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Institut de Matheḿatiques de Bourgogne

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Contents

Abstracts

KP2 real multi-line solitons: the role of combinatorics and tropical geometry in the direct and inverse spectral problems

Simonetta Abenda

University of Bologna

I will present some recent results and compare different approaches to the direct and inverse spectral problems for KP2 real multi-line soliton.

The talk is based on research in collaboration with P.G. Grinevich and with T. Özlüm Çelik, C. Fevola and Y. Mandelstam.

Numerical approximations to nonlinear dispersive equations in non-smooth regimes

Yvonne Alama Bronsard

University of Rennes

This talk deals with the numerical approximation to nonlinear dispersive equations, such as the prototypical nonlinear Schrödinger equation. We introduce novel integration techniques allowing for the construction of schemes which perform well both in smooth and non-smooth settings. We obtain symmetric low-regularity schemes with very good structure preserving properties over long times. Higher order extensions will be presented, following new techniques based on decorated trees series inspired by singular stochastic PDEs via the theory of regularity structures. We will consider both the case of deterministic and randomized initial data. Lastly, we will discuss very recent numerical advances for the approximation of some nonlinear and nonlocal integrable PDEs, such as the Benjamin-Ono equation.

Short- and long-time behavior in evolution equations: the role of the hypocoercivity index

Anton Arnold

Vienna University of Technology

The "index of hypocoercivity" is defined via a coercivity-type estimate for the self-adjoint/skew-adjoint parts of the generator, and it quantifies 'how degenerate' a hypocoercive evolution equation is, both for ODEs and for evolutions equations in a Hilbert space. We show that this index characterizes the polynomial decay of the propagator norm for short time and illustrate these concepts for the Lorentz kinetic equation on a torus.

This talk is based on joint work with F. Achleitner, E. Carlen, E. Nigsch, and V. Mehrmann.

Parametrical numerical approximation of equations of the type $\partial_t u = f(t, x, u)$

Eddy B. de Leon

TU München

We look for numerical solutions of differential equations of the type $\partial_t u = f(t, x, u)$ with $t \in \mathbb{R}$, $x \in \mathbb{R}^d$, initial conditions $u(0, x) = u_0(x)$ and the condition that u must vanish at spatial infinity for all $t\imath\mathbb{R}$. Physically interesting problems of this type include the time-dependent Schrödinger equation. In order to satisfy the condition at spatial infinity, we assume that the approximate solution lives in a manifold of functions vanishing at infinity (e.g. a manifold of Gaussians). In this context, a solution to the PDE can be regarded as a 'trajectory' inside of this manifold, whose dynamics is described by the PDE. Moreover, setting up the manifold explicitly via a set of parameters in a high-dimensional complex space, one can see the solution in terms of the actual trajectories of such parameters inside of this complex space, which in turn define the trajectory of the approximate solution inside the chosen manifold of functions.

On the preceise cusped behaviour of extreme solutions to Whitham-type equations

Mats Ehrnström
NTNU Trondheim

We prove exact leading-order asymptotic behaviour at the origin for nontrivial solutions of two families of nonlocal equations. The equations investigated include those satisfied by the cusped highest steady waves for both the uni- and bidirectional Whitham equations. The problem is in a way analogous to capturing the 120◦ interior angle at the crests of classical Stokes' waves of greatest height, but as we shall see, requires several additional steps. Our results partially settle conjectures for extreme waves posed in a series of recent papers. The methods may be generalised to solutions of other nonlocal equations, and can moreover be used to determine asymptotic behaviour of their derivatives to any order.

This is joint work with O. Mæhlen (Paris–Saclay) and K Varholm (Pittsburgh).

Methods for computing PDEs in the complex plane

Marco Fasondini
University of Leicester

We complexify one of the variables of 1+1-dimensional PDEs and consider the problem of computing solutions in the complex plane. One approach, of which several examples will be given, is numerical analytic continuation via rational and Hermite-Padé approximation. Motivated by the shortcomings of this approach, we propose two new methods: complex finite differences and a 'pole field solver' approach, inspired by the pole field solver method for the Painlevé equations.

Explicit formulas for the Benjamin-Ono equation

Louise Gassot
University of Rennes

In this talk, we show how the initial-value problem for the Benjamin-Ono equation on the line with rational initial data in $L^2(R)$ having only simple poles can be solved in closed form via a determinant formula involving contour integrals. The proof is based on an explicit formula for general solutions established by Patrick Gérard.

This is a joint work with Elliot Blackstone, Patrick Gérard and Peter Miller.

Helicity in dispersive continuum mechanics

Sergey Gavrilyuk
University of Aix-Marseilles

New conservation laws are obtained for the equations of dispersive con- tinuum mechanics describing Euler–van der Waals–Korteweg's fluids, flu- ids containing small gas bubbles, long free surface gravity waves (Serre– Green–Naghdi equations), ... The corresponding mathematical models are characterized by the dependence of the energy on spatial and temporal derivatives of unknowns. In particular, the analogues of helicity invariants are derived. This is a joint work with H. Gouin.

Global wellposedness and soliton resolution for the half-wave maps equation with rational data on the line

Patrick Gérard
University of Paris-Saclay

We show that the energy critical half-wave maps equation

$$\partial_t u = u \times |D|u,$$

where u=u(t,x) is valued to the two-dimensional sphere, admits global rational solutions in the x variable on the line. Furthermore, for a dense subset of initial data, we prove long time soliton resolution. This is a joint work with Enno Lenzmann (Basel).

On the N-wave hierarchy with constant boundary conditions

Georgi Grahovski
University of Essex

In this talk, we will present the direct scattering transform for the N-wave resonant interaction equations with non-vanishing boundary conditions. For special choices of the boundary values $Q_\pm$, we outline the spectral properties of L, the direct scattering transform and construct its fundamental analytic solutions. Then, we generalise Wronskian relations for the case of constant boundary conditions.

Finally, using the Wronskian relations we derive the dispersion laws for the N-wave hierarchy and describe the NLEE related to the given Lax operator. The results are illustrated by an example of 4-wave resonant interaction system related to the algebra $sp(4, \mathbb{C})$.

Based on a joint work with Vladimir S. Gerdjikov

Existence of prescribed L2 norm solutions for nonlinear Schrödinger equations on metric graphs: the mass supercritical case.

Louis Jeanjean

University of Franche-Comté

In this talk we discuss the existence of prescribed L2 norm solutions to nonlinear Schrödinger equations set on metric graphs. A common strategy employed to find such a solution is to search for a constrained critical point of the associated energy functional. Some geometric properties of the functional vary depending on the exponent in the nonlinear term of the equation. In the so-called mass subcritical case, the functional is bounded from below and coercive on the constraint, so one may search for a critical point as a global minimum. As such, in the last years, this case has been extensively studied. However, in the complementary case, known as the mass supercritical case, the energy functional is no longer bounded from below on the constraint and presents a lack of a priori bounds on the possible critical points. As a result, very little is yet known about this case. Through the presentation of some of the few existing results, we shall discuss the main obstacles that need to be overcome to treat this case under general assumptions. We will also present some of the tools that have already been developed for this purpose. *This talk is based on joint works with J. Borthwick, P. Carrillo, X. Chang, S. Dovetta, D. Galant, E. Serra, N. Soave, C. Troestler.*

Superfluidity and the spectrum of polaron Hamiltonians

Jonas Lampart

University of Burgundy

I will discuss how superfluidity manifests itself in the spectrum of the Hamiltonian for a test particle travelling through a Bose Einstein condensate. In the Bogoliubov-Fröhlich polaron model, a stable polaron with momentum P corresponds to a ground state of the Hamiltonian at fixed total momentum. I will explain a recent result in collaboration with Benjamin Hinrichs, which shows that a ground state exists if the momentum is less than mc, where m is the particle mass and c is the slope at zero momentum of the dispersion relation of the Bogoliubov phonons.

Observing variational gaussian wave packets

Caroline Lasser
TU München

We discuss time-dependent Schrödinger equations with smooth electric and magnetic potentials. We analyze time-dependent variational approximation with a single nonlinearly parametrized gaussian wave packet. The derived error estimates are joint work with S. Burkhard, B. Dörich and M. Hochbruck: They generalize established results on accuracy with respect to the L2-norm and strongly improve what has been known with respect to observable accuracy.

Low Regularity Time Integration of the Boussinesq Equation

Alexander Ostermann
University of Innsbruck

We study the numerical integration of the "good" Boussinesq equation with initial data in the Sobolev space H^s for $s > 0$, with a special focus on very small values of s. The main difficulty arises from the low regularity of the initial data, since for $s \leq 1/2$ standard Sobolev methods cannot be applied due to the failure of the bilinear estimate. To overcome this, we reformulate the Boussinesq equation as a first-order evolution equation, which has similarities to a nonlinear Schrödinger equation. This approach allows the use of a newly developed framework based on discrete Bourgain spaces, which finally allows us to prove convergence. We illustrate our analytical results with numerical experiments.

This is joint work with Lun Ji, Hang Li, and Chunmei Su.

Long-time asymptotics for the Kadomtsev-Petviashvili I Equation

Peter Perry
University of Kentucky

The KP-I equation is the completely integrable dispersive equation $(ut + uxxx + 6uux)x = 3uyy$ for the waveform of a dispersive wave in two space dimensions. We consider the Cauchy problem with small data and recover leading asymptotics of the solution for large times. Our work is based on previous work of Xin Zhou, Athanassios Fokas, and Li-Yeng Sung on complete integrability of the KP I equation and involves new estimates for the solutions of the associated nonlocal Riemann-Hilbert problem. To our knowledge, we obtain the first such pointwise estimates on small-data solutions in the literature.

This is joint work with Samir Donmazov (University of Kentucky) and Jiaqi Liu (University of Chinese Academy of Sciences).

On a quasilinear Schrödinger equation: the small frequency limit

Simona Rota Nodari

University Côte d'Azur

Abstract: In this talk, I will present some recent results on a quasilinear Schrödinger equation with a power nonlinearity. After showing the uniqueness and the non-degeneracy of the positive radial solution u_ω for all $\omega > 0$, I will describe its asymptotic behavior in the limit $\omega \to 0$. This gives some important information about the orbital stability of u_ω and the uniqueness of normalized ground states.

Joint work with François Genoud.

Bi-harmonic NLS: well-posedness and soliton dynamics

Svetlana Roudenko

Florida International University

The bi-harmonic NLS equation (with the double Laplacian dispersion) has been investigated in literature to some extent, however, for the semi-linear version of it the well-posedness is still not addressed for some powers of nonlinearities. In the first part of the talk, we discuss the local well-posedness, using an approach that avoids Strichartz estimates. Then we look at the more global behavior of solutions, in particular, ground state solutions and discuss their dynamics.

This talk is based on joint works with Iryna Petrenko (FIU, Miami) and Christian Klein & Nikola Stoilov (IMB, Dijon)

Shock formation for weakly dispersive perturbations of the Burgers equation

Jean-Claude Saut

University of Paris-Saclay

Polarization approach to certain singular integrals

Johannes Sjöstrand

University of Burgundy

Spectral approaches to solutions that are non-trivial at infinity

Nikola Stoilov

University of Burgundy

We present several methods for numerically finding solutions to partial differential equations as well as fractional differential equations, that have non-trivial behaviour at infinity. Generally, the compactified real line is divided into a number of intervals, thus amounting to a multidomain approach; after transformations ensuring analyticity of the respective integrands, the integrals over the different domains are computed with a Clenshaw–Curtis algorithm. As examples, we consider solutions the NLS and KdV as well as solitary waves for fractional Korteweg-de Vries equations.

This is a joint work with Christian Klein

Bloch-Floquet band gaps for water waves over a periodic bottom

Catherine Sulem

University of Toronto

A central object in the analysis of the water wave problem is the Dirichlet-Neumann operator. This study is devoted to its spectrum in the context of the water wave system linearized near equilibrium in a domain with a variable bottom, assumed to be a smooth periodic function. We use the analyticity of the Dirichlet-Neumann operator with respect to the bottom variation and combine it with general properties of elliptic systems and spectral theory for self-adjoint operators to develop a Bloch-Floquet theory and describe the structure of its spectrum. We find that under some conditions on the bottom variations, the spectrum is composed of bands separated by gaps, with explicit formulas for their sizes and locations.

This is a joint work with Christophe Lacave and Matthieu Ménard

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Recent studies on the spectral (in-)stability of periodic traveling waves.

Changzhen Sun
University of Franche-Comté

In this talk, we discuss some recent studies on the spectral (in-)stability of periodic waves, focusing on two systems from plasma physics and fluid mechanics — the Euler-Poisson system and the water wave system. First, we show that small-amplitude periodic waves in the electronic Euler-Poisson system are inherently unstable under localized perturbations, with the instability driven by perturbations with finite wavelengths (or high frequencies). Second, we examine the transverse instability of Stokes waves (one-dimensional periodic water waves) in a two-dimensional gravity water wave system with finite depth. Our key finding is that when subjected to perturbations with sufficiently small transverse frequencies, all sufficiently small Stokes waves become unstable.

The global well-posedness conjectures for 1D dispersive flows

Daniel Tataru
University of California at Berkley

Collision of two solitary waves for the Zakharov-Kuznetsov equation

Frédéric Valet
Cergy-Pontoise University

The Zakharov-Kuznetsov (ZK) equation in dimension 2 is a generalization in plasma physics of the one-dimensional Korteweg de Vries equation (KdV). Both equations admit solitary waves, that are solutions moving in one direction at a constant velocity, vanishing at infinity in space. When two solitary waves collide, two phenomena can occur: either the structure of two solitary waves is conserved without any loss of energy and change of sizes (elastic collision), or the structure is lost or modified (inelastic collision). As a completely integrable equation, KdV only admits elastic collisions. The goal of this talk is to explain the collision phenomenon for two solitary waves having almost the same size for ZK.

The talk is based on a collaboration with Didier Pilod.

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