Annual meeting of the GDR singularity in Aussois from 4th to 8th July

July 5, 2022

Schedule

Program :
4th to 8th July 2022 in Aussois

	Monday	Tuesday	Wenesday	Thursday	Friday
7h30-9h	breakfast	breakfast	breakfast	breakfast	breakfast
9h-9h30	welcome				
9h30-10h30	Mazzon 1/3	Popescu 1/3	Spicer 2/3	Spicer 3/3	9h Popescu 3/3
10h30-11h10	Coffee	Coffee	Coffee	Coffee	Coffee
11h10-12H10	Spicer 1/3	Mazzon 2/3	Mazzon 3/3	Popescu 2/3	10h20 Forey
12h30-14h	lunch	lunch	lunch	lunch	lunch
14h-16h	free	free	free	free	
16h30 -17h30	Mauri	Aceto	free	Portilla	
17h30-18h10	Coffee	Coffee	Coffee	Coffee	
18h10 -19h10	Elduque	Almiron	Sorea	Bernard	
19h30	Dinner	Dinner	Dinner	Dinner	

Mini courses

1. **Calum Spicer (King's College London)**
   Title: Minimal model program
   Abstract: We will give an introduction to the basic ideas and techniques of the Minimal Model Program (MMP). We will explain how these ideas have implications and applications to the classification theory of algebraic varieties, the study of singularities of varieties and to surgery operations in algebraic geometry. From here we will turn to some current applications of the MMP, with a particular focus on holomorphic foliations.

2. **Enrica Mazzon (University of Michigan)**
   Title: Introduction to non-archimedian geometry
   Abstract: The relation between birational geometry, non-archimedean geometry and mirror symmetry is a relatively new, rich and fascinating research subject in algebraic geometry. In this series of talks, I will describe some aspects of this connection. I will introduce the theory of Berkovich spaces and dual complexes, and I will hint at their connection with SYZ mirror symmetry.

3. **Patrick Popescu Pampu (Lille)**
   Title: Introduction to logarithmic geometry

Abstract: I will give an introduction to *log geometry* in the sense of Fontaine and Illusie, whose ideas on the subject first appeared in print in a 1989 paper of Kato. I will then focus on the *rounding* operation introduced by Kato and Nakayama in 1999, which associates functorially a new topological space to any complex log space. When the log structure is induced by a normal crossings divisor into a manifold, the rounding produces A'Campo's *real oriented blow up* of the manifold along the divisor, an operation defined in 1975. More generally, the rounding operation allows to transform any toroidal manifold into a manifold with corners, turning the toroidal boundary into a topological boundary. I will explain finally a relative version of this fact, proved in 2010 by Nakayama and Ogus. Conveniently localized, their theorem allows to give a canonical representative of a Milnor fibration of a germ of holomorphic function, once a toroidal modification of the source adapted to the function is chosen.

Title and abstract of the talks

1. **Paolo Aceto (Lille)**
   Title: Definite fillings of lens spaces
   Abstract: Motivated by the study of smoothings of cyclic quotient singularities as well as symplectic fillings of lens spaces, we consider an analogue problem in a purely topological setting. We look at smooth, definite fillings of lens spaces and consider the question of which intersection forms can be realized by such fillings. We discuss various constructions and an obstruction based on Donaldson's diagonalization theorem. Finally, we present a complete classification of the lens spaces which bound a unique negative-definite intersection form (up to stabilizations). We discuss consequences for smoothings of singularities as well as embeddings of lens spaces in certain 4-manifolds. This is joint work with Duncan McCoy and JungHwan Park.

2. **Patricio Almiron (Madrid)**
   Title: Milnor number vs Tjurina number of isolated hypersurface singularities
   Abstract: The Milnor and Tjurina numbers are two of the most important invariants of an isolated hypersurface singularity. While being of different nature, the Milnor number is a topological invariant and the Tjurina number is an analytic invariant, their quotient, or equivalently their difference, provides a measure of how far is our singularity to be quasi-homogeneous. In this talk we will adress the problem of finding sharp upper bounds for the quotient of the Milnor and Tjurina numbers.

   The first part of the talk will be devoted to introduce the motivations to study the problem of finding sharp upper bounds for the quotient of the Milnor and Tjurina numbers. We will show particular solutions in the case of plane curve singularities and surface singularities which will link our problem with other conjectures in singularity theory. In the second part of the talk, we will focus on the comparison of the Milnor and Tjurina numbers in the case of a Sebastiani-Thom type singularity. We will show the different behaviour of those invariants in that case and we will show some upper bounds for their quotient.

3. **François Bernard (Angers)**
   Title: Some variants of normalization for real affine varieties.
   Abstract: In this talk, I will present three variants of the normalization of a real affine algebraic variety: the seminormalization, the R-seminormalization and the biregular normalization. Like the normalization, they are obtained by an algebraic process and have very specific singularities in codimension 1. However, their connection with the considered variety is stronger than the finite birational morphism of normalization. We will see that they all verify a universal property and we will provide different ways to identify their coordinate rings. Then, we will compare them and give examples. Finally, we will see how they modify the complex and the real singularities of the variety.

4. **Arthur Forey (EPFL)**
   Title: Bounded motivic integral and motivic Milnor fiber
   Abstract: Building on ideas of Hrushovski and Loeser, I will present a new motivic integration morphism, the bounded integral, that interpolates Hrushovski and Kazhdan's integrals with and

without volume forms. It has applications to the motivic Milnor fiber. This is joint work with Yimu Yin.

5. **Eva Elduque (Madrid)**
   Title: Eigenspace Decomposition of Mixed Hodge Structures on Alexander Modules
   Abstract: In previous work jointly with Geske, Herradón Cueto, Maxim and Wang, we constructed a mixed Hodge structure (MHS) on the torsion part of Alexander modules, which generalizes the MHS on the cohomology of the Milnor fiber for weighted homogeneous polynomials. The cohomology of a Milnor fiber carries a monodromy action, whose semisimple part is an isomorphism of MHS. The natural question of whether this result still holds for Alexander modules was then posed. In this talk, we will talk about the solution to this question, as well as some consequences and explicit computations. Joint work with Moisés Herradón Cueto.

6. **Mirko Mauri (Michigan)**
   Title: Hodge-to-singular correspondence
   Abstract: The decomposition theorem for proper morphisms of algebraic varieties grants that the cohomology of the domain splits in elementary summands. However, in general, it is a subtle task to determine explicitly these summands. We prove that this is in fact possible in the case of Hitchin fibrations for Higgs bundles of arbitrary degree on the locus of reduced spectral curves. Surprisingly we relate the summands of the decomposition theorem to the topology of symplectic singularities on the moduli spaces of Higgs bundles in (fixed!) degree zero. This is a joint project with Luca Migliorini.

7. **Pablo Portilla (Lille)**
   Title: Characterizing the geometric monodromy group of an isolated plane curve singularity
   Abstract: In this talk we will explain an intrinsic characterisation of the geometric monodromy group of an isolated plane curve singularity as the stabiliser in the mapping class group of the Milnor fiber of the relative isotopy class of a canonical vector field. We will also discuss two interesting consequences of this result: an easy and efficient criterion for detecting whether a simple closed curve in the Milnor fiber is a geometric vanishing cycle or not, and the non-injectivity of the natural representation of the versal unfolding of the singularity. This is a joint work with Nick Salter.

8. **Miruna-Stefana Sorea (SISSA)**
   Title: Poincaré–Reeb graphs of real algebraic domains
   Abstract: Consider a real bivariate polynomial function that has a strict local minimum at the origin and that vanishes at this point. In a sufficiently small neighborhood of the origin, the non-zero level curves of this function are smooth Jordan curves.

   Whenever the origin is a Morse strict local minimum, the small enough level curves become boundaries of convex topological disks. Otherwise, the levels may be non-convex, as it was proven by M. Coste. In order to measure this non-convexity, we introduce a combinatorial object called the Poincaré-Reeb tree associated to a level curve and to a projection direction. Our goal is to characterize all topological types of Poincaré-Reeb trees. I will explain how to construct a family of polynomials that realizes a large class of these trees.

   Moreover, in a joint work with Arnaud Bodin and Patrick Popescu-Pampu, we extend the previous method of study of non-convexity to real algebraic domains.