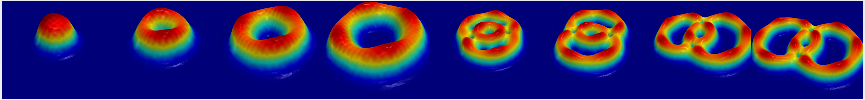
Quartic bosonic metallic state and its topological properties

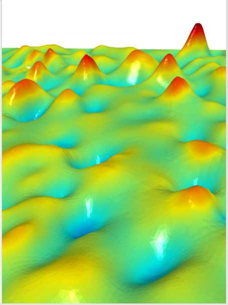
Properties of the state that breaks the time-reversal symmetry above superconducting phase transition

Julien Garaud
with Egor Babaev and Vadim Grinenko, *et al.*

Tours – March 3, 2022



Reported observation of a new state of matter in $Ba_{1-x}K_xFe_2As_2$



JG, and E. Babaev
Skyrmions and effective model of the resistive electron quadrupling state ,
Preprint (2021) [cond-mat.supr-con] arXiv:2112.01286

Grinenko, Weston, Caglieris, Wuttke, Hess, Gottschall, Maccari, Gorbunov, Zherlitsyn, Wosnitza, Rydh, Kihou, Lee, Sarkar, Dengre, **JG**, Charnukha, Hühne, Nielsch, Büchner, Klauss, and Babaev
State with spontaneously broken time-reversal symmetry above superconducting phase transition
Nature Physics **17**, 1254 (2021) [cond-mat.supr-con] arXiv:2103.17190

Reported observation of a new state of matter in $Ba_{1-x}K_xFe_2As_2$

Quartic bosonic metal: is a quantum ordered state

- which appears in a superconducting material above the critical temperature
- it shows no coherence in electronic Cooper pairs thus it is resistive
- but retains some order in pairs of pairs

Unlike the superconducting state which breaks the $\mathrm{U}(1)$ gauge symmetry

- Quartic metal spontaneously breaks the time-reversal symmetry ($\mathbb{Z}_2$)
- but here the fluctuations restored the $\mathrm{U}(1)$ gauge symmetry

Outline

- Generalities about superconductors, multi-component systems and their phases
- Time-reversal symmetry breaking mechanism and quartic metal phase
- Effective Faddeev-Skyrme theory and its topological excitations

Outline

Superconductivity – Generalities

Conventional mechanism [Bardeen, Cooper, Schrieffer 1957]

- in a metal Fermi sphere of occupied states
- states near Fermi surface can interact via phonons

Electron-phonon interaction scattering mediates attraction

- e^- moves in a potential and excites a phonon
- later absorbed by another e^-
- small attraction between electrons causes (bound) paired state with opposite momenta (Cooper pair)

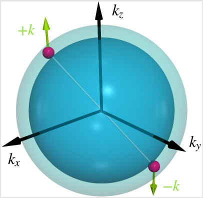
Cooper pairs are bosons

- they can undergo Bose-Einstein condensation
- macroscopic occupation of the zero momenta states

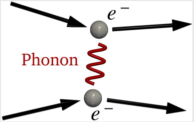
At mean field, one single macroscopic wave function

Ginzburg-Landau: effective classical mean field theory near T_c

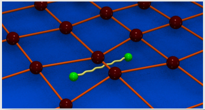
Fermi sphere



Electron-phonon



Cooper pairs



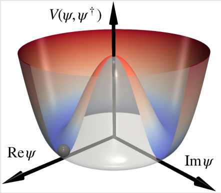
Superconductivity – Properties & Ginzburg-Landau

$$E = \int_{\mathbb{R}^3} |\nabla \times \boldsymbol{A}|^2 + D_\mu \Psi^* D^\mu \Psi + \kappa(|\Psi|^2 - 1)^2 \,, \text{ with } D_\mu = \nabla_\mu - iA_\mu$$

Classical field theory

- at the mean field level, one macroscopic wave function (density of Cooper pairs), the gap function $\Psi = |\Psi| e^{i\varphi}$
- Ψ: charged bosonic scalar field; $\boldsymbol{A}$ gauge field (photon)
- longitudinal component of the photon becomes massive
- Anderson-Higgs mechanism [Anderson 1962; Higgs 1964]

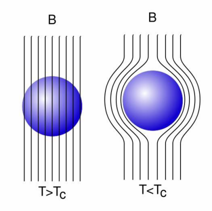
Broken $U(1)$



Properties of superconductors

- dissipationless current
- perfect diamagnetism (Meissner effect): $\boldsymbol{B}$ is screened by the superflow of Cooper pairs $\boldsymbol{J} = 2e|\Psi|^2(\nabla\varphi + \boldsymbol{A})$
- Massive photon $\Rightarrow$ London eq.: $\lambda \nabla \times \nabla \times \boldsymbol{B} = \boldsymbol{B}$ (Proca)
- Quantized flux $\Phi = \frac{\Phi_0}{2\pi} \oint \nabla\varphi \boldsymbol{d}\ell = n\Phi_0$ and $n \in \pi_1(S^1) = \mathbb{Z}$
- $\Rightarrow$ vortices [London 1948; Onsager 1949; Abrikosov 1957]

Meissner effect



Superconductors beyond mean-field

Superfluid/Superconducting state in the mean field

SC density $|\Psi|^2 \neq 0$ and massive gauge field $m_{\boldsymbol{A}} = e|\Psi|$

Superconducting states beyond mean field

$$Z = \int \mathcal{D}\Psi\mathcal{D}\boldsymbol{A} \exp\left\{ -\beta E_{\mathrm{MF}}(\Psi, \boldsymbol{A})\right\}$$

- Superconducting: ordered $\langle\Psi\rangle \neq 0$
- Normal: disordered $\langle\Psi\rangle = 0$

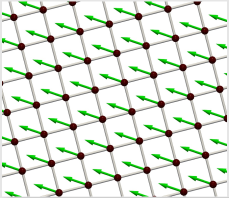
The symmetry restoration is understood as a (thermal) proliferation of topological defects (vortices)

Transitions between (dis)ordered states are measured by

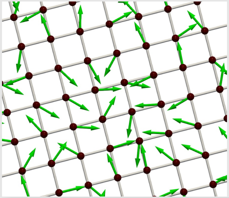
the helicity modulus/spin stiffness $\Upsilon = \left.\frac{\partial^2 E_0(\varphi)}{\partial\varphi^2}\right|_{\varphi=0}$

and gauge field correlators (dual stiffness)

Ordered: $\langle\psi\rangle \neq 0$



Disordered: $\langle\psi\rangle = 0$



What about more general models of superconductivity/superfluidity?

Multicomponent superconductors

Many recently discovered materials are multi-band

- pairing on several sheets of a Fermi Surface; Cooper pairs can tunnel btw FS [Suhl, Matthias, Walker 1959; Moskalenko 1959]
- the gap function cannot be reduced to a single complex field thus $\Psi \to \boldsymbol{\Psi} = (\psi_1, \psi_2, \cdots) \in \mathbb{C}^N$

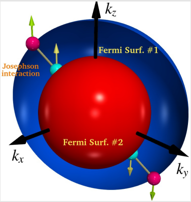
intercomponent interactions open many new physical phenomena

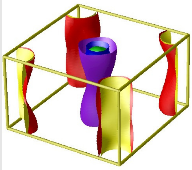
Multicomponent Ginzburg-Landau $\boldsymbol{\Psi} = (\psi_1, \psi_2, \cdots) \in \mathbb{C}^N$

$$E = \int_{\mathbb{R}^3} |\boldsymbol{\nabla} \times \boldsymbol{A}|^2 + \sum_a |\boldsymbol{D}\psi_a|^2 + V[\Psi^\dagger, \Psi] \,, \text{ with } \boldsymbol{D} = \boldsymbol{\nabla} + ie\boldsymbol{A}$$

- potential $V[\Psi^\dagger, \Psi]$, such that the ground state is $\Psi^\dagger\Psi \neq 0$
- V typically contains terms that explicitly break symmetries

Fermi surfaces





Multicomponent theories are relevant to many systems beyond the solid state

e.g. mixtures of charged condensates (LMH, LMD); metallic superfluids; nuclear SC in neutron stars; superfluid He; spinor condensates; Weinberg-Salam theory; . . .

Ginzburg-Landau theory for multicomponent superconductors

Ginzburg-Landau free energy

$\Psi = (\psi_1, \psi_2, \cdots) \in \mathbb{C}^N$ and $\psi_a = |\psi_a| e^{i\varphi_a}$

$$\mathcal{F}/\mathcal{F}_0 = \int \frac{1}{2}|\boldsymbol{\nabla}\times\boldsymbol{A}|^2 + \sum_a \frac{1}{2}|\boldsymbol{D}\psi_a|^2 + V(\Psi,\Psi^\dagger)\,, \quad \text{and} \quad \boldsymbol{D} \equiv \boldsymbol{\nabla} + ie\boldsymbol{A}$$

$$\text{and} \quad V(\Psi,\Psi^\dagger) = \alpha_{ab}\psi_a^*\psi_b + \beta_{abcd}\psi_a^*\psi_b^*\psi_c\psi_d \quad \in \mathbb{R}$$

Ginzburg-Landau and Ampère-Maxwell equations

$\boldsymbol{B} = \boldsymbol{\nabla}\times\boldsymbol{A}$

$$\boldsymbol{D}\boldsymbol{D}\psi_a = 2\delta V/\delta\psi_a^* \quad \text{and} \quad \boldsymbol{\nabla}\times\boldsymbol{B} + \boldsymbol{J} = 0\,, \quad \text{with} \quad \boldsymbol{J} = e\mathrm{Im}(\Psi^\dagger\boldsymbol{D}\Psi)$$

Meissner supercurrent

$\boldsymbol{J} := \delta\mathcal{F}/\delta\boldsymbol{A} = e\mathrm{Im}(\Psi^\dagger\boldsymbol{D}\Psi)$

$$\boldsymbol{J} = e^2\varrho^2\boldsymbol{A} + e\sum_a |\psi_a|^2\boldsymbol{\nabla}\varphi_a\,, \quad \text{with} \quad \varrho^2 = \sum_a |\psi_a|^2$$

Separation in charged and neutral modes

$\varphi_{ab} = \varphi_b - \varphi_a$

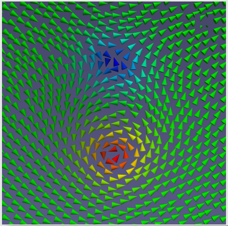
$$\mathcal{F}/\mathcal{F}_0 = \int \frac{\boldsymbol{B}^2}{2} + \sum_a \frac{1}{2}(\boldsymbol{\nabla}|\psi_a|)^2 + \frac{\boldsymbol{J}^2}{2e^2\varrho^2} + \sum_{a,b>a} \frac{|\psi_a|^2|\psi_b|^2}{2\varrho^2}(\boldsymbol{\nabla}\varphi_{ab})^2 + V(\Psi,\Psi^\dagger)\,.$$

Topological defects in multicomponent superconductors

They host a broad variety of topological defects

- fractional vortices: defects with winding in a single condensate $\oint \nabla\varphi_a d\ell = 2\pi$ $[\mathbb{S}^1 \to \mathbb{S}^1]$
  e.g. [Babaev 2002; Silaev 2011; **JG**, *et al.* 2014]
- domain-walls [broken $\mathbb{Z}_2$]
  e.g. [Sigrist, Agterberg 1999; **JG**, Babaev 2014]
- coreless vortices (skyrmions) $[\mathbb{R}^2 \to \mathbb{C}\mathrm{P}^{N-1}]$
  e.g. [**JG** & Babaev, Carlström, Speight 2011–2019; . . .]
- hopfions (knots) $[\mathbb{S}^3 \to \mathbb{S}^3]$
  [Rybakov, **JG**, Babaev 2019], also [Jäykkä 2009;+Speight 2011]

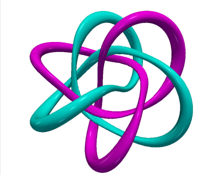
Skyrmion texture



But also modified properties like

- electrodynamics *e.g.* [Jäykkä, Babaev, Speight 2009; . . .]
- type-1.5 physics *e.g.* [Babaev, Speight 2009; . . .]
- thermoelectric *e.g.* [Silaev, **JG**, Babaev 2015; 2016]

Hopfions

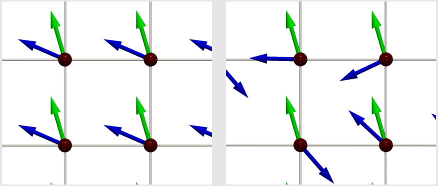


. . . and also new phases beyond mean-field

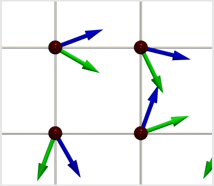
New phases beyond mean-field (*e.g.* [Babaev, Subø, Aschcroft 2004])

$$\mathcal{F}/\mathcal{F}_0 = \int \frac{\boldsymbol{B}^2}{2} + \frac{1}{2e^2\varrho^2}\left(|\psi_1|^2\boldsymbol{\nabla}\varphi_1 + |\psi_2|^2\boldsymbol{\nabla}\varphi_2 + e^2\varrho^2\boldsymbol{A}\right)^2 + \frac{|\psi_1|^2|\psi_2|^2}{2\varrho^2}(\boldsymbol{\nabla}\varphi_{12})^2$$

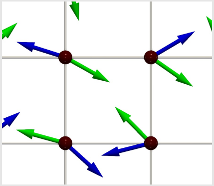
Superconducting states



Metallic superfluid



Normal fluid



Superconducting states

- Superconducting superfluid: ordered $\langle\psi_1\rangle \neq 0$, $\langle\psi_2\rangle \neq 0$ and $\langle\psi_1^*\psi_2\rangle \neq 0$
- One-gap superconductor: $\langle\psi_1\rangle \neq 0$, $\langle\psi_2\rangle = 0$ and $\langle\psi_1^*\psi_2\rangle = 0$

New state: Metallic superfluid

- Disordered indiv. phases: $\langle\psi_a\rangle = 0$ Ordered relative phase : $\langle\psi_1^*\psi_2\rangle \neq 0$
- Resistive to co-flow (charged) but dissipationless counter-flow (neutral)

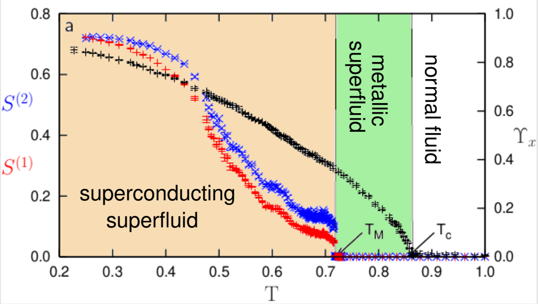
Symmetry restoration occurs by proliferation of different vortex loops

New phases beyond mean-field

[Smørgrav, Babaev, Smiseth, Subø 2005]

Liquid metallic hydrogen: mixture of electronic and protonic Cooper pairs

LHM:
$\mathrm{U}(1)\times\mathrm{U}(1)$
superconductor



Splitting
of transitions

"ordinary" states

- Superconducting superfluid: ordered $\langle\psi_1\rangle \neq 0$, $\langle\psi_2\rangle \neq 0$ and $\langle\psi_1^*\psi_2\rangle \neq 0$
- Normal liquid: disordered $\langle\psi_1\rangle = 0$, $\langle\psi_2\rangle = 0$ and $\langle\psi_1^*\psi_2\rangle = 0$

New state: Metallic superfluid

- Disordered total phases: $\langle\psi_a\rangle = 0$
  Ordered relative phase : $\langle\psi_1^*\psi_2\rangle \neq 0$
- Resistive to co-flow (charged) but dissipationless counter-flow (neutral)

Outline

States that Break the Time-Reversal Symmetry (BTRS)

Several materials break the time-reversal symmetry $p+ip$, $s+is$, $s+id$, ...

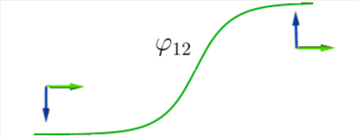
Observation of spontaneous bulk magnetic field below T_c, via muon spin relaxation (μSR) or/and the Kerr effect Sr_2RuO_4, UPt_3, $Ba_{1-x}K_xFe_2As_2$, ...

The time-reversal symmetry is broken if $\mathcal{T}(\Psi_0) \neq \Psi_0 e^{i\chi}\ \forall\chi$ $\mathcal{T}(\Psi_0) \equiv \Psi_0^*$

Two-component toy model (London limit)

$$\mathcal{F} \propto \frac{1}{2}(\nabla\varphi_{12})^2 + m^2\cos 2\varphi_{12}$$

- Minima: $\varphi_{12} = \pm\pi/2$ φ_{ab} is neither 0 nor π
- Energy is invariant under $\varphi_{ab} \to -\varphi_{ab}$
- The discrete $\mathbb{Z}_2$ symmetry is broken



Spontaneous breakdown of the discrete $\mathbb{Z}_2$

$\Rightarrow$ the theory allows for domain-walls

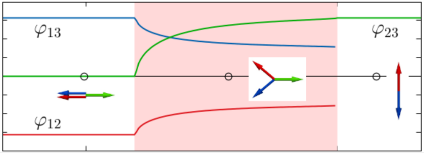
Domain-walls formation

via Kibble-Zurek mechanism, see *e.g.* [Vadimov, Silaev 2013; **JG**, Babaev 2014]

States that Break the Time-Reversal Symmetry (BTRS)

$s+is$ state in iron based superconductors

- due to competing pairing channels
- preserves point group symmetries
- relevant for $Ba_{1-x}K_xFe_2As_2$



Three-component with competing phase-locking terms

$$V(\Psi) = \cdots + \sum_{b>a}^{3} \eta_{ab} |\psi_a||\psi_b| \cos\varphi_{ab} + \dots \quad \text{and} \quad \eta_{ab} > 0$$

- each Josephson term anti-locks the phases ($\varphi_{ab} = \pi$)
- they cannot be simultaneously satisfied $\Rightarrow$ frustration.

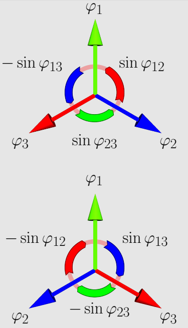
A simple example if $\eta_{ab} = 1$ φ_{ab} is neither 0 nor π

$(\varphi_{12}, \varphi_{13})$ is either $(2\pi/3, -2\pi/3)$ or $(-2\pi/3, 2\pi/3)$

The potential $V(\Psi, \Psi^\dagger)$ is invariant under $\varphi_{ab} \to -\varphi_{ab}$

there is an additional discrete $\mathbb{Z}_2$ degeneracy of the ground state

Phase-lockings



Phase diagram for the $s+is$ state

Breakdown of the time-reversal symmetry

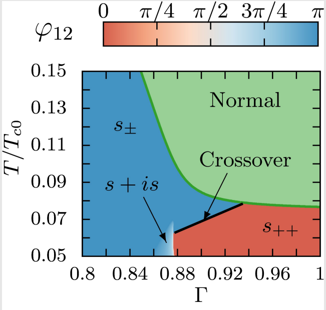
Due to competing pairing channels: depends on the doping level or on impurities

- in clean three-band superconductors [Stanev, Tesanovic 2010; **JG**, *et al.* 2011; 2014; Maiti Chubukov 2013;...]
- in dirty two-band superconductors [Gurevich 2003; **JG**, *et al.* 2017; 2018;...]

In $Ba_{1-x}K_xFe_2As_2$ ($0.7 \lesssim x \lesssim 0.85$)

μSR experiments [Grinenko, Sarkar, *et al.* 2020]

$s+is$ dome in the SC state



Mean-field time-reversal symmetry breaking transition occurs at $T_c^{\mathbb{Z}_2} < T_c$

- typically a dome of BTRS state inside the superconducting state
- the BTRS transition is associated with a divergent length-scale (2nd order)
- $\Rightarrow$ can there be paired states that break the TRS beyond mean-field?

Phase diagram for the $s+is$ state

Breakdown of the time-reversal symmetry

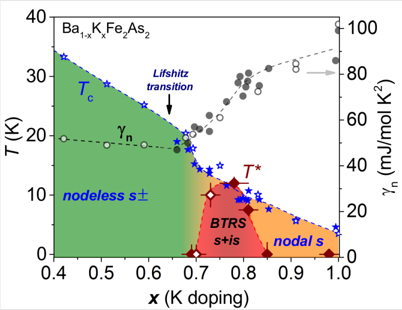
Due to competing pairing channels: depends on the doping level or on impurities

- in clean three-band superconductors [Stanev, Tesanovic 2010; **JG**, *et al.* 2011; 2014; Maiti Chubukov 2013;. . .]
- in dirty two-band superconductors [Gurevich 2003; **JG**, *et al.* 2017; 2018;. . .]

In $Ba_{1-x}K_xFe_2As_2$ ($0.7 \lesssim x \lesssim 0.85$)

μSR experiments [Grinenko, Sarkar, *et al.* 2020]

$s+is$ dome in the SC state



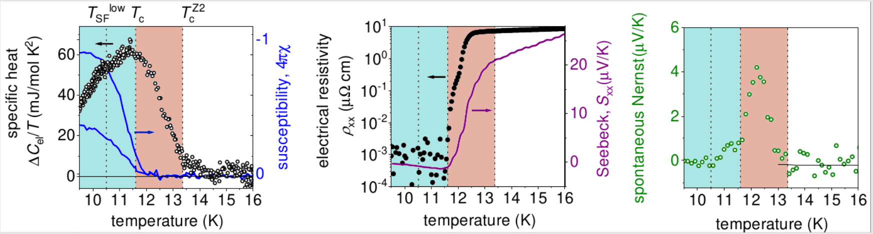
Mean-field time-reversal symmetry breaking transition occurs at $T_c^{\mathbb{Z}_2} < T_c$

- typically a dome of BTRS state inside the superconducting state
- the BTRS transition is associated with a divergent length-scale (2nd order)
- $\Rightarrow$ can there be paired states that break the TRS beyond mean-field?

Observation of the quartic metallic state

[Grinenko *et al.* 2021]

Splitting of the $\mathrm{U}(1)$ and $\mathbb{Z}_2$ transitions in $Ba_{1-x}K_xFe_2As_2$ at $x = 0.77$, $B = 0T$



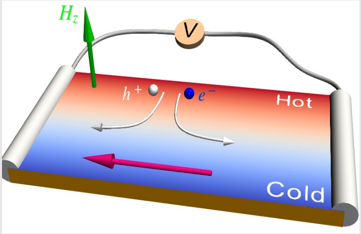
Susceptibility, specific heat, thermoelectric

- Susceptibility $\chi \propto$ gauge field mass
- non-zero electric resistivity
- Spontaneous Nernst $\Rightarrow$ BTRS

Quartic bosonic metallic state

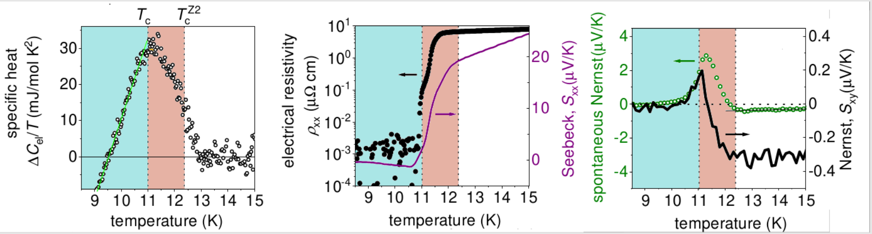
- Unbroken $\mathrm{U}(1)$: No SC ($\rho \neq 0$, $\chi = 0$)
- Broken $\mathbb{Z}_2$: Spontaneous Nernst $\Leftrightarrow$ BTRS

Thermoelectric Nernst effect



Observation of the quartic metallic state [Grinenko *et al.* 2021]

Splitting of the $\mathrm{U}(1)$ and $\mathbb{Z}_2$ transitions in $Ba_{1-x}K_xFe_2As_2$ at $x = 0.77$, $B = 1T$



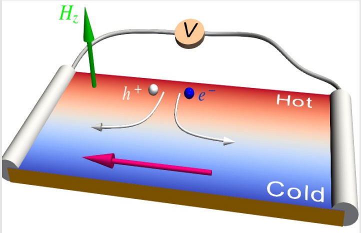
Susceptibility, specific heat, thermoelectric

- Susceptibility $\chi \propto$ gauge field mass
- non-zero electric resistivity
- Spontaneous Nernst $\Rightarrow$ BTRS

Quartic bosonic metallic state

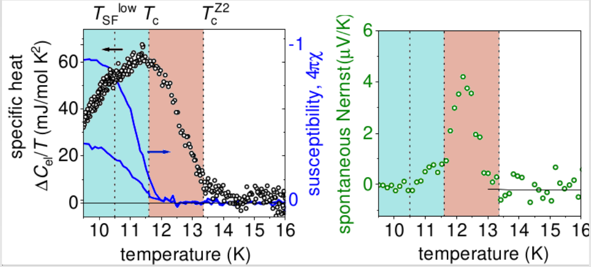
- Unbroken $\mathrm{U}(1)$: No SC ($\rho \neq 0$, $\chi = 0$)
- Broken $\mathbb{Z}_2$: Spontaneous Nernst $\Leftrightarrow$ BTRS

Thermoelectric Nernst effect

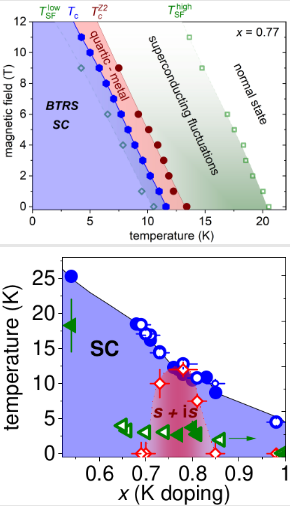


Quartic bosonic metallic state

[Grinenko *et al.* 2021]



Phase diagrams



Experimental evidences

- Spontaneous Nernst effect is not due by magnetism since it disappears at larger T $\Leftrightarrow$ BTRS
- related to the existence of Cooper pairs $\Delta C \neq 0$

Quartic bosonic metallic state (fluctuation-induced)

- Resistive: unbroken $\mathrm{U}(1)$ gauge symmetry
- Breaks the time-reversal symmetry: broken $\mathbb{Z}_2$

Quartic bosonic metallic state

[Grinenko *et al.* 2021]

Experimental, numerical, and theoretical

- Muon spin rotation (μSR), conductivity, diamagnetic response, thermoelectric ultrasound measurements
- Monte-Carlo simulations
- Effective field theory above T_c

$\Rightarrow$ all consistent with a new state above T_c

New, fluctuation-induced, ordered state

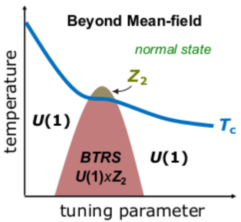
- Disordered total phases: $\langle\psi_a\rangle = 0$
- Ordered relative phase : $\langle\psi_1^*\psi_2\rangle \neq 0$

Thermal proliferation of topological defects

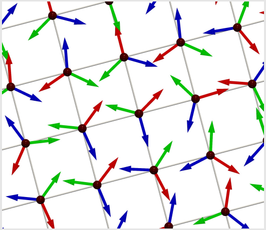
- proliferation of composite vortices
- but not of fractional vortices or domain walls

effective description of the electron quadruplet state?

Phase diagram



$\langle\psi_a\rangle = 0$ and $\langle\psi_a^*\psi_b\rangle \neq 0$



Outline

Proposed mechanisms for TRSB in pnictides

Pairing dominated by the competition between different interband repulsive channels

Clean SC with three overlapping bands at the Fermi level

Eilenberger eqs. for the quasi-classical propagators

$$\boldsymbol{v}_F^{(a)} \cdot (\boldsymbol{\nabla} + ie\boldsymbol{A}) f_a + 2\omega_n f_a - 2\Delta_a g_a = 0,$$

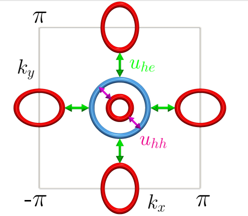
$$\boldsymbol{v}_F^{(a)} \cdot (\boldsymbol{\nabla} - ie\boldsymbol{A}) f_a^\dagger - 2\omega_n f_a^\dagger + 2\Delta_a^* g_a = 0\,, \quad g_a^2 + f_a f_a^\dagger = 1$$

Fermi vel.: $\boldsymbol{v}_F^{(a)}$; Matsubara freq.: $\omega_n = (2n+1)\pi T$

The self-consistency equation for the gaps is

$$\Delta_a(\boldsymbol{p}, \boldsymbol{r}) = T \sum_{n, \boldsymbol{p}'} \Lambda_{ab}(\boldsymbol{p}, \boldsymbol{p}') f_b(\boldsymbol{p}, \boldsymbol{r}, \omega_n)$$

$Ba_{1-x}K_xFe_2As_2$



$$\hat{\Lambda} = - \begin{pmatrix} 0 & u_{hh} & u_{eh} \\ u_{hh} & 0 & u_{eh} \\ u_{eh} & u_{eh} & 0 \end{pmatrix}$$

Derivation of the Ginzburg-Landau theory [JG, Silaev, Babaev 2016; 2017]

$\Rightarrow$ expansion by powers of the gap functions amplitudes Δ_a and their gradients

$$\left[(G_0 + \tau - \hat{\Lambda}^{-1})\boldsymbol{\Delta}\right]_a = -K_{ij}^{(a)} D_i D_j \Delta_a + |\Delta_a|^2 \Delta_a$$

Ginzburg-Landau model for the $s + is$ state (1/2)

$\hat{\Lambda}^{-1}$ has only 2 positive eigenvalues, so the GL theory has to be reduced to a two-component GL model in terms of $\eta_{1,2}$:

$$\boldsymbol{\Delta} = \eta_1 \boldsymbol{\Delta}_1 + \eta_2 \boldsymbol{\Delta}_2$$

Ginzburg-Landau model for $s + is$ states

$\eta_a = |\eta_a| \mathrm{e}^{i\phi_a}$

$$\mathcal{F} = \frac{\boldsymbol{B}^2}{2} + \frac{1}{2}\sum_{a,b=1}^{2} k_{ab} (\boldsymbol{D}\eta_a)^* \boldsymbol{D}\eta_b + V(\boldsymbol{\eta}) \,,$$

$$\text{where } V(\boldsymbol{\eta}) = \sum_{a,b=1}^{2}\left(\tilde{\alpha}_{ab}\eta_a^*\eta_b + \frac{\tilde{\beta}_{ab}}{2}|\eta_a|^2|\eta_b|^2 \right) + \frac{\tilde{\gamma}_{12}}{2}\left(\eta_1^{*2}\eta_2^2 + c.c.\right)$$

all coefficients are obtained from the microscopic model $(\tilde{\gamma}_{12} > 0)$

BTRS follows from the term $\propto \tilde{\gamma}_{12} \cos\left(2(\phi_2 - \phi_1)\right) \quad \Rightarrow \quad \phi_2 - \phi_1 = \pm\pi/2$

Kinetic term contains mixed-gradients (positive definite $\Rightarrow \det \hat{k} > 0$)

$$\mathcal{F}_k = \frac{1}{2}\sum_{a,b=1}^{2} k_{ab} (\boldsymbol{D}\eta_a)^* \boldsymbol{D}\eta_b := \frac{1}{2}(\boldsymbol{D}\boldsymbol{\eta})^\dagger \hat{k} \boldsymbol{D}\boldsymbol{\eta} \,, \quad \text{where } \boldsymbol{\eta}^\dagger = (\eta_1^*, \eta_2^*)$$

Ginzburg-Landau model for the $s + is$ state (2/2)

Elimination of the mixed-gradients: since $\hat{k}$ is a positive definite square matrix

$$\hat{k} = \mathcal{R}^\dagger \mathcal{R}\,, \text{ where } \mathcal{R} = \left(\hat{k} + \mathbb{1}\sqrt{\det \hat{k}}\right) \Big/ \sqrt{\operatorname{tr}\hat{k} + 2\sqrt{\det \hat{k}}}$$

Reparametrization of the superconducting *d.o.f.* with diagonal kinetic term

$$\mathcal{F} = \frac{\boldsymbol{B}^2}{2} + \frac{1}{2}(\boldsymbol{D}\Psi)^\dagger \boldsymbol{D}\Psi + V(\Psi)\,, \text{ where } \Psi = \mathcal{R}\boldsymbol{\eta}\,,$$

Separation of charged and neutral modes $\boldsymbol{J} = e^2\Psi^\dagger\Psi\boldsymbol{A} + e\,\mathrm{Im}\left(\Psi^\dagger\boldsymbol{\nabla}\Psi\right)$

$$\boldsymbol{A} = \frac{\boldsymbol{J}}{e^2\varrho^2} - \frac{1}{e\varrho^2}\mathrm{Im}\left(\Psi^\dagger\boldsymbol{\nabla}\Psi\right), \quad \text{where } \varrho^2 = \Psi^\dagger\Psi$$

$$B_k = \epsilon_{kij}\left\{\nabla_i\left(\frac{J_j}{e^2\varrho^2}\right) + i\frac{\varrho^2\boldsymbol{\nabla}_i\Psi^\dagger\boldsymbol{\nabla}_j\Psi + (\Psi^\dagger\boldsymbol{\nabla}_i\Psi)(\boldsymbol{\nabla}_j\Psi^\dagger\Psi)}{e\varrho^4}\right\}$$

$$|\boldsymbol{D}\Psi|^2 = \frac{\boldsymbol{J}^2}{e^2\varrho^2} + \boldsymbol{\nabla}\Psi^\dagger\cdot\boldsymbol{\nabla}\Psi + \frac{\left(\Psi^\dagger\boldsymbol{\nabla}\Psi - \boldsymbol{\nabla}\Psi^\dagger\Psi\right)^2}{4\varrho^2}$$

Effective model (1/3)

[JG, Babaev 2021]

Free energy in terms of charged and neutral modes

$$\mathcal{F} = \frac{1}{2}\left[\epsilon_{kij}\left\{\nabla_i\left(\frac{J_j}{e^2\varrho^2}\right) + i\frac{\varrho^2\nabla_i\Psi^\dagger\nabla_j\Psi + (\Psi^\dagger\nabla_i\Psi)(\nabla_j\Psi^\dagger\Psi)}{e\varrho^4}\right\}\right]^2 + \frac{\boldsymbol{J}^2}{2e^2\varrho^2}$$
$$+ \boldsymbol{\nabla}\Psi^\dagger\cdot\boldsymbol{\nabla}\Psi + \frac{1}{4\varrho^2}\left(\Psi^\dagger\boldsymbol{\nabla}\Psi - \boldsymbol{\nabla}\Psi^\dagger\Psi\right)^2 + V(\Psi)$$

Electron quadrupling order parameter: depends on relative phase φ_{12} and densities

$$\boldsymbol{m} = \Psi^\dagger\boldsymbol{\sigma}\Psi = \left(2|\psi_1||\psi_2|\cos\varphi_{12}, 2|\psi_1||\psi_2|\sin\varphi_{12}, |\psi_1|^2 - |\psi_2|^2\right)$$

Free energy in terms of charged and neutral modes

$\varrho^2 \equiv \|\boldsymbol{m}\|$

$$\mathcal{F} = \frac{1}{2}\left[\epsilon_{kij}\left\{\nabla_i\left(\frac{J_j}{e^2\varrho^2}\right) - \frac{1}{4e\varrho^6}\boldsymbol{m}\cdot\partial_i\boldsymbol{m}\times\partial_j\boldsymbol{m}\right\}\right]^2 + \frac{\boldsymbol{J}^2}{2e^2\varrho^2} + \frac{1}{8\varrho^2}(\boldsymbol{\nabla}\boldsymbol{m})^2 + V(\boldsymbol{m})$$

In the resistive state the Meissner current vanishes

$(\boldsymbol{J} = 0)$

currents associated with the gradients of the quadrupling order parameter $\boldsymbol{m}$ do not

Effective model (2/3)

[JG, Babaev 2021]

Assuming a mean-field approximation for the fields that are fourth-order in fermions

The resistive state ($J = 0$) is described by the Faddeev-Skyrme-like model

$$\mathcal{F} = \frac{\left(\boldsymbol{m}\cdot\partial_i\boldsymbol{m}\times\partial_j\boldsymbol{m}\right)^2}{16e^2\|\boldsymbol{m}\|^6} + \frac{\left(\boldsymbol{\nabla}\boldsymbol{m}\right)^2}{8\|\boldsymbol{m}\|} + V(\boldsymbol{m}) \quad \text{and} \quad \boldsymbol{B} = -\frac{\epsilon_{\alpha\beta\gamma}m_\alpha\boldsymbol{\nabla}m_\beta\times\boldsymbol{\nabla}m_\gamma}{4e\|\boldsymbol{m}\|^3}$$

Topological invariants in 2d

degree of the maps $\boldsymbol{m}/\|m\| : \mathbb{S}^2 \mapsto \mathbb{S}^2_{\boldsymbol{m}}$

$$\mathcal{Q}(\boldsymbol{m}) = \frac{1}{4\pi}\int_{\mathbb{R}^2}\frac{\boldsymbol{m}\cdot\partial_x\boldsymbol{m}\times\partial_y\boldsymbol{m}}{\|\boldsymbol{m}\|^3}\,dxdy$$

$\mathcal{Q}(\boldsymbol{m}) \in \mathbb{Z}$ whenever $\|\boldsymbol{m}\| \neq 0$

Quantized magnetic flux: $\int B_z = \Phi_0\mathcal{Q}(\boldsymbol{m})$

Potential term $m_0 := \|\boldsymbol{m}\|$

$$V(\boldsymbol{m}) = \sum_{a=0,x,y,z}\alpha_a^{\boldsymbol{m}}m_a + \frac{1}{2}\sum_{a,b=0,x,y,z}\beta_{ab}^{\boldsymbol{m}}m_am_b$$

Time-reversal operations

$$\mathcal{T}(\Psi) = \Psi^* \quad \Leftrightarrow \quad \mathcal{T}(\boldsymbol{m}) = (m_x, -m_y, m_z)$$

Broken time-reversal symmetry for the resistive electron quadrupling state $\boldsymbol{m}$

When the phase transitions temperatures $T_c < T_c^{\mathbb{Z}_2}$, in the ground-state $\boldsymbol{m}_y \neq 0$

Effective model (3/3)

[JG, Babaev 2021]

The resistive state ($J=0$) is described by the Faddeev-Skyrme-like model

$$\mathcal{F} = \frac{(\boldsymbol{m}\cdot\partial_i\boldsymbol{m}\times\partial_j\boldsymbol{m})^2}{16e^2\|\boldsymbol{m}\|^6} + \frac{(\boldsymbol{\nabla}\boldsymbol{m})^2}{8\|\boldsymbol{m}\|} + V(\boldsymbol{m}) \quad \text{and} \quad \boldsymbol{B} = -\frac{\epsilon_{\alpha\beta\gamma}m_\alpha\boldsymbol{\nabla}m_\beta\times\boldsymbol{\nabla}m_\gamma}{4e\|\boldsymbol{m}\|^3}$$

Degree of the maps $\boldsymbol{m}/\|\boldsymbol{m}\| : \mathbb{S}^2 \mapsto \mathbb{S}^2_{\boldsymbol{m}}$

$$\mathcal{Q}(\boldsymbol{m}) = \frac{1}{4\pi}\int_{\mathbb{R}^2}\frac{\boldsymbol{m}\cdot\partial_x\boldsymbol{m}\times\partial_y\boldsymbol{m}}{\|\boldsymbol{m}\|^3}\,dxdy \in \mathbb{Z}$$

Easy-plane potential $m_0 := \|\boldsymbol{m}\|$

$$V(\boldsymbol{m}) = \sum_{a=0,x,y,z}\alpha_a^{\boldsymbol{m}}m_a + \frac{1}{2}\sum_{a,b=0,x,y,z}\beta_{ab}^{\boldsymbol{m}}m_am_b$$

Anisotropic Faddeev-Skyrme model (when $\|\boldsymbol{m}\|=1$) is known to host

(baby-)skyrmions and hopfions. See *e.g.* [Foster 2010; Jäykka, Speight 2010; Jäykka, Speight Sucliffe 2012; Kobayashi, Nitta 2013; Harland 2014; Samoilenka, Shnir 2014; . . .]

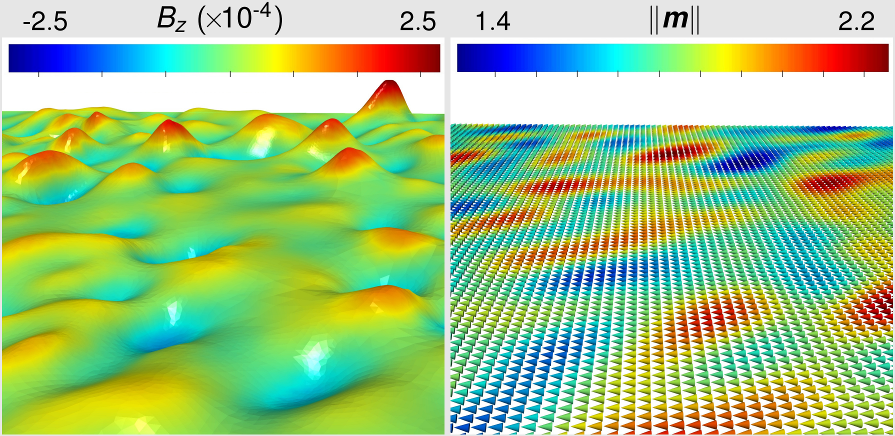
Numerical construction: Finite Elements + Non Linear Conjugate Gradients

- Minimize from an initial configuration with desired topological property (energy barriers prevent the change of topological sectors)
- why NLCG? It is lightweight and faster than other gradient descents

Spontaneous magnetic fields

[JG, Babaev 2021]

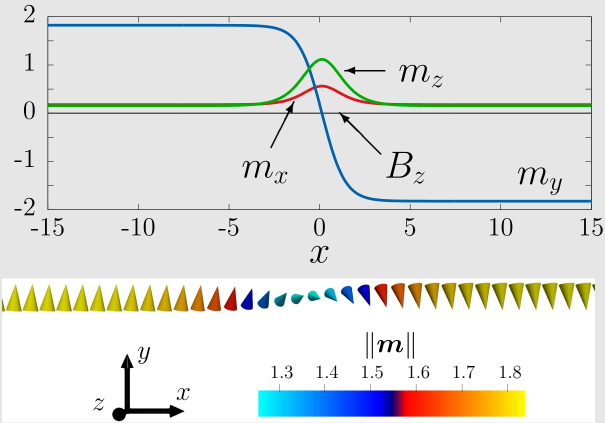
Inhomogeneities: $a_{ii} \equiv a_{ii}(\tau, \boldsymbol{x}) = a_{ii}^0[\tau(\boldsymbol{x}) - 1]$ **where** $\tau(\boldsymbol{x}) = \tau_0[1 + \delta\tau\, \mathrm{ran}(\boldsymbol{x})]$



Magnetic field $B_z \propto (\boldsymbol{m} \cdot \partial_x \boldsymbol{m} \times \partial_y \boldsymbol{m})/\|\boldsymbol{m}\|^3$

Domain-walls between time-reversal states [JG, Babaev 2021]

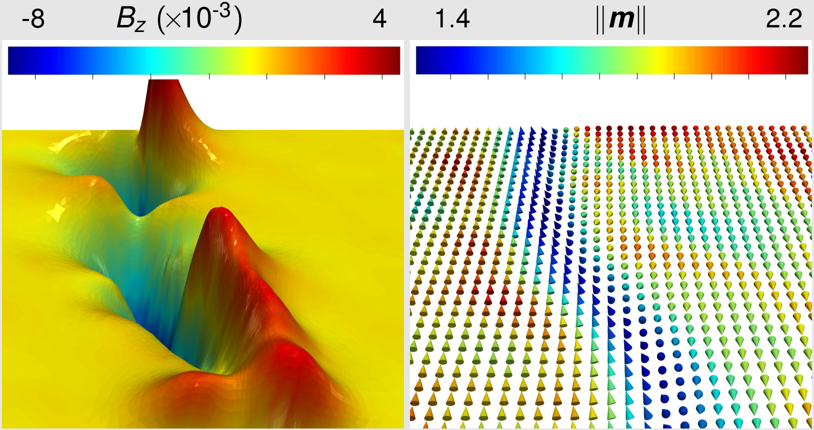
Since the discrete $\mathbb{Z}_2$ is broken (BTRS), there are domain-walls



No magnetic field, since $B_z \propto (\boldsymbol{m} \cdot \partial_x \boldsymbol{m} \times \partial_y \boldsymbol{m})/\|\boldsymbol{m}\|^3$

Domain-walls with inhomogeneities [JG, Babaev 2021]

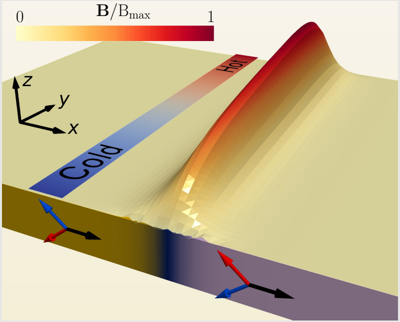
Inhomogeneities: $a_{ii} \equiv a_{ii}(\tau, \boldsymbol{x}) = a^0_{ii}[\tau(\boldsymbol{x}) - 1]$ **where** $\tau(\boldsymbol{x}) = \tau_0[1 + \delta\tau\,\mathrm{ran}(\boldsymbol{x})]$

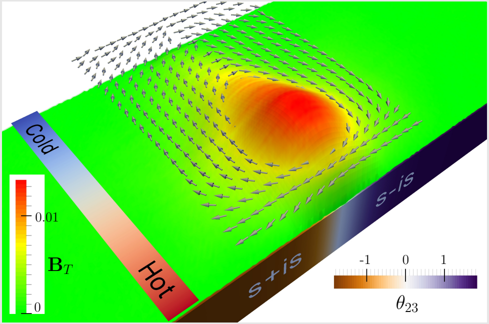


Magnetic field $B_z \propto (\boldsymbol{m} \cdot \partial_x \boldsymbol{m} \times \partial_y \boldsymbol{m})/\|\boldsymbol{m}\|^3$

Domain-walls with thermal gradients [Grinenko, *et al.* 2021]

Thermal gradients along domain-walls: $a_{ii} \equiv a_{ii}(\tau, \boldsymbol{x}) = a^0_{ii}[\tau(\boldsymbol{x}) - 1]$





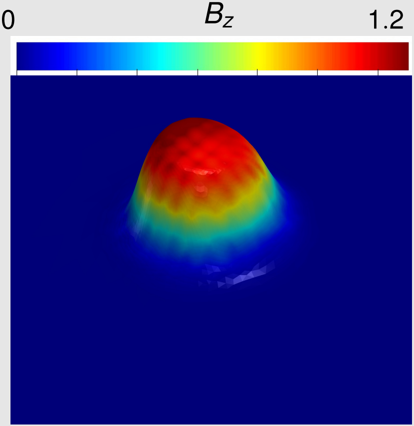
Similar response to the superconducting case [JG, Silaev, Babaev 2015; 2016]

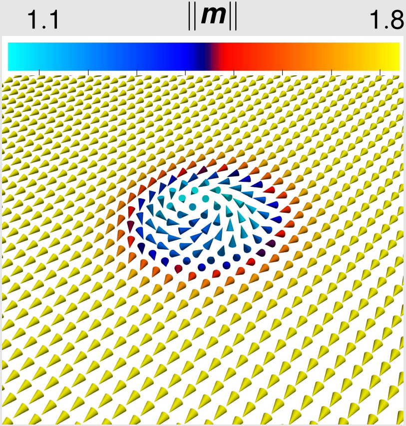
$$\boldsymbol{B} = \boldsymbol{\nabla} \times \left(\frac{\boldsymbol{J}}{e^2 \varrho^2} \right) + \sum_{a,b>a} \boldsymbol{\nabla} \left(\frac{|\psi_a|^2 - |\psi_b|^2}{N e \varrho^2} \right) \times \boldsymbol{\nabla} \varphi_{ab}$$

Skyrmion textures

[JG, Babaev 2021]

Topological charge: $\mathcal{Q}(\boldsymbol{m}) = 1$ **flux is quantized** $\int B_z = \Phi_0 \mathcal{Q}(\boldsymbol{m})$



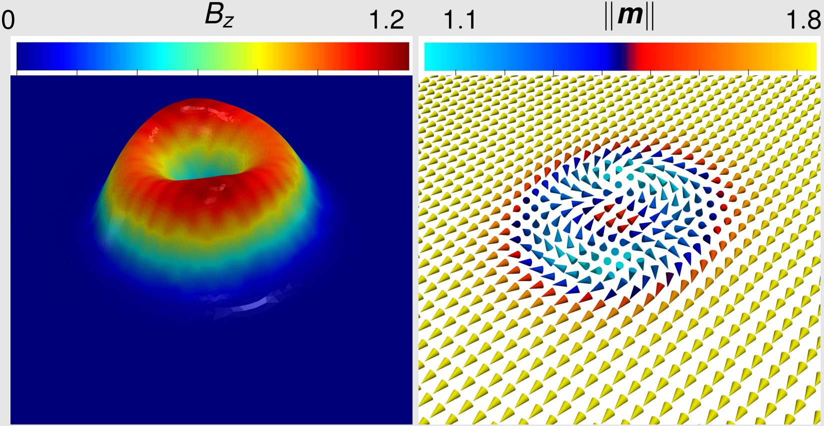


Magnetic field $B_z \propto (\boldsymbol{m} \cdot \partial_x \boldsymbol{m} \times \partial_y \boldsymbol{m})/\|\boldsymbol{m}\|^3$

Skyrmion textures

[JG, Babaev 2021]

Topological charge: $\mathcal{Q}(\boldsymbol{m}) = 2$ **flux is quantized $\int B_z = \Phi_0 \mathcal{Q}(\boldsymbol{m})$**

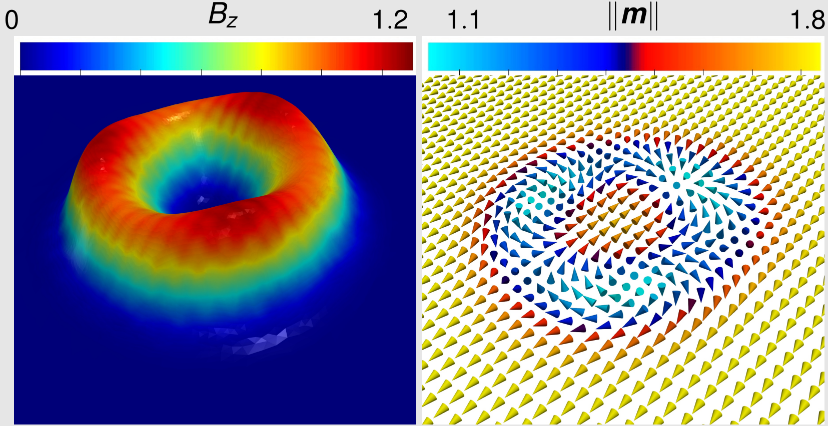


Magnetic field $B_z \propto (\boldsymbol{m} \cdot \partial_x \boldsymbol{m} \times \partial_y \boldsymbol{m}) / \|\boldsymbol{m}\|^3$

Skyrmion textures

[JG, Babaev 2021]

Topological charge: $\mathcal{Q}(\boldsymbol{m}) = 3$ **flux is quantized** $\int B_z = \Phi_0 \mathcal{Q}(\boldsymbol{m})$

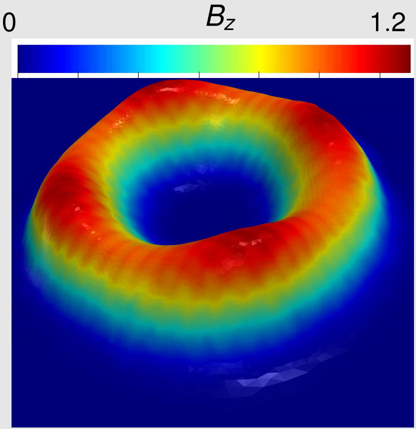


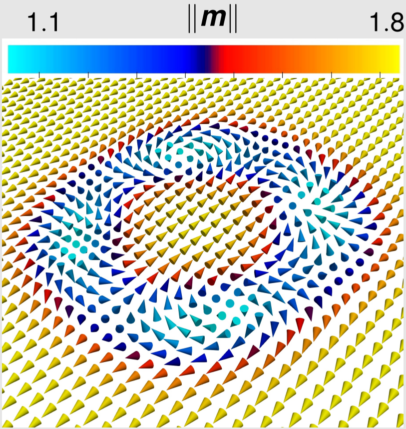
Magnetic field $B_z \propto (\boldsymbol{m} \cdot \partial_x \boldsymbol{m} \times \partial_y \boldsymbol{m})/\|\boldsymbol{m}\|^3$

Skyrmion textures

[JG, Babaev 2021]

Topological charge: $\mathcal{Q}(\boldsymbol{m}) = 4$ **flux is quantized $\int B_z = \Phi_0 \mathcal{Q}(\boldsymbol{m})$**



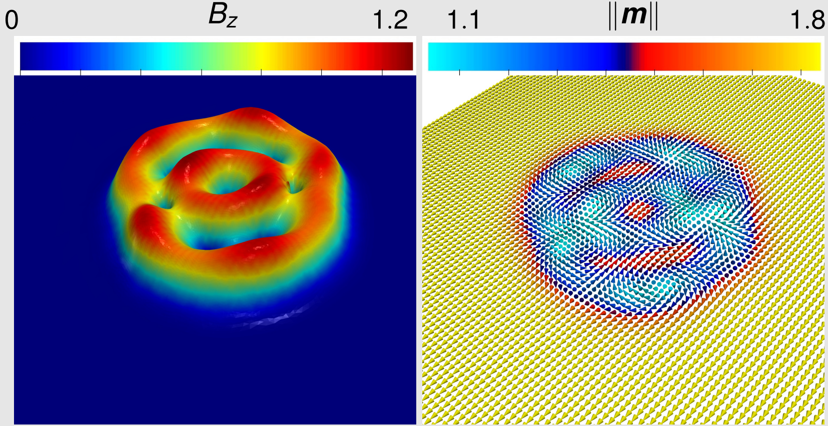


Magnetic field $B_z \propto (\boldsymbol{m} \cdot \partial_x \boldsymbol{m} \times \partial_y \boldsymbol{m}) / \|\boldsymbol{m}\|^3$

Skyrmion textures

[JG, Babaev 2021]

Topological charge: $\mathcal{Q}(\boldsymbol{m}) = 7$ flux is quantized $\int B_z = \Phi_0 \mathcal{Q}(\boldsymbol{m})$

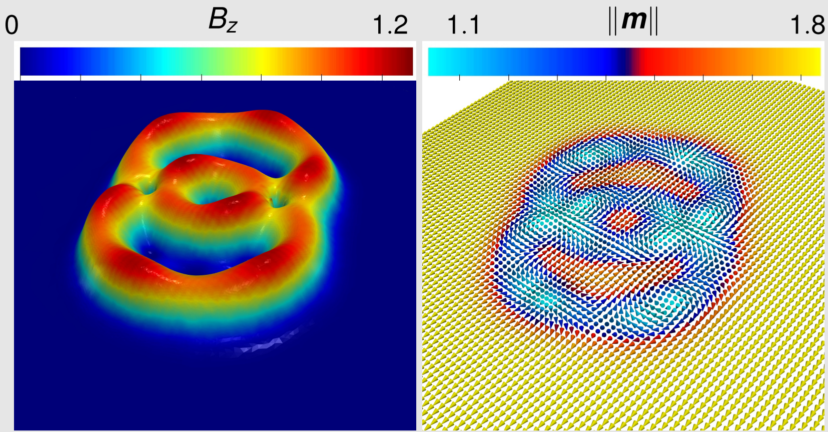


Magnetic field $B_z \propto (\boldsymbol{m} \cdot \partial_x \boldsymbol{m} \times \partial_y \boldsymbol{m}) / \|\boldsymbol{m}\|^3$

Skyrmion textures

[JG, Babaev 2021]

Topological charge: $\mathcal{Q}(\boldsymbol{m}) = 8$ **flux is quantized $\int B_z = \Phi_0 \mathcal{Q}(\boldsymbol{m})$**

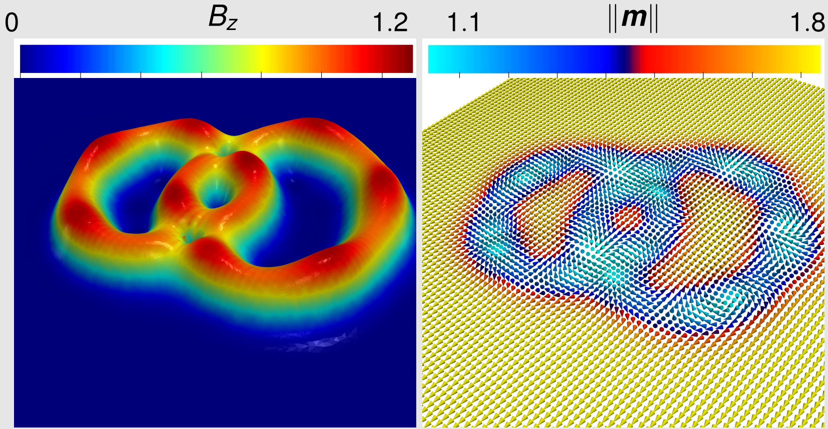


Magnetic field $B_z \propto (\boldsymbol{m} \cdot \partial_x \boldsymbol{m} \times \partial_y \boldsymbol{m}) / \|\boldsymbol{m}\|^3$

Skyrmion textures

[JG, Babaev 2021]

Topological charge: $\mathcal{Q}(\boldsymbol{m}) = 9$ — flux is quantized $\int B_z = \Phi_0 \mathcal{Q}(\boldsymbol{m})$



Magnetic field $B_z \propto (\boldsymbol{m} \cdot \partial_x \boldsymbol{m} \times \partial_y \boldsymbol{m})/\|\boldsymbol{m}\|^3$

Conclusion

Quartic bosonic metallic state (fluctuation-induced)

- Resistive: unbroken $\mathrm{U}(1)$ gauge symmetry
- Breaks the time-reversal symmetry: broken $\mathbb{Z}_2$
- fourth-order in fermion fields
- precedes the $s+is$ state

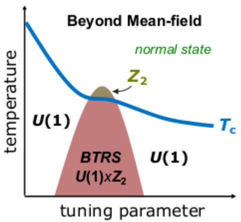
Effective theory: Faddeev-Skyrme-like model

$$\mathcal{F} = \frac{(\boldsymbol{m}\cdot\partial_i\boldsymbol{m}\times\partial_j\boldsymbol{m})^2}{16e^2\|\boldsymbol{m}\|^6} + \frac{(\nabla\boldsymbol{m})^2}{8\|\boldsymbol{m}\|} + V(\boldsymbol{m})$$

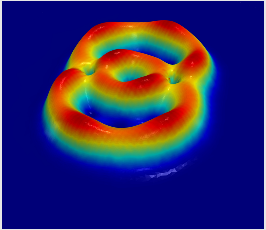
- four fermion order parameter: $\boldsymbol{m} = \Psi^\dagger\boldsymbol{\sigma}\Psi$ with no information about the total phase
- spontaneous fields: $B_z \propto (\boldsymbol{m}\cdot\partial_x\boldsymbol{m}\times\partial_y\boldsymbol{m})/\|\boldsymbol{m}\|^3$
- topological excitations: domain-walls and skyrmions

should definitely host hopfions

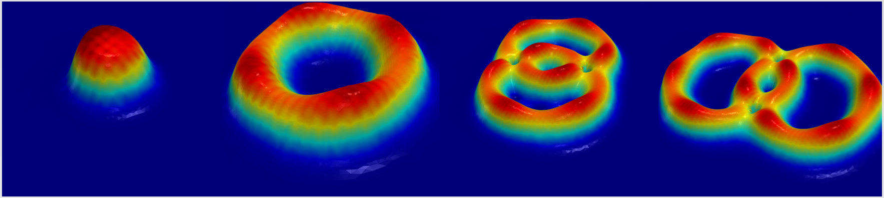
Phase diagram



Skyrmions



Thank you for your attention!



based on

JG, and E. Babaev
Skyrmions and effective model of the resistive electron quadrupling state ,
Preprint (2021) [cond-mat.supr-con] arXiv:2112.01286

Grinenko, Weston, Caglieris, Wuttke, Hess, Gottschall, Maccari, Gorbunov, Zherlitsyn, Wosnitza, Rydh, Kihou, Lee, Sarkar, Dengre, **JG**, Charnukha, Hühne, Nielsch, Büchner, Klauss, and Babaev
State with spontaneously broken time-reversal symmetry above superconducting phase transition
Nature Physics **17**, 1254 (2021) [cond-mat.supr-con] arXiv:2103.17190