Light Propagation in Massive, Non-Linear, (SuSy) Standard-Model Extension theories

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Highlights of the talk

It is not an organised exposition on non-Maxwellian electromagnetism but rather an account (à vol d'oiseau) on the activities (and what I like most).

- Context and motivations
- Massive theories (de Broglie-Proca, Bopp-Podolsky, Stueckelberg,...)
- Standard-Model Extension (SME) and Lorentz(-Poincaré) Symmetry Violation (LSV).
- Non-linear (Born-Infeld, Heisenberg-Euler....)
- Application and results:
  - (SME-LSV effective) photon mass, dispersion, sub-super luminal velocities, birefringence, non-conservation and frequency shifts.
  - Observational and experimental photon mass upper limits, magnetars, reinterpretation of dark energy.
- Heisenberg principle at cosmological scales (just mention).

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Since 2016 Non-Maxwellian EM (before GR)

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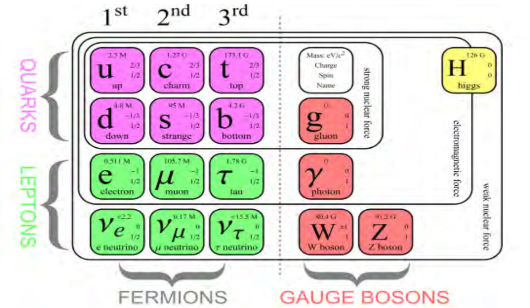
Motivations and context: is light the solution?

- GW detection 2015, but universe understanding is based on EM observations.
- As photons are the main messengers, fundamental physics has a concern in testing the foundations of electromagnetism.
- 96% universe dark (unknown), only part of 4% is known: yet precision cosmology.
- Dark matter and energy: *ad hoc* suppositions and experimentally undetected.
- Proposals of alternative theories of gravity, but GR works.
- Striking contrast: complex and multi-parameterised cosmology - linear electromagnetism from the 19^{th} century.
- There is no theoretical prejudice against a photon small mass: all radiative corrections are proportional to mass ('t Hooft).
- Electromagnetic radiation has zero rest mass ($v = c$). Since it carries momentum and energy, it has non-zero inertial mass. Hence, for EP, it has non-zero gravitational mass: $\rightarrow$ light must be heavy ('t Hooft).
- The Einstein demonstration of the equivalence of mass and energy (wagon at rest on frictionless rails, photon shot *inside* end to end) implies a massive photon.
- Special Relativity and Quantum Mechanics were born by reinterpreting light.

Motivations and context

- The photon is the only free massless particle of the Standard Model.
- The SM successful but shortcomings: neutrinos are massive, unbalance matter-antiamtter, Higgs is too light (for some),no gravitons..., no dark energy or dark matter particles.



Some considerations on non-Maxwelian electromagnetism

- non-linear Born-Infeld (for renormalisation of singularities); Heisenberg-Euler (2nd order QED as photon splitting, merging, photon-photon interaction, birefringence) or massive (de Broglie-Proca).
- Massive photons evoked for dark matter, inflation, magnetic monopoles, red-shifts, superconductors and "light shining through walls" exp.
- The dBP theory is not gauge invariant, but others are (quantisable Stueckelberg theory presents a scalar compensating field. Boulware showed the renormalisability and unitarity of QED with a dBP photon). If mass rises from the spontaneous symmetry $U(1)$ breaking, gauge invariance is insured also after breaking, possibly determined by the Higgs mechanism (but see Guendelman).
- For charge conservation (dBP Gauss law) the coupling of the photon mass to the scalar potential implies a density of "pseudo-charge" proportional to the squared mass, added to the ordinary charges. The two kinds of charges are conserved separately (but see Nussinov).
- Impact on relativity? Difficult answer: variety of the theories; removal of ordinary landmarks and rising of interwoven implications (TLP and dBP).

Massive theories: de Broglie-Proca

- The concept of a massive photon has been vigorously pursued by Louis de Broglie from 1922 throughout his life. Through dispersions in 1923 he defines the upper limit as 10^{-53} kg (PDG value 10^{-54} after many experiments and observations). In 1936 dB writes the modified Maxwells equations in a non-covariant form.
- Insted, the original aim of A. Proca, de Broglie's student, was the description of electrons and positrons. Despite Proca's assertions on the photons being massless, his Lagrangian has been used.

Massive theories: de Broglie-Proca

$$\mathcal{L} = -\frac{1}{4\mu}F_{\alpha\beta}F^{\alpha\beta} - \frac{\mathcal{M}^2}{2\mu}A_\alpha A^\alpha - j^\alpha A_\alpha \tag{1}$$

$F_{\mu\nu} = \partial_\mu A^\nu - \partial_\nu A^\mu$. Minimal action (Euler-Lagrange) $\rightarrow$ inhomogeneous eqs.
Ricci Curbastro-Bianchi identity $\partial^\lambda F^{\mu\nu} + \partial^\nu F^{\lambda\mu} + \partial^\mu F^{\nu\lambda} = 0 \rightarrow$ homogeneous eqs.

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} - \mathcal{M}^2\phi \ , \tag{2}$$

$$\nabla \times \vec{B} = \mu_0\vec{j} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t} - \mathcal{M}^2\vec{A} \ , \tag{3}$$

$$\nabla \times \vec{E} = -\frac{\partial\vec{B}}{\partial t} \ , \tag{4}$$

$$\nabla \cdot \vec{B} = 0 \ , \tag{5}$$

ϵ_0 permittivity, μ_0 permeability, ρ charge density, $\vec{j}$ current, ϕ and $\vec{A}$ potential.
$\mathcal{M} = m_\gamma c/\hbar = 2\pi/\lambda$, $\hbar$ reduced Planck (or Dirac) constant, c speed of light, λ Compton wavelength, m_γ photon mass.

Eqs. (2, 3) are Lorentz-Poincaré transformation but not Lorenz gauge invariant, though in static regime they are not coupled through the potential.

From the Lagrangian we get $\partial_\alpha F^{\alpha\beta} + \mathcal{M}^2 A^\beta = \mu j^\beta$. With the Lorentz subsidiary condition $\partial_\gamma A^\gamma = 0$,

$$\left[\partial_\mu \partial^\mu + \mathcal{M}^2\right] A^\nu = 0 \tag{6}$$

Through Fourier transform, at high frequencies (photon rest energy < the total energy; $\nu \gg 1$ Hz), the positive difference in velocity for two different frequencies ($\nu_2 > \nu_1$) is

$$\Delta v_g = v_{\mathrm{g}2} - v_{\mathrm{g}1} = \frac{c^3 \mathcal{M}^2}{8\pi^2}\left(\frac{1}{\nu_1^2} - \frac{1}{\nu_2^2}\right) \ , \tag{7}$$

being v_g the group velocity. For a single source at distance d, the difference in the time of arrival of the two photons is

$$\Delta t = \frac{d}{v_{\mathrm{g}1}} - \frac{d}{v_{\mathrm{g}2}} \simeq \frac{\Delta v_g d}{c^2} = \frac{dc\mathcal{M}^2}{8\pi^2}\left(\frac{1}{\nu_1^2} - \frac{1}{\nu_2^2}\right)$$
$$\simeq \frac{d}{c}\left(\frac{1}{\nu_1^2} - \frac{1}{\nu_2^2}\right) 10^{100} m_\gamma^2 \ . \tag{8}$$

Massive theories: Stueckelberg

- The Stueckelberg Lagrangian

$$\mathcal{L} = -\frac{1}{2}F^{\mu\nu}F_{\mu\nu} + m^2\left(A_\mu - \frac{\partial_\mu B}{m}\right)^2 - (\partial^\mu A_\mu + mB)^2 \qquad (9)$$

  where B is a scalar field to render the dBP *manifestly* gauge invariant.

- We have two fields and two equations of motion. The wave equations are

$$\partial_\mu\partial^\mu A^\nu + m^2 A^\nu = 0 \qquad (10)$$

$$\partial_\mu\partial^\mu B + m^2 B = 0 \qquad (11)$$

- First massive photon theory, gauge invariant

$$A_\mu \to A_\mu + \partial_\mu\Lambda \qquad B \to B + m\Lambda \qquad (\partial^2 + m^2)\Lambda = 0$$

Massive theories: Podolsky

- The Podolsky Lagrangian

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{b^2}{4}\left(\partial^\nu F^{\mu\nu}\right)\partial_\nu F_{\mu\nu} + j^\mu A_\mu \qquad (12)$$

where b has the dimension of m^{-1}.

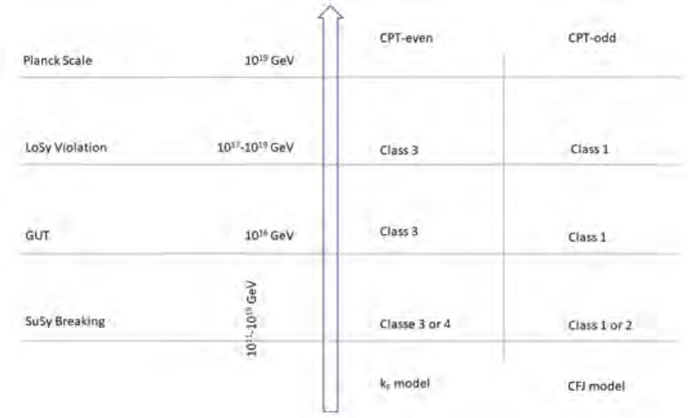
- The equations are

$$-b^2\partial_\mu\partial^\mu\left(\vec{\nabla}\cdot\vec{E}\right) + \vec{\nabla}\cdot\vec{E} - \rho = 0 \qquad (13)$$

$$-b^2\partial_\mu\partial^\mu\left[\frac{\partial\vec{E}}{\partial t} - \vec{\nabla}\times\vec{B}\right] + \frac{\partial\vec{E}}{\partial t} - \vec{\nabla}\times\vec{B} + \vec{j} = 0 \qquad (14)$$

- Gauge invariant $A_\mu \longrightarrow A_\mu + \partial_\mu\Lambda$
- Magnetic monopoles? and massive photons.
- Cut-off for short distances $\phi = \frac{e}{4e\pi}(1 - e^{-\frac{r}{b}})$

(SuSy and) LoSy breaking (LSV)



LoSy breaking (LSV)

- 4 models involving (Super and) Lorentz symmetries breaking. Dispersion relations show a non-Maxwellian behaviour for CPT even and odd sectors. Birefringence. Sub-super luminal velocities.
- An effective mass photon behaviour for both odd and pair CPT. In the odd CPT classes, f^{-2} in the group velocities emerges.
- A massive and gauge invariant Carroll-Field-Jackiw term in the Lagrangian is extracted and shown to be proportional to the background vector (or tensor).
- Effective or real mass? Higgs for charged leptons and quarks, the W and Z Bosons, while the Chiral Symmetry (Dynamical) Breaking (CSB) for (mostly) composite hadrons (baryons and mesons). Is it epistemologically evident what is effective mass and what is real?
- Frame dependency renders the SME-LSV mass unusual, but still the dimension is that of a mass.
- The effective mass upper value is compatible with experimental data.

LoSy breaking

The Lagrangian L_1 reads

$$L_1 = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}\epsilon^{\mu\nu\sigma\rho}k_\mu^{\mathrm{AF}}A_\nu F_{\sigma\rho} \ . \tag{15}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ are the covariant and contravariant forms, respectively, of the EM tensor; $\epsilon^{\mu\nu\sigma\rho}$ is the contravariant form of the Levi-Civita pseudo-tensor, and A_μ the potential covariant four-vector.
We observe the coupling between the EM field and the breaking vector k_α^{AF}.

The Lagrangian L_4 reads

$$L_4 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{r}{2}\chi_{\mu\nu}F^\mu_\kappa F^{\nu\kappa} + \frac{s}{2}\chi_{\mu\nu}\partial_\alpha F^{\alpha\mu}\partial_\beta F^{\beta\nu} \ , \tag{16}$$

The $\chi^{\alpha\beta}$ tensor is linearly related to the breaking tensor $k^{\alpha\beta\rho\sigma}$.
The s - mass^{-2} - parameter and the r - dimensionless - coefficient come from SuSy.

Effective mass and dispersion in SME

In Class 1 (CFJ) for $k_0^{\mathrm{AF}} = 0$ and $\vec{k}^{\mathrm{AF}}$, time and space components of the LSV vector, we get

$$m_\gamma = \frac{\hbar|\vec{k}^{\mathrm{AF}}|}{c}x \ , \tag{17}$$

where x is an angular factor (difference between the preferred frame and observer directions). In the photon rest frame, the angular dependence disappears.

$$\Delta t_{CFJ} = \frac{dc|\vec{k}^{\mathrm{AF}}|^2}{2}\left(\frac{1}{\omega_1^2} - \frac{1}{\omega_2^2}\right)x \ . \tag{18}$$

Comparing with dBP theory

$$\Delta t_{dBP} = \frac{d\ m_\gamma^2 c^3}{2h^2}\left(\frac{1}{\omega_1^2} - \frac{1}{\omega_2^2}\right) \ . \tag{19}$$

In Class 4, the integration of the photino leads

$$m_\gamma = \left(\frac{1 - r\chi}{s\chi}\right)^{1/2} \ . \tag{20}$$

Observational - experimental limits SME parameters

Table: Upper limits of the LSV parameters (the last value is in SI units): [a]Energy shifts in the spectrum of the hydrogen atom; [b]Rotation of the polarisation of light in resonant cavities; [c,e]Astrophysical observations. Such estimates are close to the Heisenberg limit on the smallest measurable energy or mass or length for a given time t, set equal to the age of the universe; [d]Rotation in the polarisation of light in resonant cavities. [f]Typical value.

$\lvert\vec{k}^{\rm AF}\rvert$ [a]	$< 10^{-10}$ eV $= 1.6 \times 10^{-29}$ J; 5.1×10^{-4} m^{-1}
$\lvert\vec{k}^{\rm AF}\rvert$ [b]	$< 8 \times 10^{-14}$ eV $= 1.3 \times 10^{-32}$ J; 4.1×10^{-7} m^{-1}
$\lvert\vec{k}^{\rm AF}\rvert$ [c]	$< 10^{-34}$ eV $= 1.6 \times 10^{-53}$ J; 5.1×10^{-28} m^{-1}
$k_0^{\rm AF}$ [d]	$< 10^{-16}$ eV $= 1.6 \times 10^{-35}$ J; 5.1×10^{-10} m^{-1}
$k_0^{\rm AF}$ [e]	$< 10^{-34}$ eV $= 1.6 \times 10^{-53}$ J; 5.1×10^{-28} m^{-1}
$k_{\rm F}$ [f]	$\simeq 10^{-17}$

Experimental mass limits: Particle Data Group

Citation: M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018)

γ (photon) $I(J^{PC}) = 0,1(1^{--})$

γ MASS

Results prior to 2008 are critiqued in GOLDHABER 10. All experimental results published prior to 2005 are summarized in detail by TU 05.

The following conversions are useful: $1\ \text{eV} = 1.783 \times 10^{-33}\ \text{g} = 1.957 \times 10^{-6}\ m_e$; $\bar{\lambda}_C = (1.973 \times 10^{-7}\ \text{m}) \times (1\ \text{eV}/m_\gamma)$.

VALUE (eV)	CL%	DOCUMENT ID		COMMENT
$<1 \times 10^{-18}$		[1] RYUTOV	07	MHD of solar wind
• • • We do not use the following data for averages, fits, limits, etc. • • •				
$<2.2 \times 10^{-14}$		[2] BONETTI	17	Fast Radio Bursts, FRB 121102
$<1.8 \times 10^{-14}$		[3] BONETTI	16	Fast Radio Bursts, FRB 150418
$<1.9 \times 10^{-15}$		[4] RETINO	16	Ampere's Law in solar wind
$<2.3 \times 10^{-9}$	95	[5] EGOROV	14	Lensed quasar position
		[6] ACCIOLY	10	Anomalous magn. mom.
$<1 \times 10^{-26}$		[7] ADELBERGER	07A	Proca galactic field
no limit feasible		[7] ADELBERGER	07A	γ as Higgs particle
$<1 \times 10^{-19}$		[8] TU	06	Torque on rotating magnetized toroid
$<1.4 \times 10^{-7}$		ACCIOLY	04	Dispersion of GHz radio waves by sun
$<2 \times 10^{-16}$		[9] FULLEKRUG	04	Speed of 5-50 Hz radiation in atmosphere
$<7 \times 10^{-19}$		[10] LUO	03	Torque on rotating magnetized toroid
$<1 \times 10^{-17}$		[11] LAKES	98	Torque on toroid balance
$<6 \times 10^{-17}$		[12] RYUTOV	97	MHD of solar wind
$<8 \times 10^{-16}$	90	[13] FISCHBACH	94	Earth magnetic field
$<5 \times 10^{-13}$		[14] CHERNIKOV	92	Ampere's Law null test
$<1.5 \times 10^{-9}$	90	[15] RYAN	85	Coulomb's Law null test
$<3 \times 10^{-27}$		[16] CHIBISOV	76	Galactic magnetic field
$<6 \times 10^{-16}$	99.7	[17] DAVIS	75	Jupiter's magnetic field
$<7.3 \times 10^{-16}$		HOLLWEG	74	Alfven waves
$<6 \times 10^{-17}$		[18] FRANKEN	71	Low freq. res. circuit
$<2.4 \times 10^{-13}$		[19] KROLL	71A	Dispersion in atmosphere
$<1 \times 10^{-14}$		[20] WILLIAMS	71	Tests Coulomb's Law
$<2.3 \times 10^{-15}$		GOLDHABER	69	Satellite data

Experimental mass limits: the graviton

- LIGO upper limit 2×10^{-58} kg (classical dispersion effect check).
- Often graviton mass upper limit supposes massless photons.

3. Graviton mass limits:

Gravitation wave dispersion (Finn and Sutton, 2002)	3×10^{12}	8×10^{-20}	10^{-55}	Question mark for scalar graviton
Pulsar timing (Baskaran *et al.*, 2008)	2×10^{16}	9×10^{-24}	2×10^{-59}	Fluctuations due to graviton phase velocity
Gravity over cluster sizes (Goldhaber and Nieto, 1974)	2×10^{22}	10^{-29}	2×10^{-65}	
Near field constraints (Gruzinov, 2005)	3×10^{24} (10^{8} pc)	6×10^{-32}	10^{-67}	For DGP model
Far field constraints (Dvali, Gruzinov, and Zaldarriaga, 2003)	3×10^{26} (10^{10} pc)	6×10^{-34}	10^{-69}	For DGP model

Non-linear theories: Born-Infeld

- The Born-Infeld Lagrangian

$$\mathcal{L} = \sqrt{1+F} - 1 + j^\mu A_\mu \tag{21}$$

- The equations are

$$\partial_\mu \left(\frac{F^{\mu\nu} \left(1+F\right)^{-\frac{1}{2}}}{2} \right) = j^\nu \tag{22}$$

- Electromagnetic field gives origin to the mass of the charge.
- Avoidance of infinities out of self-energy $\phi(0) = 1.8541 \frac{e}{r_0}$.
- The parameter r_0 is computed out of analytic expressions.

- The Heisenberg-Euler Lagrangian

$$\mathcal{L} = -\frac{F_{\mu\nu}F^{\mu\nu}}{4} + \frac{e^2}{\hbar c}\int_0^\infty d\eta \frac{e^{-\eta}}{\eta^3}\cdot\left\{ i\frac{\eta^2}{2}F^{\mu\nu}F^*_{\mu\nu}\cdot \right.$$
$$\cdot\frac{\cos\left[\frac{\eta}{\mathfrak{E}_k}\sqrt{\frac{-F_{\mu\nu}F^{\mu\nu}}{2}+iF^{\mu\nu}F^*_{\mu\nu}}\right]+\cos\left[\frac{\eta}{\mathfrak{E}_k}\sqrt{\frac{-F_{\mu\nu}F^{\mu\nu}}{2}-iF^{\mu\nu}F^*_{\mu\nu}}\right]}{\cos\left[\frac{\eta}{\mathfrak{E}_k}\sqrt{\frac{-F_{\mu\nu}F^{\mu\nu}}{2}+iF^{\mu\nu}F^*_{\mu\nu}}\right]-\cos\left[\frac{\eta}{\mathfrak{E}_k}\sqrt{\frac{-F_{\mu\nu}F^{\mu\nu}}{2}-iF^{\mu\nu}F^*_{\mu\nu}}\right]}$$
$$\left. + |\mathfrak{E}_k|^2 + \frac{\eta^3}{6}\cdot F_{\mu\nu}F^{\mu\nu}\right\} \tag{23}$$

$$F^*_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma} \tag{24}$$

- Photon-Photon interaction and Photon splitting since HE theory relates to second order QED.
- Vacuum polarisation occurs for $E_c > 1.3 \times 10^{18}$ V/m or $B_c > 4.4 \times 10^{13}$ G.

Heisenberg-Euler on magnetars overcritical magnetic field. Blue or red shift depending on polarisation for a photon emitted up to similar values to the gravitational redshift.

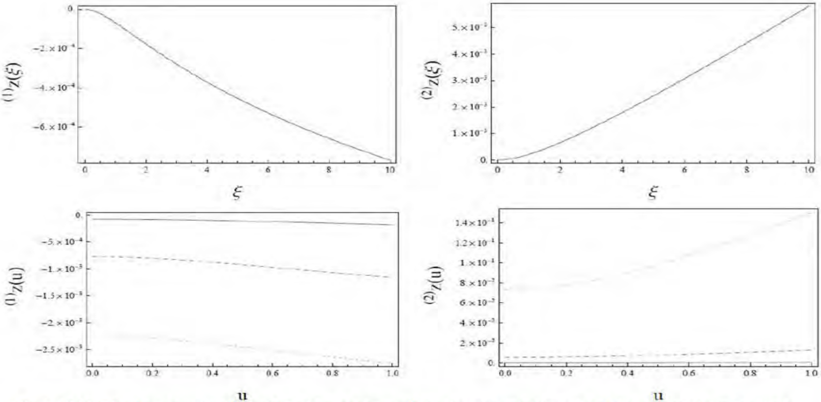


Fig.1. EMS (Electromagnetic shift) of the two photon polarisations versus the ratio of the magnetic/overcritical fields (upper panel), and the azimuthal angle (lower panel). The EMS can reach comparable values to the gravitational Einstein shift. The figure is taken from [Bonetti, Perez Bergliaffa, Spallicci, 2016].

Experimental mass limits: warnings

- Quote "Quoted photon-mass limits have at times been overly optimistic in the strengths of their characterisations. This is perhaps due to the temptation to assert too strongly something one knows to be true. A look at the summary of the Particle Data Group (Amsler et al.. 2008) hints at this. In such a spirit, we give here our understanding of both secure and speculative mass limits." Goldhaber and Nieto, Rev. Mod. Phys., 2000
- The lowest theoretical limit on the measurement of any mass is dictated by the Heisenberg's principle $m \geq \hbar/2\Delta t c^2$, and gives 1.35×10^{-69} kg, where Δt is the supposed age of the Universe.
- Photon mass reproduces plasma dispersion for the frequency f^{-2} dependence of the group velocity. There is not the possibility to disentangle the two effects, unless a different z dependence.

de Broglie-Proca

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} - \mathcal{M}^2 \vec{A} = \mu_0 \vec{j} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} - \mathcal{M}^2 \vec{A} \ , \tag{25}$$

where $\mathcal{M} = m_\gamma c/\hbar$.

SME-LSV

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{j} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} + k_0^{\rm AF} \vec{B} - \vec{k}^{\rm AF} \times \frac{\vec{E}}{c} \ , \tag{26}$$

where $k_0^{\rm AF}$ and $\vec{k}^{\rm AF}$ are the time and space components of the LSV vector.

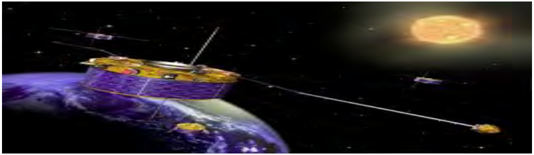
NLEM generalised Lagrangian, encompassing Born-Infeld and Euler-Heisenberg: polynomial, function of integer powers of the field and its dual $\mathcal{L} = \mathcal{L}(\mathcal{F}, \mathcal{G})$ where

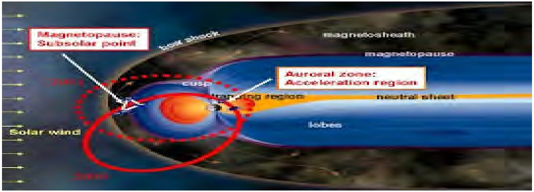
$$\mathcal{F} = \frac{1}{2\mu_0} \left(\frac{\vec{E}^2}{c^2} - \vec{B}^2 \right) \qquad \mathcal{G} = \frac{1}{\mu_0} \frac{\vec{E}}{c} \cdot \vec{B} \ .$$

The modified Ampère-Maxwell equation becomes

$$\nabla \times \left(\frac{\partial \mathcal{L}}{\partial \mathcal{F}} \vec{B} - \frac{1}{c} \frac{\partial \mathcal{L}}{\partial \mathcal{G}} \vec{E} \right) = \mu_0 \vec{j} + \frac{1}{c^2 \partial t} \left(\frac{\partial \mathcal{L}}{\partial \mathcal{F}} \vec{E} + c \frac{\partial \mathcal{L}}{\partial \mathcal{G}} \vec{B} \right) \ . \tag{27}$$

Experimental mass limits: Cluster





- Highly elliptical evolving orbits in tetrahedron: perigee 4 $R_\oplus$ apogee 19.6 $R_\oplus$, visited a wide set of magnetospheric regions. Inter-spacecraft separation ranging from 10^2 to 10^4 km.
- Small mass → precise experiment or very large apparatus (Compton wavelength). The largest-scale magnetic field accessible to *in situ* spacecraft measurements, *i.e.* the interplanetary magnetic field carried by the solar wind.

Experimental mass limits: Cluster

- $j_P = 1.86 \cdot 10^{-7} \pm 3 \cdot 10^{-8}$ A m^{-2}, while $j_B = |\nabla \times \vec{B}|/\mu_0$ is $3.5 \pm 4.7 \cdot 10^{-11}$ A m^{-2}. A_H is an estimate, not a measurement.

$$A_H{}^{\frac{1}{2}}\left(m_\gamma + \Delta m_\gamma\right) = A_H{}^{\frac{1}{2}}\left(m_\gamma + \left|\frac{\partial m_\gamma}{\partial j_P}\right|\Delta j_P + \left|\frac{\partial m_\gamma}{\partial j_B}\right|\Delta j_B\right) =$$

$$k\left[(j_P - j_B)^{\frac{1}{2}} + \frac{\Delta j_P + \Delta j_B}{2(j_P - j_B)^{\frac{1}{2}}}\right] \quad . \qquad (28)$$

Considering j_P and Δj_P of the same order, $j_P = 0.62\ \Delta j_P$, and both much larger than j_B and Δj_B, Eq. (28), after squaring, leads to

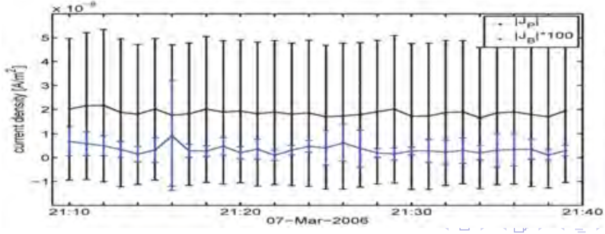
$$A_H{}^{\frac{1}{2}}\left(m_\gamma + \Delta m_\gamma\right) \sim k\left(j_P + \Delta j_P\right)^{1/2} \quad . \qquad (29)$$

Table: The values of m_γ (according to the estimate on A_H).

A_H [T m]	0.4	29 (Z)	637
m_γ [kg]	1.4×10^{-49}	1.6×10^{-50}	3.4×10^{-51}

Experimental mass limits: Cluster

- The particle current density $\vec{j} = \vec{j}_P = ne(\vec{v}_i - \vec{v}_e)$ from ion and electron currents; n is the number density, e the electron charge and $\vec{v}_i$, $\vec{v}_e$ the velocity of the ions and electrons, respectively.
- An accurate assessment of the particle current density in the solar wind is difficult due to inherent instrument limitations.
- $j_P >> j_B$ (up to four orders of magnitude), mostly due to the differences in the i, e velocities, while the estimate of density is reasonable. While we can't exclude that this difference is due to the dBP massive photon, the large uncertainties related to particle measurements hint to instrumental limits.



Experimental mass limits: dispersion

- Photon mass reproduces plasma dispersion, the frequency f^{-2} dependence of the group velocity of the pulsar or FRB radiation through the ionised components of the interstellar medium. Again, pulses at lower radio frequencies arrive later than those at higher frequencies.
- In absence of an alternative way to measure plasma dispersion, there is no way to disentangle plasma effects from a dBP photon

$$\frac{m_\gamma}{\sqrt{n}} \left[\mathrm{kg\ m}^{3/2}\right] = 6.62 \times 10^{-50} \ , \tag{30}$$

implies that for this ratio, a massive photon and the average electron density along the line of sight determine the same dispersion.
- Data on FRB 150418 indicate $m_\gamma \lesssim 1.8 \times 10^{-14}$ eV c^{-2} (3.2×10^{-50} kg), for a red-shift $z = 0.492$, while for FRB 121102 $m_\gamma \lesssim 2.2 \times 10^{-14}$ eV c^{-2} (3.9×10^{-50} kg). The different red-shift dependences of the plasma and photon mass contributions to DM can be used to improve the sensitivity to m_γ.
- OLFAR swarm of small satellites around the Moon, or Moon station

SI units, photon energy-momentum tensor density $[\mathrm{Jm}^{-3}]$
${\theta^0}_0 =$ energy density, ${\theta^0}_k =$ energy flux divided by c along the k direction, ${\theta^k}_0 =$ momentum density through the orthogonal surface to k, multiplied by c.
The derivative of the energy-momentum density tensor $[\mathrm{Jm}^{-4}]$.

$$\partial_\alpha {\theta^\alpha}_\tau \longrightarrow \Delta\nu \text{ .wave} - \text{particle correspondence : red or blue shift} \tag{31}$$

The scalar fields $\mathcal{F}$ and its dual $\mathcal{G}$ are

$$\mathcal{F} = -\frac{1}{4\mu_0}F^2 = -\frac{1}{4\mu_0}F_{\sigma\tau}F^{\sigma\tau} = \frac{1}{2\mu_0}\left(\frac{\vec{E}^2}{c^2} - \vec{B}^2\right) , \tag{32}$$

and

$$\mathcal{G} = -\frac{1}{4\mu_0}FG = -\frac{1}{4\mu_0}F_{\sigma\tau}G^{\sigma\tau} = \frac{1}{\mu_0}\frac{\vec{E}}{c}\cdot\vec{B} , \tag{33}$$

where $F_{\sigma\tau}$ is the electromagnetic field tensor and $G^{\sigma\tau} = \frac{1}{2}\epsilon^{\sigma\tau\alpha\beta}F_{\alpha\beta}$ is its dual;
$\mu_0 = 4\pi \times 10^{-7} \approx 1.256$ H m^{-1} or V s A^{-1} m^{-1} is the vacuum permeability.

We now associate the fields above to a background field and imagine a photon crossing such a background. Thereby, we split the total (T) electromagnetic tensor field $F_{\rm T}$ and the total (T) electromagnetic 4-potential $A_{\rm T}$ in the background (capital letters) and photon (small letters)

$$A_{\rm T}^{\beta} = A^{\beta} + a^{\beta} \ , \qquad F_{\rm T}^{\alpha\beta} = F^{\alpha\beta} + f^{\alpha\beta} \ , \qquad G_{\rm T}^{\alpha\beta} = G^{\alpha\beta} + g^{\alpha\beta} \ . \quad (34)$$

It is known, although seldom mentioned, that a photon may exchange energy-momentum (density), represented by the tensor $\theta^{\alpha}{}_{\tau}$, with the background, even in the Maxwellian theory, if the background field is space-time dependent. The energy-momentum density tensor variation is

$$\partial_\alpha \theta^{\alpha}{}_{\tau} = \underbrace{j^\alpha f_{\alpha\tau} - \frac{1}{\mu_0}(\partial_\alpha F^{\alpha\beta}) f_{\beta\tau}}_{\text{Maxwellian terms}} \ , \tag{35}$$

being j^β a possibly existing external 4-current. In conclusion, a frequency shift may exist even in the framework of the standard electromagnetic theory.

Stepping into the dBP formalism, the photon therein interacts with the background through the potential even when the background field is constant. Indeed, if a field is constant, its associated potential is not. For $\mathcal{M} = m_\gamma c/\hbar$, being m_γ the photon mass, the energy-momentum density tensor variation becomes

$$\partial_\alpha \theta^\alpha{}_\tau = \underbrace{j^\alpha f_{\alpha\tau} - \frac{1}{\mu_0}(\partial_\alpha F^{\alpha\beta}) f_{\beta\tau}}_{\text{Maxwellian terms}} + \underbrace{\frac{1}{\mu_0}\mathcal{M}^2(\partial_\tau A^\beta) a_\beta}_{\text{de Broglie-Proca term}} \ . \qquad (36)$$

Incidentally, the dBP photon does not display energy changes in absence of a background, unless invoking imaginary masses and frequencies [Thiounn, 1960, Yourgrau, Woodward, 1974].

LSV present in the photonic sector only. The space-time metric, spin connection and curvature are unaffected by the LSV, and we stick to the Minkowski space-time. In contrast to the LSV tensor, the LSV vector does violate the CPT theorem. The frequency shift is thereby an observable of CPT violation,

$$\partial_\alpha \theta^\alpha{}_\tau = \underbrace{j^\nu f_{\nu\tau} - \frac{1}{\mu_0}\left(\partial_\alpha F^{\alpha\nu}\right) f_{\nu\tau}}_{\text{Maxwellian terms}} -$$

$$\frac{1}{\mu_0}\left[\underbrace{\frac{1}{2}\left(\partial_\alpha k_\tau^{\rm AF}\right) g^{\alpha\nu} a_\nu - \frac{1}{4}\left(\partial_\tau k_{\rm F}^{\alpha\nu\kappa\lambda}\right) f_{\alpha\nu} f_{\kappa\lambda}}_{\text{EM background independent terms}} + \underbrace{\partial_\alpha\left(k_{\rm F}^{\alpha\nu\kappa\lambda} F_{\kappa\lambda}\right) f_{\nu\tau}}_{\text{non-constant term}} + \right.$$

$$\left.\underbrace{k_\alpha^{\rm AF} G^{\alpha\nu} f_{\nu\tau}}_{\text{constant term}}\right] . \tag{37}$$

The LSV 4-vector and the rank-4 tensor are the vacuum condensation in the context of string models [Kostelecký, Samuel, 1998]. They determine the space-time anisotropies and their presence reveals that vacuum effects are responsible for the energy variation of light-waves. Anisotropies are under considerations in cosmology, e.g., [Migkas et al, 2021].

For NLEM, we have set a generalised Lagrangian, encompassing the formalisms of Born-Infeld and Euler-Heisenberg, as a polynomial, function of integer powers of the field and its dual. Indeed, the generalised Lagrangian is written as

$$\mathcal{L} = \mathcal{L}\left(\mathcal{F}, \mathcal{G}\right) \ . \tag{38}$$

The photon energy-momentum density tensor variation $\partial_\alpha \theta^\alpha_\tau$ [Jm^{-4}] is given by

$$\partial_\alpha \theta^\alpha_\tau = -\partial_\alpha \left(C_1 F^{\alpha\nu} + C_2 G^{\alpha\nu}\right) f_{\nu\tau} + \frac{1}{4}\left(\partial_\tau C_1\right) f^2 + \frac{1}{4}\left(\partial_\tau C_2\right) gf - \frac{1}{8}\left(\partial_\tau s^{\nu\alpha\kappa\lambda}\right) f_{\nu\alpha} f_{\kappa\lambda} - \frac{1}{4}\left(\partial_\tau t^{\nu\alpha\rho\sigma}\right) f_{\nu\alpha} f_{\rho\sigma} \ , \tag{39}$$

where the coefficients are computed on the background and are

$$\left.\frac{\partial\mathcal{L}}{\partial\mathcal{F}}\right|_{\mathrm{B}} = C_1 \quad \left.\frac{\partial\mathcal{L}}{\partial\mathcal{G}}\right|_{\mathrm{B}} = C_2 \quad \left.\frac{\partial^2\mathcal{L}}{\partial\mathcal{F}^2}\right|_{\mathrm{B}} = D_1 \quad \left.\frac{\partial^2\mathcal{L}}{\partial\mathcal{G}^2}\right|_{\mathrm{B}} = D_2 \quad \left.\frac{\partial^2\mathcal{L}}{\partial\mathcal{F}\partial\mathcal{G}}\right|_{\mathrm{B}} = D_3 \ , \tag{40}$$

$$s^{\mu\nu\kappa\lambda} = D_1 F^{\mu\nu} F^{\kappa\lambda} + D_2 G^{\mu\nu} G^{\kappa\lambda} \qquad t^{\mu\nu\kappa\lambda} = D_3 F^{\mu\nu} F^{\kappa\lambda} \ , \tag{41}$$

Whether this shift is accompanied necessarily by a photon with an effective mass, it is subject of on-going investigations.

Superposing the shifts.

- $z = \Delta\nu/\nu_o$ where $\Delta\nu = \nu_\mathrm{e} - \nu_o$ is the difference between the observed ν_o and emitted ν_e frequencies, or else $z = \Delta\lambda/\lambda_\mathrm{e}$ for the wavelengths.
- Expansion causes λ_e to stretch to λ_c that is $\lambda_\mathrm{c} = (1 + z_\mathrm{C})\lambda_\mathrm{e}$. The wavelength λ_C could be further stretched or shrunk for the ETE shift to $\lambda_\mathrm{o} = (1 + z_\mathrm{S})\lambda_\mathrm{c} = (1 + z_\mathrm{S})(1 + z_\mathrm{C})\lambda_\mathrm{e}$. But since $\lambda_\mathrm{o} = (1 + z)\lambda_\mathrm{e}$, we have $1 + z = (1 + z_\mathrm{C})(1 + z_\mathrm{S})$.

$$z = z_\mathrm{C} + z_\mathrm{S} + z_\mathrm{C} z_\mathrm{S} \ . \tag{42}$$

The second order is not negligible for larger z_C.

Impact on cosmology: dark energy

Behaviour of the ETE shift with distance.

Type	1	2	3	4
$d\nu$	$k_1 \nu dr$	$k_2 \nu_e dr$	$k_3 dr$	$k_4 \nu_o dr$
ν_o	$\nu_e e^{k_1 r}$	$\nu_e(1 + k_2 r)$	$\nu_e + k_3 r$	$\dfrac{\nu_e}{1 - k_4 r}$
$z_{\rm S}$	$e^{-k_1 r} - 1$	$-\dfrac{k_2 r}{1 + k_2 r}$	$-\dfrac{k_3 r}{\nu_e + k_3 r}$	$-k_4 r$
k_i	$-\dfrac{\ln(1 + z_{\rm S})}{r}$	$-\dfrac{z_{\rm S}}{r(1 + z_{\rm S})}$	$-\dfrac{\nu_{\rm e} z_{\rm S}}{r(1 + z_{\rm S})}$	$-\dfrac{z_{\rm S}}{r}$
r	$-\dfrac{\ln(1 + z_{\rm S})}{k_1}$	$-\dfrac{z_{\rm S}}{k_2(1 + z_{\rm S})}$	$-\dfrac{\nu_{\rm e} z_{\rm S}}{k_3(1 + z_{\rm S})}$	$-\dfrac{z_{\rm S}}{k_4}$

Table: The different variations of the frequency ν can be summarised by four different cases of proportionality: 1. to the instantaneous frequency and the distance; 2. to the emitted frequency and the distance; 3. to the distance only; 4. to the observed frequency and the distance. These variations determine the frequency observed ν_o, the shift $z_{\rm S}$, the parameters k_i and the distance r. The positiveness of the distance r constraints $z_{\rm S} > 0$ for $k_1 < 0$, and $-1 < z_{\rm S} < 0$ for $k_1 > 0$; $z_{\rm S} > 0$ for $k_2 < 0$, and $-1 < z_{\rm S} < 0$ for $k_2 > 0$; $z_{\rm S} > 0$ for $k_3 < 0$, and $-1 < z_{\rm S} < 0$ for $k_3 > 0$; $z_{\rm S} > 0$ for $k_4 < 0$, and $z_{\rm S} < 0$ for $k_4 > 0$.

Impact on cosmology: dark energy

1. The total red-shift z is the combination of the expansion red-shift $z_{\rm C}$ and of a static, red or blue shift $z_{\rm S}$, due to the energy non-conservation of the photon propagating through EM fields (host galaxy, intergalactic and Milky Way) and possibly LSV fields.
2. Then, $z_{\rm S}$ is a manifestation of an effective dark energy caused by the expectation values of the vacuum under LSV. If so, dark energy, *i.e.* vacuum energy, is not causing an accelerated expansion but a frequency shift.
3. The *single* $z_{\rm S}$ shift from a *single* SNIa may be small or large, red or blue, depending on the orientations of the LSV (vector or tensor) and of the EM fields, as well as the distance of the source. Anyway, the colour of $z_{\rm LSV}$ is the final output of a series of shifts, both red and blue, encountered along the path.
4. If the $z_{\rm S}$ shift is blue, the photon gains energy; it implies that the real z, traditionally the red-shift, is larger than the measured z, as $z_{\rm S}$ is subtracted from $z_{\rm C}$, the expansion red-shift. If red, $z_{\rm S}$ corresponds to dissipation along the photon path; it implies that the real z is smaller than the measured z, as $z_{\rm LSV}$ is added to $z_{\rm C}$.

Impact on cosmology: dark energy

- Cosmology model A: we set $\Omega_M = 0.3$ and consider $\Omega_K = 0$, implying a flat universe where the "cosmic triangle" relation $\Omega_M + \Omega_K + \Omega_\Lambda = 1$, is not satisfied *ab initio*. Nevertheless, the dark energy effect could be replaced *a posteriori* by the effect of z_{S}. This approach supposes that z_{S} is a manifestation of the LSV vacuum energy in string models, in case of the SME.
- Cosmology model B: we take into account an open universe model, where $\Omega_M = 0.3$ and $\Omega_K = 0.7$, so that $\Omega_K + \Omega_M = 1$.
- Cosmology model C: we return to the Einstein-de Sitter conception, that is a flat, matter dominated universe with $\Omega_M = 1$. This was one of the most popular cosmological model before the advent of the Dark Energy hypothesis.

Impact on cosmology: dark energy

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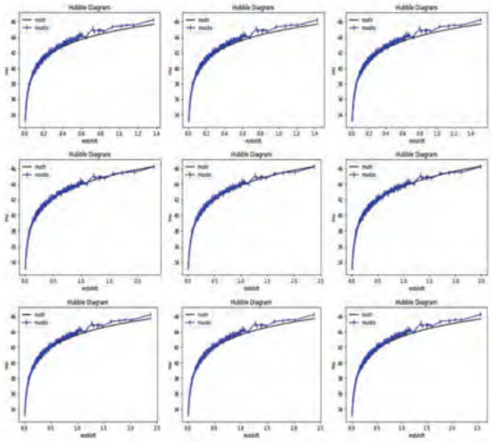

Figure: Through the Hubble diagrams, we compare the three cosmological models A, B, C, each for row, based upon the best fit values of k_1 with data from the Pantheon Sample (1084 SNe Ia), with three values of H_0 (67, 70, 74 km s^{-1} per Mpc), each for column. The black lines represent the models, while the blue marks trace the SNe Ia data with their errors.

Conclusions and perpectives, on-going work

- Departing from the Maxwellian opens rich and various large avenues for reinterpretation of the universe and the foundations of physics.
- MMS satellites: we are working on a very large 7 years set of data, analysing in extreme detail the Ampère's law. Exciting intermediate outcomes are emerging.
- Interferometry proposal: $H_0 = 70$ km s^{-1} per Mpc $= 2.3 \times 10^{-18}$ m s^{-1} per metre. This corresponds to a static relative shift upper limit $\frac{H_0}{c} = \frac{\Delta f}{f}$ per metre $= 7.7 \times 10^{-27}$. This appears measurable with a laboratory optical length larger than the Earth-Moon distance.

Heisenberg at cosmological scales

For an observation time equal to the universe age, the Heisenberg principle fixes the value of the smallest measurable mass at $m_{\rm H} = 1.35 \times 10^{-69}$ kg and prevents to probe the masslessness for any particle using a balance. The corresponding reduced Compton length to $m_{\rm H}$ is $\lambdabar_{\rm H}$, and represents the length limit beyond which masslessness cannot be proved using a metre ruler. In turns, $\lambdabar_{\rm H}$ is equated to the luminosity distance $d_{\rm H}$ which corresponds to a red shift $z_{\rm H}$. When using the Concordance-Model parameters, we get $d_{\rm H} = 8.4$ Gpc and $z_{\rm H} = 1.3$. Remarkably, $d_{\rm H}$ falls quite short to the radius of the *observable* universe. According to this result, tensions in cosmological parameters could be nothing else but due to comparing data inside and beyond $z_{\rm H}$. Finally, in terms of quantum quantities, the expansion constant H_0 reveals to be one order of magnitude above the smallest measurable energy, divided by the Planck constant.

$$H_0 = \frac{1}{\Delta t} = \frac{2\Delta E}{\hbar} = \frac{2m_{\rm H}c^2}{\hbar} = \frac{4\pi m_{\rm H}c^2}{h} = 70 \text{ km/s per Mpc} . \qquad (43)$$

$70 \text{ km/s per Mpc}/4\pi =$
$5.3 \text{ km/s per Mpc which corresponds to the Hubble tension}$

Grazie per la vostra attenzione