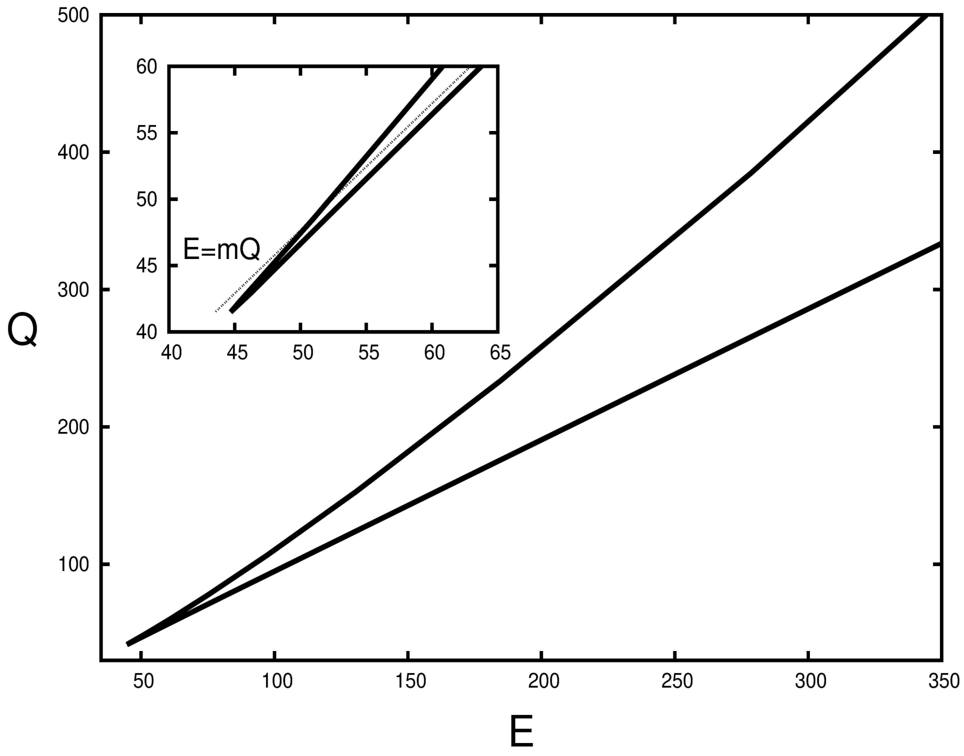
Q-balls stability

Condition of classical stability
(Vakhitov-Kolokolov criteria):

$$\frac{\omega}{Q}\,\frac{dQ}{d\omega} < 0$$

Solution is classically stable if it is a minimum of energy at a fixed value of charge:

$$E \leq mQ$$



Fission condition: the energy of a single Q-ball must be less than the total energy of smaller Q-balls that it could fragment to.

Quantum stablity: there are no negative modes in the linearized spectrum of fluctuations aroung the Q-ball

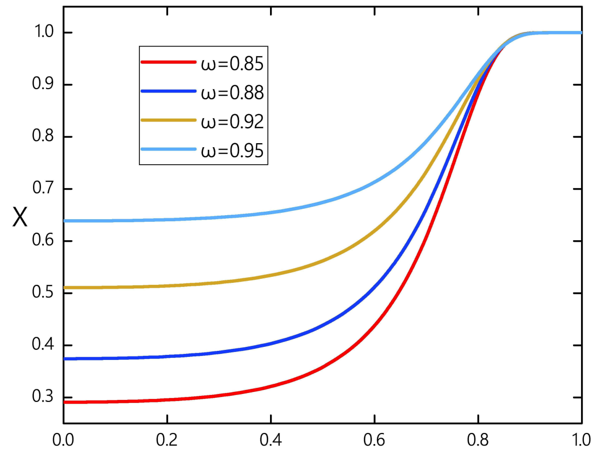
Friedberg-Lee-Sirlin Q-balls

$$L = (\partial_\mu \xi)^2 + |\partial_\mu \phi|^2 - m^2 \xi^2 |\phi|^2 - \mu^2 (1-\xi^2)^2$$

$$Q = i \int d^3x \, (\phi \partial_t \phi^* - \phi^* \partial_t \phi)$$

$$\xi = X(r)\,; \qquad \phi = Y(r) e^{i\omega t}$$

Spherically symmetric FLS Q-ball

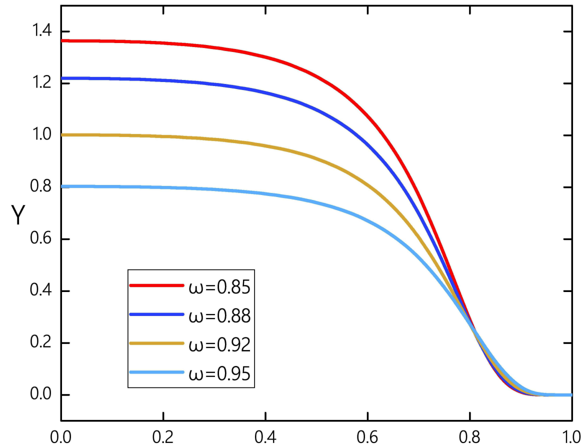


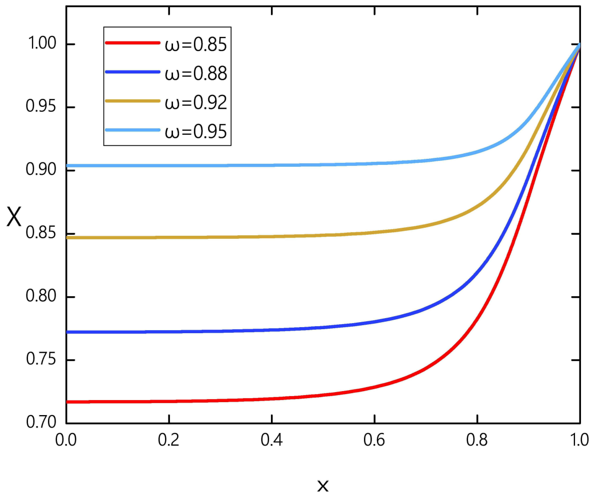
- **massive**

$$\mu^2 = 1/4$$

$$m^2 = 1$$

$$X \sim 1 - e^{-\sqrt{\mu^2 - \omega^2}\ r}$$



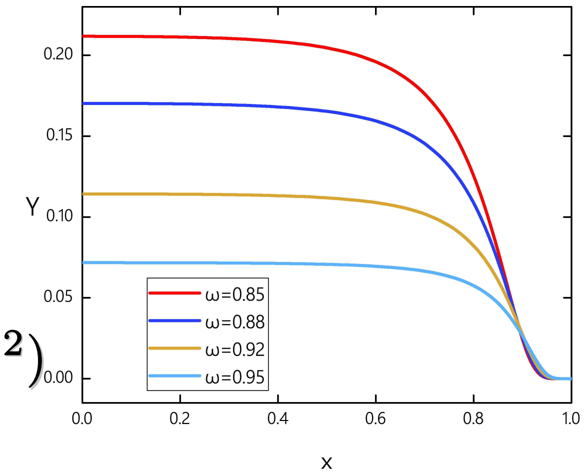


- **massless**

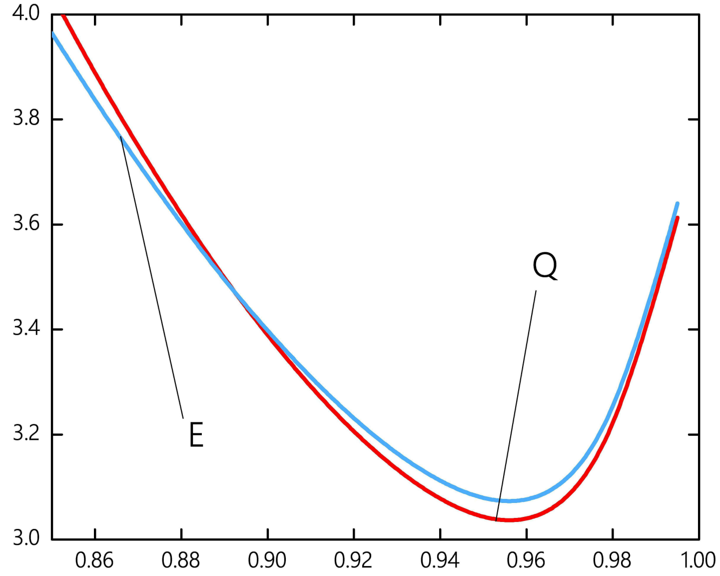
$$\mu^2 = 0$$

$$m^2 = 1$$

$$X(r) \sim 1 - \frac{C}{r} + O(r^{-2})$$



Friedberg-Lee-Sirlin Q-balls



- **massive**

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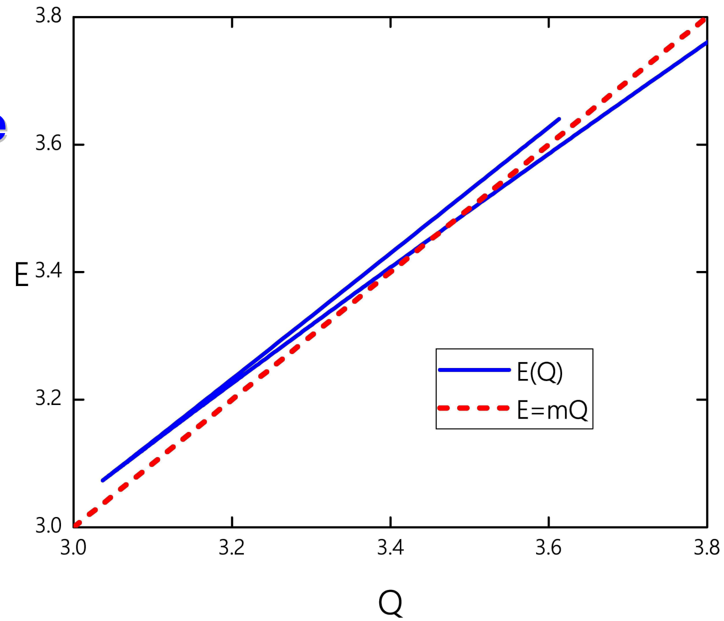
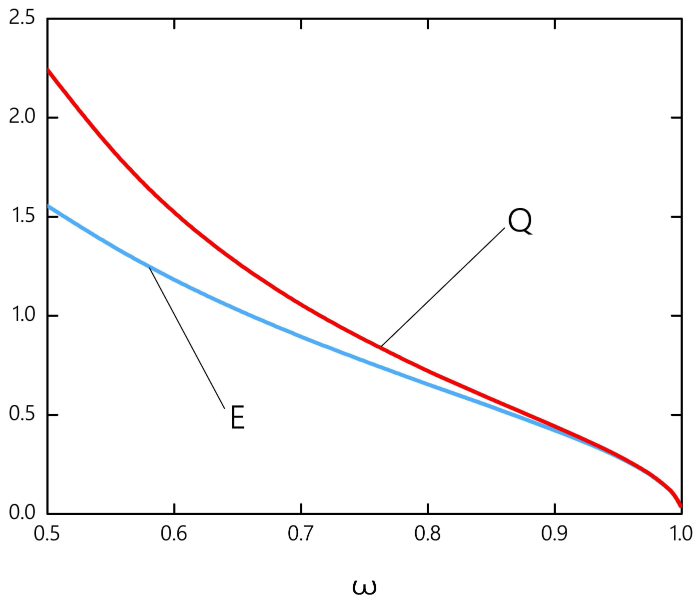
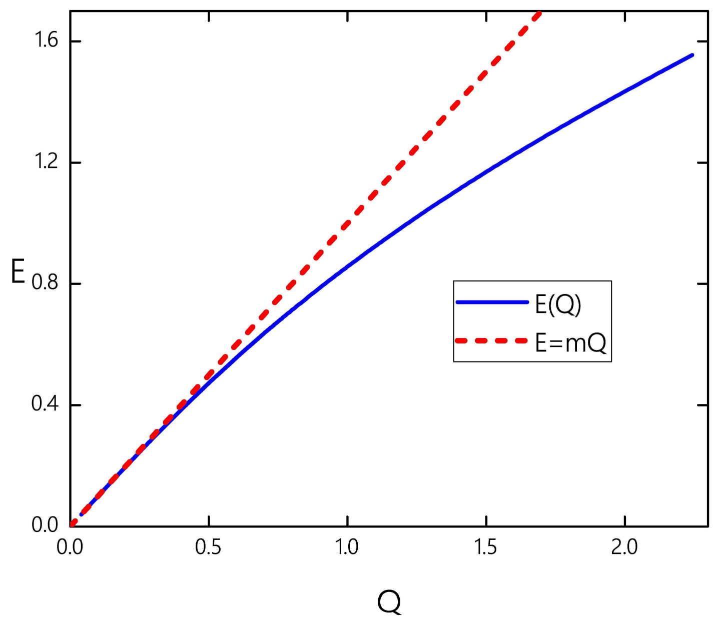




- **massless**

$$\mu^2 = 0$$

$$m^2 = 1$$



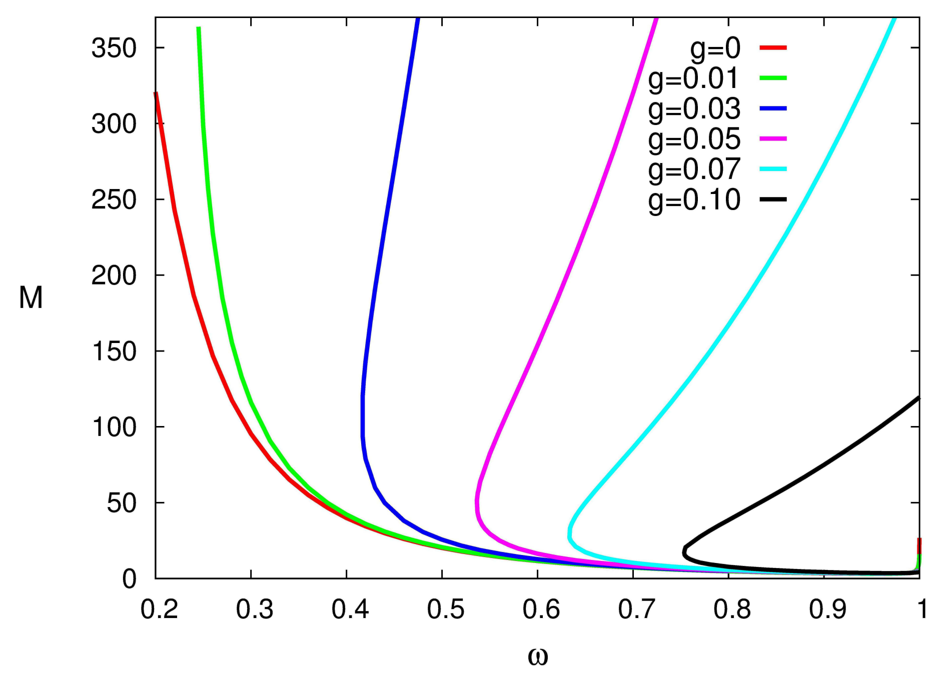
U(1) gauged Friedberg-Lee-Sirlin Q-balls

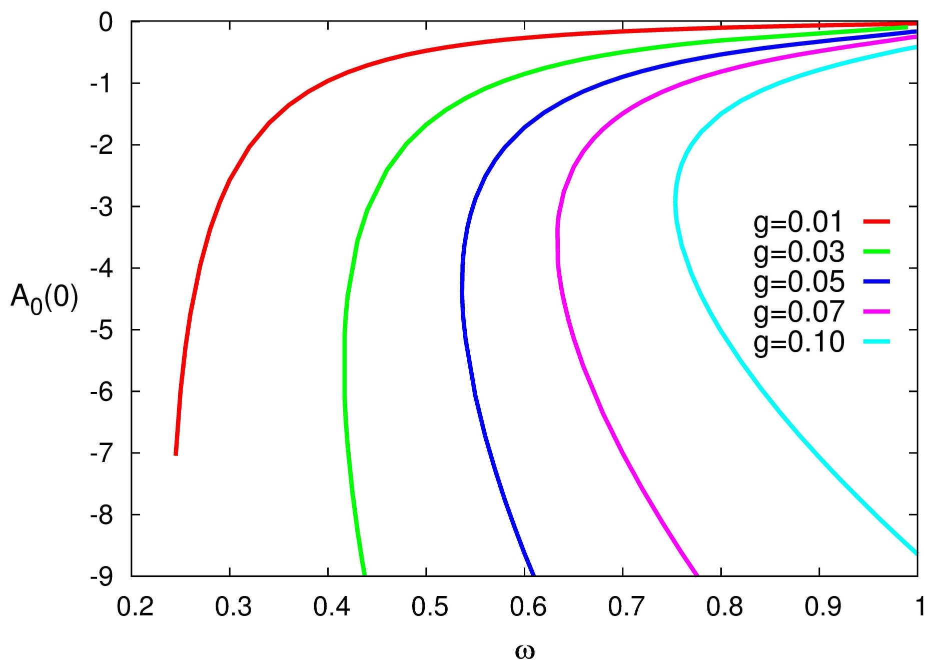
$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (\partial_\mu \xi)^2 + |D_\mu \phi|^2 - m^2\xi^2|\phi|^2 - \mu(1-\xi^2)^2$$

$$D_\mu = \partial_\mu + igA_\mu, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- **U(1) current:** $j_\mu = i(\phi D_\mu \phi^* - \phi^* D_\mu \phi)$

$$Q = \int d^3x (gA_0 + \omega)|\phi|^2$$

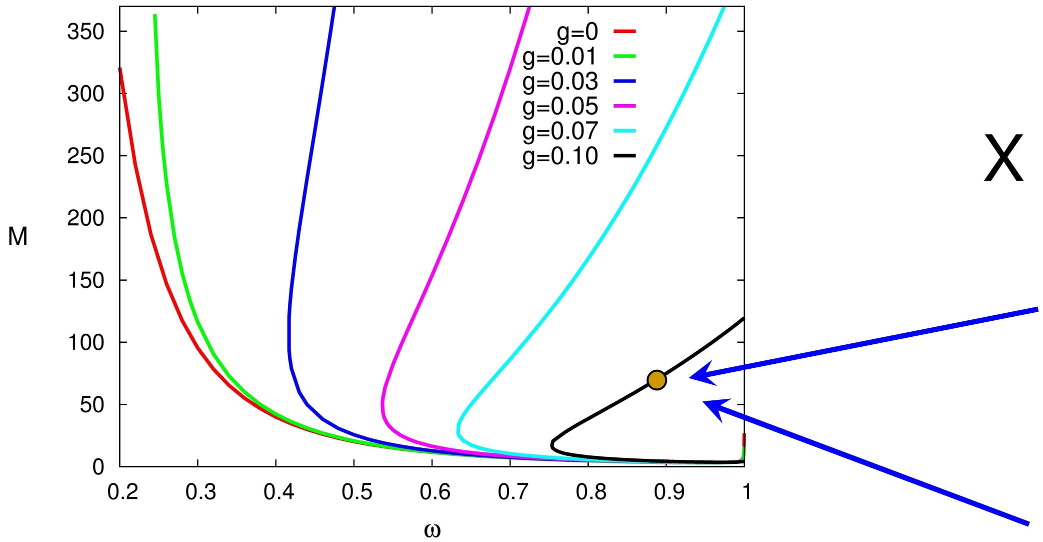


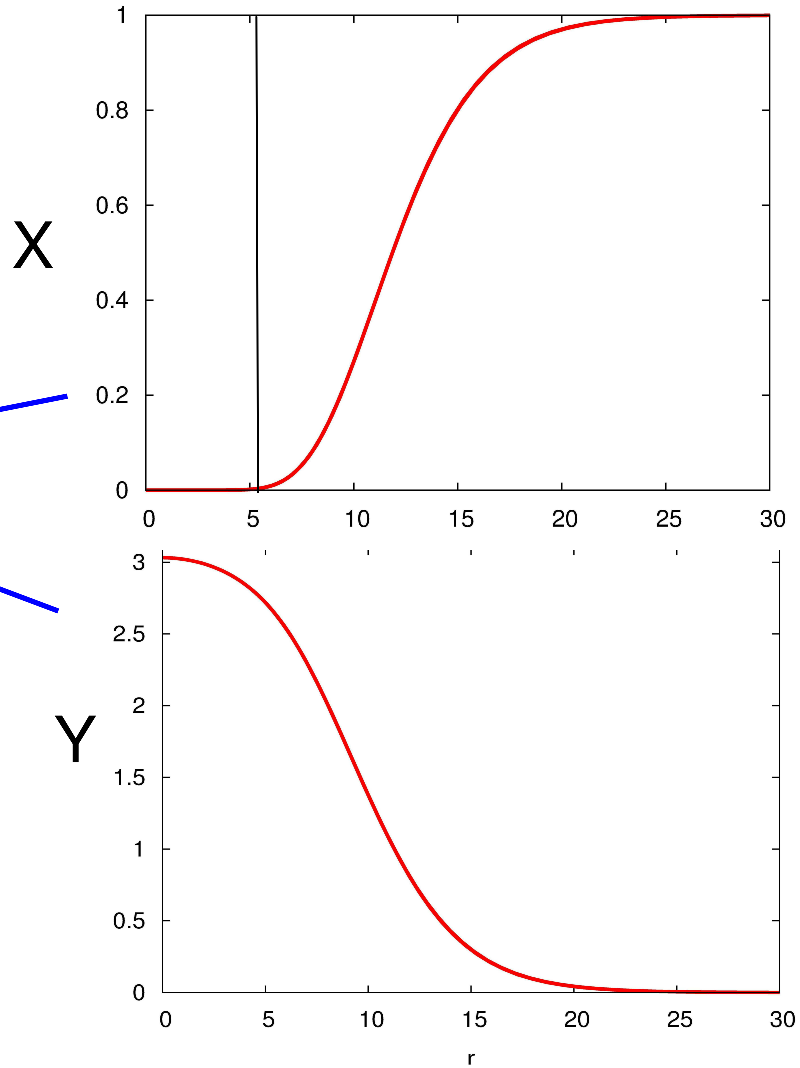


U(1) gauged Friedberg-Lee-Sirlin Q-balls

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (\partial_\mu \xi)^2 + |D_\mu \phi|^2 - m^2\xi^2|\phi|^2 - \mu(1-\xi^2)^2$$

$$\xi = X(r)\,; \qquad \phi = Y(r)e^{i\omega t}$$





Bubble of the massless charged complex scalar field ϕ

Spinning Q-balls

$$\phi = f(r,\theta)e^{i(\omega t + n\varphi)}, \quad n \in \mathbb{Z}$$

Volkov & Wohnert (2002)

Axially symmetric Q-balls → $J = \int d^3x T^0_\varphi = 2n\omega N^2 = nQ$

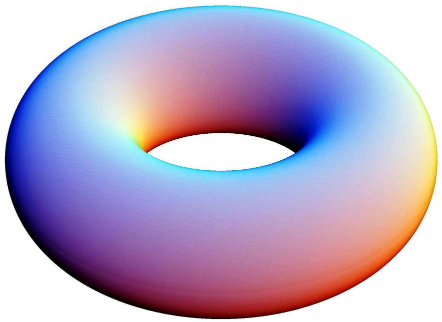
$$\phi(r,\theta,\varphi) \propto \frac{1}{\sqrt{r}} J_{l+1/2}(\omega r) Y_l^n(\theta,\varphi)$$

- ***Parity-even solutions:***

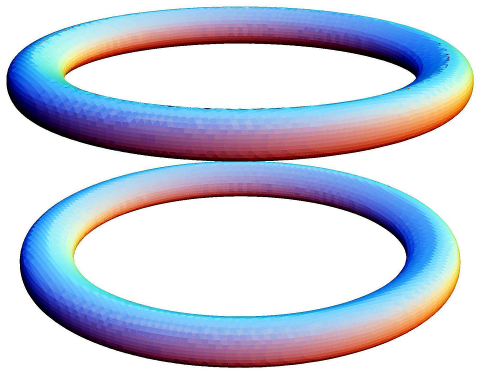
  $f(r,\theta) = f(r,\pi-\theta)$

- ***Parity-odd solutions:***

  $f(r,\theta) = -f(r,\pi-\theta)$



n=1 P-even



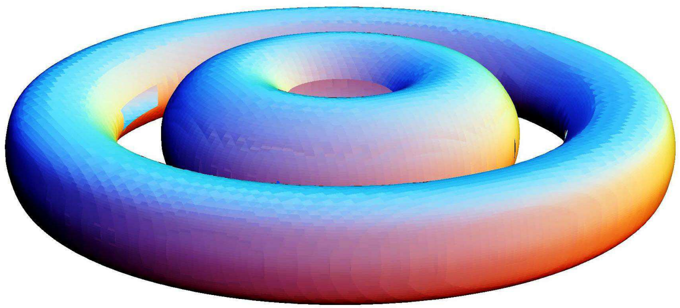
n=1 P-odd

- ***There are zeros of***

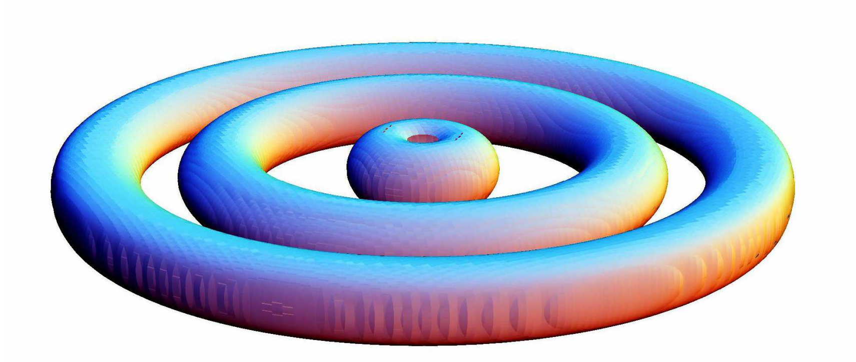
$$J_{l+1/2}(\omega r), \qquad Y_l^n(\theta,\varphi)$$

Radially and angularly excited Q-balls

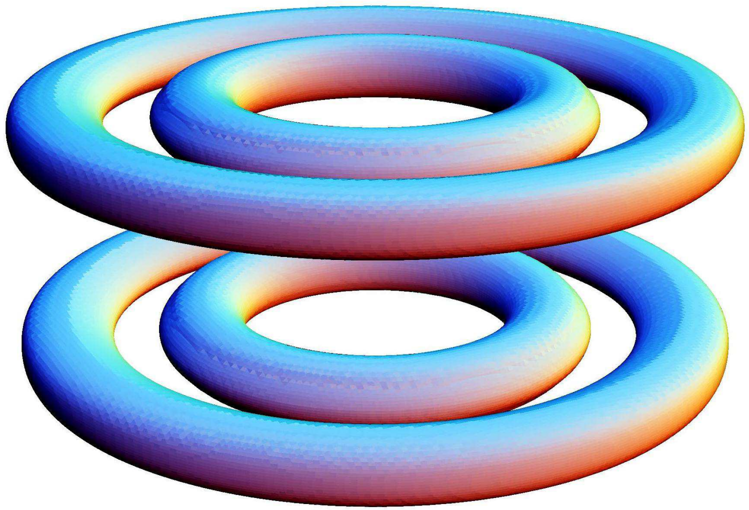
Spinning Q-balls



n=1,k=1 P-even



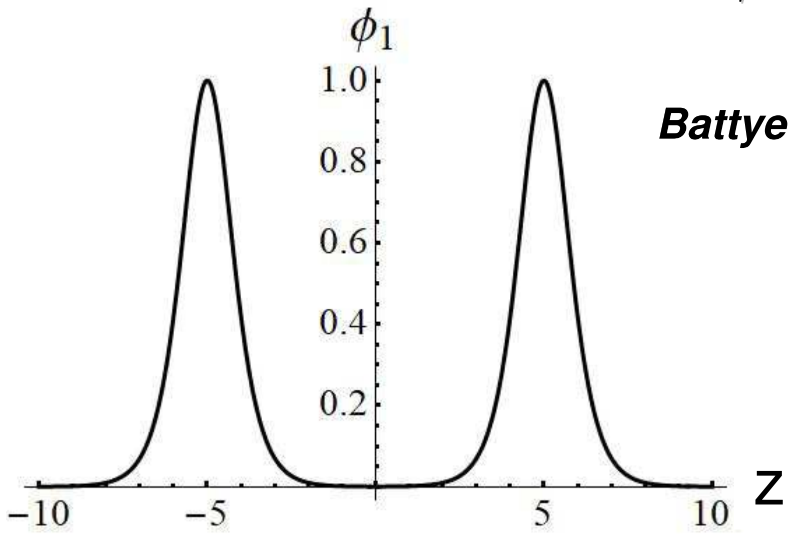
n=1,k=2 P-even



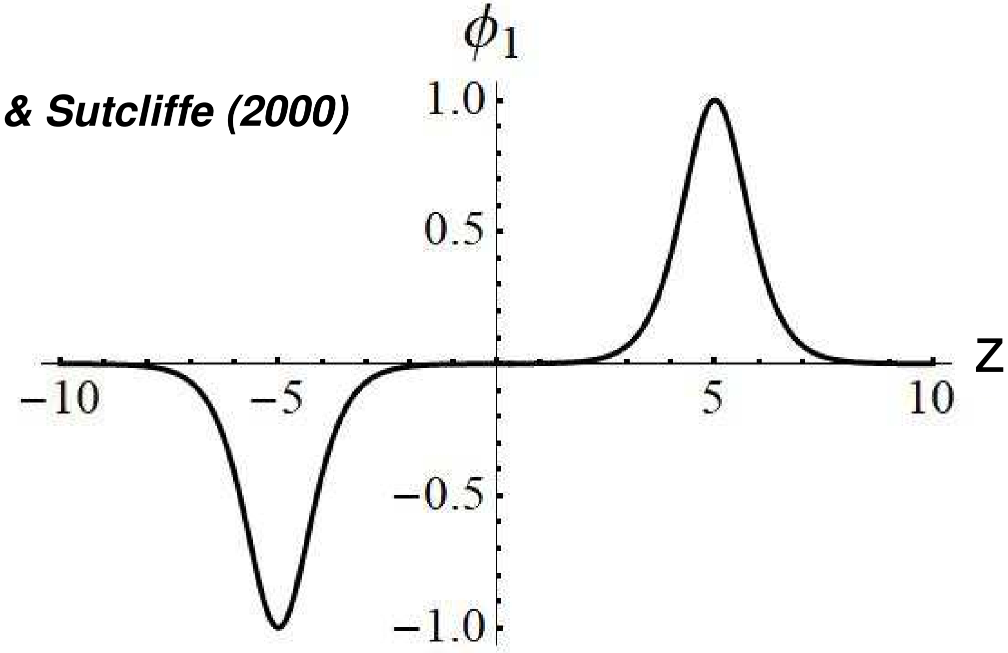
n=1,k=2 P-odd

Q-balls interaction

$$\phi = \phi_1 + i\phi_2$$



Battye & Sutcliffe (2000)



Bowcock, Foster & Sutcliffe (2009)

Asymptotic force of interaction:

$$F = -16\omega^4 \cos\alpha e^{-2d}$$

The interaction is attractive if the Q-balls are in phase ($\alpha = 0$), it is repulsive if the Q-balls are out of phase, ($\alpha = \pi$)

Localised solitons: Gravity vs Klein-Gordon

Pure gravity (attraction)

$$L = -\frac{R}{16\pi G}$$

Lichnerowicz (1955): there are black holes but there are no gravitational solitons, the only globally regular, asymptotically flat, static vacuum solution to the Einstein eqs with finite energy is Minkowski space.

Klein-Gordon massive theory

$$L = |\partial_\mu \phi|^2 + \mu^2 |\phi|^2$$

Derrek theorem: Complex Klein-Gordon theory in 3+1 dim do not admit localised soliton solutions.

Stationary spinning configurations: a way to evade Derrick's theorem

Kaup (1968): Dispersion can be balanced by the gravitational attraction

The Boson Stars: $\phi(\mathbf{r}, t) = f(\mathbf{r}) e^{i\omega t}$

Boson Stars

$$S = \int d^4x\sqrt{-g}\left\{\frac{R}{16\pi G} - |\partial_\mu\phi|^2 - \mu^2|\phi|^2\right\}$$

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 2\alpha^2 T_{\mu\nu}; \qquad \left(\Box - \mu^2\right)\phi = 0 \qquad \alpha^2 = 4\pi G$$

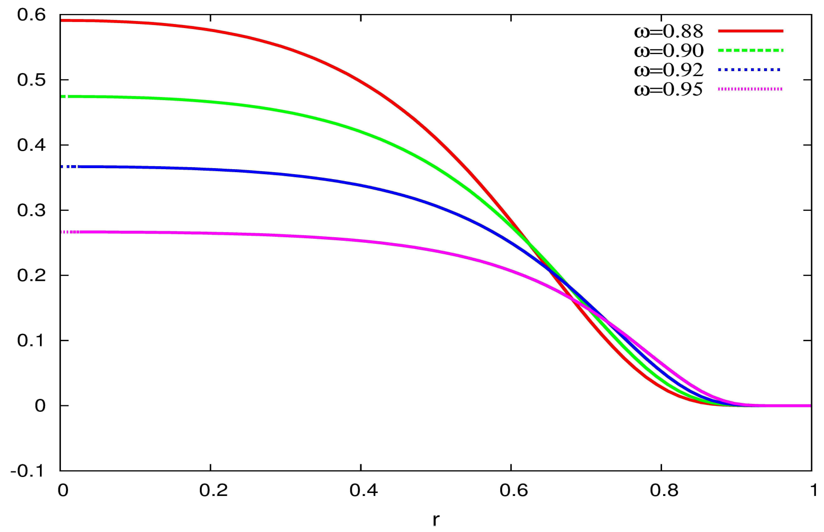
- **U(1) current:** $j_\mu = i(\phi\partial_\mu\phi^* - \phi^*\partial_\mu\phi)$

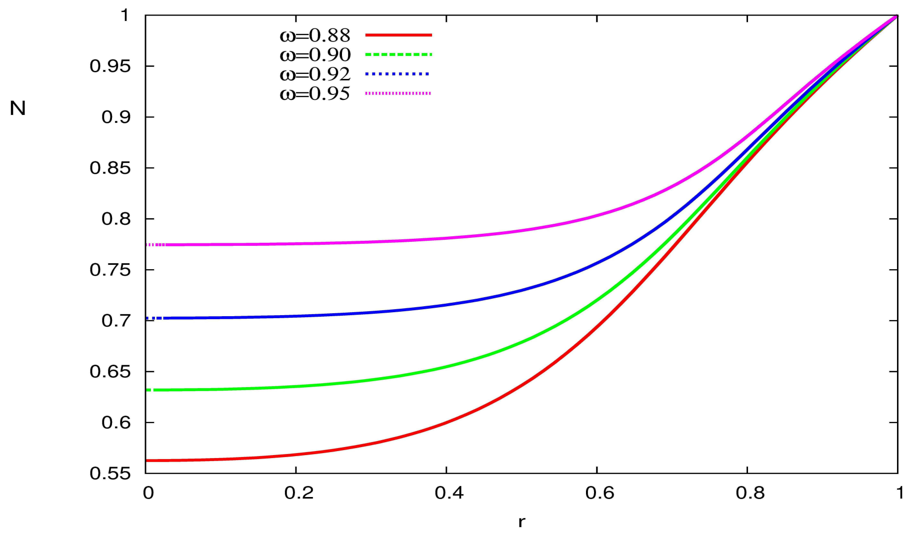
$$Q = \int \sqrt{-g}j^t d^3x$$

Spherical symmetry:

$$\phi = f(r)e^{-i\omega t}$$

$$ds^2 = -\sigma^2(r)N(r)dt^2 + \frac{1}{N(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$





Boson Stars

$$S = \int d^4x\sqrt{-g}\left\{\frac{R}{16\pi G} - |\partial_\mu \phi|^2 - \mu^2|\phi|^2\right\}$$

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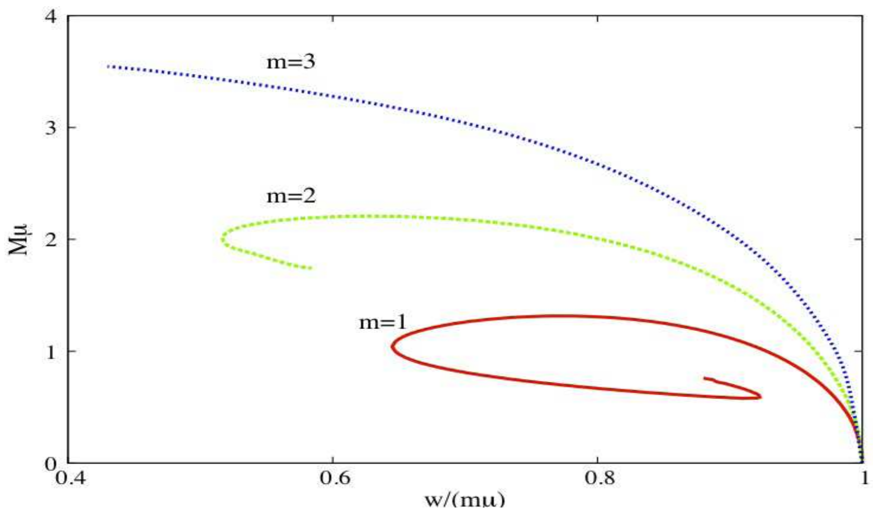
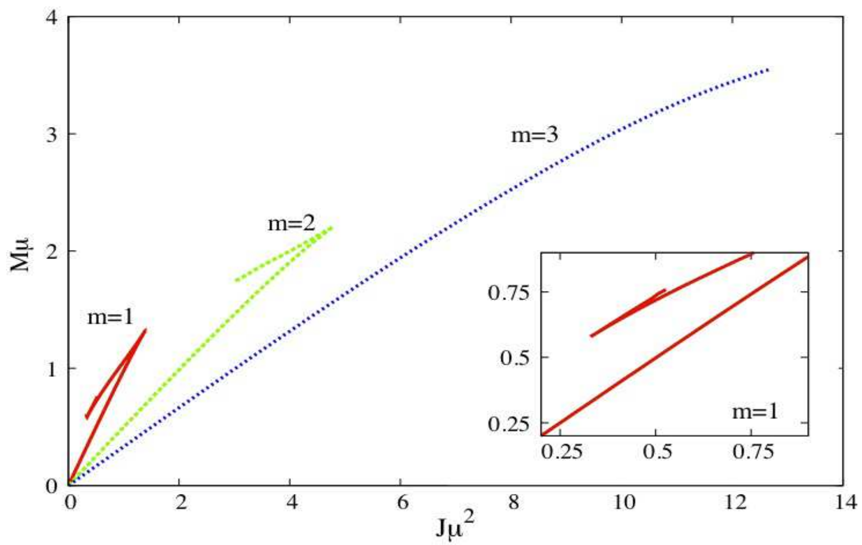
- **U(1) current:** $j_\mu = i(\phi\partial_\mu\phi^* - \phi^*\partial_\mu\phi)$

$$Q = \int \sqrt{-g} j^t d^3x$$

Axial symmetry: $\phi = f(r,\theta)e^{i(m\varphi - \omega t)}$

Volkov & Wohnert (2002), Kleihaus, Kunz and List (2005)





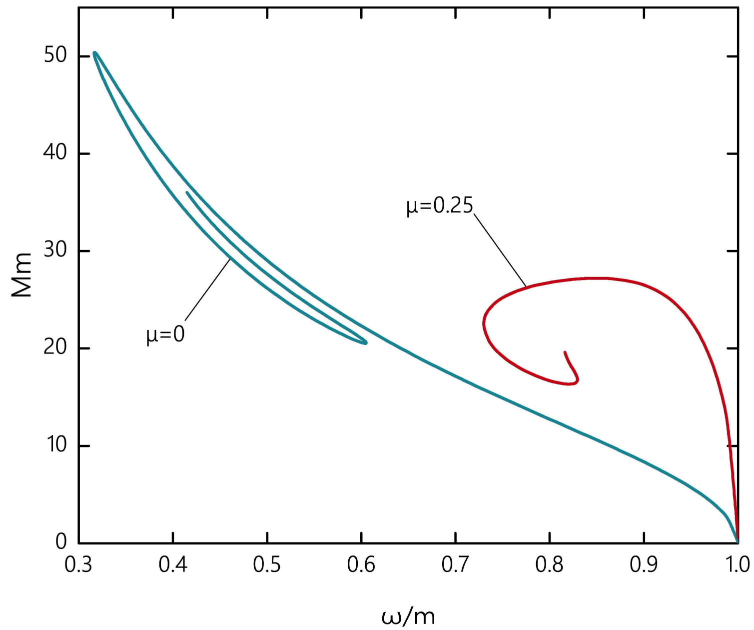
FLS Boson Stars

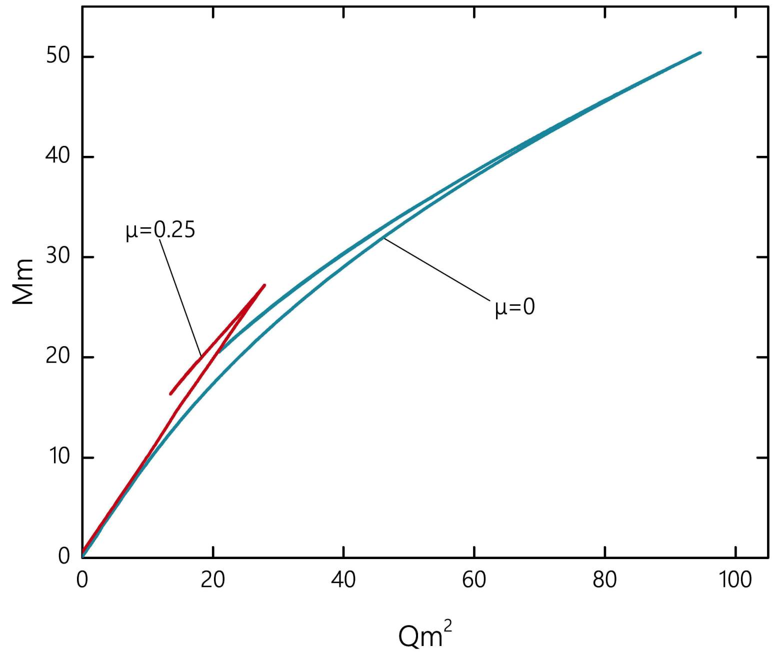
Friedberg-Lee-Sirlin model (1976):

$$\mathcal{L}_{\rm m} = \frac{1}{2}\left(\partial_\mu \xi\right)^2 + |\partial_\mu \phi|^2 + m^2 \xi^2 |\phi|^2 - \mu^2 \left(1-\xi^2\right)^2$$

$$ds^2 = -F_0 dt^2 + F_1\left(dr^2 + r^2 d\theta^2\right) + r^2 \sin^2\theta F_2 \left(d\varphi - \frac{W}{r} dt\right)^2$$

$$\xi = X(r,\theta), \quad \phi = Y(r,\theta) e^{i\omega t + n\varphi}$$





Head-on collision of the EKG boson stars (full 3d simulations)

Repulsive channel

Reproduced by courtesy of
Carlos Herdeiro and Pedro de Alfonso

Boson Constellations

$$S = \int d^4x\sqrt{-g}\left\{\frac{R}{16\pi G} - |\partial_\mu\phi|^2 - \mu^2|\phi|^2\right\}$$

Herdeiro, Kunz, Perapechka, Radu & YS (2020)

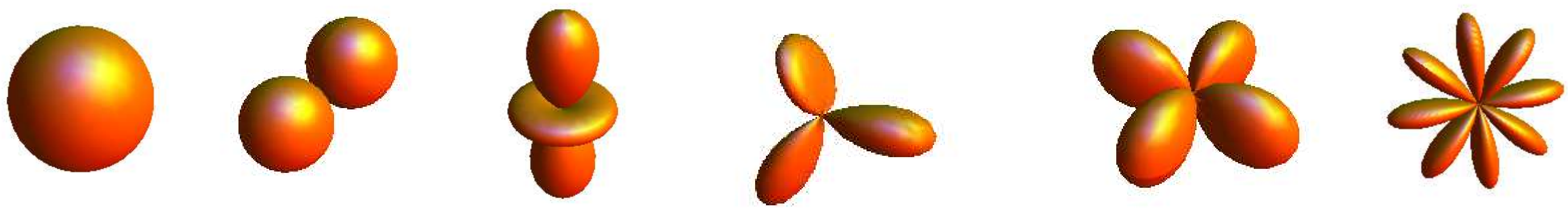
$$\phi = f(r,\theta,\varphi)e^{i\omega t}$$

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial r}\left(g^{rr}\sqrt{-g}\frac{\partial f}{\partial r}\right) + \frac{1}{\sqrt{-g}}\frac{\partial}{\partial \theta}\left(g^{\theta\theta}\sqrt{-g}\frac{\partial f}{\partial \theta}\right) \left(n^2 g^{\varphi\varphi} - 2g^{\varphi t} + \omega^2 g^{tt}\right) f = \mu^2(f)$$

$$f(r,\theta,\varphi) \sim \frac{1}{\sqrt{r}} J_{1/2+l}(-ir\sqrt{\mu^2 - w^2})\, Y_{lm}(\theta,\varphi)$$

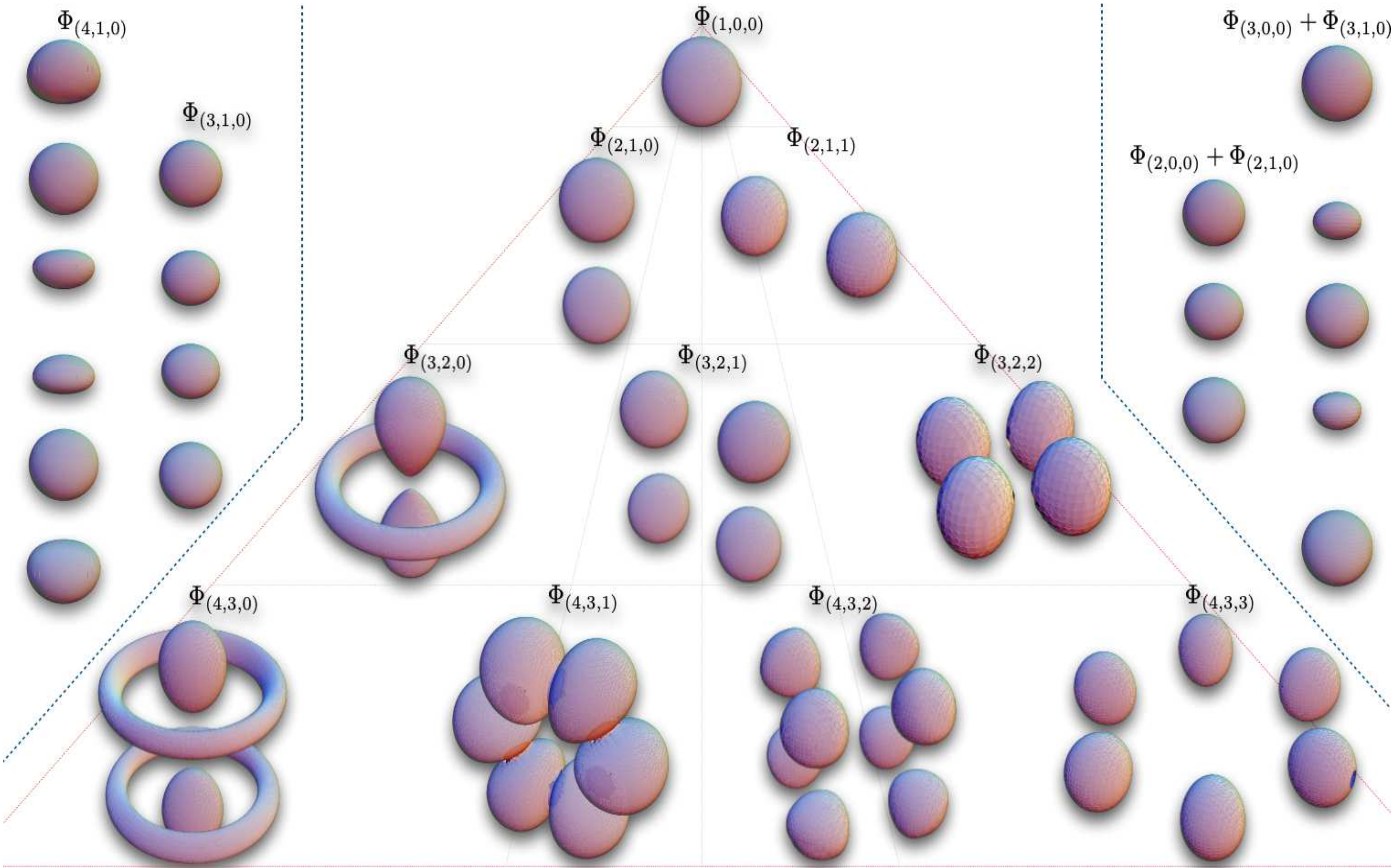
Real spherical harmonics

$$Y_{lm}(\theta,\varphi) = \sqrt{2}\sqrt{\frac{(2l+1)}{4\pi}\frac{(l-m)!}{(l+m)!}}P_l^m(\cos\theta)\cos m\varphi$$



Perturbative excitations of the scalar field are seeds for bounded configurations of the multiboson stars

Boson Constellations



C.Herdeiro,J.Kunz, E Radu, Perapechka & Y Shnir (2020)

N. Boulle et al,
Phys. Rev. A 102, 053307 (2020)

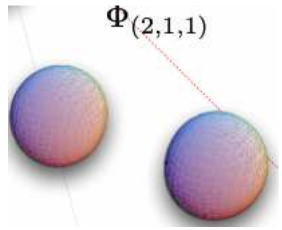
- **Gross-Pitaevskii equation with localizing potential:**

$$i\Psi_t = -\Psi + \Psi|\Psi|^2 + V(\vec{r})\Psi; \quad \Psi(\vec{r},t) = e^{-i\mu t}\psi(\vec{r})$$

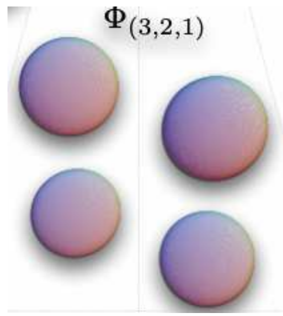
$$\phi_{\ell+1,\ell,1}$$

Parity-even solutions: even l

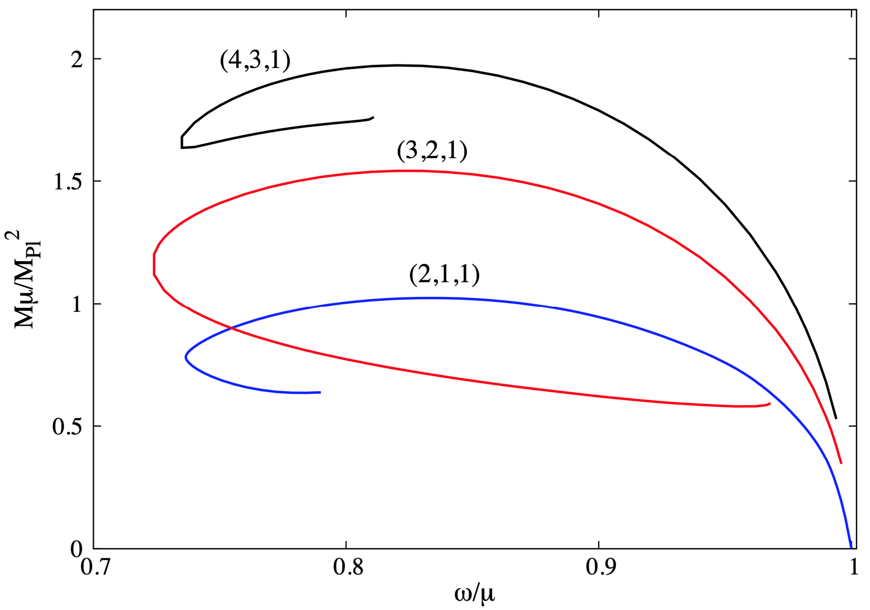
Parity-odd solutions: odd l

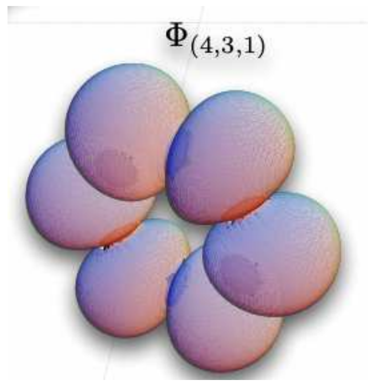


dipole



quadrupole





sextipole

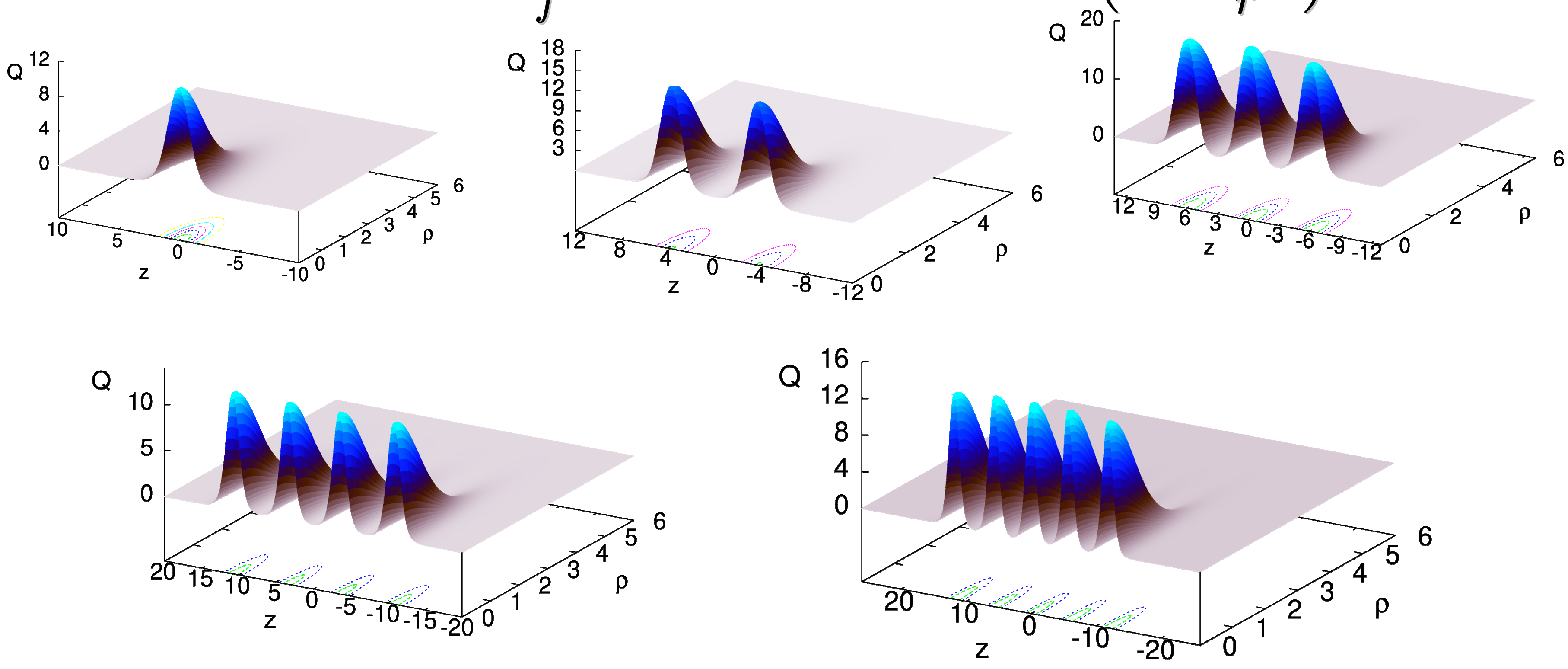
Chains of Boson Stars: $\phi_{N,1,0}$

$$S = \int d^4x\sqrt{-g}\left\{\frac{R}{16\pi G} - |\partial_\mu\phi|^2 - \lambda^2|\phi|^2(|\phi|^4 - a|\phi^2| + \mu^2)\right\}$$

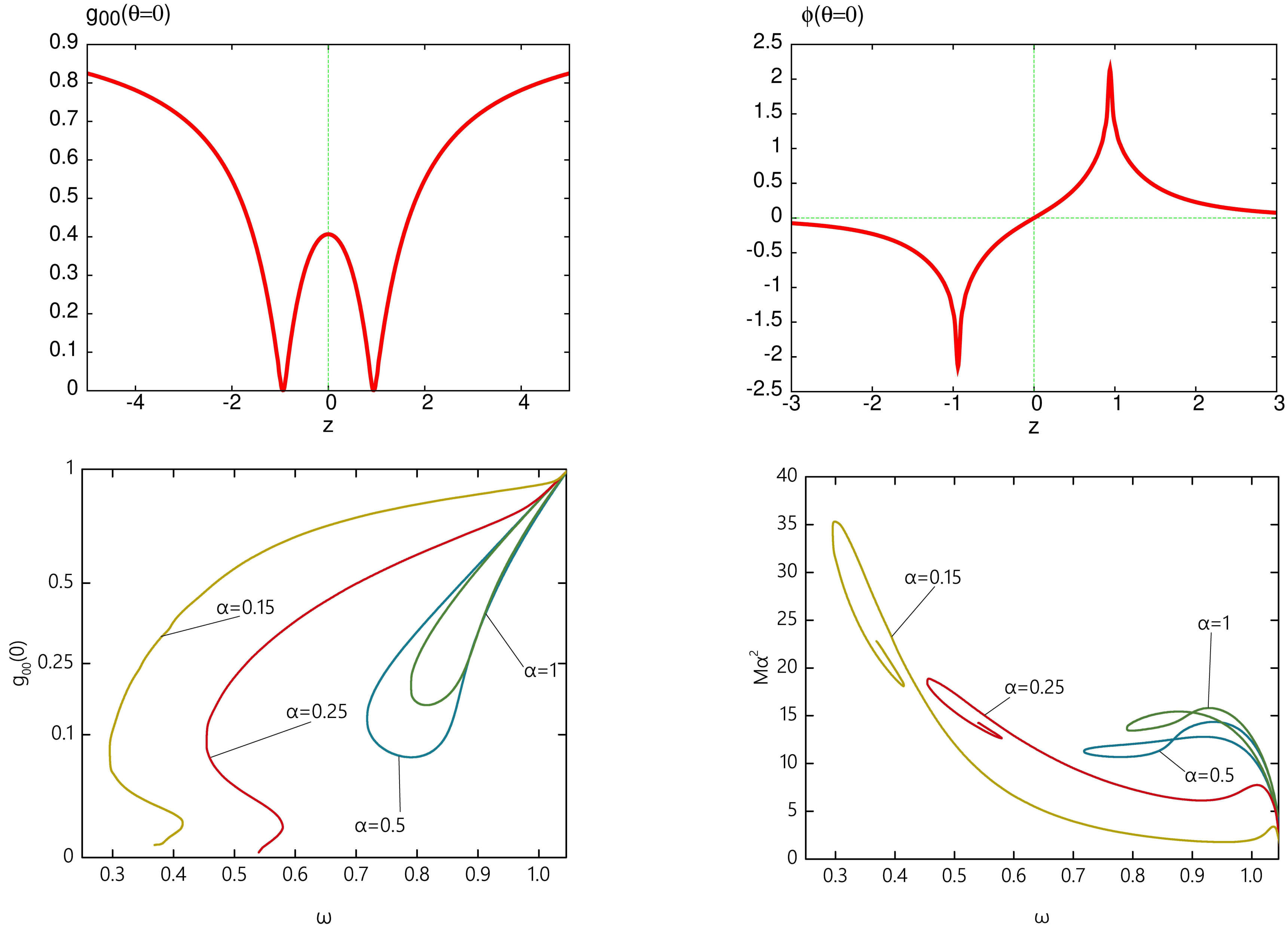
- **Lewis-Papapetrou parametrization:**

Herdeiro, Kunz, Perapechka, Radu & YS (2021)

$$ds^2 = -f dt^2 + \frac{m}{f}\left(dr^2 + r^2 d\theta^2\right) + l r^2 \sin^2\theta \left(d\varphi - \frac{o}{r}dt\right)^2$$



Chains of BSs: critical behavior



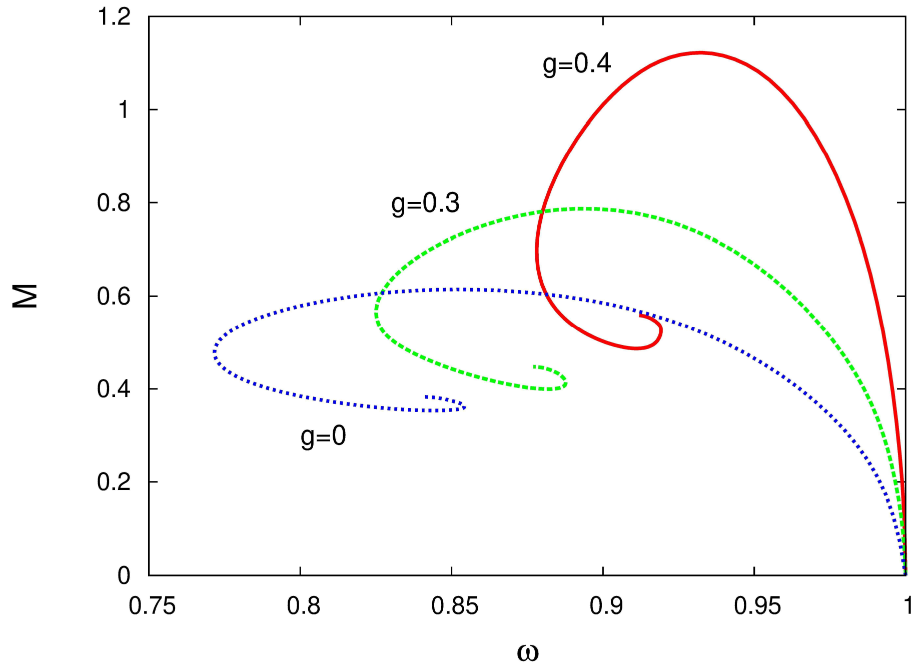
U(1) gauged Boson Stars

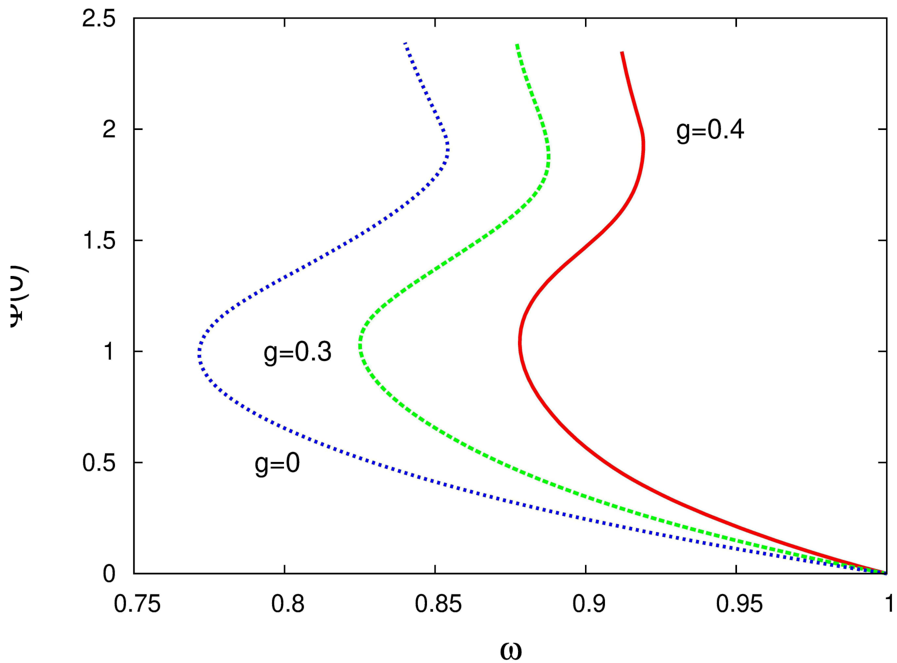
P.Jetzer and J.J.van der Bij (1989), D.Pugliese, H.Quevedo, J.Rueda and R.Ruffini (2013):
Boson stars in the Einstein-Klein-Gordon-Maxwell model:

$$S = \int d^4x\sqrt{-g}\left\{\frac{R}{16\pi G} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |D_\mu\phi|^2 - \mu^2|\phi|^2\right\}; \quad D_\mu\phi = \partial_\mu\phi - igA_\mu\phi$$

- **U(1) current:** $j_\mu = i(\phi D_\mu\phi^* - \phi^* D_\mu\phi)$

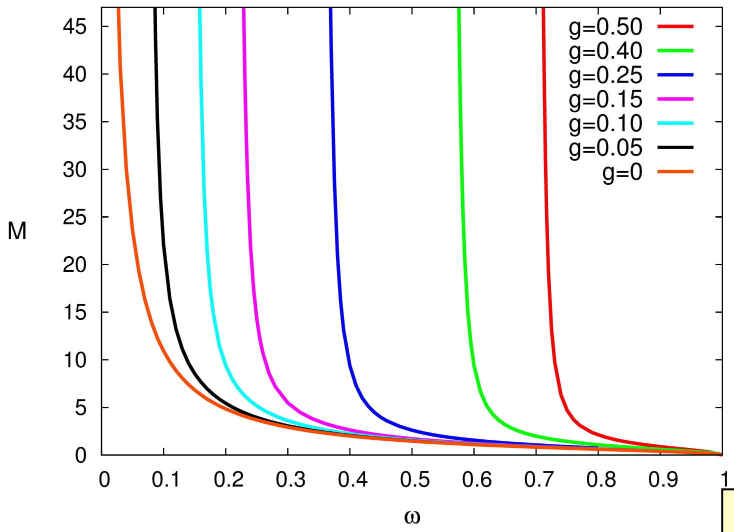
$$Q = \int d^3x(gA_0 + \omega)|\phi|^2$$



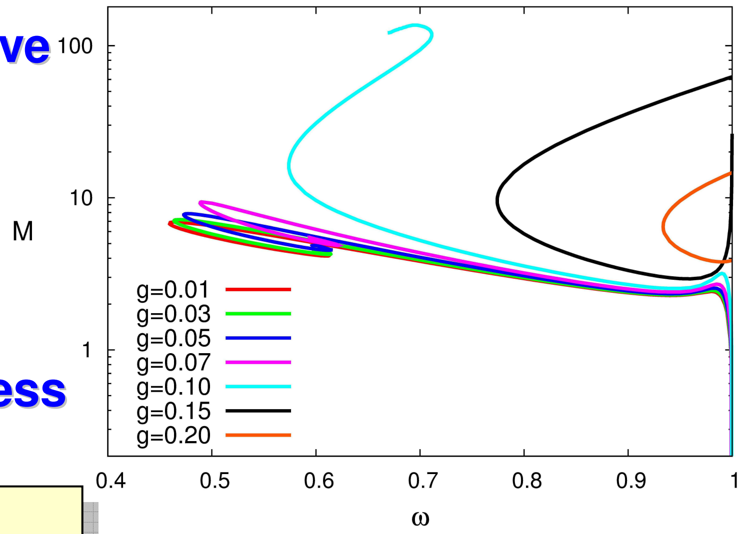


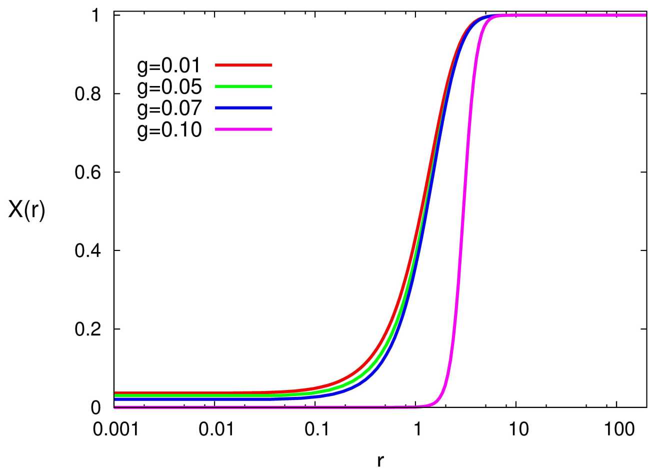
U(1) gauged FLS Boson Stars

$$S = \int d^4x\sqrt{-g}\left\{\frac{R}{16\pi G} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (\partial_\mu\xi)^2 + |D_\mu\phi|^2 - m^2\xi^2|\phi|^2 - \mu(1-\xi^2)^2\right\}$$



- **massive** →
- ← **massless**

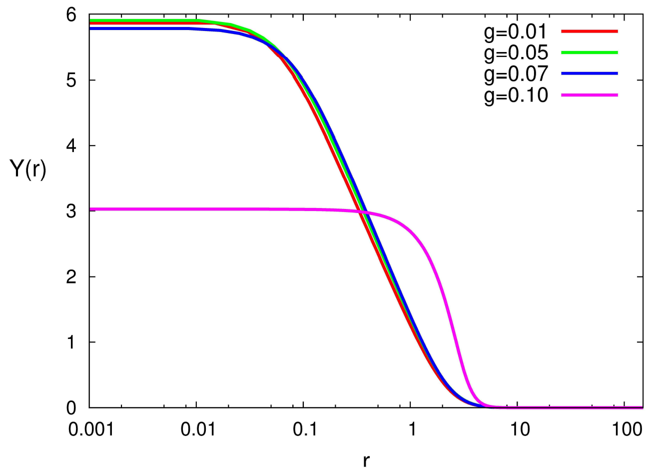




Gravity (attraction)

Electrostatic (repulsion)

2 Scalars (attraction & repulsion)

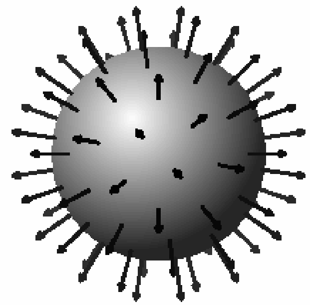


"How the Universe Works"
(Discovery channel, 2018)

S Coleman: “Can a *black hole* have *colored hair*?”

J Wheeler: “Black holes have no hair”

S Hawking: “Black holes have hair “



1970s- 1980s :

Israel’s theorem: Static Einstein-Maxwell black holes are spherically symmetric

‘No-hair’ theorem: Stationary black holes are completely characterized by their mass **M**, charge **Q** and angular momentum **J**

From 1990s…

Black holes may have hairs!

The hairs are:

Other observables apart the mass **M**, charge **Q** and angular momentum **J**

Examples:

- **Einstein-Skyrme theory**
- **Einstein gravity coupled to Yang-Mills fields**
- **Self-gravitating U(1) gauged scalar field with non-linearity**
- Modified models of gravity
- Higher dimensional theories
- Models in AdS spacetime
- **Spinning black holes with matter fields**
- **etc..**

From Boson Stars to Black Holes

no-scalar-hair theorem (Pena & Sudarsky, 1997): there are no static black hole analogues of the spherically symmetric regular boson stars

Я Зельдович, (1971): Генерация волн вращающимся телом, Письма ЖЭТФ, 14, 270

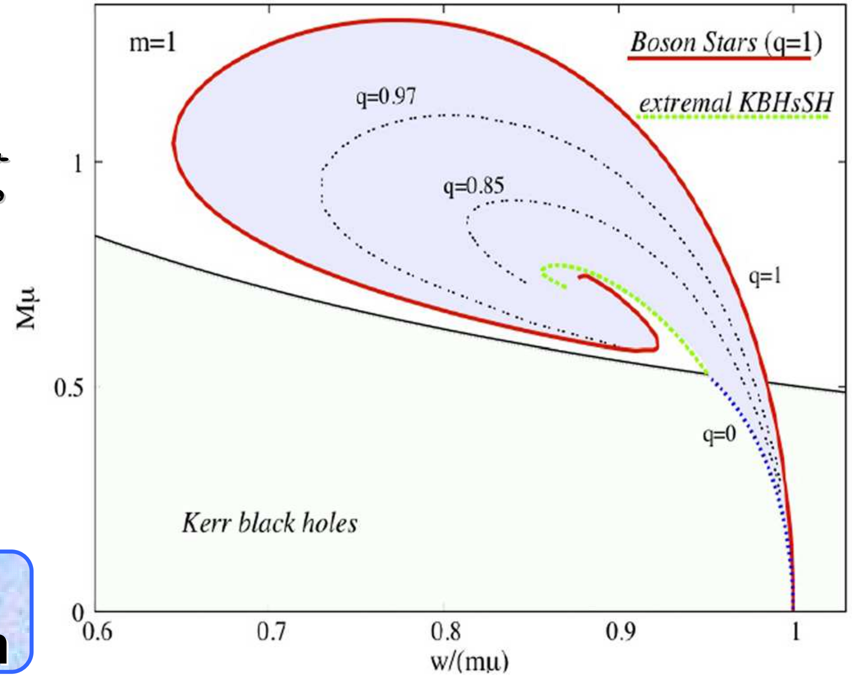
Hod (2012), Herdeiro and Radu (2014): ***Kerr BHs with scalar hair***

Synchronisation condition: $w = m\Omega_H$

- **Two Killing vectors:c** $\zeta = \partial_\varphi; \quad \xi = \partial_t$
- **Symmetry of the solution:** $\chi = \xi + \frac{\omega}{m}\zeta$

there is no flux of scalar field into the BH: $\chi^\mu \partial_\mu \phi = 0$

Superradiant instability of the Kerr spacetime: Black hole bomb mechanism



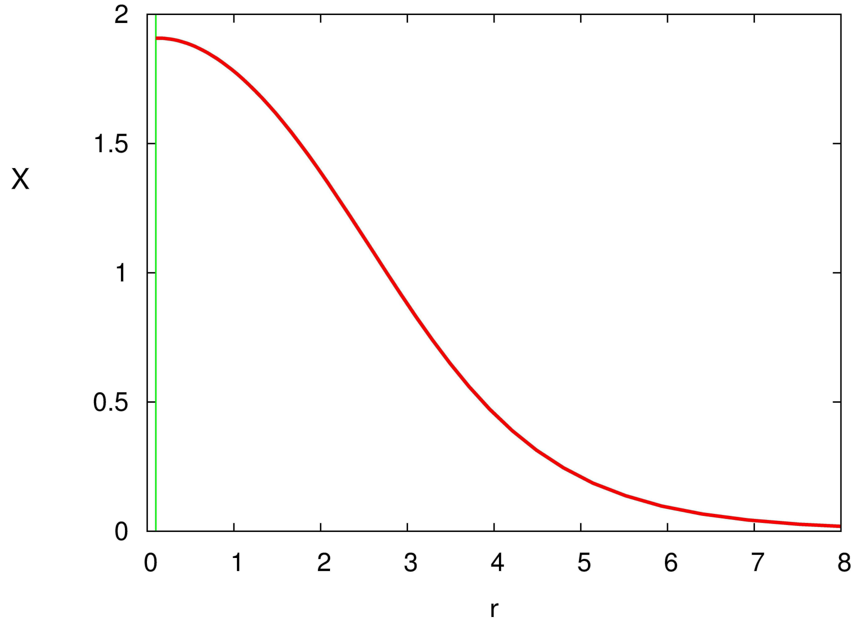
From Boson Stars to Black Holes

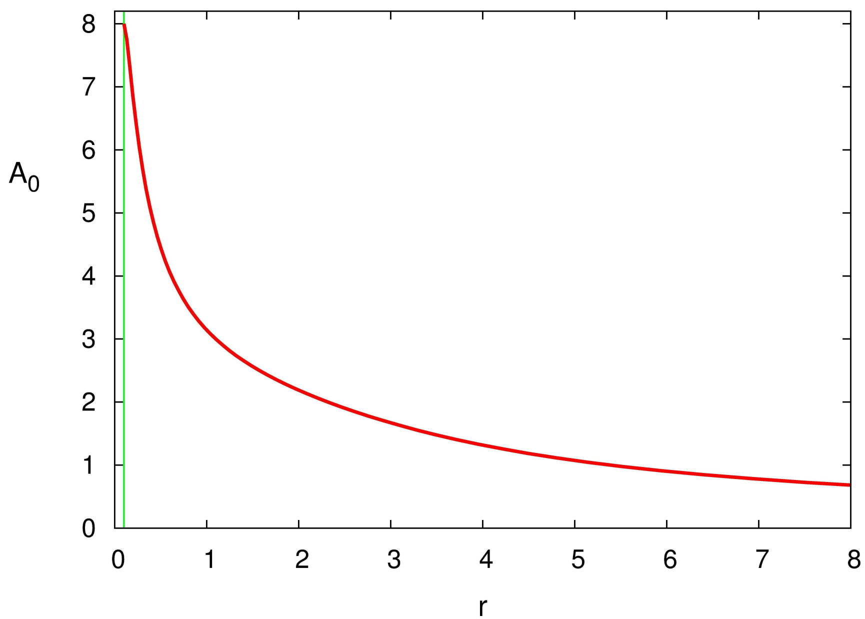
J.P. Hong et al (2020), Herdeiro and Radu (2020): ***RN BHs with charged scalar hair***

$$S = \int d^4x\sqrt{-g}\left\{\frac{R}{16\pi G} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |D_\mu\phi|^2 - V(|\phi|)\right\}; \quad V(|\phi| = \mu^2\phi^2 - \lambda\phi^4 + \beta\phi^6$$

- **Gauge fixing:** $A_0(\infty) = 0$

Resonance condition: $gA_0(r_h) + w = 0$





Kerr black holes with parity odd hairs

(Herdeiro & Radu 2014)

(Kunz, Perapechka & Ya S 2019)

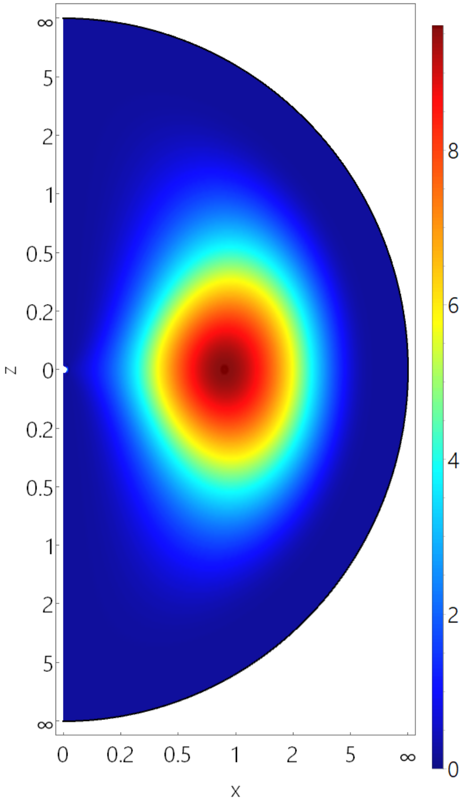
$$\mathcal{L}_m = |\partial_\mu \phi|^2 + \mu^2 |\phi|^2$$

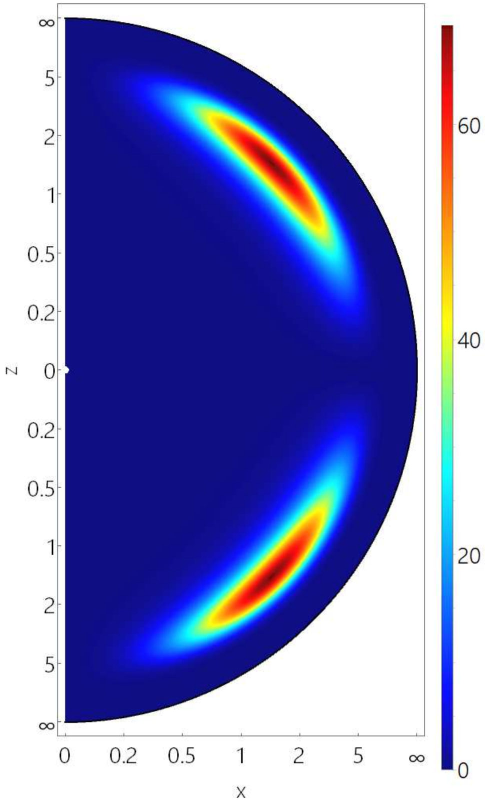
$$(\Box - \mu^2)\phi = 0$$

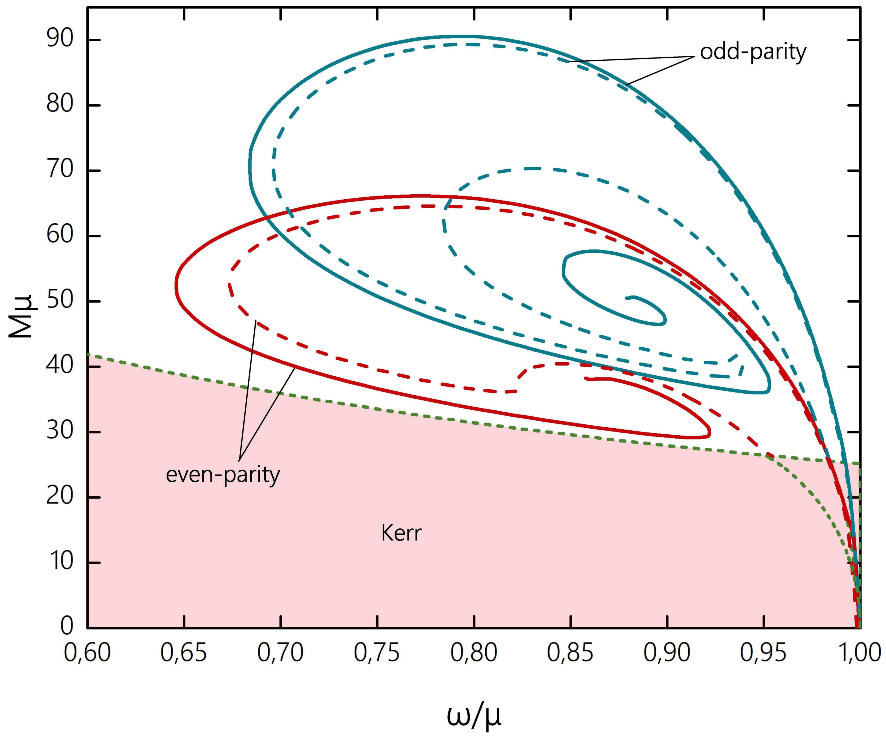
- **U(1) current:** $j_\mu = i(\phi \partial_\mu \phi^* - \phi^* \partial_\mu \phi)$

$$\phi = f(r,\theta) e^{i(\omega t + n\varphi)}$$

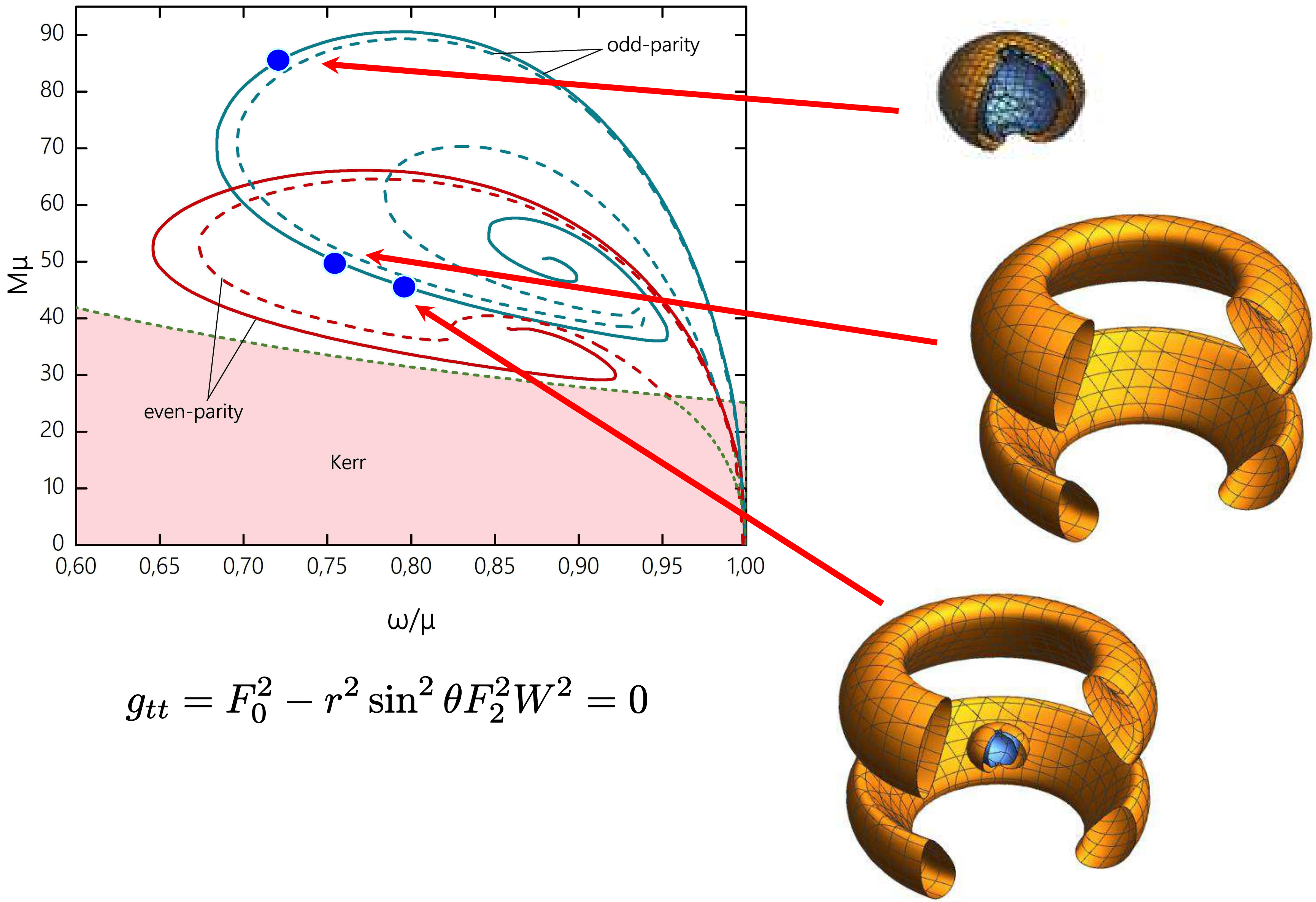
- *Parity-even solutions:* $f(r,\theta) = f(r, \pi - \theta)$
- *Parity-odd solutions:* $f(r,\theta) = -f(r, \pi - \theta)$







Ergosurfaces



$$g_{tt} = F_0^2 - r^2 \sin^2\theta F_2^2 W^2 = 0$$

Boson Stars and hairy BHs in the O(3) sigma-model

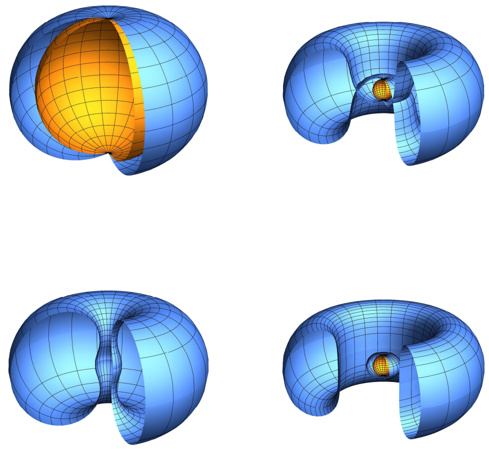
$$S=\int d^4x\sqrt{-g}\left\{\frac{R}{8\pi G}-(\partial_\mu\phi_a)^2-\mu(1-\phi_3)\right\};\quad (\phi_a)^2=1$$

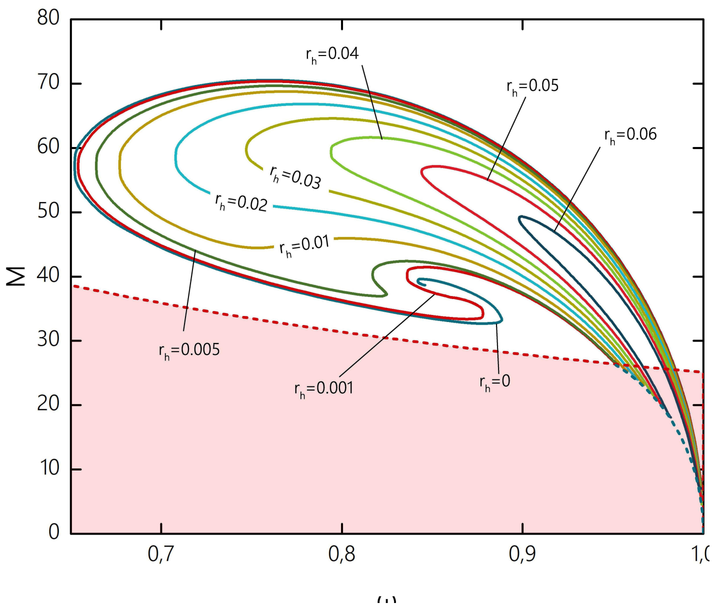
C. Herdeiro, E. Radu, I.Perapechka and Ya.Shnir, JHEP 02 (2019) 111

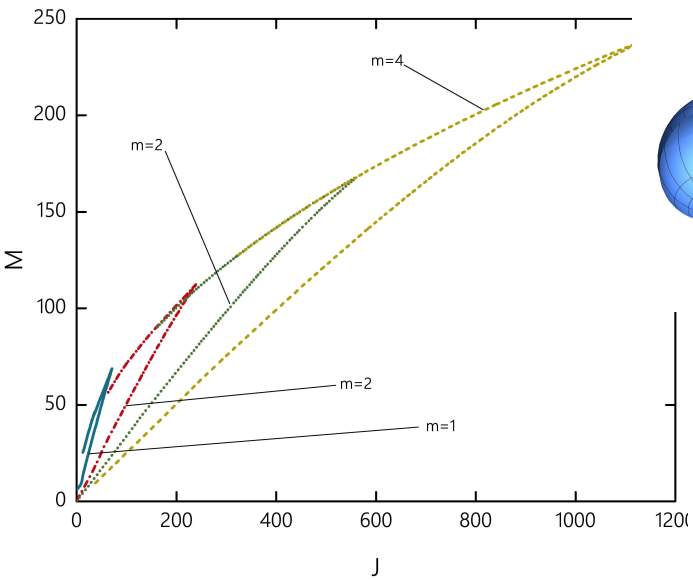
- **Spinning Q-lump:** $\phi_1=\sin f\cos(n\varphi+\omega t);\quad \phi_2=\sin f\sin(n\varphi+\omega t);\quad \phi_3=\cos f$

$$ds^2=-F_0(r,\theta)dt^2+F_1(r,\theta)\left(dr^2+r^2d\theta^2\right)+F_2(r,\theta)r^2\sin^2\theta\left[d\varphi-W(r,\theta)dt\right]^2$$

- **SO(2) current:** $j_\mu=-\phi_1\partial_\mu\phi_2+\phi_2\partial_\mu\phi_1$







Gravitating Skyrmions

$$S = \int \left\{ \frac{R}{\alpha^2} + \mathcal{L}_{Sk} \right\} \sqrt{-g}\, d^4x$$

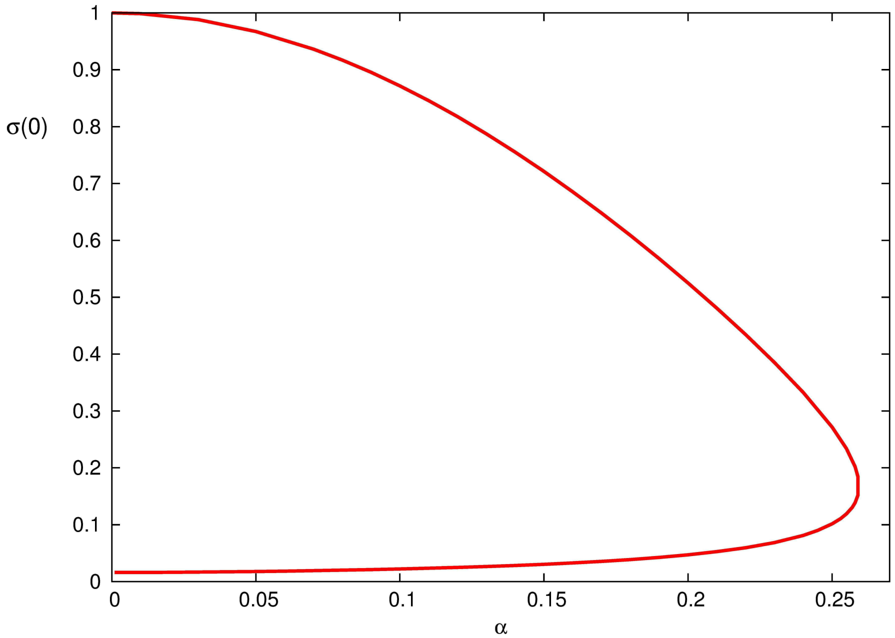
- **The Skyrme field:** $U(\vec{r}, t) \xrightarrow[r\to\infty]{} \mathbb{I}$

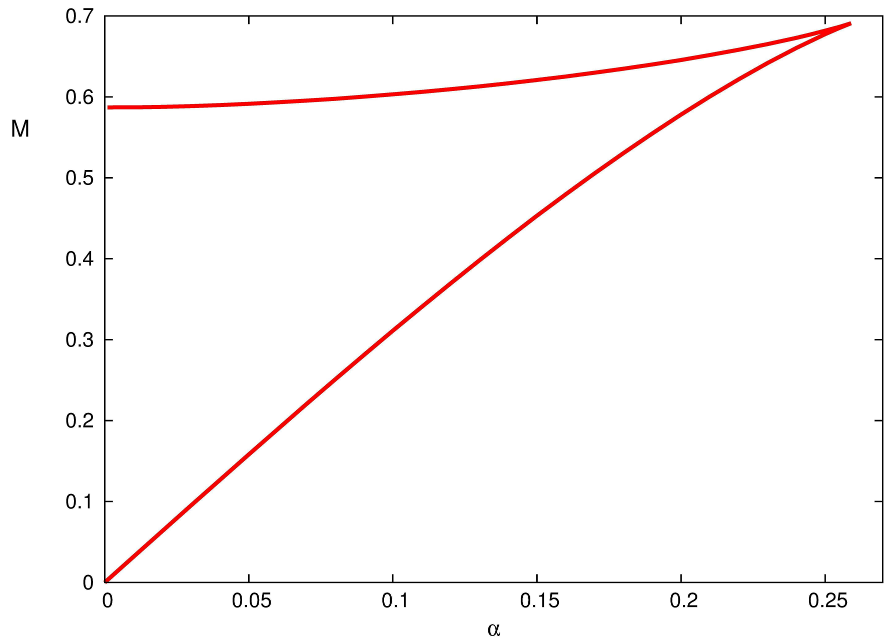
  $U: \; S^3 \to S^3$

Spherical symmetry:

$$ds^2 = -\sigma^2(r) N(r) dt^2 + \frac{1}{N(r)} dr^2 + r^2 (d\theta^2 + \sin^2\theta\, d\phi^2)$$

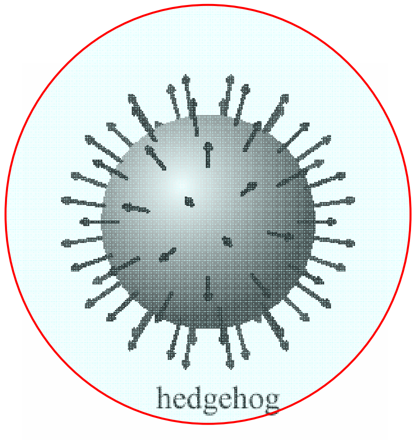
$$\mathcal{L}_{Sk} = \frac{1}{2} \mathrm{Tr}\, \left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{4} \mathrm{Tr}\, \left(\left[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U \right]^2 \right) + \mu^2 \mathrm{Tr}\, \left(U - \mathbb{I}\right)$$





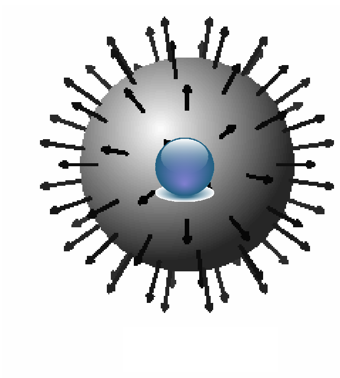
Black holes with skyrmionic hairs

(Luckock and Moss, 1986, Bison 1992, Shiiki and Sawado, 2005)

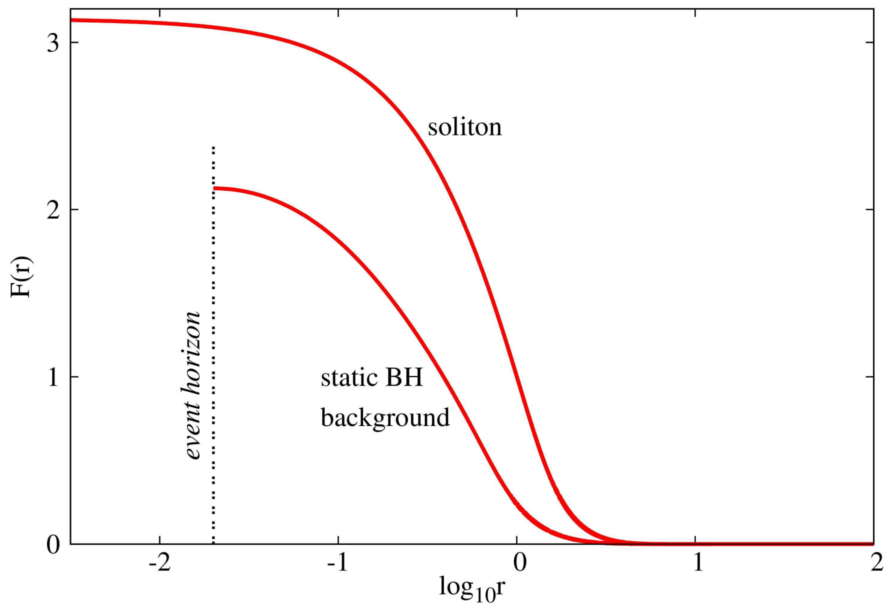


Skyrmion size $R_{Sk} \sim (eF_\pi)^{-1}$ **vs Schwarzschild radius** $R_{Sch} = 2M_{Sk}G;$

$$M_{Sk} \sim F_\pi e^{-1} \longrightarrow R_{Sk} \sim R_{Sch} \text{ as } F_\pi \sim M_{Pl} = G^{-1/2}$$



Hairy black hole – event horizon ***inside*** Skyrmion



Gravitating isospinning Skyrmions

$$U(r) = \sigma + \pi^a \cdot \tau^a$$

T.Ioannidou, B.Kleihaus and J.Kunz
Phys.Lett. B643 (2006) 213

$$\pi_1 = \phi_1 \cos(n\varphi + \omega t); \quad \pi_2 = \phi_1 \sin(n\varphi + \omega t); \quad \pi_3 = \phi_2; \quad \sigma = \phi_3$$

Q=1

Pion clouds:

$$\phi_1 = \sin H(r,\theta); \quad \phi_2 = 0; \quad \phi_3 = \cos H(r,\theta)$$

Q=0

- **Lewis-Papapetrou parametrization:**

$$ds^2 = -f dt^2 + \frac{m}{f}\left(dr^2 + r^2 d\theta^2\right) + l r^2 \sin^2\theta \left(d\varphi - \frac{o}{r} dt\right)^2$$

- **Generalized Einstein-Skyrme model:**

I.Perapechka and Ya.Shnir
Phys.Rev. D96 (2017) 125006

$$S = \int \left\{ \frac{R}{\alpha^2} + \mathcal{L}_{Sk} \right\} \sqrt{-g} d^4x$$

$$\mathcal{L}_{Sk} = L_2 + L_4 + cL_6 + L_0$$

Potential $\mu^2(1-\sigma^2)$

- **Asymptotic expansion:**

$$f \approx 1 - \frac{2MG}{r} + O\left(\frac{1}{r^2}\right), \qquad o \approx -\frac{2JG}{r^2} + O\left(\frac{1}{r^3}\right)$$

Skerrmions

$$U = \sigma\, \mathbb{I} + \tau^a \cdot \pi^a$$

$$\pi^1 + i\pi^2 = \phi^1(r,\theta) e^{i(m\varphi - wt)}, \ \pi^3 = \phi^2(r,\theta), \ \sigma = \phi^3(r,\theta)$$

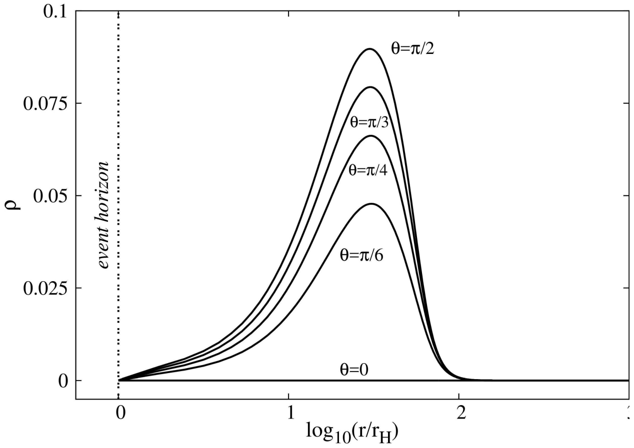
The event horizon angular velocity:

$$\Omega_H = \frac{\sqrt{M_{\rm Kerr}^2 - 4r_H^2}}{2M_{\rm Kerr}(M_{\rm Kerr} + 2r_H)}$$

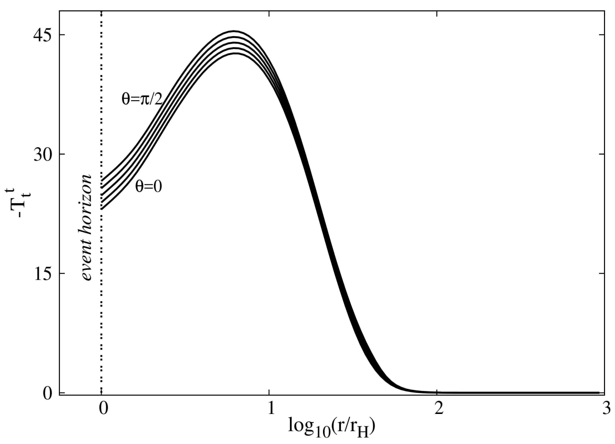
Synchronisation condition: $w = m\Omega_H$

$$L_m = L_2 + L_4$$

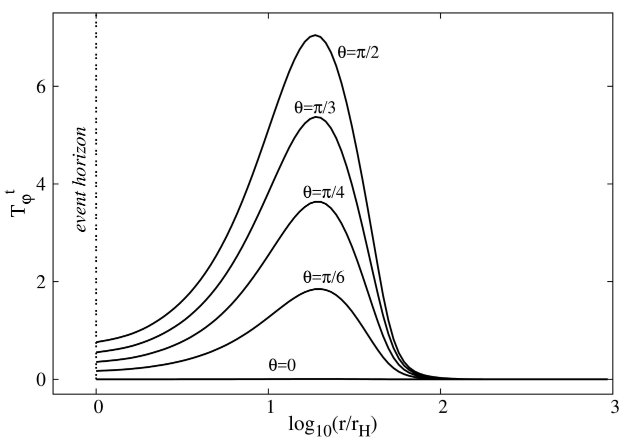
$$\Omega_H = 0.95, \quad M_{\rm Kerr} = 0.04$$



Q density



E density



J density

Skerrmions (Topological sector)

(Herdeiro, Perapechka, Radu & Ya S 2018)

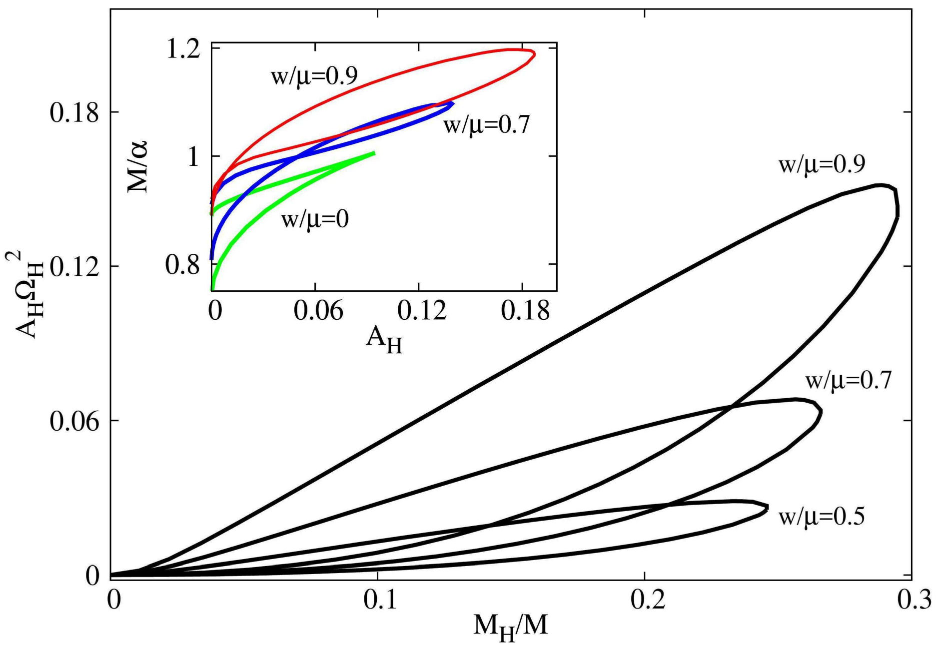
$$S = \int \left\{ \frac{R}{\alpha^2} + \mathcal{L}_{Sk} \right\} \sqrt{-g} d^4x$$

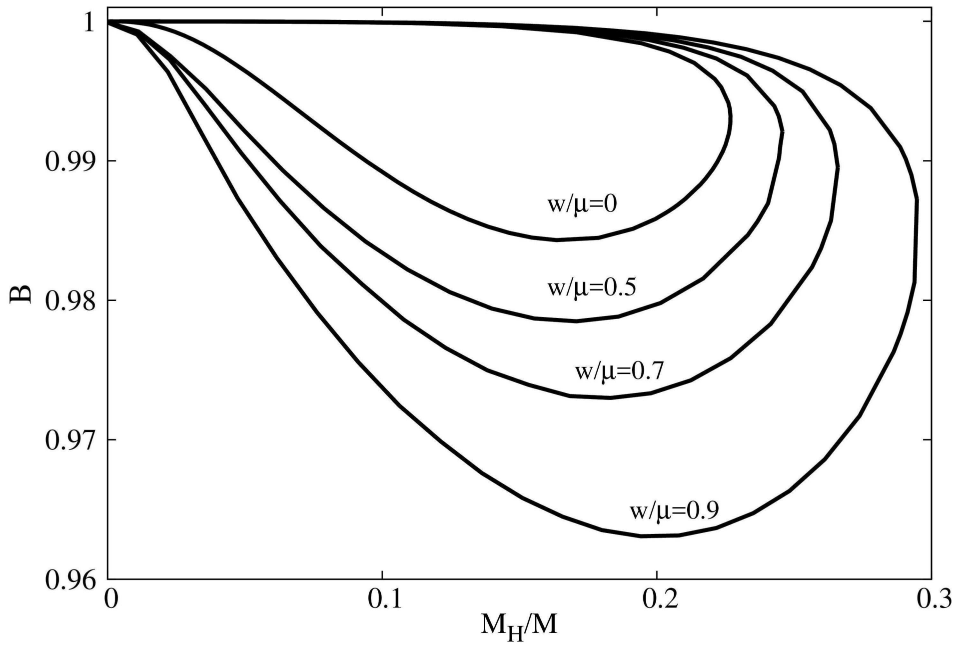
Line element (with backreaction):

$$ds^2 = -F_0(r,\theta)dt^2 + F_1(r,\theta)\left(dr^2 + r^2 d\theta^2\right) + F_2(r,\theta)r^2 \sin^2\theta \left[d\varphi - W(r,\theta)dt\right]^2$$

BH hairiness: $p = M_H/M$

- **p=1 ⟶ Kerr BH**
- **p=0 ⟶ GraviSkyrme**





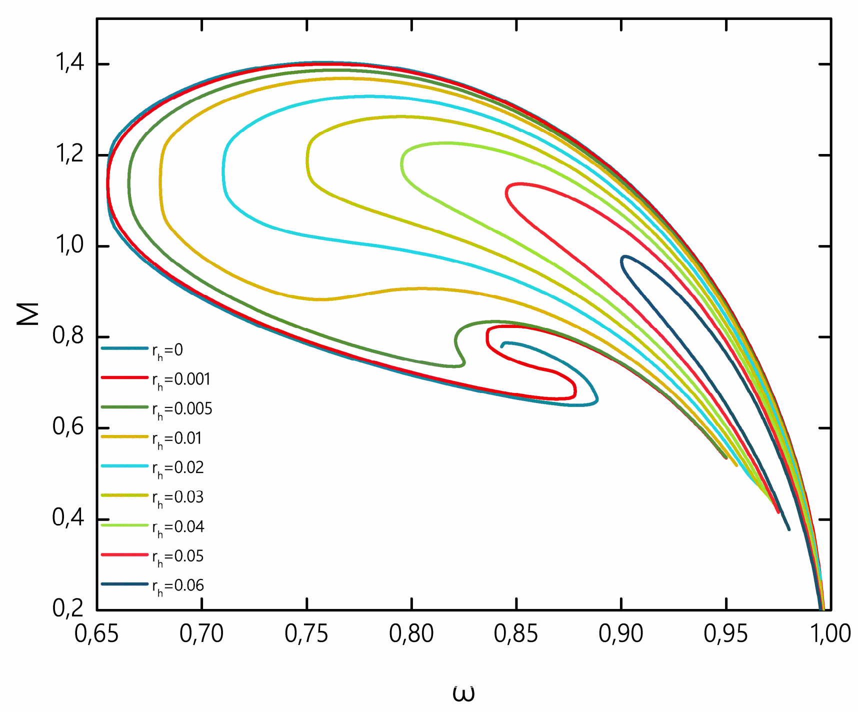
Skerrmions (Pion clouds)

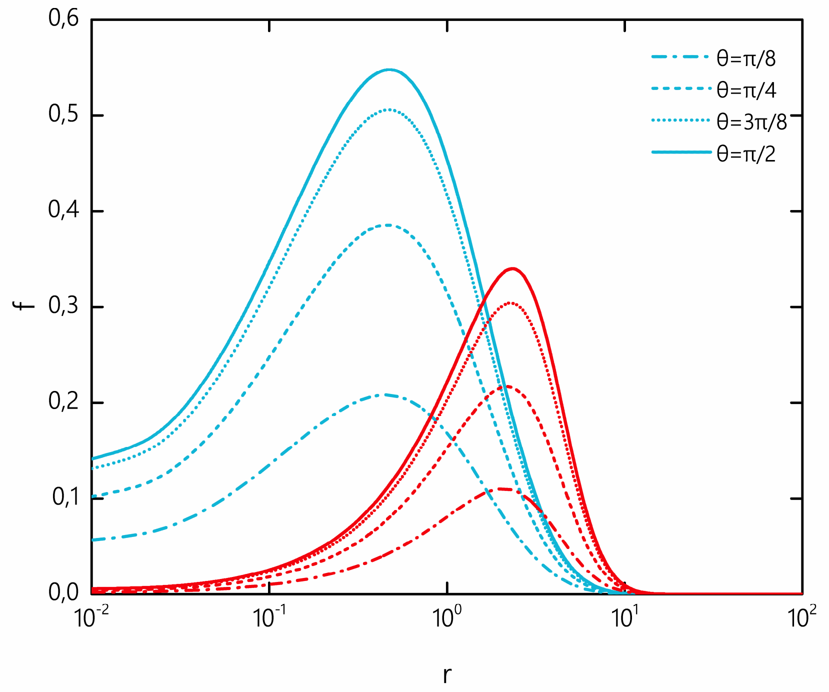
$$U = \sigma\,\mathbb{I} + \tau^a \cdot \pi^a$$

$$\pi^1 + i\pi^2 = \sin f(r,\theta) e^{i(m\varphi - wt)},\ \ \pi^3 = 0,\ \ \sigma = \cos f(r,\theta)$$

- **U(1) Noether charge:** $J = mQ$

$$\alpha = 0.5$$



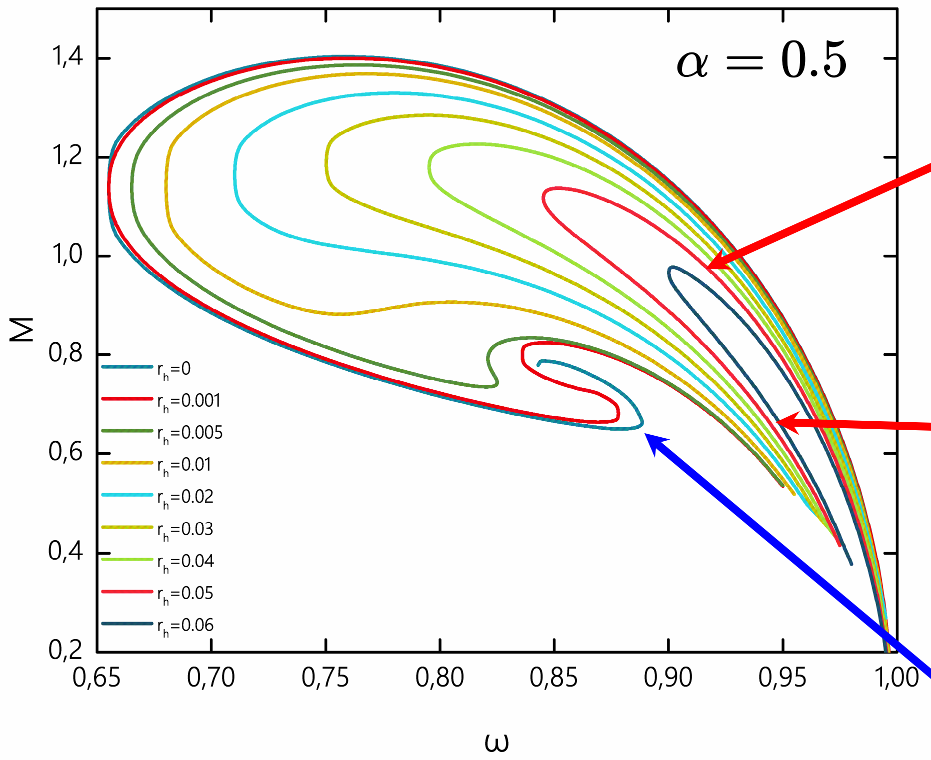


Skerrmions (Pion clouds)

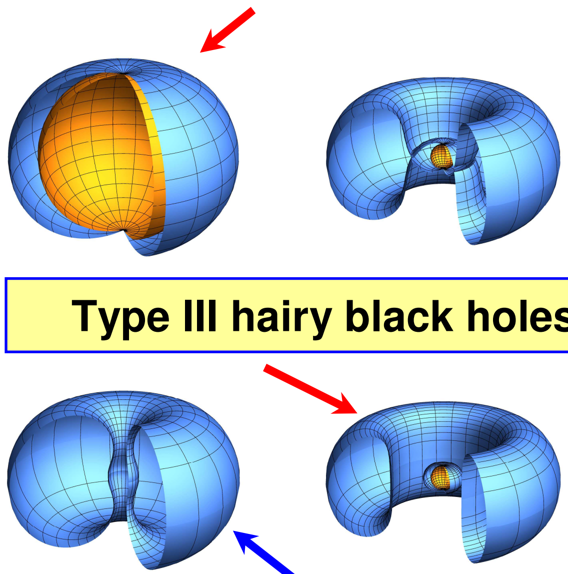
$$U = \sigma\,\mathbb{I} + \tau^a \cdot \pi^a$$

$$\pi^1 + i\pi^2 = \sin f(r,\theta) e^{i(m\varphi - wt)}, \;\; \pi^3 = 0, \;\; \sigma = \cos f(r,\theta)$$

- **Ergosurfaces:** $g_{tt} = F_0^2 - r^2 \sin^2\theta F_2^2 W^2 = 0$



Type I hairy black holes



Type III hairy black holes

Type II boson stars with ergoregion

Dirac stars

$$\mathcal{L}_m = -i\frac{1}{2}\left(\gamma^\mu D_\mu \bar{\Psi}\Psi - \bar{\Psi}\gamma^\mu D_\mu \Psi\right) + \mu\bar{\Psi}\Psi$$

(Herdeiro, Perapechka, Radu & Ya S 2019)

$$D_\mu \Psi = (\partial_\mu - \Gamma_\mu)\Psi$$

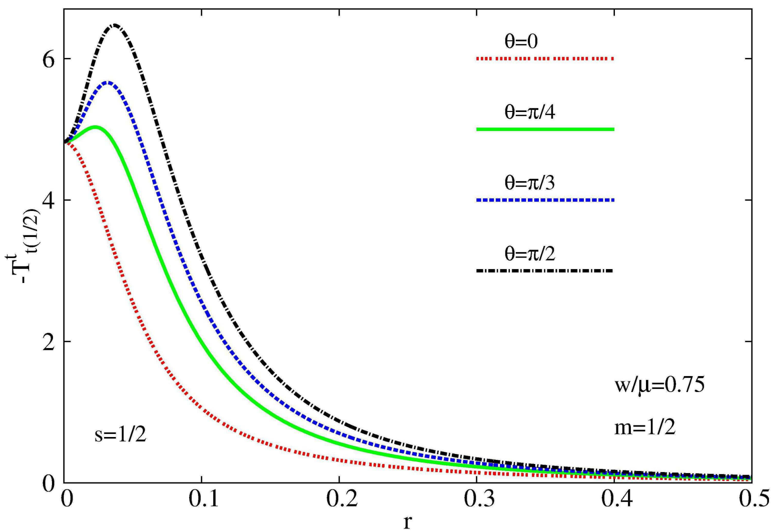
$$ds^2 = \eta_{ab}(\mathbf{e}^a_\mu dx^\mu)(\mathbf{e}^b_\nu dx^\nu)$$

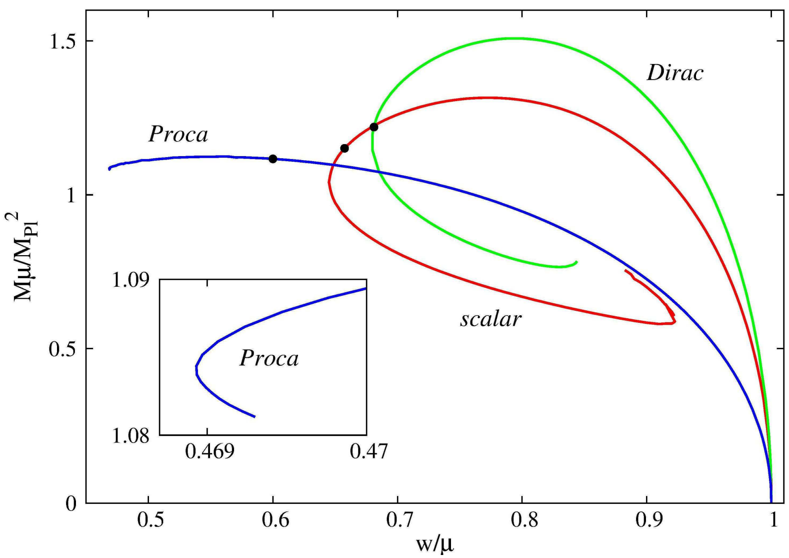
- **Fermionic current:** $j_\mu = \bar{\Psi}\gamma_\mu\Psi$ $\qquad \bar{\Psi} = e^{i(m\varphi - \omega t)}(\psi_1, \psi_2, -i\psi_1^*, -i\psi_2^*)$

- **Metric tetrad:** $\mathbf{e}^0_\mu dx^\mu = e^{F_0} dt\ ,\quad \mathbf{e}^1_\mu dx^\mu = e^{F_1} dr\ ,$

$$\mathbf{e}^2_\mu dx^\mu = e^{F_1} r d\theta\ ,\qquad \mathbf{e}^3_\mu dx^\mu = e^{F_2} r \sin\theta \left(d\varphi - \frac{W}{r} dt\right)$$

$$\gamma^\alpha = \mathbf{e}^\alpha_\mu \gamma^\mu$$





U(1) gauged Dirac stars

(Herdeiro, Perapechka, Radu & Ya S 2022)

$$\mathcal{L}_m = -i\frac{1}{2}\left(\gamma^\mu \mathcal{D}_\mu \bar{\Psi}\Psi - \bar{\Psi}\gamma^\mu \mathcal{D}_\mu \Psi\right) + \mu\bar{\Psi}\Psi$$

$$ds^2 = \eta_{ab}(\mathbf{e}^a_\mu dx^\mu)(\mathbf{e}^b_\nu dx^\nu)$$

$$\gamma^\alpha = \mathbf{e}^\alpha_\mu \gamma^\mu$$

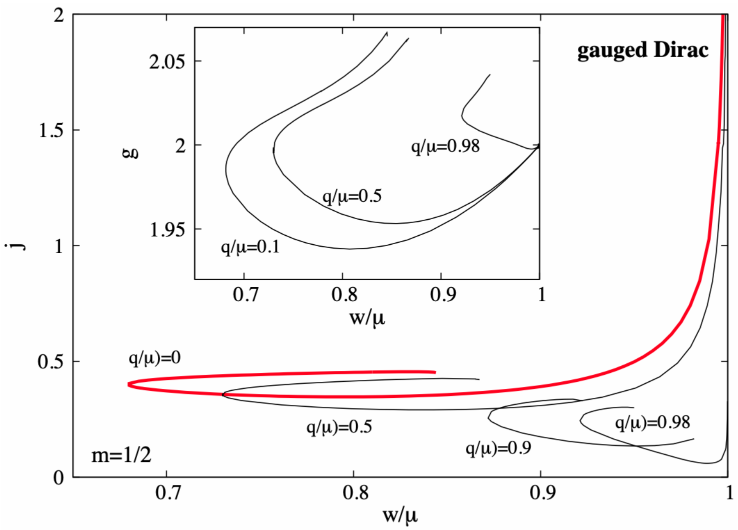
$$S = \int \left\{ \frac{R}{\alpha^2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathcal{L}_m \right\} \sqrt{-g}d^4x$$

$$\mathcal{D}_\mu \Psi = (\partial_\mu - \Gamma_\mu + igA_\mu)\Psi$$

- **Fermionic current:** $j_\mu = \bar{\Psi}\gamma_\mu \Psi$
- **Angular momentum:** $J = mQ$
- **magnetic dipole moment:** $\mu = g\frac{QJ}{2M}$

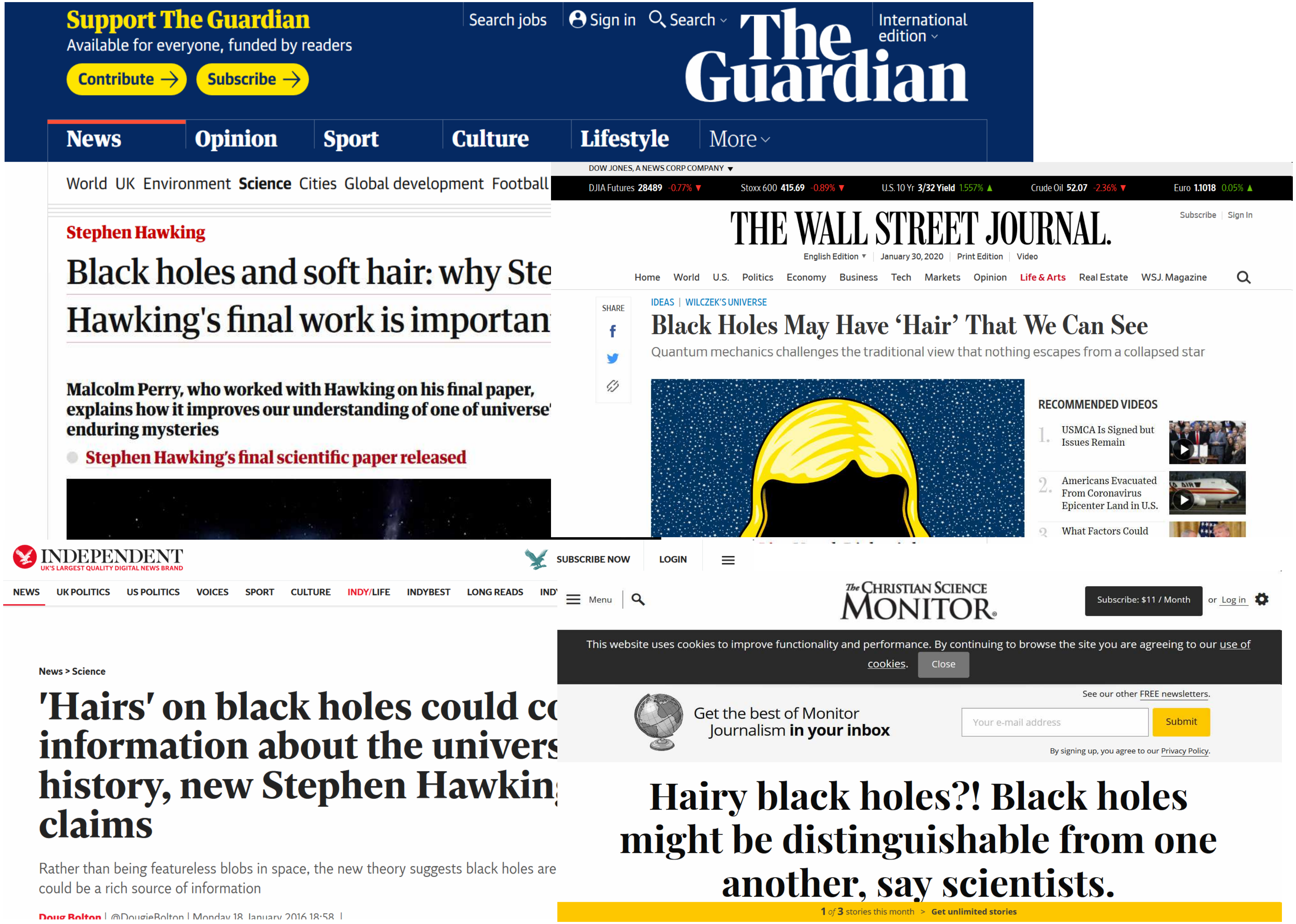
gyromagnetic ratio

One particle condition: Q=1



Summary

- **There are new regular solutions of the EKG model which represent multipolar BSs with a well defined multicomponent structure.**
- **There is certain similarity with multicomponent configurations in the macroscopic Bose-Einstein condensates.**
- **The morphologies of the energy density of the multipolar boson stars is similar to those of the probability density of the hydrogen atomic orbitals.**
- **The hairy black holes are necessarily spinning, the internal rotation (isorotation) must be synchronous with the rotational angular velocity of the event horizon.**
- **We constructed spinning Dirac stars, they possess non-zero angular momentum J=nQ with half-integer n.**
- **U(1) gauged multicomponent boson stars, AdS_4 spacetime, possible link to the flat space solutions, BEC… etc**



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Thank you!

Einstein-deTurck Equations

(M. Headrick, S. Kitchen and T. Wiseman, Class. Quant. Grav. 27 (2010) 035002)

Elliptic Einstein-de Turck equations:

$$R_{\mu\nu} - \nabla_{(\mu}\xi_{\nu)} - \Lambda g_{\mu\nu} = 2\alpha^2 T_{\mu\nu}$$

DeTurck choice of ξ :

$$\xi^\mu = g^{\nu\rho}\left(\Gamma^\mu_{\nu\rho}(g) - \bar{\Gamma}^\mu_{\nu\rho}(\mathbf{g})\right)$$

← Reference metric

Spacetime metric:

$$ds^2 = f_1(r,\theta)\frac{dr^2}{N(r)} + S_1(r,\theta)(rd\theta + S_2(r,\theta)dr)^2$$
$$f_2(r,\theta)r^2\sin^2\theta d\phi^2 - f_0(r,\theta)N(r)dt^2$$

Reference metric:
(e.g. Schwarzschield)

$$ds^2 = \frac{dr^2}{N(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) - N(r)dt^2$$

**Have to verify *a posteriori* that ξ= 0,
to get a solution to Einstein equation**