# WHAT DID BELL REALLY PROVE?

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Abstract. The goal of this paper is to give a pedagogical introduction to Bell's theorem and its implication for our view of the physical world, in particular how it establishes the existence of non-local effects or of actions at a distance. We also discuss several misunderstandings of Bell's result and we will explain how the de Broglie-Bohm theory allows us to understand, to some extent, what non-locality is.

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Based on a talk given at the Conference: Quantum Theory Without Observers III ZIF, Bielefeld, April 23, 2013.

# 1. **INTRODUCTION**

Although the goal of this paper is *not* to discuss the usual foundational issues of quantum mechanics, namely how to understand the quantum state[1] and its “collapse”, it is useful to begin by recalling the problem. As is well known, when there are no measurements, the quantum state $\Psi$ evolves according to the Schrödinger evolution:

$$i\partial_t\Psi = H\Psi = (H_0 + V)\Psi, \tag{1.1}$$

where $H_0$ is the free Hamiltonian and $V$ the potential (in this paper, we set $\hbar = 1$). For a system of $N$ particles, the function $\Psi = \Psi(x_1, x_2, \ldots, x_N, t)$ is defined on the space $\mathbb{R}^{3N}$ of all possible configurations of those particles and depends also on time. However, suppose that one measures an observable represented by an operator $\mathcal{A}$, having eigenvalues $\lambda_i$ and eigenvectors $\Psi_i$, when the quantum state is $\Psi$. Then, one writes $\Psi$ in the basis of the $\Psi_i$'s: $\Psi = \sum_i c_i\Psi_i$, the measurement yields the value $\lambda_i$ with probability $|c_i|^2$ and the quantum state becomes, after the measurement, $\Psi_i$. This is called the collapse of the quantum state.

Thus, we have two different types of evolutions for the quantum state: the Schrödinger evolution between measurements and the collapse during measurements.

How to understand this dual evolution? One can have at least two different sorts of answers to that question: either the quantum state represent the *information* that we have about a physical system, in which case its “collapse” simply means that we learn something about the system, or it represent something “physical”, in which case the collapse may be seen as the result of a physical interaction between a measuring device and the system.

Each of these views has its own difficulties, but we will not go into that. We simply note that these two views are at least suggested by the quantum formalism, which treats the “measurement process” as special. According to the first view, that process reveals something about the system; according to the second, it affects the system in some way and modifies its state.

In classical probability, if we throw a coin, it either lands head or tail. If we don't look at the result, we may assign probabilities one half to each outcome. This assignment is clearly related to our ignorance. Indeed, if we look at the state of the coin, after it has landed, our probabilities (if we still want to use this term) will become one for head or for tail, depending on the result, and zero for the other possibility. In that case, we are in the first situation discussed above, and probabilities are related to information. When our information changes, our probabilities change also, but nothing physical happens to the system.

To contrast this with the situation in quantum mechanics, consider a spin measurement. Let us assume that one measures the spin of an electron, via a Stern-Gerlach apparatus, in a given direction and that the initial quantum state is in a superposed state: spin up + spin down (normalized). Of course, the probability of either result is again one-half. But what does it mean here? According to the view that the quantum state reflects our information, it means that the electron is either up or down *before* the measurement and that the latter simply reveals which is the case. It would be like the coin that is either head or tail, before one looks at it. Let us emphasize that this view does *not* mean that the quantum state would be up or down –the quantum state

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[1] We use this word to designate the wave function times possible spin states.

is what it is, namely a superposition– but it would mean that the spin of the electron itself is up or down. In other words, the electron would have properties that are simply not encoded in the quantum state. These properties are sometimes called "hidden variables": "variables", because they characterize a given system, but vary from one system to another, even with the same quantum state, and "hidden" because they are not included in the description of the system by its quantum state. Many physicists are convinced that "hidden variable theories" have been ruled out, on logical or empirical grounds, either by von Neumann in 1935 [46] or by others later, maybe even by Bell in 1964, but this is a misconception to which we will return later.

If, on the other hand, one thinks of the measurement as a physical process that affects the system, then the measurement would be rather analogous to throwing the coin rather than looking at it after it has fallen on one side. The measurement would, in that view, not be a measurement of anything, but rather a "random creation" of the result.

The reader may think that the discussion is pointless, or "metaphysical", because it is impossible to know which of the two views above is the correct one. Maybe, but as we shall see, merely raising that question (is the measurement simply revealing pre-existing properties or is it acting on the system?) has led to an extraordinary discovery.

Indeed, Einstein firmly held the first view, and that view is often expressed by saying that quantum mechanics is not complete, which is a natural terminology, since it means that each individual system has properties (like having a spin up or down) that are not reflected by its quantum state. This is like saying that describing a coin, once it has fallen on one of its faces, by a probability "one-half, one-half", is not a complete description of the state of the coin, since the latter is either head or tail.

It is not entirely clear what people who maintained, in contradiction with Einstein, that quantum mechanics *is* a complete description of the system meant by that. Coming back to the analogy with the coin, one may imagine that the coin is inside a sealed box that nobody can open. It that case, the probabilistic description "one-half, one-half" could be called "complete" in the sense that we humans have no way to *know* more about the state of the coin. Nevertheless, one might also want to say that the word complete here means "relative to us", and that *in reality* the coin is either head or tail.

It should be emphasized that the issue here is not whether one can manipulate or predict the value of those "hidden variables", but whether one can merely think of them as existing. It is possible that what the people who think that quantum mechanics is "complete" mean by this expression is simply that, given that those extra variables are impossible to predict or to manipulate, why bother with them? That may be a natural reaction, but it should not prevent us from trying to see what speculating about the existence of those variables may lead us to.

Einstein developed several ingenious arguments (one of them, in 1935, together with Podolsky and Rosen [25], to which we refer below by the letters EPR) in order to show indirectly that quantum mechanics *must* be incomplete, in the sense given above. These were based on thought experiments, but that were later performed, in a modified form. Contrary to another popular misconception, the arguments were not faulty, at least if they are stated as follows: if a certain assumption of locality or of no "action at a distance" is granted, *and nothing else, in particular no assumption whatsoever* *about "realism" or "determinism"* (more on that later) then quantum mechanics is incomplete, in the sense introduced here. Einstein, Podolsky and Rosen did not state their result like that, because, for them, the locality assumption was too obvious to be stated explicitly as a genuine assumption[2]. Moreover, for various reasons that we will discuss in Section 5, their argument was not generally understood, and certainly not in the form stated here.

Bell did show, almost thirty years later (in 1964), that the locality assumption *alone*, in the context of the EPR experiment, leads to a contradiction with quantum mechanical predictions, that were later verified experimentally [4]. Unfortunately, Bell assumed, at the beginning of his own reasoning, the argument of EPR to be well known; and since the EPR argument was misunderstood, Bell's argument was also widely misunderstood.

We will start by a little known, but very simple, thought experiment, the one of Einstein's boxes. This example already allows us to raise the issue of locality. Then we will give a simple derivation of Bell's argument (due to [21]), *combined* with the one of EPR; that is the simplest and clearest way to arrive at the conclusion, namely that the world is, in some sense, non-local. Next, we will review and discuss some of the misunderstandings of both EPR and Bell among physicists. Finally, we will explain how de Broglie-Bohm's theory illustrates and explains, as far as one can, what non-locality is.

## 2. EINSTEIN's BOXES

Consider the following thought experiment[3]. A single particle is in a box $B$ (see Figure 1 below), and its quantum state is: $\mid \text{state} > = \mid B >$, meaning that its quantum state is distributed over the box[4]. One then cuts the box in two half-boxes, $B_1$ and $B_2$, and the two half-boxes are then separated and sent as far apart as one wants:

According to ordinary quantum mechanics, the state becomes

$$\frac{1}{\sqrt{2}}(|B_1 > + |B_2 >)$$

where the state $|B_i >$ means that the particle "is" in box $B_i$, $i = 1, 2$ (and again, is distributed in that box). Here, we put quotation mark around the verb "is", because of the ambiguity inherent in the meaning of the quantum state: if it reflects our knowledge of the system, then the particle *is* in one of the boxes $B_i$, without quotation marks.

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[2] However, in his 1949 "Reply to criticisms", Einstein did pose the question in the form of a dilemma; speaking of the EPR 'paradox', he wrote: "[. . . ]the paradox forces us to relinquish one of the following two assertions:

(1) the description by means of the $\psi$-function is complete.

(2) the real states of spatially separated objects are independent of each other." ([28] p. 682).

[3] We base ourselves in this Section on [41], where the description of the experiment is due to de Broglie [18, 17]. The original idea of Einstein was expressed in a letter to Schrödinger, written on June 19, 1935, soon after the EPR paper was published [30], p.35. Figure 1 below is reproduced with permission from [41]; copyright 2005 American Association of Physics Teachers.

[4] The precise distribution does not matter, provided it is spread over $B_1$ and $B_2$ defined below; it could be distributed according to the square of the ground state wave function of a particle in the box $B$.

But, if one thinks of the quantum state as being physical and of the position of the particle as being created or realized only when one measures it, then the quotation marks are necessary and "is" means: "would be found in box $B_i$ after measurement".

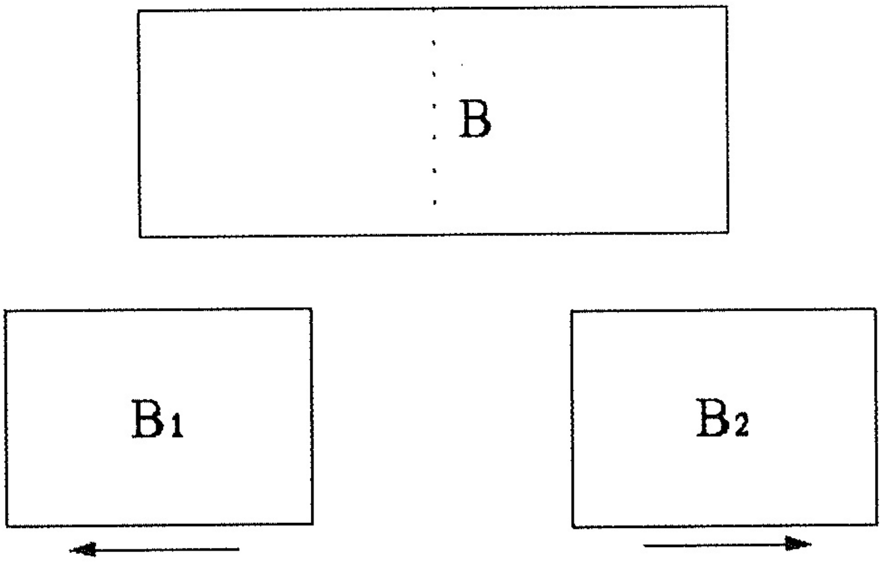

Figure 1: Einstein's Boxes

Now, if one opens one of the boxes (say $B_1$) and that one does *not* find the particle in it, one *knows* that it is in $B_2$. Therefore, the state "collapses" instantaneously: state $\longrightarrow |B_2>$ (and, if one opens the box $B_2$, one will find the particle in it !).

Since $B_1$ and $B_2$ are as far apart as one wants, if we reject the notion of action at a distance, then it follows that acting on $B_1$, namely opening that box, cannot have any physical effect whatsoever on $B_2$. However, if opening the box $B_1$ leads to the collapse of the quantum state into one where the particle is necessarily in $B_2$, it must be that the particle was in $B_2$ all along. That is of course the common sense view and also the one that one would reach if the particle was replaced by any large enough object.

But then, one must admit that quantum mechanics is not complete, in the sense that Einstein gave to that word: there exists other variables than the quantum state that describe the system, since the quantum state does not tell us in which box the particle is and we just showed, assuming no action at a distance, that the particle *is* in one of the two boxes, before one opens either of them.

In any case, with his argument of the boxes, Einstein had at least proven the following dilemma: either there exists some sort of action at a distance in nature (opening the box $B_1$ changes the physical situation in $B_2$) or quantum mechanics is incomplete. Since actions at a distance were anathema for him (and probably for everybody else at that time[5]), he thought that he had shown that quantum mechanics is incomplete.

There are many examples, at a macroscopic level, that would raise a similar dilemma and where one would side with Einstein in making assumptions, even very unnatural ones, that would preserve locality. Suppose that two people are located far away, each

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[5] Bohr's position was ambiguous on this issue; we will discuss it in section 5.

of whom tosses coins and the results are always either heads or tails, randomly, but are the same for both throwers. Or suppose that in two casinos, again far away from each other, the roulette always ends up on the red or black color, again randomly but always the same in both casinos. Or imagine twins far apart that behave exactly in the same fashion. In all these examples (and in many others that are easy to imagine) one would naturally assume (even if it would sound very surprising) that the two coin throwers or the casino owners are able to manipulate their apparently random results and have coordinated them in advance or that genetic determinism is much stronger than one usually thinks. Who would suppose that one coin tosser immediately affects the result of the other one, far away, or that the spinning of the ball in one casino affects the motion of the other ball, or that the action of one twin affects the behavior of the other twin? In all these cases, one would assume a locality hypothesis; denying it would sound even more surprising than whatever one would have to assume to explain those odd correlations.

But one thing should be a truism, namely that those correlations pose a dilemma: the results are either coordinated in advance or there exists a non-local action.

Note that, compared to those examples Einstein's assumption in the case of the boxes (incompleteness of quantum mechanics) was actually very natural.

As an aside, let us mention that the example of the boxes also raises a serious question about the quantum-classical transition. Indeed, if the quantum particle is replaced by a "classical" one, meaning a large enough object, nobody denies that the particle *is* in one of the boxes before one opens any of them. But where is the quantum/classical division? Usually the passage from quantum to classical physics is thought as some kind of limit; but a limit is something that one gets closer and closer to when a parameter varies. Here, we are supposed to go from the statement "the particle is in neither of the boxes" to the "the particle is in one of them, but we do not know which one". This is an ontological jump and not the sort of continuous change that the notion of limit expresses[6].

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[6] As we will see in section 6, this problem does not arise in the de Broglie Bohm theory.

Actually, a somewhat similar argument was already put forward, by Einstein, in 1927, at the Solvay Conference[7]. Einstein considered a particle going through a hole as in Figure 2.

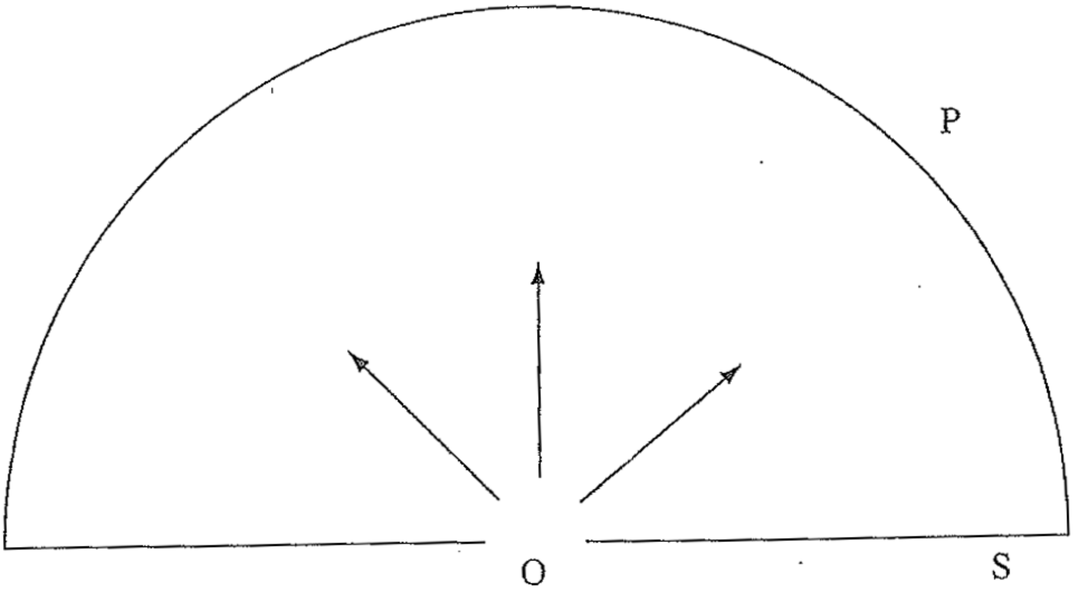

Figure 2: Einstein's objection at the 1927 Solvay Conference

In the situation described in the picture, the quantum state spreads itself on the half circle, but one always detects the particle at a given point. If the particle *is not* localized anywhere before its detection (think of it as a sort of "cloud" as spread out as the quantum state), then it must condense itself at a given point, again in a non-local fashion, since the part of the particle that is far away from the detection point must "jump" there instantaneously.

## 3. **WHAT IS NON-LOCALITY?**

Let us consider what sort of non-locality or actions at a distance would be necessary to deny Einstein's conclusion about the incompleteness of quantum mechanics.

1. That action should be instantaneous, since the particle has to be entirely in box $B_2$, once we open box $B_1$. Of course, instantaneity is not a relativistic notion, so let us say, instantaneous in the reference frame where both boxes are at rest.

2. a. The action extends arbitrarily far, since the particle is entirely in box $B_2$, once we open box $B_1$, it is created there (since it was, by assumption in neither box before opening $B_1$) and that fact does not change with the distance between the boxes.

b. The effect of that action does not decrease with the distance: indeed, the effect is the "creation" of the particle in box $B_2$ and that effect is the same irrespective of the distance between the boxes.

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[7] See [5], p. 486 or [43] for the "original" (published in French translation at the time of the Solvay Conference). In fact Einstein raised a similar issue as early as in 1909, at a meeting in Salzburg, see [5], p. 198. Figure 2 below is taken from [5] p. 440. Originally in [43] p. 254.

3. This effect is individuated: suppose we send a thousand of pairs of half-boxes, each coming from the splitting in two of one box with a single particle in it. Then opening one half box will affect the state in the other half box but not in any other box.

4. That action cannot be used to transmit messages: if we open one box, we learn what the state becomes in the other box, but we cannot use that to transmit a message from where one box is to where the other box is. Indeed, since we do not have any way, by acting on one box, to choose in which of the two boxes the particle will be, there is no way to use that experiment to send messages.

The impossibility to send messages is sometimes taken to mean that there is nothing non-local going on. But non-locality refers here to causal interactions as described (in principle) by physical theories. Messages are far more anthropocentric than that and require that humans be able to control these interactions to communicate. As remarked by Maudlin, the Big Bang and earthquakes cannot be used to send messages, but they have causal effects nevertheless ([35], p. 136-137).

Let us now compare that sort of non-locality with the one in Newton's gravity. The latter also allows actions at a distance: since the gravitational force depends on the distribution of matter in the universe, changing that distribution, say by moving my body, instantaneously affects all other bodies in the universe. That action at a distance has property 1 and 2a, but not the others; of course that effect decreases with the distance, because of the inverse square law, and it affects all bodies at a given distance equally (it is not individuated). On the other hand, it can in principle be used to transmit messages: if I decide to choose, at every minute, to wave my arm or not to wave it, then one can use that choice of movements to encode a sequence of zeros and ones and, assuming that the gravitational effect can be detected, one can therefore transmit a message instantaneously and arbitrarily far (but the further away one tries to transmit it, the harder the detection). Of course, all this refers to Newton's *theory*. There were no experiment performed or suggested that could test whether gravitational forces really acted instantaneously (and, as we will see, this is a major difference with the situation in quantum mechanics)[8].

Post-Newtonian physics has tried to eliminate property 1, and classical electromagnetism or the general theory of relativity have kept only property 2a and the negation of 4. And, due to special relativity, the combination of 1 and the negation of 4 allows in principle the sending of messages into one's own past, so that, if 1 holds, 4 must hold also.

One may ask: does quantum mechanics prove that there are physical effects displaying properties 1-4 ? The example of the boxes does not allow that conclusion, because one can consistently think that the particle is always in one of the boxes. Indeed, in de Broglie-Bohm's theory (explained in Section 6), particles do have positions and trajectories and the predictions of the theory are the same as those of ordinary quantum mechanics. It is not the place here to discuss the merits of de Broglie-Bohm's theory,

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[8] It is well known that Newton did not like that aspect of his own theory. He said: "that one body may act upon another at a distance through a vacuum without the mediation of any thing else [...] is to me so great an absurdity that I believe no man who has in philosophical matters any competent faculty of thinking can ever fall into it." [40, 37]. Newton thought that gravitation was mediated by particles moving at a finite speed, so that the effect of gravitation could not be instantaneous, see [36] for more details.

but its mere existence and consistency implies that, in the example of the boxes, nothing forces us to accept non-locality. It is true that de Broglie-Bohm's theory itself is non-local, but not when that theory is applied only to the situation of Einstein's boxes. In order to prove non-locality in the sense introduced here, i.e. a phenomenon having properties 1-4 above, we have to turn to a more sophisticated situation.

## 4. A SIMPLE PROOF OF NON-LOCALITY

4.1. **An anthropomorphic thought experiment.** Let us start with an anthropomorphic thought experiment (which can be realized in principle by using quantum mechanics, as we will see in subsection 4.2 below); two people, $A$ (for Alice) and $B$ (for Bob) are together in the middle of a room and go towards two different doors, located at $X$ and $Y$. At the doors, each of them is given a number, $1, 2, 3$ (let's call them "questions", although they do not have any particular meaning) and has to say "Yes" or "No" (the reason why we introduce 3 questions will be clear in the Theorem below). That experience is repeated a large number of times, with $A$ and $B$ gathering together each time in the middle of the room, and the questions and the answers vary apparently at random. When $A$ and $B$ are together in the room, they can decide to follow whatever "strategy" they want in order to answer the questions, but the statistics of their answers, although they look random, have nevertheless to satisfy two striking properties.

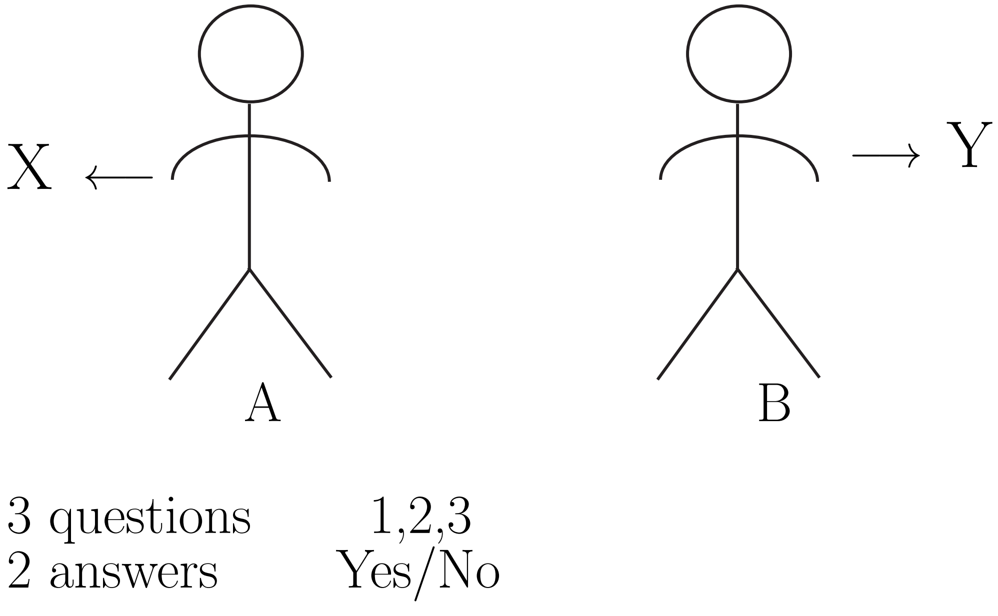

Figure 3: The anthropomorphic experiment.

The first property is that, when the same question is asked at $X$ and $Y$, one always gets the same answer. How can that be realized? One obvious possibility is that $A$ and $B$ agree before moving towards the doors which answers they will give: they may decide, for example, to say both Yes if the question is 1, No, if it is 2 and Yes if it is 3.

They can choose different strategies at each repetition of the experiment and choose those strategies "at random" so that the answers will look random.

Another possibility is that, when $A$ reaches door $X$, she calls $B$ and tells him which question was asked and the answer he gave; then, of course, $B$ can juste give the same answer as $A$ if he is asked the same question and any other answer if the question is different.

But let us assume that the answers are given simultaneously (in the reference frame in which the experience takes place), so that the second possibility is ruled out unless there exists some sort of action at a distance between $A$ at $X$ and $B$ at $Y$. Maybe $A$ and $B$ communicate by telepathy .... Of course, that is not to be taken seriously, but that is the sort of interaction that Einstein had in mind when he spoke of "spooky actions at a distance" ([15] p.158).

The question that the reader should ask herself at this point is whether there is *any other possibility*: either the answers are predetermined or (assuming simultaneity of the answers) there is a "spooky action at a distance", namely a communication of some sort takes place between $A$ and $B$ *after* they are asked the questions. This is similar to the dilemma about the boxes: either the particle is in one of the boxes or there is some sort of physical action between the two boxes.

Of course, this dilemma already arises if one asks only *one* question and the answers are perfectly correlated (the usefulness of having three questions will appear below).

A third possibility, which is sometimes suggested (probably out of desperation) is that the perfect correlations between the answers when the same questions are asked is simply a coincidence that does not need to be explained. Or, in the same vein, one sometimes claims that science limits itself to predictions, not to explanations. But the whole of science can be seen as an attempt to account for correlations or empirical regularities: the theory of gravitation for example accounts for the regularities in the motion of planets, moons, satellites etc. The atomic theory of matter accounts for the proportions of elements in chemical reactions. The effects of medicines account for the cure of diseases etc. To refuse to account for correlations, without giving any particular reason for doing so, is, in general, a very unscientific attitude.

In any case, refusing to face a question is not the same thing as answering it. So, let us proceed by accepting the dilemma, which is actually the EPR part of the argument, except that EPR did not formulate the issue with people but with quantum particles (we will come to that later) and not as a real dilemma since, for them, non-locality was ruled out (see, however, footnote 2).

Now, there is a second peculiarity of the statistics of the answers: when the two questions asked to $A$ and $B$ are *different*, then the answers are the same only in one-quarter of the cases (the number $1/4$ being the result of a quantum mechanical calculation, as we shall see in the next subsection).

But that property, combined with the idea that the properties are predetermined leads to a contradiction, which is a (very simple) version of *Bell's theorem*[9].

**THEOREM (Bell)**

We cannot have these two properties together:

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[9] The argument given here is taken from [21]. In the original paper by Bell [7], the proof, although fundamentally the same, looked more complicated.

-The answers are determined before the questions are asked and are the same on both sides.

-The frequency of having the same answers on both sides when the questions are different is $\frac{1}{4}$.

**PROOF**

There are 3 questions 1 2 3,
and 2 answers Yes/No.
If the answers are given in advance, there exists $2^3 = 8$ possibilities :

| 1 | 2 | 3 |
|---|---|---|
| $Y$ | $Y$ | $Y$ |
| $Y$ | $Y$ | $N$ |
| $Y$ | $N$ | $Y$ |
| $Y$ | $N$ | $N$ |
| $N$ | $Y$ | $Y$ |
| $N$ | $Y$ | $N$ |
| $N$ | $N$ | $Y$ |
| $N$ | $N$ | $N$ |

In *each case* there are at least *two questions* with the same answer.
Therefore,

Frequency (answer to 1 = answer to 2)
+ Frequency (answer to 2 = answer to 3)
+ Frequency (answer to 3 = answer to 1) $\geq 1$

But if

Frequency (answer to 1 = answer to 2)
= Frequency (answer to 2 = answer to 3)
= Frequency (answer to 3 = answer to 1)
$= \dfrac{1}{4}$

we get $\Rightarrow \dfrac{3}{4} \geq 1$, which is a contradiction. □

The inequality $\frac{3}{4} \geq 1$ is an example of *Bell's inequalities*.

Before drawing the conclusions of this theorem, let us see how the two people in the preceding section could realize these "impossible" statistics.

4.2. **The real quantum experiment.** Let us first describe the situation in the previous section in a non anthropomorphic manner. $A$ and $B$ are replaced by particles with spin 1/2 (in reality, the experiments are made with polarized photons, but conceptually the two situations are similar, see [4] for some of the experiments).

At $X$ and $Y$, there are Stern-Gerlach apparatuses that "measure the spin" along some direction.

The "questions" 1, 2, 3 are 3 possible directions for that "measurement" (we put quotation marks here because, as we shall see below, there is no intrinsic property of the particles that is really measured in these experiments).

The answer Yes/No correspond to results Up/Down for the spin (we will define this correspondence more precisely below).

One sends the particles towards the apparatuses and the initial state of the two particles is[10]:

$$
\begin{aligned}
&|\Psi > \\
&= \frac{1}{\sqrt{2}}(|A\ 1 \uparrow> |B\ 1 \downarrow> - |A\ 1 \downarrow> |B\ 1 \uparrow>) \\
&= \frac{1}{\sqrt{2}}(|A\ 2 \uparrow> |B\ 2 \downarrow> - |A\ 2 \downarrow> |B\ 2 \uparrow>) \\
&= \frac{1}{\sqrt{2}}(|A\ 3 \uparrow> |B\ 3 \downarrow> - |A\ 3 \downarrow> |B\ 3 \uparrow>),
\end{aligned}
\tag{4.1}
$$

where $|A\ 1 \uparrow>$ is the state in which the particle $A$ has its spin up in direction 1 (meaning that a particle in that state will have its spin up with certainty after a spin measurement in direction 1, or in other words, that state is the "up" eigenstate of the spin operator in direction 1) , and the other symbols are defined analogously. We leave aside the issue of how to create in practice such a state and note only that it can be done (at least, in an analogous situation with photons and polarization instead of particles and spin). We also accept without proof the fact that this state has three similar representations in each of the directions 1, 2 or 3.

We also leave aside the "spatial" part of that quantum state: we assume implicitly that the state $|A\ 1 \uparrow>$ is coupled to a wave function moving towards the door located at $X$, while the state $|B\ 1 \downarrow>$ is coupled to a wave function moving towards the door located at $Y$, and similarly for the other parts of the state $|\Psi >$.

Consider now the standard quantum mechanical description of a measurement of the spin in direction 1 of A at $X$, without measuring anything on particle B, for the moment. If one sees $\uparrow$, the state becomes $|A\ 1 \uparrow> |B\ 1 \downarrow>$ (by the collapse rule). If one sees $\downarrow$, the state becomes $|A\ 1 \downarrow>\ |B\ 1 \uparrow>$. And, of course, we get similar results if one measures the spin in direction 2 or 3 of $A$.

But then the state changes *non-locally* for particle $B$, because, if one sees $\uparrow$, the state "collapses", i.e. becomes $|A\ 1 \uparrow> |B\ 1 \downarrow>$ and the part $|A\ 1 \downarrow> |B\ 1 \uparrow>$ of the state has been suppressed by the collapse; another way to say this is that, after the measurement of A, any measurement of B at $Y$ is guaranteed to yield the result opposite to what was found about A, while, before the measurement on A, the result of B was undetermined. This gives rise to the same dilemma as for Einstein's boxes : either the measurement on A affects the physical situation of B, or the particle B had its spin determined in advance, and anti-correlated with the one of A, which must also have been pre-determined. Since we could of course measure the spin of B first and then of A, or do both simultaneously, one is led to the dilemma:

- either the spin values, up or down, were pre-determined before the measurement, and in all three directions, because the same reasoning can be done for each one of them.
- or there is some form of action at a distance between X and Y (that can, in principle, be arbitrarily far from each other).

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[10] The fact that the three representations below are equal follows from the rotation invariance in spin space of that particular state $|\Psi >$.

But, the first assumption leads to a contradiction with observations made when the directions in which the spin is "measured" are *different* for $A$ and $B$. To see this, denote by $v_A(\mathbf{a})$, $v_B(\mathbf{b})$ the pre-existing values of the results of measurements on $A$ and $B$ (assuming that they exist), where $\mathbf{a}, \mathbf{b}$ denote unit vectors in the directions $1, 2, 3$, to be specified below, in which the spin is measured at X or Y. Let us make the following conventions at $X$ et $Y$: $v_A(\mathbf{a}) = +1$ means that the answer is "Yes", $v_A(\mathbf{a}) = -1$ means that the answer is "No", but $v_B(\mathbf{b}) = +1$ means that the answer is "No", $v_A(\mathbf{b}) = -1$ means that the answer is "Yes". With that convention, we see that we always get the same answer when the same questions are asked on both sides. The contradiction comes from the fact that we get the same answer only $\frac{1}{4}$ of the time when one asks different questions at $X$ and $Y$ and our Theorem shows that this is impossible.

Of course, one has to run the experiment a large number of times in order to get the "impossible" statistics (impossible without accepting the existence of actions at a distance).

Let us show how to obtain this factor $\frac{1}{4}$. This is an elementary quantum mechanical calculation. Compute first $\mathbf{E_{a,b}} \equiv < \Psi|\sigma^A_{\mathbf{a}} \otimes \sigma^B_{\mathbf{b}}|\Psi >$, where $\mathbf{a}$, $\mathbf{b}$ are the unit vectors in the directions (1, 2 or 3), and $\sigma^A_{\mathbf{a}} \otimes \sigma^B_{\mathbf{b}}$ is a tensor product of matrices, each one acting on the $A$ or $B$ part of the quantum state (with $\sigma_{\mathbf{a}} = a_1\sigma_1 + a_2\sigma_2 + a_3\sigma_3$, where, for $i = 1, 2, 3$, $a_i$ are the components of $\mathbf{a}$ and $\sigma_i$ the usual Pauli matrices.). This quantity is bilinear in $\mathbf{a}$, $\mathbf{b}$ and rotation invariant, so it must be of the form $\lambda \mathbf{a} \cdot \mathbf{b}$, for a certain $\lambda \in \mathbf{R}$.

For $\mathbf{a} = \mathbf{b}$, the result must be $-1$, because of the anti-correlations (if the spin is up at A, it must be down at B and vice versa). So, $\lambda = -1$, and thus $\mathbf{E_{a,b}} = -\cos\theta$, where $\theta$ is the angle between the directions $\mathbf{a}$ and $\mathbf{b}$ . We know that $v_A(\mathbf{a}), v_B(\mathbf{b}) = \pm 1$; thus:

$$\mathbf{E_{a,b}} = P(v_A(\mathbf{a}) = v_B(\mathbf{b})) - P(v_A(\mathbf{a}) = -v_B(\mathbf{b})) = 1 - 2P(v_A(\mathbf{a}) = -v_B(\mathbf{b})),$$

and $P(v_A(\mathbf{a}) = -v_B(\mathbf{b})) = \frac{1-\mathbf{E_{a,b}}}{2} = \frac{1+\cos\theta}{2}$.

One chooses then the directions:
$1 \Rightarrow \theta = 0$ degree,
$2 \Rightarrow \theta = 120$ degree,
$3 \Rightarrow \theta = 240$ degree.

Since $\cos 120 = \cos 240 = -1/2$, we get $P(v_A(\mathbf{a}) = -v_B(\mathbf{b})) = \frac{1}{4}$. Thus we have perfect anti-correlations only $\frac{1}{4}$ of the time when $\mathbf{a}$ and $\mathbf{b}$ are different. This means, with our convention, that one gets the same answer when one asks different questions on both sides only $\frac{1}{4}$ of the time.

Finally, if one wants to reproduce the anthropomorphic experiment described in the previous subsection, one simply sends the particles towards the two doors once Alice ($A$) and Bob ($B$) have reached them. At each door, there is a Stern-Gerlach instrument, with which Alice and Bob "measure the spin" along the angles corresponding to the numbers given to them; then, they answer the "questions" according to the results of their "measurements". In that way the "impossible" statistics mentioned in the Theorem above will be reproduced.

4.3. **Consequences.** Let us first summarize what has been shown: assuming only locality, one derives from the perfect anti-correlations when the angles of detection are the same, that the spin values pre-exist to their measurement. Then, the theorem in

Section 4.1 shows that this fact, combined with the $\frac{1}{4}$ value for the frequency of unequal results when the measurement angles are different, leads to a contradiction.

Therefore the assumption of locality is false [11].

To repeat: the EPR part of the argument shows that, if there are no pre-existing values, then the perfect correlations when the angles are the same imply some sort of action at a distance. The Bell part of the argument, i.e. the theorem of the previous subsection, shows that the mere assumption that there are pre-existing values leads to a contradiction when one takes into account the statistics of the results when the angles are different. That is why we used quotation marks in “measurement”: there is no real property of the particle that is being “measured”, since there are no spin values existing before the interaction with the “measuring” device (why this is so will become clearer in section 6).

But what does it mean? It means that some sort of action at a distance exists in Nature, but it does not tell us which sort. And we cannot answer that question without having a theory that goes beyond ordinary quantum mechanics. In ordinary quantum mechanics, what is non-local is the collapse of the quantum state, as we see in the transformation of (4.1) into $|A\ 1 \uparrow> \ |B\ 1 \downarrow>$ or $|A\ 1 \downarrow> \ \ |B\ 1 \uparrow>$, depending on the result of a measurement at A. This affects the state at B, since now the second part of $|\Psi>$ has been suppressed.

Since the meaning of the quantum state and of its collapse is ambiguous in ordinary quantum mechanics, it is not clear that this is a real physical effect. But, as we have emphasized, if there are no physical effect whatsoever or, if one interprets the collapse of the quantum state as a mere gain of information, then, it means that we must have those pre-determined values that lead to a contradiction.

It is important to notice that one cannot use this non-local effect to send messages. This is what contradicts all the pseudo-scientific uses of Bell's result: there is no telepathy of any sort that can be based on that result. And, if one could send messages, the theory of relativity implies that one could send messages into one's own past, which is certainly something that nobody is ready do accept.

The reason for the impossibility of sending messages is similar to what it was for Einstein's boxes. Each side sees a perfectly random sequence of results “spin up” or “spin down”. Since there is no mechanism that allows, given the initial quantum state (4.1), to control or affect that result by acting on one side of the experiment, there is no way to send a message from one side to the other (see [24] for a more general proof of the impossibility of sending messages via EPR-Bell experiments).

But if each person tells the other which “measurements” have been made (1, 2 or 3), *without* telling which results are obtained, they both know which result has been

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[11] As an aside, we note that an inequality similar to Bell's was derived later, in 1982, by Richard Feynman (with no reference made to Bell's work), who drew the following conclusion (what Feynman calls a local classical computer means more or less what we call locality here):

> That's all. That's the difficulty. That's why quantum mechanics can't seem to be imitable by a local classical computer. I've entertained myself always by squeezing the difficulty of quantum mechanics into a smaller and smaller place, so as to get more and more worried about this particular item. It seems to be almost ridiculous that you can squeeze it to a numerical question that one thing is bigger than another.
>
> Richard Feynman [29] p. 485.

obtained on the other side when the same measurement is made on both sides. Then, they both share a common sequence of Yes/No or up/down, which is a form of "information". This information was not transmitted directly, by hypothesis, and it cannot come from the source (the one that has emitted the two particles), because of the non-existence of pre-existing spin values. Thus, some non-local transfer of information must have taken place[12]. This is one of the basis of the field of "quantum information", whose development will hopefully will lead to a better appreciation of the radical consequences of Bell's discovery.

But the "better appreciation", if it ever comes, will be in the future. Indeed, for the time being, it is mostly massive misunderstandings of Bell and of EPR that prevails and that we will now discuss[13] .

## 5. MISUNDERSTANDINGS

5.1. **Misunderstandings of Einstein-Podolsky-Rosen.** The EPR article was not using the spin variables as above (this extension is due to Bohm [10]) but position and momentum. They considered two particles, starting from the same place, moving in opposite directions, and so that their total momentum was conserved and equal to zero. Thus, by measuring the momentum of one particle, one could know the momentum of the other particle. But, if one measured instead the position of that one particle, one would know the position of the other particle (since they move at the same speed in opposite directions). However, if the two particles are far apart and, if locality holds, the choice that we make of the quantity to measure on one particle cannot affect the state of the other particle. Thus, that second particle must possess a well defined momentum *and* a well defined position, before any measurement, and this shows that quantum mechanics is incomplete, since the quantum state does not include those variables.

One should emphasize (as is done by Maudlin in [36]) that the concern of Einstein with "determinism" may not have been due to determinism itself but to his attachment to locality and to the fact that he understood (unlike most of his critics) that locality required determinism, at least in the sense that variables such as the spin values or positions and momenta *must* exist if measures on one side (on A) do not affect the physical situation on the other side (on B). Indeed, if the measurement on side A is truly "random", then it produces for state like (4.1) a definite result at B, which is therefore no longer random (which it was also, of course, by symmetry, before the measurement at A); this means that, if we assume indeterminism of the results, we must accept that measurements on one side affects non-locally the physical situation on the other side. The sentence of Einstein "God does not play dice" is often quoted, but a more precise expression of his thought can be found in a 1942 letter:

> It seems hard to sneak a look at God's cards. But that he plays dice and uses "telepathic"methods (as the present quantum theory requires of him) is something that I cannot believe for a moment.

---

[12] See the book on relativity and non-locality by Maudlin [35] for a detailed discussion of the differences between messages and information and of what exactly is compatible or not with relativity.

[13] The following section has a lot in common with a paper by Maudlin [36]. In particular, Maudlin starts by quoting a video from *Physics World* [3] that nicely summarizes all the misunderstandings discussed below.

Albert Einstein [26] quoted in [36].

So, the problem that bothered Einstein was not simply determinism, but the fact that the latter implies "telepathy", i.e. actions at a distance.

Coming back to the EPR article, one may think that their reasoning was not as transparent as one might have liked it to be[14]. But Einstein was completely clear when he wrote, in 1948:

> If one asks what, irrespective of quantum mechanics, is characteristic of the world of ideas of physics, one is first of all struck by the following : the concepts of physics relate to a real outside world, that is, ideas are established relating to things such as bodies, fields, etc., which claim a 'real existence' that is independent of the perceiving subject - ideas which, on the other hand, have been brought into as secure a relationship as possible with the sense-data. It is further characteristic of these physical objects that they are thought of as arranged in a space time continuum. An essential aspect of this arrangement of things in physics is that they lay claim, at a certain time, to an existence independent of one another, provided these objects 'are situated in different parts of space'. [. . . ]
>
> The following idea characterizes the relative independence of objects far apart in space (A and B) : external influence on A has no direct influence on B.
>
> Albert Einstein [27] (reproduced in [15] p.170-171.)

But Born, for example, missed Einstein's argument; he wrote, when he edited the Born-Einstein correspondence:

> The root of the difference between Einstein and me was the axiom that events which happens in different places A and B are independent of one another, in the sense that an observation on the states of affairs at B cannot teach us anything about the state of affairs at A.
>
> Max Born [15] p.176

As Bell says:

> Misunderstanding could hardly be more complete. Einstein had no difficulty accepting that affairs in different places could be correlated. What he could not accept was that an intervention at one place could influence, immediately, affairs at the other.
>
> John Bell [9] p. 144

What Born said was that making an experiment at one place teaches us something about what is happening at another place, which is unsurprising. If, in the anthropomorphic example above, both people had agreed on a common strategy, one would learn what $B$ will answer to question 1, 2 or 3, by asking that same question to $A$. In fact, in his comments about Einstein, Born gives the following example:

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[14] The article, according to Einstein, was written by Podolsky "for reasons of language", meaning that Einstein's English was far from perfect, and was not written with optimal clarity; see his June 19, 1935 letter to Schrödinger in [30] p.35.

> When a beam of light is split in two by reflection, double-refraction, etc., and these two beams take different paths, one can deduce the state of one of the beams at a remote point B from an observation at point A.
>
> Max Born [15] p. 176.

But here Born is giving a classical example, where the polarization does pre-exist to its measurement, which is exactly contrary to the idea that the quantum state is a complete description, since the latter does not, in general specify any value of the position, momentum, spin, angular momentum, energy, etc., i.e. when it is a superposition of different eigenstates of the operator corresponding to the given physical quantity (as, for example in the state (4.1)).

This shows that Born thought that, in our language, there are pre-existing answers, namely that the spins are up or down before the measurements, which means that, in fact, he agreed with Einstein that quantum mechanics is incomplete, but simply did not understand what Einstein meant by that.

There is also a rather widespread belief among physicists that Bohr adequately answered the EPR paper in [13]. But, unlike Born, whose misunderstanding of Einstein is clearly stated, Bohr's answer is hard to understand. EPR had written that:

> If, without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity,
>
> Albert Einstein, Boris Podolsky, Nathan Rosen [25], reprinted in [47], pp. 138-141.

which again means that if, by doing something at A, we can learn something about the situation at B (for example, the value of the result of a spin measurement in a given direction[15]), then what we learn must exists before we learn it, since A and B can be far apart. Here of course, EPR assume locality.

Bohr replied :

> [...] the wording of the above mentioned criterion ... contains an ambiguity as regards the meaning of the expression "without in any way disturbing a system". Of course there is in a case like that just considered no question of a mechanical disturbance of the system under investigation during the last critical stage of the measuring procedure. But even at this stage there is essentially the question of *an influence on the very conditions which define the possible types of predictions regarding the future behavior of the system* ... their argumentation does not justify their conclusion that quantum mechanical description is essentially incomplete ... This description may be characterized as a rational utilization of all possibilities of unambiguous interpretation of measurements, compatible with the finite and uncontrollable interaction between the objects and the measuring instruments in the field of quantum theory.

---

[15] In the EPR paper, they considered position and momentum instead of spin, that can also be predicted at B once they are measured at A, due to conservations laws.

Niels Bohr [13], quoted in [9], p. 155 (italics in the original).

Bell dissects that passage as follows:

> Indeed I have very little idea what this means. I do not understand in what sense the word 'mechanical' is used, in characterizing the disturbances which Bohr does not contemplate, as distinct from those which he does. I do not know what the italicized passage means - 'an influence on the very conditions ...'. Could it mean just that different experiments on the first system give different kinds of information about the second ? But this was just one of the main points of EPR, who observed that one could learn *either* the position *or* the momentum of the second system. And then I do not understand the final reference to 'uncontrollable interactions between measuring instruments and objects', it seems just to ignore the essential point of EPR that in the absence of action at a distance, only the first system could be supposed disturbed by the first measurement and yet definite predictions become possible for the second system. Is Bohr just rejecting the premise -'no action at a distance' - rather than refuting the argument ?
>
> John Bell [9], p. 155-156

One rather common misconception is to think that the goal of EPR was to beat the uncertainty principle and to show that one could measure, say, the spin in direction 1 at $X$ and in direction 2 at $Y$ and therefore know the values of the spin of both particles in directions associated to operators that do not commute (and, therefore, cannot be simultaneously measured, according to standard quantum mechanics)[16].

The main point of the EPR paper was not to claim that one could *measure* quantities that are impossible to measure simultaneously according to quantum mechanics, but rather that, if one can learn something about a physical system by making a measurement on a distant system, then, barring actions at a distance, that "something" must already be there before the measurement is made on the distant system.

Einstein explicitly denied that his goal was to beat the uncertainty principle in his letter to Schrödinger of June 19, 1935. His argument, expressed in our language, was that, if one measures at A the spin in direction 1, when the state is given by (4.1), one may get, for example the state $|A\ 1 \uparrow>\ |B\ 1 \downarrow>$ and if one measures the spin in direction 2, one may get, say, the state $|A\ 2 \downarrow>\ |B\ 2 \uparrow>$. But then at B, one gets two different states $|B\ 1 \downarrow>$ and $|B\ 2 \uparrow>$, which leads to different predictions for the future behavior of the system. So that, by choosing which quantity to measure at A, one changes in different ways the state at B: action at a distance! And that is what Einstein objected to. However, Einstein emphasized that "he couldn't care less" whether the collapsed states at A and B would be eigenstates of incompatible observables, like the spin in directions 1 and 2 ([30], p. 38).

In fact, the reason why one cannot measure simultaneously the spin in direction 1 at $X$ and in direction 2 at $Y$ is that any one of those measurements affects the quantum state (by "collapsing" it) and affects it at both places, i.e. non-locally, so that a measurement of the spin in direction 1 at $X$ will change, in general, the probabilities of

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[16] Of course, since EPR were speaking of position and momentum instead of spin, it is those quantities that would have been simultaneously measured.

the results of a spin measurement in direction 2 at $Y$. In the EPR situation, it is because of the non-local character of the collapse that one cannot perform these simultaneous measurements. But, if things were perfectly local, then one could measure the spin in direction 1 at $X$ (or the position of the particle at $X$) and the spin in direction 2 at $Y$ (or the momentum of the particle at $Y$); since none of these measurements would, by the locality assumption, affect the state of the other particle, one would therefore know, because of the perfect correlations, the spin in both directions (or the position and the momentum) at $X$ and at $Y$.

And that is why Bohr, in order to reply to EPR had to "reject the premise -'no action at a distance' - rather than refute the argument"[17].

5.2. **Misunderstandings of Bell.** The result of Bell, taken by itself and forgetting about the one of EPR, can be stated as a "no hidden variable theorem": the pre-existing values of the spins are by definition called hidden variables, because they are not part of the description provided by the quantum state. This expression is a misnomer, because, as we shall see in the next section, some "hidden variables" are not hidden at all. But, accepting this terminology for the moment, what Bell showed is that the mere supposition that the values of the spin pre-exist to their "measurement", combined with the perfect anti-correlation when the axes along which measurements are made are the same and the $\frac{1}{4}$ result for measurements along different axes lead to a contradiction. Since the last two claims are empirical, this means that these hidden variables or pre-existing values cannot exist. There have been other "no hidden variable theorems", most notably one due to John von Neumann [46], but the latter, unlike Bell's result, relied on some arbitrary assumptions, that we will not discuss (see [6] and [39] for a discussion of this theorem).

But Bell of course, always presented his result *in combination with* the one of EPR, which shows that the mere assumption of locality, combined with the perfect correlation when the directions of measurement (or questions) are the same, implies the existence of those hidden variables that are "impossible". So, for Bell, his result, combined with the one of EPR was not a "no hidden variable theorem", but a non-locality theorem, the result on the impossibility of hidden variables being only one step in a two step argument. Here is a quote of Bell that states it clearly (here EPRB means EPR and Bohm, who reformulated the EPR argument in term of spins [10]):

> Let me summarize once again the logic that leads to the impasse. The EPRB correlations are such that the result of the experiment on one side immediately foretells that on the other, whenever the analyzers happen to be parallel. If we do not accept the intervention on one side as a causal influence on the other, we seem obliged to admit that the results on both sides are determined in advance anyway, independently of the intervention on the other side, by signals from the source and by the local magnet setting. But this has implications for non-parallel settings which conflict with those of quantum mechanics. So we *cannot* dismiss intervention on one side as a causal influence on the other.

---

[17] This was also Einstein's analysis of Bohr's reply to the EPR paper; Einstein thought that Bohr rejected the premise of EPR that "the real situation of B could not be influenced (directly) by any measurement taken on A", see [28], p. 681-682.

John Bell [9], p. 149-150

He was also conscious of the misunderstandings of his results :

> It is important to note that to the limited degree to which determinism[18] plays a role in the EPR argument, it is not assumed but inferred. What is held sacred is the principle of "local causality" - or "no action at a distance" . . . It is remarkably difficult to get this point across, that determinism is not a presupposition of the analysis.

John Bell [9], p. 143

And he added, unfortunately only in a footnote:

> My own first paper on this subject (*Physics* **1**, 195 (1965))[19] starts with a summary of the EPR argument *from locality to* deterministic hidden variables. But the commentators have almost universally reported hat it begins with deterministic hidden variables.

John Bell [9], p. 157, footnote 10

A famous physicist who is also such a commentator is Murray Gell-Mann, who wrote:

> Some theoretical work of John Bell revealed that the EPRB experimental setup could be used to distinguish quantum mechanics from hypothetical hidden variable theories. . . After the publication of Bell's work, various teams of experimental physicists carried out the EPRB experiment. The result was eagerly awaited, although virtually all physicists were betting on the corrections of quantum mechanics, which was, in fact, vindicated by the outcome.

Murray Gell-Mann [31] p. 172

So, Gell-Mann opposes hidden variable theories to quantum mechanics, but the only hidden variables that Bell considered were precisely those that were needed, because of the EPR argument, in order to "save" locality. So, if there is a contradiction between the existence of those hidden variables and experiments, it is not just quantum mechanics that is vindicated, but locality that is refuted.

In one of his most famous paper, "Bertlmann's socks and the nature of reality" [8], Bell gave the example of a person (Mr Bertlmann) who always wears socks of different colors (see the picture below, taken from [8]). If we see that one sock is pink, we know automatically that the other sock is not pink (let's say it is green). That would be true even if the socks were arbitrarily far away. So, by looking at one sock, we learn something about the other sock and there is nothing surprising about that, because socks *do have* a color whether we look at them or not. But what would we say if we were told that the socks have no color before we look at them? That would be surprising, of course, but the idea that quantum mechanics is complete means exactly that (if we replace the color of the socks by the values of the spin before measurement). But then, looking at one sock would "create" non only the color of that sock but also the one of the other sock. And that would be even more surprising, because it implies the existence of actions at a distance, if the socks are far apart.

---

[18] Here, "determinism" refers the idea of pre-existing values (note of J.B.)

[19] Reprinted as Chapter 2 in [9] (note of J.B.).

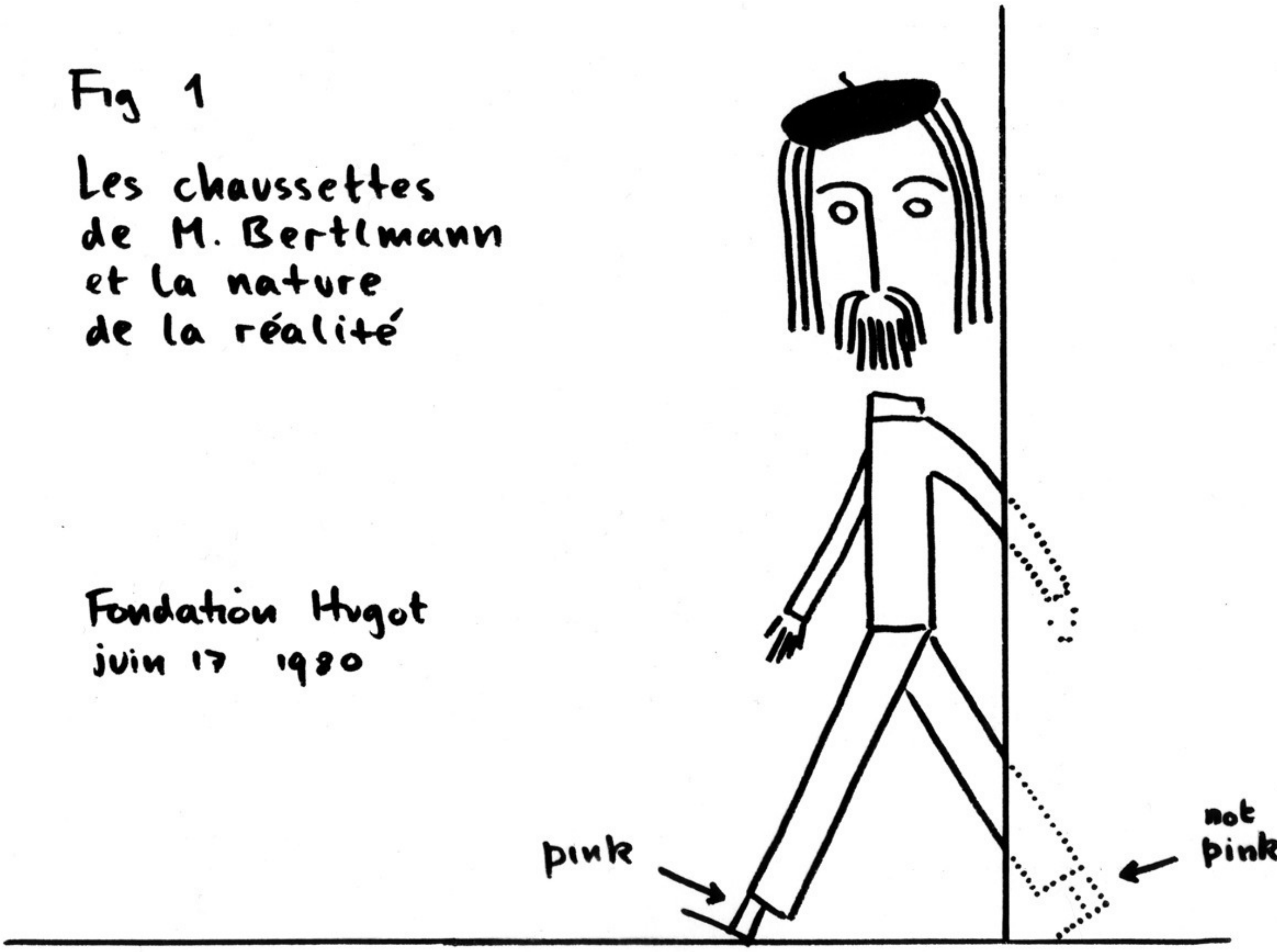

Figure 4: Bertlmann's socks; taken from [9] p. 139.

However, Murray Gell-Mann makes the following comment on the Bertlmann's socks paper:

> The situation is like that of Bertlmann's socks, described by John Bell in one of his papers. Bertlmann is a mathematician who always wears one pink and one green sock. If you see just one of his feet and spot a green sock, you know immediately that his other foot sports a pink sock. Yet no signal is propagated from one foot to the other. Likewise no signal passes from one photon to the other in the experiment that confirms quantum mechanics. No action at a distance takes place.
>
> Murray Gell-Mann [31] p. 172-173

This is not correct: it is true that, because of the random nature of the results, the experimental setup of EPRB cannot be used to send messages (or signals), as we saw in subsection 4.3. But nevertheless, some action at a distance does take place. Of course, the goal of Gell-Mann in the passage quoted here is to dismiss pseudo-scientific exploitations of Bell's result, but his defense of science is misdirected: the behavior of quantum particles is *not* like the one of Bertlmann's socks (which is indeed totally unsurprising), and that was the whole point of Bell's paper.

Eugene Wigner also saw Bell's result solely as a no hidden variable result:

> The proof he [von Neumann] published... though it was made much more convincing later on by Kochen and Specker[20], still uses assumptions which, in my opinion, can quite reasonably be questioned... In my opinion, the most convincing argument against the theory of hidden variables was presented by J.S. Bell.
>
> Eugene Wigner [48], p.291

This is misleading, because Wigner considers only Bell's argument, which indeed shows that pre-existing spin values (or "hidden variables") cannot exist, but forgets the EPR part of the argument, which was the starting point of Bell, and which shows that these variables must exist if the world is local (see [32] for a further discussion of Wigner's views).

David Mermin summarized the situation described here in a amusing way:

> Contemporary physicists come in two varieties.
>
> Type 1 physicists are bothered by EPR and Bell's theorem.
>
> Type 2 (the majority) are not, but one has to distinguish two subvarieties.
>
> Type 2a physicists explain why they are not bothered. Their explanations tend either to miss the point entirely (like Born's to Einstein) or to contain physical assertions that can be shown to be false.
>
> Type 2b are not bothered and refuse to explain why. Their position is unassailable. (There is a variant of type 2b who say that Bohr straightened out the whole business, but refuse to explain how.)
>
> David Mermin [38]

Yet, the same David Mermin also wrote (Bohm theory, which we call de Broglie-Bohm's theory, will be explained in the next section):

> Bell's theorem establishes that the value assigned to an observable must depend on the complete experimental arrangement under which it is measured, even when two arrangements differ only far from the region in which the value is ascertained – a fact that Bohm theory exemplifies, and that is now understood to be an unavoidable feature of any hidden-variables theory.
>
> To those for whom nonlocality is anathema, Bell's Theorem finally spells the death of the hidden-variables program.
>
> David Mermin [39], p. 814

But Bell's theorem shows that non-locality, whether we consider it anathema or not, is an unavoidable feature of the world. What is even more surprising is that this comment comes at the end of a remarkably clear paper on Bell's theorem.

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[20] Who proved another "no hidden variable" result [34]; see [39] for a discussion (note of J.B.).

## 6. WHY IS THE THEORY OF DE BROGLIE-BOHM COMPATIBLE WITH EPR-BELL?

To understand non-locality, we need a theory that goes beyond ordinary quantum mechanics. In quantum mechanics, non-locality is manifested by the collapse of the quantum state in situations such as the one of EPR-Bell. But since the meaning of the quantum state is unclear, so is its collapse. By "going beyond ordinary quantum mechanics", we mean a theory that gives a clear meaning to the quantum state, and in particular "decides" whether the latter represents our information about the system or is rather something physical or both. One such theory is the one of de Broglie-Bohm and, in that theory, non-locality also acquires a clear meaning[21] .

Our goal here is not to discuss the de Broglie-Bohm's theory in any detail, but only to explain enough of that theory so as to show how it allows us to understand, to some extent, non-locality. In a nutshell, the de Broglie-Bohm's theory is a "hidden variable theory", that eliminates the special role of "observations" in quantum mechanics and that is non-local, but that does not contain the hidden variables that are shown to be impossible by Bell (i.e. the spin values). Obviously, if certain variables are such that one cannot even conceive that they exist, they cannot be part of a consistent theory. And, by EPR + Bell, we know that any theory describing the world has to be non-local.

It cannot be emphasized enough that this is a quality of the de Broglie-Bohm's theory: obviously if the theory was local, it could not be true! In fact, one of the motivations of Bell, when he derived his result, was to see if one could obtain an alternative to ordinary quantum mechanics that removes the central role of observations (as the de Broglie-Bohm's theory does) while remaining local.

Moreover, as we will see, the "hidden variables" in that theory are not hidden at all.

All this sounds impossible, but here is how the theory goes:

In the de Broglie-Bohm's theory, the state of system is a pair $(\Psi, X)$, where $\Psi$ is the usual quantum state and $X$ denotes the actual positions of all the particles in the system under consideration, $X = (X_1, \ldots, X_N)$. $X$ are the hidden variables in his theory; this is obviously a misnomer, since particle positions are the only things that we ever directly observe (think of the double-slit experiment for example). As we shall see below, this is also true when one "measures" the spin (and, in fact, any other quantum mechanical "observable").

A first remark about the de Broglie-Bohm's theory is that, in the "Einstein boxes" experiment, the particle is always in one of the boxes, since it always has a position, so there is no paradox and no non-locality.

The dynamics of the de Broglie-Bohm's theory is as follows: both objects $(\Psi, X)$ evolve in time; $\Psi$ follows the usual Schrödinger's equation (1.1):

$$i\partial_t \Psi = H\Psi = (H_0 + V)\Psi, \tag{6.1}$$

The evolution of the positions is guided by the quantum state: writing $\Psi = Re^{iS}$

$$\dot{X}_k = \frac{1}{m_k} \quad \frac{\mathrm{Im}(\Psi^* \nabla_k \Psi)}{\Psi^* \Psi}(X_1, \ldots, X_N) = \frac{1}{m_k} \nabla_k S(X_1, \ldots, X_N) \tag{6.2}$$

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[21] For the original papers of de Broglie see his contribution to the 1927 Solvay Conference in [43, 5]; the earlier work by de Broglie is also discussed in [5]. For the original papers of Bohm, see [11]. For a pedagogical introduction to that theory, see [1, 2]. For a more detailed exposition, see [9, 12, 19, 20, 22, 23, 33].

for $k = 1, \ldots, N$, where $X_1, \ldots, X_N$ are the actual positions of the particles. For multi-components quantum states, with spin, the products $\Psi^* \nabla_k \Psi$ and $\Psi^* \Psi$ are replaced by scalar products is spin space.

We are not going to discuss here in detail how the de Broglie-Bohm's theory reproduces the results of ordinary quantum mechanics (see e.g. [45] for a good discussion of the double slit experiment), but we will focus on one example: a spin measurement. Let us consider a particle whose initial state is: $\Psi = \Phi(x,z)(|\uparrow> + |\downarrow>)$. $\Phi(x,z)$ denotes the spatial part of the quantum state and $(|\uparrow> + |\downarrow>)$ its (normalized) spin part (we consider here a two dimensional motion, where $z$ denotes the "vertical" direction in which the spin is "measured", by a Stern-Gerlach apparatus and $x$ the transverse direction in which the particle moves towards the apparatus)[22]. Assume that $\Phi(x,z)$ is a function whose support is concentrated around zero along both axis. As far as the free motion of the particle is concerned, we have a rectilinear motion along the $x$ axis (there is also some spreading of the wave function, but we will neglect that, assuming that the time of the experiment is short enough); we will furthermore neglect that motion and replace $\Phi(x,z)$ by $\Phi(z)$. One can associate a measurement of spin with the introduction of an inhomogeneous magnetic field in the Hamiltonian, at some time, say 0 (see [22] section 8.4 for a detailed discussion). The solution of Schrödinger's equation is then, with much simplification:

$$\Phi(z-t)|\uparrow> + \Phi(z+t)|\downarrow> \tag{6.3}$$

This means that the particle has a state which is composed of two parts, one localized near $t$, the other near $-t$, along the $z$ axis. If $t$ is not too small, those two regions are far apart, and, by detecting the particle, we can see in which region it is. If it is near $t$, we say that the spin is up and if it is near $-t$, we say that it is down.

The first thing to notice is that all we see in that "measurement" is the position of the particle. We never literally see the spin. In the de Broglie-Bohm's theory, the particle has a position and therefore a trajectory; it will either go up or down.

One may ask how the de Broglie-Bohm's theory reproduces the quantum statistics. Since the theory is deterministic, all "randomness" must be in the initial conditions. So, one has to assume that, if we start with an ensemble of particles with the state $\Phi(x,z)(|\uparrow> + |\downarrow>)$, they will distributed at the initial time according to the usual $|\Psi(x,z)|^2 = |\Phi(x,z)|^2$ distribution. The dynamics (6.1, 6.2) preserves that distribution, in the sense that the particles, evolving according to (6.2) will be distributed at a later time $t$ according to the $|\Psi(x,z,t)|^2$ distribution, where $\Psi(x,z,t)$ is the solution of Schrödinger's equation (6.1) at time $t$. Then obviously, the usual quantum predictions are recovered. We will not discuss here or try to justify this statistical assumption on the initial conditions (see [19]), but we note that the de Broglie-Bohm's theory answers the question about the status of the quantum state (does it represents our information about the system or is it rather something physical or both?) by "both" : it has a clear physical role, through the guiding equation, (6.2), but also a statistical role (that is, it reflects our ignorance or our partial information) through the assumption on initial conditions.

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[22] See [2], pp. 145-160 for a more detailed discussion of these experiments within the theory of de Broglie-Bohm.

Now, coming back to the spin measurement, suppose that $\Phi$ is symmetric in $z$: $\Phi(z) = \Phi(-z)$. Then the derivative $\partial_z \Phi(x, z)$ vanishes at $z = 0$ and, by (6.2), the particle velocity is zero for $z = 0$. Therefore, the particle never crosses the line $z = 0$. From this we conclude that the particle will go up if its initial condition satisfies $z > 0$ and down otherwise.

But, and here is the surprising point, suppose that we reverse the direction of the magnetic field. Then, the solution of Schrödinger's equation becomes (again, with much simplification):

$$\Phi(z+t)| \uparrow> +\Phi(z-t)| \downarrow> \tag{6.4}$$

So, now, if the particle is near $-t$, we say that the spin is up and if it is near $t$, we say that it is down.

But, if we do the experiment starting with the same quantum state and the same particle position, the particle will go in the upward direction again if its initial condition satisfies $z > 0$ and downward otherwise, since it cannot cross the line $z = 0$ . But now, as we just saw, we declare that its spin is down if it goes in the positive $z$ direction, and we declare that its spin is up if the particle goes in the negative $z$ direction.

In other words, the value up or down of the spin is "contextual": that value does not depend only on the quantum state and the original particle position but on the concrete arrangement of the "measuring" device. Here the quotation marks are introduced, because we see that there is no intrinsic property of the particle that is being "measured". Of course, since the system is deterministic, once we fix the initial state (quantum state and position) of the particle *and* the experimental device, the result of the experiment is pre-determined. But that does not mean that the spin value that we "observe" is pre-determined, because we can measure the spin along a given axis by orienting the magnetic field in one direction or the opposite one along that same axis. So, the value of the spin depends on our conventions, which means that it does not exists as an intrinsic property of the particle. The same remark also holds for other quantum mechanical "observables", like energy or momentum, see [20].

So, not only is the word "hidden variable" misleading, but so are the words "observable" or "measurement", since, except when one measures positions, one does not observe or measure any intrinsic property of the system being observed or measured.

This vindicates in some sense Bohr's emphasis on :

> [...] the *impossibility of any sharp distinction between the behaviour of atomic objects and the interaction with the measuring instruments which serve to define the conditions under which the phenomena appear.*
>
> Niels Bohr [14], p. 210, quoted in [9], p. 2 (italics in original).

But in the de Broglie-Bohm's theory, this follows from the equations of the theory and not from some more or less a priori statement.

Besides, this is good news, since it explains how the de Broglie-Bohm's theory can introduce hidden variables (the positions of the particles) without introducing the hidden variables (the spin values) that would lead to a contradiction, because of Bell's theorem.

Now, let us see why the de Broglie-Bohm's theory is non-local. If we look at equation (6.2), we see that, if the quantum state factorizes: $\Psi = \prod_{i=1}^{N} \psi_i(x_i)$, then each particle is guided by its quantum state $\psi_i$ and does not depend on the other $\psi_j$'s, $j \neq i$. But if the state does not factorize, which is the case for the state (4.1) (such states are called "entangled"), then the motion of each $X_i$ is influenced by the value of the quantum state $\Psi$ evaluated at the actual value $(X_1, \ldots, X_N)$ of the positions of all the particles of the system. So, in the example of the state (4.1), if we change the position of the particle $A$, we influence immediately the motion of particle $B$, through equation (6.2).

Of course, one might say that in a deterministic theory, changing the position of a particle would involve counterfactual reasonings, since all the motions are determined anyway. But a Schrödinger equation may involve "external" potentials (like a magnetic field) and the latter can be viewed as representing an external action on the system described by that equation. In Figure 5, we consider two particles, we replace (to simplify matters) $\mathbb{R}^3 \times \mathbb{R}^3$ by $\mathbb{R} \times \mathbb{R}$ and denote by $(X_1, X_2)$ the actual position of the two particles.

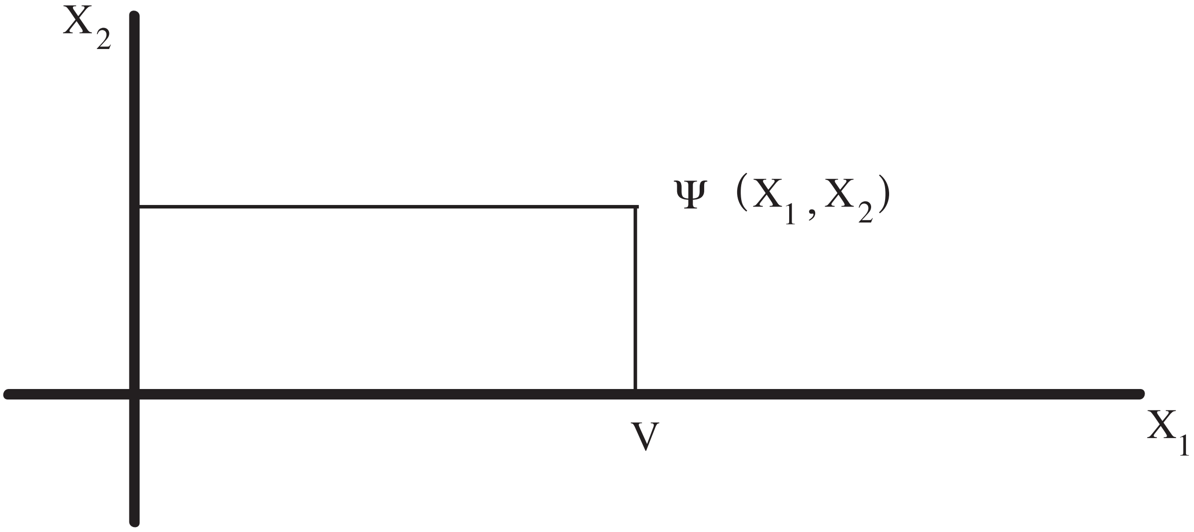

Figure 5: Non-locality in de Broglie-Bohm's theory

Now suppose we "act" on the first particle, by introducing near the place where that particle is located a potential $V$ (in equation (6.1)), for example a magnetic field that is used to "measure" the spin. This of course does not affect directly the motion of either the first or the second particle, but it affects the behavior of the quantum state, through Schrödinger's equation (6.1), and, through the guiding equation (6.2), this affects simultaneously the behavior of both particles, (when $\Psi$ is not factorizable as a product of functions of $X_1$ and $X_2$, as is the case for the state (4.1)).

If we go back to the state (4.1), when one introduces a magnetic field as in the Hamiltonian to "measure" the spin in one of the three directions, let's say first at A, the particle at A follows an evolution like in (6.3) and, once the particle is coupled to a detector, that induces an effective collapse of the quantum state (see for example [22] chapters 8, 9, for more details). In the de Broglie-Bohm theory, the quantum state never really collapses and always follows Schrödinger's equation. But, once the state of a microscopic system is coupled to the one of a macroscopic system, interference effects become, in practice, impossible (this is also called decoherence). Then, one can take the microscopic state coupled to the one that is seen for the macroscopic system, as being the quantum state for all the future behavior of the microscopic system. Thus, there is an effective, but not fundamental, collapse (this is somewhat similar to the phenomenon of irreversibility in statistical mechanics). Suppose that the state collapses, if the magnetic field is along direction 1, on $|A\ 1\uparrow>\ |B\ 1\downarrow>$. This means that the state changes also at B (and instantaneously) and the future behavior of the particle at B (on which one may apply a field in direction 1, 2 or 3) will be guided by the state $|B\ 1\downarrow>$. This of course allows the de Broglie-Bohm theory to recover the usual quantum predictions in the EPR-Bell situation.

A well-known problem is that whether the measurement is done first at A or at B is not a relativistically invariant notion- it will depend on our frame of reference. But this problem lies outside the scope of this paper. It is a problem caused by the existence of non-local effects, and has nothing to do with the de Broglie-Bohm theory per se. Ordinary quantum mechanics has also this problem, but simply ignores it: for example, in relativistic quantum theories, including quantum field theories, the "collapse" operation is never treated in a relativistic way, and, because it is simultaneous, poses the same problem as the one raised here. The only way out would be to to say that the collapse simply reveals preexisting properties of the system, which is exactly the idea of preexisting spin values which, as Bell showed, is inconsistent with quantum mechanical predictions. So, the problem of making relativity and quantum mechanics truly compatible is deep and unsolved (irrespective of what one thinks of the de Broglie-Bohm theory).

## 7. **SUMMARY AND CONCLUSIONS**

During the debates around the "Copenhagen" interpretation of quantum mechanics, Einstein held an heterodox and minority view. He maintained that the quantum state is not a complete description of physical reality and that particles have properties beyond what is included in their quantum state. In other words, he thought that quantum mechanics provides a very accurate statistical description of quantum systems, but not a complete physical description of individual systems.

Einstein tried to give indirect arguments proving that quantum mechanics is incomplete and, in 1935, together with Podolsky and Rosen, he made the following reasoning: suppose that there is a quantity that I can measure on one system and that, because of conservation laws, immediately tell me the value of the corresponding quantity for another system which is far away. Then, if we assume that the measurement on the first system cannot affect the physical state of the second system, because of their spatial separation, the second system must have had that property all along.

The EPR argument was made unnecessarily complicated by considering two quantities (position and momentum) instead of one, each of which can be measured on the first system and then predicted for the second system. But, if all one wants to show is incompleteness of quantum mechanics, then considering one quantity is enough since the quantum state does not assign a definite value to that quantity for the second system, while the EPR argument shows that, barring non-local effects, there must be one.

The EPR argument was generally ignored (except mostly by Schrödinger [42]) and most physicists thought that it had been countered by Bohr (although Bohr's argument is far less clear than the EPR paper).

In 1964, almost thirty years after the EPR paper, and when their argument had been essentially forgotten, Bell showed that merely assuming that the quantities that EPR showed must exist if the world is local, leads to a contradiction with quantum predictions, that were later verified experimentally.

Thus, the question raised by EPR, whether the quantum mechanical description is complete, was answered in a way that Einstein would probably have liked least, as Bell said: by showing that the "obvious" assumption of locality made by EPR is actually false ([6], p. 11).

But since most people had forgotten or misunderstood the EPR paper, Bell's argument was taken to be just one more proof that quantum mechanics is complete in the sense that no other variables, that would complete the quantum description, can be introduced without running into contradictions.

Of course, that was the opposite of what Bell showed and explicitly said: the conclusion of his argument, combined with the one of EPR, is rather that there are non-local physical effects (and not just correlations) in Nature.

As for the completeness of quantum mechanics, while it is true that the EPR argument does not settle that issue, a more complete theory was proposed by de Broglie in 1927 and developed by Bohm in 1952. That theory of course is non-local, which is taken by people who misunderstand the EPR-Bell result as an argument against that theory, while, in reality, if that theory was local, it could not be empirically adequate.

The speculations of Einstein, looked to many physicists as being purely philosophical or even "metaphysical"; for example, Pauli wrote:

> As O. Stern said recently, one should no more rack one's brain about the problem of whether something one cannot know anything about exists all the same, than about the ancient question of how many angels are able to sit on the point of a needle. But it seems to me that Einstein's questions are ultimately always of this kind.
>
> Wolfang Pauli [15], p. 223

But those speculations have led to what is probably "the most profound discovery of science", to use Henri Stapp's phrase ([44] p. 271). They have also, because they lie at the basis of quantum information, led to various technological applications.

This should be a lesson for "pragmatists".

## Acknowledgments

I have benefitted from a great number of discussions about the meaning of Bell's theorem with many people, but specially with Detlef Dürr, Sheldon Goldstein and Tim Maudlin. I thank Dominique Lambert and Serge Dendas for useful remarks on this manuscript.

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