Workshop *Shape optimization and related topics*

Roscoff, June 13th-17th, 2022

SCHEDULE

	Monday 13	Tuesday 14	Wednesday 15	Thursday 16	Friday 17
9:00	Bucur	Maggi	Oudet	Harbrecht	Polterovich
10:00	Pratelli	Terracini	Masnou	Henrot	Lemenant
11:00	*Coffee Break*	*Coffee Break*	*Coffee Break*	*Coffee Break*	*Coffee Break*
11:30	Paoli	Saracco	Rozanova-Pierrat	Nardulli	Tortone
12:00	Ognibene	Piscitelli	Bevilacqua	Djitte	*Lunch*
12:30	Das	König	Benatti	Pegon	
13:00	*Lunch*	*Lunch*	*Lunch*	*Lunch*	
15:00	Buttazzo	Freitas		Iurlano	
16:00		Bogosel		Carazzato	
16:30	*Coffee break*	*Coffee break*		*Coffee break*	
17:00	Ftouhi	*Poster Session*		Buet	
17:30	Michetti				

- On Tuesday 14th, after the Poster session, a tasting of selected Breton beers will be offered to all participants.
- On Thursday 16th, we will have a special Banquet !
- On Friday 17th, lunch will be served a bit earlier so that people can leave around 1:30pm.

Abstracts

- *L. Benatti*: Minkowski Inequality in Riemannian manifolds with nonnegative Ricci curvature

  The aim of this talk is to present the sharp Minkowski Inequality, proved with M. Fogagnolo and L. Mazzieri. In the setting of complete Riemannian manifold of dimension $n \geq 3$, with nonnegative Ricci curvature and Euclidean Volume Growth we show that the inequality

$$\left(\frac{|\partial\Omega^*|}{|\mathbb{S}^{n-1}|}\right)^{\frac{n-2}{n-1}} \mathrm{AVR}(g)^{\frac{1}{n-1}} \leq \frac{1}{|\mathbb{S}^{n-1}|} \int\limits_{\partial\Omega} \left|\frac{\mathrm{H}}{n-1}\right| \, \mathrm{d}\sigma$$

  holds for every bounded open subset Ω with smooth boundary, where H is the mean curvature of the boundary, Ω^* is the striclty outward minismising hull of Ω and $\mathrm{AVR}(g)$ is the Asymptotic Volume Ratio of (M, g). We also characterise strictly outward minimising subsets $\Omega \subseteq M$ with strictly mean-convex boundary that saturate the inequality.

- *M. van den Berg*: On some variational problems involving capacity, torsional rigidity, perimeter and measure

  We investigate the existence of a maximiser among open, bounded, convex sets in $\mathbb{R}^d$, $d \geq 3$ for the product of torsional rigidity and Newtonian capacity, with constraints involving Lebesgue measure or a combination of Lebesgue measure and perimeter. Joint work with Andrea Malchiodi.

- *G. Bevilacqua*: Critical points for elastic curves: variational and numerical approaches

  We characterize critical points for the elastic curve deriving the first-order necessary conditions applying the infinite dimensional version of the Lagrange multipliers' method. Then, we derive, using analytical and numerical techniques, Euler-Lagrange equations to study a toy model to describe the physics of cellular membranes as biomembrane which spans an elastic component, like the case of the adsorption of a protein.

- *B. Bogosel*: Numerical shape optimization among convex sets

  The optimization of shape functionals under convexity, diameter or constant width constraints is challenging from a numerical point of view. The support function allows to characterize these constraints from a functional point of view. On the practical side, the discretization of the support function can be achieved using a truncated spectral decomposition or by working directly with the values at a uniform angular discretization. The description of these numerical frameworks will be presented, together with various applications from convex geometry and spectral optimization.

- *D. Bucur*: Maximization of Neumann Eigenvalues

  We discuss the maximization of the k-th eigenvalue of the Laplace operator with Neumann boundary conditions among domains of $\mathbb{R}^N$ with prescribed measure. We relax the problem to the class of (possibly degenerate) densities in $\mathbb{R}^N$ with prescribed mass and prove the existence of an optimal density. For $k = 1, 2$ the two problems are equivalent and the maximizers are known to be one and two equal balls, respectively. For $k \geq 3$ this question remains open, except in one dimension of the space where we prove that the maximal densities correspond to a union of k equal segments. This result provides sharp upper bounds for Sturm-Liouville eigenvalues and proves the validity of the Pólya conjecture in the class of densities in $\mathbb{R}$. Based on the relaxed formulation, we provide numerical approximations of optimal densities for $k = 1, \ldots, 8$ in $\mathbb{R}^2$. This is a joint work with E. Martinet and E. Oudet.

- *B. Buet*: A varifold perspective on discrete surfaces

  Joint work with: Gian Paolo Leonardi (Trento), Simon Masnou (Lyon) and Martin Rumpf (Bonn).

  We propose a natural framework for the study of surfaces and their different discretizations based on varifolds. Varifolds have been introduced by Almgren to carry out the study of minimal surfaces. Though mainly used in the context of rectifiable sets, they turn out to be well suited to the study of discrete type objects as well. While the structure of varifold is flexible enough to adapt to both regular and discrete objects, it allows to define variational notions of mean curvature and second fundamental form based on the divergence theorem. Thanks to a regularization of these weak formulations, we propose a notion of discrete curvature (actually a family of discrete curvatures associated with a regularization scale) relying only on the varifold structure. We prove nice convergence properties involving a natural growth assumption: the scale of regularization must be large with respect to the accuracy of the discretization. We performed numerical computations of mean curvature and Gaussian curvature on point clouds in $\mathbb{R}^3$ to illustrate this approach. Building on the explicit expression of approximate mean curvature we propose, we perform mean curvature flow of point cloud varifolds beyond the formation of singularities and we recover well-known soap films.

- *G. Buttazzo*: Variations on a theme by Cheeger

  We study a general version of the Cheeger inequality by considering the shape functional $F_{p,q}(\Omega) = \lambda_p^{1/p}(\Omega)/\lambda_q^{1/q}(\Omega)$, where $\lambda_p(\Omega)$ denotes the principal eigenvalue of the Dirichlet p-Laplacian. The infimum and the supremum of $F_{p,q}$ are studied in the class of *all* domains Ω of $\mathbb{R}^d$ and in the subclass of *convex* domains. In the latter case the issue concerning the existence of an optimal domain for $F_{p,q}$ is deeply discussed.

  Joint work with:
  Luca Briani - Department of Mathematics, University of Pisa, Italy;

Francesca Prinari - Department of Agriculture, Food and Environment, University of Pisa, Italy.

- *D. Carazzato*: A minimization problem with a concave-convex energy profile

  In this talk I will treat a one-dimensional variational problem connected to a Gamow-like functional. Given a relevant one-dimensional profile $\mathcal{E}$ which is concave close to the origin and convex elsewhere, I am interested in the minimization of $\sum_i \mathcal{E}(m_i)$ with $\sum m_i = m$. From the structure of $\mathcal{E}$, I will deduce some strong conditions on the optimal families and a particular dependence on m. This talk is based on a joint work with Aldo Pratelli.

- *U. Das*: On the optimization of the first weighted eigenvalue

  For $N \geq 2$, a bounded smooth domain Ω in $\mathbb{R}^N$, and $g_0, V_0 \in L^1_{loc}(\Omega)$, we study the minimization of the first eigenvalue for the following weighted eigenvalue problem:

  $$-\Delta_p \phi + V|\phi|^{p-2}\phi = \lambda g|\phi|^{p-2}\phi \text{ in } \Omega, \quad \phi = 0 \text{ on } \partial\Omega,$$

  where g and V vary over the rearrangement classes of g_0 and V_0, respectively. We prove the existence of a minimizing pair $(\underline{g}, \underline{V})$ for g_0 and V_0 lying in certain Lebesgue spaces. We will discuss various qualitative properties of the minimizers as well as the associated eigenfunctions for the case $p = 2$.

- *S.M. Djitte*: Fractional Hadamard formula and applications

  We computed the one-sided shape derivative of the best constant in the subcritical fractional Sobolev embedding theorem. In particular, we obtained the nonlocal version of the classical variational Hadamard formula for the first eigenvalue of the Dirichlet Laplacian. As an application, we consider the maximization of the first and second fractional Dirichlet eigenvalues in simply connected domains bounded by two spheres and prove that the maximum is attained in both cases by concentric spheres.

  The talk is based on three papers: "A fractional Hadamard formula and applications" joint work with M.M. Fall (African Institut for Mathematical Sciences in Senegal) and Tobias. Weth (Goethe University, Frankfurt am Main); "Symmetry of odd solutions to equations with the fractional Laplacian" joint work with Sven Jarohs (Goethe University, Frankfurt am Main) and "Nonradiality of second fractional eigenfunctions of thin annuli", joint work with Sven Jarohs.

- *P. Freitas*: The importance of being second

  We consider the effect the second term (or its "absence") in the Weyl asymptotics for the spectrum of the Laplacian has on extremal problems for eigenvalues, Plya's conjecture, and Plya-type inequalities. This will be illustrated by the behaviour in different geometric settings and for different types of boundary conditions. Based on joint work with James Kennedy, and Jing Mao and Isabel Salavessa.

- *I. Ftouhi*: About some new sharp estimates of the Cheeger constant

  In this talk, we present some new sharp estimates of the Cheeger constant via various geometric functionals for the class of planar convex sets. We then show how such inequalities allow to describe the Blaschke–Santaló diagrams involving the functionals under study. Finally, we provide some applications to illustrate the importance of such results. This work is in collaboration with Alba Masiello and Gloria Paoli.

- *H. Harbrecht*: Shape Optimization for Time-dependent Domains

  This talk is concerned with the solution of time-dependent shape optimization problems. Specifically, we consider the heat equation in a domain which might change over time. We compute Hadamard's shape gradient in case of both, domain integrals and boundary integrals. As particular examples, we consider the one-phase Stefan problem and the detection of a time-dependent inclusion. Numerical results are given.

  This is a joint work with Rahel Brügger

- *A. Henrot*: Quantitative isoperimetric inequality with the barycentric asymmetry

  The classical quantitative isoperimetric inequalities usually involve the Fraenkel asymmetry. In this work we study what happens if we replace it by the barycentric asymmetry (the L^1 distance with the ball centered at the barycenter of the set) that is much easier to compute. This will lead us to a problem in the calculus of variations: a Poincaré type inequality with three constraints, that has its own interest. This is a joint work with Chiara Bianchini and Gisella Croce

- *T. König*: Brezis–Peletier revisited – Multibubble analysis near criticality in three dimensions

  For a smooth bounded domain $\Omega \subset \mathbb{R}^3$ and smooth functions a and V, we consider the asymptotic behavior of a sequence of positive solutions u_ϵ to $-\Delta u_\epsilon + (a + \epsilon V)u_\epsilon = u_\epsilon^5$ on Ω with zero Dirichlet boundary conditions, which blow up as $\epsilon \to 0$. We derive the sharp blow-up rate and characterize the location of concentration points in the general case where no a-priori bound on the energy or the number of concentration points is assumed. Our analysis yields a complete picture of blow-up phenomena in dimension N=3, which is well-known to be the most intricate case for Brezis–Nirenberg-type problems.

  This is work in preparation, joint with Paul Laurain (IMJ-PRG Paris and ENS Paris).

- *F. Iurlano*: Shape optimization of light structures and the vanishing mass conjecture

  We present rigorous results about the vanishing-mass limit of the classical problem to find a shape with minimal elastic compliance. Contrary to all previous results in the mathematical literature, which utilize a soft mass constraint by introducing a Lagrange multiplier, we here consider the hard mass constraint. Our results are the first to establish the convergence of approximately optimal shapes of (exact) size $\varepsilon \to 0$ to a limit generalized shape represented by a (possibly diffuse) probability measure. This limit generalized shape is a minimizer of the limit compliance, which involves a new integrand, namely the one conjectured by Bouchitté in 2001 and predicted heuristically before in works of Allaire & Kohn and Kohn & Strang from the 1980s and 1990s. This integrand gives the energy of the limit generalized shape understood as a fine oscillation of (optimal) lower-dimensional structures. Its appearance is surprising since the integrand in the original compliance is just the Euclidean norm and the non-convexity of the problem is not immediately obvious. In fact, it is the interaction of the mass constraint with the requirement of attaining the loading (in the form of a divergence-constraint) that gives rise to this new integrand. Our proofs rest on compensated compactness arguments applied to an explicit family of (symmetric) divquasiconvex quadratic forms, computations involving the Hashin-Shtrikman bounds for the Kohn-Strang integrand, and the characterization of limit minimizers due to Bouchitté & Buttazzo. This is a joint work with J.-F. Babadjian and F. Rindler.

- *A. Lemenant*: A shape optimisation problem with 2 distinct maximizers

  I will talk about a recent work in collaboration with Antoine Henrot and Ilaria Lucardesi, in which we try to maximize the first Neumann eigenvalue with a perimeter constraint, among all convex bodies in the 2d-plane. A quite surprising conjecture says that there should be at least two different maximizers for this problem. We have solved the question (positively) adding a further assumption that the shape admits 2 axis of symetry (not necessarily perpendicular). The proof uses a nice inegality on mu1 that seems to be new.

- *F. Maggi*: A mesoscale flatness criterion and its application to exterior isoperimetry

  We introduce a "mesoscale flatness criterion" for hypersurfaces with bounded mean curvature, discussing its relation and its differences with classical blow-up and blow-down theorems, and then we exploit this tool for a complete resolution of relative isoperimetric sets with large volume in the exterior of a compact obstacle. This is joint work with Michael Novack at UT Austin.

- *S. Masnou*: Learning mean curvature flows with neural networks

  The mean curvature flow is an emblematic geometric flow which is very naturally related to various numerical and physical applications, e.g. in image / data processing or in material sciences. The talk will be devoted to new, effective numerical methods based on neural networks for the approximation of the mean curvature flow of either oriented or non-orientable surfaces. To learn the correct interface evolution law, the neural networks are trained on phase field representations of exact evolving interfaces. The structure of the networks draws inspiration from splitting schemes used for the discretization of the Allen-Cahn equation. But when the latter approximates the mean curvature motion of oriented interfaces only, the proposed approach extends very naturally to the non-orientable case. In addition, although trained on smooth flows only, the proposed networks can handle singularities as well. Furthermore, they can be coupled easily with additional constraints. Various applications will be shown to illustrate the flexibility and effectivity of our approach: mean curvature flows with volume constraint, multiphase mean curvature flows, numerical approximation of Steiner trees, numerical approximation of minimal surfaces.

- *M. Michetti*: Optimization of Strum-Liouville eigenvalues and applications

  In this talk, we present results concerning optimization problems for (non uniformly elliptic) Strum-Liouville eigenvalues, using abstract optimality conditions we will explicitly solve the maximization problem. As an a application of these results we study the Neumann eigenvalues under diameter constraint. This is a joint work with Prof. Antoine Henrot.

- *S. Nardulli*: Multiplicity of solutions to the multiphasic Allen-Cahn-Hilliard system with a small volume constraint on closed parallelizable manifolds

  We prove the existence of multiple solutions to the Allen–Cahn–Hilliard (ACH) vectorial equation involving a multi-well (multiphasic) potential with a small volume constraint on a closed parallelizable Riemannian manifold. More precisely, we find a lower bound for the number of solutions depending on topological invariants of the underlying manifold. The phase transition potential is considered to have a finite set of global minima, where it also vanishes, and a subcritical growth at infinity. Our strategy is to employ the Lusternik–Schnirelmann and infinite-dimensional Morse theories for the vectorial energy functional. To this end, we exploit that the associated ACH energy Γ-converges to the vectorial perimeter for clusters, which combined with some deep theorems from isoperimetric theory yields the suitable setup to apply the photography method. Along the way, the lack of a closed analytic expression for the Euclidean multi-isoperimetric function for clusters imposes a delicate issue. Furthermore, using a transversality theorem, we also show the genericity of the set of metrics for which solutions to our geometric system are nondegenerate.

- *R. Ognibene*: Sharp spectral stability under perturbation of the boundary conditions

  In this talk I will discuss the behavior of the spectrum of the Laplacian on bounded domains, subject to varying mixed boundary conditions. More precisely, let us assume the boundary of the domain to be split into two parts, on which homogeneous Neumann and Dirichlet boundary conditions are respectively prescribed; let us then assume that, alternately, one of these regions "disappears" and the other one tends to cover the whole boundary. In this framework, I will first describe under which conditions the eigenvalues of the mixed problem converge to the ones of the limit problem (where a single kind of boundary condition is imposed); then, I will sharply quantify the rate of this convergence by providing an explicit first-order asymptotic expansion of the "perturbed" eigenvalues. These results have been obtained in collaboration with Veronica Felli and Benedetta Noris.

- *E. Oudet*: TBA

- *G. Paoli*: Sharp and quantitative estimates for the p-Torsion of convex sets

  Let $\Omega \subset \mathbb{R}^n$, $n \geq 2$, be an open, bounded and convex set and let f be a positive and non-increasing function depending only on the distance from the boundary of Ω. We consider the $p-$torsional rigidity associated to Ω for the Poisson problem with Dirichlet boundary conditions, denoted by $T_{f,p}(\Omega)$. Firstly, we prove a Pólya type lower bound for $T_{f,p}(\Omega)$ in any dimension; then, we consider the planar case and we provide two quantitative estimates in the case $f \equiv 1$.
  This is a joint work with V. Amato, A.L. Masielllo and R. Sannipoli

- *M. Pegon*: An isoperimetric problem involving the competition between the perimeter and a nonlocal perimeter

  In this talk, I will present an isoperimetric problem in which the perimeter is replaced by the difference between the classical perimeter and a nonlocal energy P_ε which converges to a fraction of the perimeter when ε vanishes. This problem is derived from Gamow's liquid drop model for the atomic nucleus in the case where the repulsive potential is sufficiently decaying at infinity and in the large mass regime. I will discuss the existence, and characterization of minimizers for small ε. This is joint work with Michael Goldman and Benoît Merlet.

- *G. Piscitelli*: Sharp estimates for the first Robin eigenvalue of nonlinear elliptic operators

  The aim of this paper is to obtain optimal estimates for the first Robin eigenvalue of the anisotropic p-Laplace operator, namely:

$$\lambda_1(\beta,\Omega) = \min_{\psi \in W^{1,p}(\Omega)\setminus\{0\}} \frac{\displaystyle\int_\Omega F(\nabla\psi)^p dx + \beta \int_{\partial\Omega} |\psi|^p F(\nu_\Omega) d\mathcal{H}^{N-1}}{\displaystyle\int_\Omega |\psi|^p dx},$$

  where $p \in]1,+\infty[$, Ω is a bounded, mean convex domain in $\mathbb{R}^N$, ν_Ω is its Euclidean outward normal, β is a real number, and F is a sufficiently smooth norm on $\mathbb{R}^N$. The estimates we found are in terms of the first eigenvalue of a one-dimensional nonlinear problem, which depends on β and on geometrical quantities associated to Ω. More precisely, we prove a lower bound of λ_1 in the case $\beta > 0$, and a upper bound in the case $\beta < 0$. As a consequence, we prove, for $\beta > 0$, a lower bound for $\lambda_1(\beta,\Omega)$ in terms of the anisotropic inradius of Ω and, for $\beta < 0$, an upper bound of $\lambda_1(\beta,\Omega)$ in terms of β. Based on a joint work with Francesco Della Pietra.

- *I. Polterovich*: Stability of isoperimetric eigenvalue inequalities on surfaces

Optimisation of Laplace eigenvalues on Riemannian manifolds is a fascinating topic in spectral geometry. In the past decade, significant progress has been achieved on maximisation of eigenvalues on surfaces under the area constraint. I will discuss some recent advances on this subject, with an emphasis on the stability estimates for sharp isoperimetric inequalities. The talk is based on a joint work with M. Karpukhin, M. Nahon and D. Stern.

- *A. Pratelli*: Minimization problems for general energies with attractive-repulsive behaviour

  The celebrated liquid drop model by Gamow is one of the oldest and most studied energies of attractive-repulsive type, and it has gathered a huge interest among physicists and mathematicians. In the last years several generalisations of the model have been studied, and now many important properties are known, though still some fundamental questions are open, even in the original model. In addition, people have started to consider the minimization in the class of L^1 positive functions, instead than in the class of sets. In this talk, we will describe the main features of the problem, and we will concentrate ourselves in the even more general case of minimization among positive measures, already considered by some authors but largely open. We will briefly present some properties, proven very recently, and some open questions. Some of the results have been proved in collaboration with Carazzato, Fusco, Novaga.

- *A. Rozanova-Pierrat*: Existence of optimal shapes in linear acoustics in the Lipschitz and non-Lipschitz classes

  To find the most efficient shape of a noise-absorbing wall to dissipate the acoustical energy of a sound wave, we consider a frequency model described by the Helmholtz equation with damping on the boundary modeled by a complex-valued Robin boundary condition. We introduce a class of admissible Lipschitz boundaries, in which an optimal shape of the wall ensures the infimum of the acoustic energy. Then we also introduce a larger compact class of (ε, ∞)- or uniform domains with possibly non-Lipschitz (for example, fractal) boundaries on which an optimal shape exists, giving the minimum of the energy. The boundaries are described as the supports of Radon measures ensuring their Hausdorff dimension in the segment $[n-1, n)$. A by-product of our proof is that the class of bounded (ε, ∞)-domains with fixed ε is stable under Hausdorff convergence. Another related result is the Mosco convergence of Robin-type energy functionals on converging domains.

- *G. Saracco*: Isoperimetric sets, sets with prescribed curvature, and p-Cheeger sets: three faces of the same coin

  Let $\Omega \subset \mathbb{R}^2$ be a planar, open set. The isoperimetric problem in Ω amounts to determining the sets minimizing

  $$\mathcal{J}(V) := \inf\{\, P(E) \,:\, E \subset \Omega,\, |E| = V \,\},$$

for $V \leq |\Omega|$. Denoting by R the radius of the greatest ball contained in Ω, the characterization of minimizers is trivial for $V \leq \pi R^2$.

For a rather large class of planar sets, containing for instance all convex sets, Leonardi and myself ("The isoperimetric problem in 2d domains without necks" *Calc. Var. Partial Differential Equations*, 61(2):56, 2022) proved a fine geometric characterization of minimizers for volumes above πR^2, using the dual problem of the prescribed curvature functional

$$\mathcal{F}(\kappa) := \inf\{ P(E) - \kappa|E| \ : \ E \subset \Omega, \ |E| \geq \pi\kappa^{-2}\},$$

for curvatures $\kappa \geq R^{-1}$. Jointly with Caroccia ("Relations between p-Cheeger sets, isoperimetric sets, and sets with prescribed curvature", forthcoming), we showed a further, alternative approach, that is, to consider the p-Cheeger problem

$$h(p) := \inf \left\{ \frac{P(E)}{|E|^p} \ : \ E \subset \Omega, \ |E| > 0 \right\},$$

for powers $p \geq 1/2$.

Under some geometrical assumptions, we can show that volumes $V \geq \pi R^2$ and curvatures $\kappa \geq R^{-1}$ are in bijection; powers $1/2 \leq p \leq 1$ are as well with an interval of volumes and curvatures. It is object of current investigation whether, under some suitable condition, such a bijection can be extended to values $p > 1$.

- *S. Terracini*: Rotating Spirals in segregated reaction-diffusion systems

  We give a complete characterization of the boundary traces φ_i ($i = 1, \ldots, K$) supporting spiraling waves, rotating with a given angular speed ω, which appear as singular limits of competition-diffusion systems of the type

$$\begin{cases} \partial_t u_i - \Delta u_i = \mu u_i - \beta u_i \sum_{j \neq i} a_{ij} u_j & \text{in } \Omega \times \mathbb{R}^+ \\ u_i = \varphi_i & \text{on } \partial\Omega \times \mathbb{R}^+ \\ u_i(\mathbf{x}, 0) = u_{i,0}(\mathbf{x}) & \text{for } \mathbf{x} \in \Omega \end{cases}$$

  as $\beta \to +\infty$. Here Ω is a rotationally invariant planar set and $a_{ij} > 0$ for every i and j. We tackle also the homogeneous Dirichlet and Neumann boundary conditions, as well as entire solutions in the plane. As a byproduct of our analysis we detect explicit families of eternal, entire solutions of the pure heat equation, parameterized by $\omega \in \mathbb{R}$, which reduce to homogeneous harmonic polynomials for $\omega = 0$.

  It is a joint work with A. Salort, G. Verzini and A. Zilio

- *G. Tortone*: Liouville Theorems and Optimal regularity in elliptic equations

  The objective of this talk is the connection between the problem of optimal regularity among solutions to elliptic divergence equations with measurable coefficients with the Liouville property at infinity. Initially, we address the two-dimensional case by proving an Alt-Caffarelli-Friedman type monotonicity formula and then we discuss the higher-dimensional problem by focusing on the role of the monotonicity formula in the characterization of the least growth at infinity and the exponent of regularity as well. Finally, we establish this connection by combining blow-up and G-convergence argument.

Poster Session

- *D.G. Afonso*: Partially overdetermined problems in general containers: first steps

  A generalization of Serrin's classical overdetermined problem consists in considering domains Ω inside some unbounded set $\mathcal{C}$ of $\mathbb{R}^N$ and prescribing both Dirichlet and Neumann conditions only on the part of $\partial\Omega$ inside $\mathcal{C}$. While the case when $\mathcal{C}$ is a cone has been widely studied, this poster presents some partial general results. A remarkable point is that a notion of relative (to $\mathcal{C}$) Cheeger set of Ω appears, so we also introduce this concept and show some related results. This is the theme of my ongoing PhD and joint work with Alessandro Iacopetti and Filomena Pacella.

- *F. Bianchi*: The fractional Makai-Hayman inequality

  We consider the classical Makai-Hayman inequality for the first eigenvalue of the Dirichlet-Laplacian and we discuss its extension to the case of fractional Sobolev spaces. In particular, we ask if there exists a lower bound for the first eigenvalue of the fractional Dirichlet-Laplacian $(-\Delta)^s$ in terms of the inradius for the class of planar simply connected sets. We also discuss the dependence of such an estimate with respect to s and show that this is optimal, in a sense. Some of the results presented are contained in a paper in collaboration with Lorenzo Brasco.

- *M. Ghosh*: Monotonicity of the first Dirichlet eigenvalue of the p-Laplacian w.r. to dihedral symmetry

  Let $\Omega = B \setminus \overline{P}$, where $B \subset \mathbb{R}^2$ is an open disk, and $P \subset B$ is a domain having dihedral symmetry ($\mathbb{D}_n$-symmetry) for some $n \geq 2,\ n \in \mathbb{N}$. We established that among all the rotations of P w.r. to its center, the first Dirichlet eigenvalue $\lambda_1(\Omega)$ of the p-Laplacian is optimal when Ω is symmetric with respect to the line containing both the centers of B and P. We also deduce monotonicity properties of $\lambda_1(\Omega)$ w.r. to rotations of P, and as a consequence, we show that if the nodal set of a second eigenfunction of the p-Laplacian possesses a dihedral symmetry (of the same order), then it can not enclose P.

- *R. Resende*: Density of the boundary regular set of 2d area minimizing currents with arbitrary codimension and multiplicity

  In the present work, we consider area minimizing currents in the general setting of arbitrary codimension and arbitrary boundary multiplicity. We study the boundary regularity of 2d area minimizing currents, beyond that, several results are stated in the more general context of (C_0, α_0, r_0)-almost area minimizing currents of arbitrary dimension m and arbitrary codimension taking the boundary with arbitrary multiplicity. Furthermore, we do not consider any type of convex barrier assumption on the boundary, in our main regularity result which states that the regular set, which includes one-sided and two-sided points, of any 2d area minimizing current T is an open dense set in Γ.

- *C. Tajani*: Application of metaheuristic Algorithms for reconstruction of missing boundary conditions

  In this study, we are interested in the data completion problem for Laplace's equation which consists of reconstructing the unknown data on the inaccessible part, which cannot be evaluated because of physical difficulties or geometric inaccessibility, from measurements on the accessible part of the boundary. The considered inverse problem is known to be severely ill-posed in Hadamard sense, since the existence or uniqueness or the continuous dependence on the data of their solutions may not be ensured. Therefore, the inverse problem is formulated as an optimization one adding a regularization term to the functional, then, several metaheuristic optimization methods including the genetic algorithm, particle swarm optimization and bat algorithm are explored to solve the considered problem. The efficiency and accuracy of the proposed methods are assessed by their ability of finding and reconstructing of the results. The effects of some metaheuristic parameters on the optimum solutions are examined.

- *Y. Teplitskaya*: About maximal distance minimizers

  A *maximal distance minimizer* for a given compact set $M \subset \mathbb{R}^n$ and a given positive real r is a connected set Σ of minimal length (one-dimensional Hausdorff measure) such that $M \subset \overline{B_r(\Sigma)}$ (i.e. every point from M has a point from Σ in its closed r-neighbourhood).

  It appears that for every compact set M a maximal distance minimizer Σ should be 'good': for every point $x \in \Sigma$ there exists one, two or three tangent rays and the pairwise angles between these rays are at least $2\pi/3$. Moreover for a planar set M a maximal distance minimizer Σ has a finite number of points with three tangent rays (a non-planar case remains open problem).

  We also are looking for the solutions for the certain set M. We know maximal distance minimizers for a "big" circle (or other sets with "'big" radius of curvature) and we know the topology of maximal distance minimizers for a "big" rectangle.

- *A. C. Zagati*: A *comparison principle* for Lane-Emden equation: applications to geometric estimates

  We introduce a comparison principle for positive supersolutions and subsolutions to the Lane-Emden equation for the $p-$Laplacian. Then we apply such a comparison principle to obtain a variety of results, as sharp pointwise double-side estimates for positive solutions in convex sets, a "hierarchy" of sign-changing solutions of the equation and a sharp geometric estimate on the Sobolev-Poincaré constants of convex sets. The results are obtained in collaboration with Lorenzo Brasco (Ferrara) and Francesca Agnese Prinari (Pisa).