WHAT DID BELL REALLY PROVE?

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THE CLASSICAL OR EPISTEMIC VIEW OF PROBABILITIES:

Laws are deterministic

$\Rightarrow$ probabilities are ONLY due to our ignorance.

Think of coin tossing.

WHAT ABOUT QUANTUM MECHANICS?

$|\Psi|^2$ has a probabilistic interpretation

$|\Psi(x)|^2$ = density of probability of the particle "being observed" at x.

More generally, if

$$\Psi = \sum_i c_i \Psi_i,$$

where, for some self-adjoint operator A,

$$A\Psi_i = \lambda_i \Psi_i,$$

and the $\Psi_i' s$ form a basis, then the measurement of the "observable" associated with the operator A, will give the result λ_i with probability $|c_i|^2$.

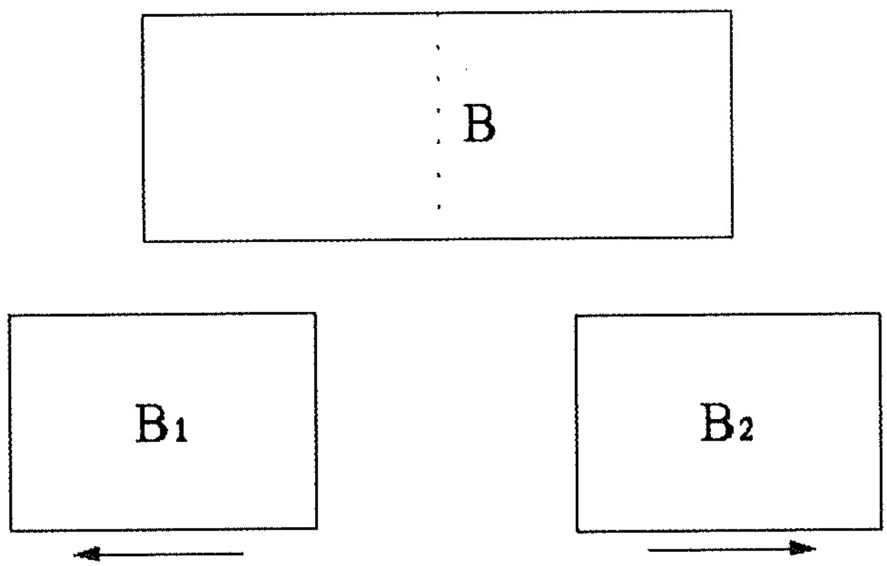
But is probability here only reflecting our ignorance ?

If so, why does Ψ evolve according to a physical law ?

What about the reduction or collapse of Ψ due to a measurement (Ψ becomes Ψ_i if the result λ_i is observed)? Is it like coin tossing?

Let us consider a simple example:

Einstein's boxes



A single particle is in Box B.

The $|$ state $>=|\ B>$,

One cuts the box in two half-boxes,

The state becomes

$\longrightarrow \frac{1}{\sqrt{2}}(|B_1>+|B_2>)$

where $|B_i>=$ particle "is" in box B_i, $i=1,2$.

The two half-boxes B_1 and B_2 are then separated and sent as far apart as one wants.

If one opens one of the boxes (say B_1) and that one does *not* find the particle, one *knows* that it is in B_2. Therefore, the state "collapses" instantaneously and in a non local way, since the two boxes are as far part as one wants.

One opens box $B_1 \longrightarrow$ nothing

This is a "measurement", therefore state $\longrightarrow |B_2>$

(and, if one opens the box B_2, one will find the particle !).

DILEMMA:

Is the reduction or collapse of the

$|$ state $>$ a real (= physical) operation

or does it represent only our knowledge (= epistemic), as in the example of coin tossing ?

<u>If</u> physical $\longrightarrow$ A non local form of causality exists

<u>If</u> epistemic $\longrightarrow$ QM "incomplete" : there exists other variables than the quantum state that describe the system.

These variables would tell in which half-box the particle **IS** *before one opens either of them.* This is exactly what the "incompleteness" of QM **means**.

Of the two branches of the dilemma, incompleteness is by far the most reasonable one!

However, it turns out that, putting aside the issue of completeness, one can *prove* non locality.

What is non locality ?

Non local causality (causality NOT mere correlation)

Properties:

1. Instantaneous
2. a. Extends arbitrarily far

   b. The effect does not decrease with the distance
3. Individuated
4. Could be used to transmit messages

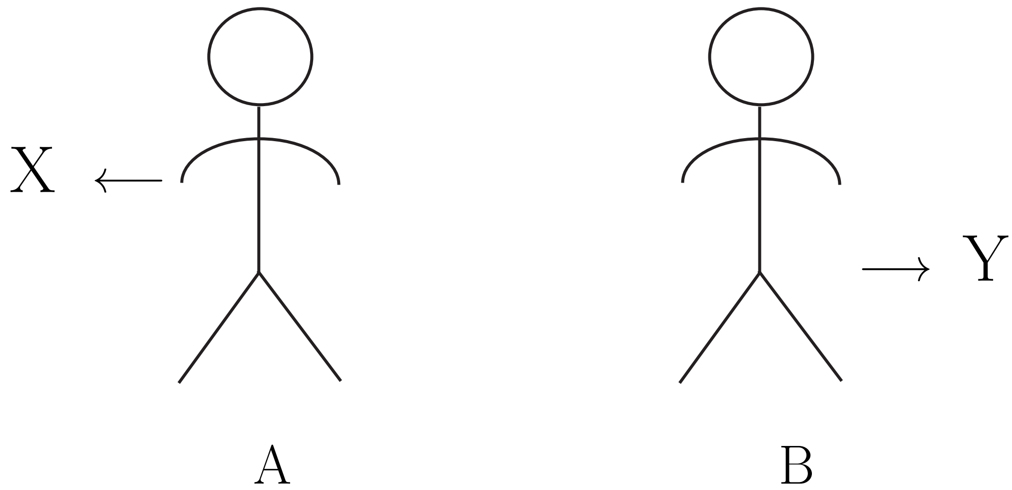
Newton's gravity : 1, 2a and 4

Post-Newtonian physics (e.g. field theories) : 2a and 4

Is there a phenomenon with properties : 1-3 ?

(Not 4 $\rightarrow$ pseudoscience).

HOW TO PROVE NON LOCALITY ?



3 questions 1,2,3

2 answers yes/no

Questions and answers vary. But when the same question is asked at X and Y, one always gets the same answer.

Only one possibility : either the answers are predetermined *or* there exists a form of causality at a distance *after* one asks the questions.

This is the Einstein Podolsky and Rosen (EPR-1935) argument (in Bohm's formulation).

BUT

This assumption

(alone)

leads to a contradiction with observations made when the questions are different.

Bell (1964)

PROOF

3 Questions 1 2 3

2 Answers Yes/No

If the answers are given in advance, there exists $2^3 = 8$ possibilities :

1	2	3
Y	Y	Y
Y	Y	N
Y	N	Y
Y	N	N
N	Y	Y
N	Y	N
N	N	Y
N	N	N

In *each case* there are at least *two questions* with the same answer.

Therefore,

$$
\begin{aligned}
&\quad \text{Frequency (answer to 1 = answer to 2)} \\
&+ \text{Frequency (answer to 2 = answer to 3)} \\
&+ \text{Frequency (answer to 3 = answer to 1)} \geq 1
\end{aligned}
$$

BUT,

in some experiments,

$$
\begin{aligned}
&\quad \text{Frequency (answer to 1 = answer to 2)} \\
&= \text{Frequency (answer to 2 = answer to 3)} \\
&= \text{Frequency (answer to 3 = answer to 1)} \\
&= \frac{1}{4}
\end{aligned}
$$

$$\Rightarrow \frac{3}{4} \geq 1$$

FALSE !

$\Rightarrow$ CONTRADICTION

That's all. That's the difficulty.

I've entertained myself always by squeezing the difficulty of quantum mechanics into a smaller and smaller place, so as to get more and more worried about this particular item. It seems to be almost ridiculous that you can squeeze it to a numerical question that one thing is bigger than another.

R. FEYNMAN, in "Simulating physics with computers" (1982)

QUANTUM DESCRIPTION

(NOT needed, but most people insist)

A and B are replaced by particles

X and Y are Stern-Gerlach apparatuses that "measure the spin" along some direction.

1, 2, 3 = 3 possible directions for that "measurement".

Yes/No = Up/Down.

$|$ state of the two particles $>$

$= \frac{1}{\sqrt{2}}(|A\ 1\uparrow>|B\ 1\downarrow> - |A\ 1\downarrow>|B\ 1\uparrow>)$
$= \frac{1}{\sqrt{2}}(|A\ 2\uparrow>|B\ 2\downarrow> - |A\ 2\downarrow>|B\ 2\uparrow>)$
$= \frac{1}{\sqrt{2}}(|A\ 3\uparrow>|B\ 3\downarrow> - |A\ 3\downarrow>|B\ 3\uparrow>)$

These three representation follow from rotation invariance (in the "spin space")-that is an elementary fact about QM.

Let us consider one representation:

$|$ state of the two particles $>$

$= \frac{1}{\sqrt{2}}(|A\ 1 \uparrow> |B\ 1 \downarrow> - |A\ 1 \downarrow> |B\ 1 \uparrow>)$

If one measures the spin in direction 1 at X, and one sees $\uparrow$, the state becomes $|A\ 1 \uparrow> |B\ 1 \downarrow>$. A later measurement of the spin at Y will yield $| \downarrow>$ with certainty.

If one sees $\downarrow$, the state becomes $|A\ 1 \downarrow>\ \ |B\ 1 \uparrow>$ and a later measurement of the spin at Y will yield $| \uparrow>$ with certainty.

Similar result if one measures the spin in direction 2 or 3 at X.

But then the state changes *non locally* at Y.

Same dilemma as for Einstein's boxes :

reduction of the $|$ state $>$ = physical or epistemic ?

If physical $\longrightarrow$ non locality

If epistemic $\longrightarrow$ "answers" are given in advance, i.e. the particle B *is* $1\uparrow$ or $1\downarrow$, $2\uparrow$ or $2\downarrow$, $3\uparrow$ or $3\downarrow$, *before* any measurement at X. These answers would be "hidden variables".

BUT (Bell 1964) this leads to a contradiction with observations made when the directions in which the spin is "measured" are *different* at X and Y (the 1/4 is the result of standard quantum mechanical computations) .

In particular, this shows that "spin" does not exist, meaning its value is not determined prior to measurement!

This is sometimes called a no hidden variable result, because it shows that one cannot introduce those pre-existing answers (the spin values) that would "save " locality. But the significance of the result is that, combined with the EPR argument, it refutes locality, not merely that it rejects (certain) "hidden variables".

To summarize: the perfect correlations (here, we have perfect anti-correlations, but that is a matter of conventions for YES/NO) are not merely correlations, but the result of a subtle form of non-locality. In other words, the reduction of the quantum state, which is non local, is not merely epistemic, but related to something physical.

One cannot use this to send messages

If one could, then relativity implies that one could send messages into one's own past.

— Each side sees a perfectly random sequence of YES/NO

— BUT if each person tells the other which "measurements" have been made (1, 2 or 3), then, they both know which result has been obtained on the other side when the same measurement is made on both sides.

$\Rightarrow$ Then, they both share a common sequence of YES/NO, which is form of "information". Since that information cannot possibly come from the source (Bell), some sort of nonlocal transmission of information has taken place.

This is the basis of quantum information theory $\rightarrow$ may lead to a better understanding of nonlocality.

BELL WAS QUITE EXPLICIT ABOUT WHAT THIS MEANS

Let me summarize once again the logic that leads to the impasse. The EPRB correlations are such that the result of the experiment on one side immediately foretells that on the other, whenever the analyzers happen to be parallel. If we do not accept the intervention on one side as a causal influence on the other, we seem obliged to admit that the results on both sides are determined in advance anyway, independently of the intervention on the other side, by signals from the source and by the local magnet setting. But this has implications for non-parallel settings which conflict with those of quantum mechanics. So we cannot dismiss intervention on one side as a *causal* influence on the other.

J. BELL

BUT BELL WAS WIDELY MISUNDERSTOOD

Bell was also conscious of the misunderstandings of his results : "It is important to note that to the limited degree to which determinism plays a role in the EPR argument, it is not assumed but inferred. What is held sacred is the principle of "local causality" - or "no action at a distance" ...It is remarkably difficult to get this point across, that determinism is not a presupposition of the analysis." And he added, unfortunately only in a footnote: "My own first paper on this subject (*Physics* **1**, 195 (1965)) starts with a summary of the EPR argument *from locality to* deterministic hidden variables. But the commentators have almost universally reported that it begins with deterministic hidden variables."

One example of such a commentator is Murray Gell-Mann:

> Some theoretical work of John Bell revealed that the EPRB experimental setup could be used to distinguish quantum mechanics from hypothetical hidden variable theories... After the publication of Bell's work, various teams of experimental physicists carried out the EPRB experiment. The result was eagerly awaited, although virtually all physicists were betting on the corrections of quantum mechanics, which was, in fact, vindicated by the outcome.
>
> M. GELL-MANN

The proof he [von Neumann] published… though it was made much more convincing later on by Kochen and Specker, still uses assumptions which, in my opinion, can quite reasonably be questioned… In my opinion, the most convincing argument against the theory of hidden variables was presented by J.S. Bell.

E. WIGNER

EINSTEIN WAS ALSO MISUNDERSTOOD

An essential aspect of this arrangement of things [physical objects] in physics is that they lay claim, at a certain time, to an existence independent of one another, provided these objects "are situated in different parts of space". The following idea characterizes the relative independence of objects far apart in space (A and B) : external influence on A has no direct influence on B.

A. EINSTEIN

Here is how Born "understood" Einstein:

The root of the difference between Einstein and me was the axiom that events which happens in different places A and B are independent of one another, in the sense that an observation on the states of affairs at B cannot teach us anything about the state of affairs at A.

M. BORN

Bell comments this passage as follows:

"Misunderstanding could hardly be more complete. Einstein had no difficulty accepting that affairs in different places could be correlated. What he could not accept was that an intervention at one place could influence, immediately, affairs at the other."

Physicist David Mermin has an amusing summary of the situation:

Contemporary physicists come in two varieties. Type 1 physicists are bothered by EPR and Bell's theorem. Type 2 (the majority) are not, but one has to distinguish two subvarieties. Type 2a physicists explain why they are not bothered. Their explanations tend either to miss the point entirely (like Born's to Einstein) or to contain physical assertions that can be shown to be false. Type 2b are not bothered and refuse to explain why. Their position is unassailable. (There is a variant of type 2b who say that Bohr straightened out the whole business, but refuse to explain how.)

D. MERMIN

Yet, the same David Mermin also wrote:

"Bell's theorem establishes that the value assigned to an observable must depend on the complete experimental arrangement under which it is measured, even when two arrangements differ only far from the region in which the value is ascertained – a fact that Bohm theory exemplifies, and that is now understood to be an unavoidable feature of any hidden-variables theory.

To those for whom nonlocality is anathema, Bell's Theorem finally spells the death of the hidden-variables program."

HOW DID BELL ARRIVE AT HIS RESULT?

He started from the de Broglie-Bohm's theory, which is non local and wondered if one could do "better", that is, have a theory about the world, not just about "results of experiments", but a local one.

In the de Broglie-Bohm's theory, the state of system is a pair (Ψ, X), where $X = (X_1, \ldots, X_N)$ denotes the actual positions of all the particles in the system under consideration, and $\Psi = \Psi(x_1, \ldots, x_N)$ is the usual quantum state, $(x_1, \ldots, x_N)$ denoting the arguments of the function Ψ. X are the hidden variables in his theory; this is obviously a misnomer, since particle positions are the only things that we ever directly observe (think of the double-slit experiment for example).

A first remark about the de Broglie-Bohm's theory is that, in the "Einstein boxes" experiment, the particle is always in one of the boxes, since it always has a position, so there is no paradox and no non locality.

The dynamics of the de Broglie-Bohm's theory is as follows: both objects (Ψ, X) evolve in time; Ψ follows the usual Schrödinger's equation:

$$i\hbar\partial_t\Psi(x_1,\ldots,x_N,t) = (H\Psi)(x_1,\ldots,x_N) \qquad (1)$$

where H is the Hamiltonian.

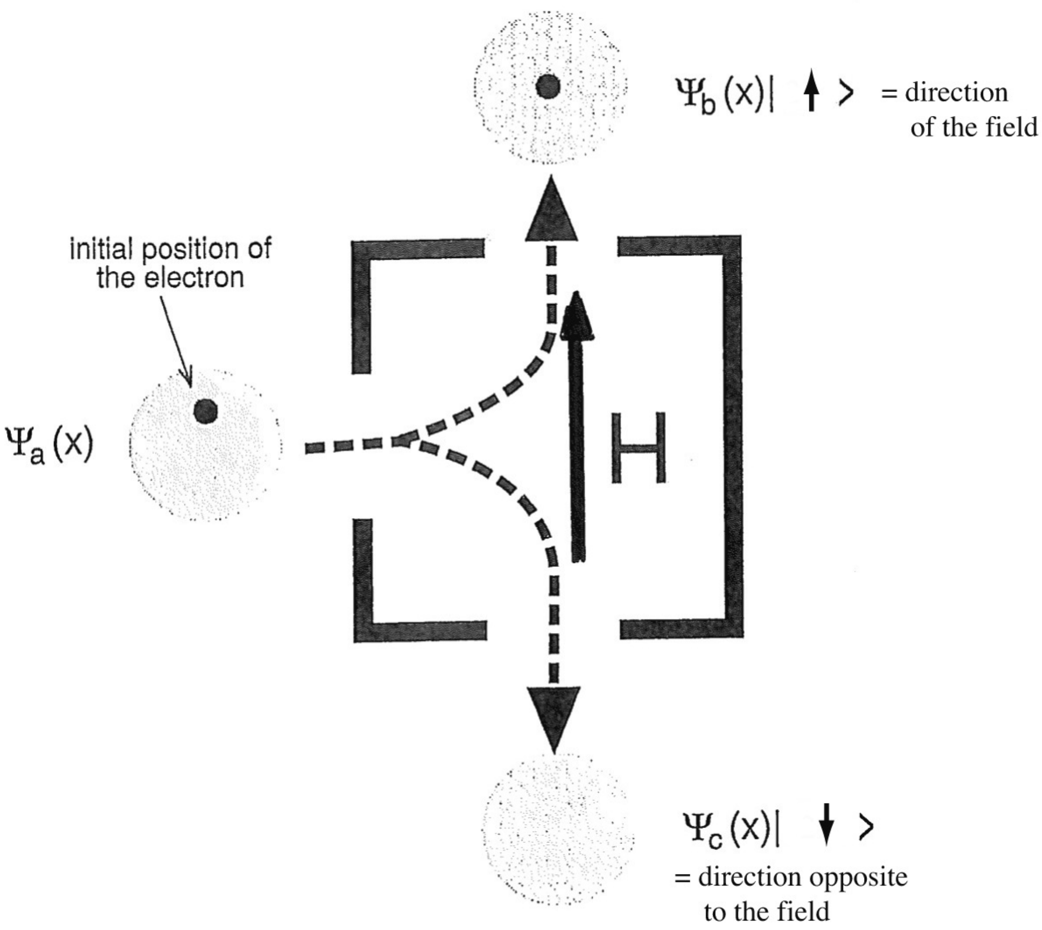
The evolution of the positions is guided by the quantum state: writing $\Psi = Re^{iS}$

$$\dot{X}_k(t) = \frac{\hbar}{m_k}\nabla_k S(X_1(t),\ldots,X_N(t)) \qquad (2)$$

for $k = 1,\ldots,N$, where $X_1,\ldots,X_N$ are the actual positions of the particles. The theory is non local because the evolution of the position of each particle, say X_k, depends on the positions of all the other particles, $X_1,\ldots,X_N$, because the value of the guiding field $S(X_1,\ldots,X_N)$ depends on them. In that way, the de Broglie-Bohm's theory is compatible with Bell's theorem.

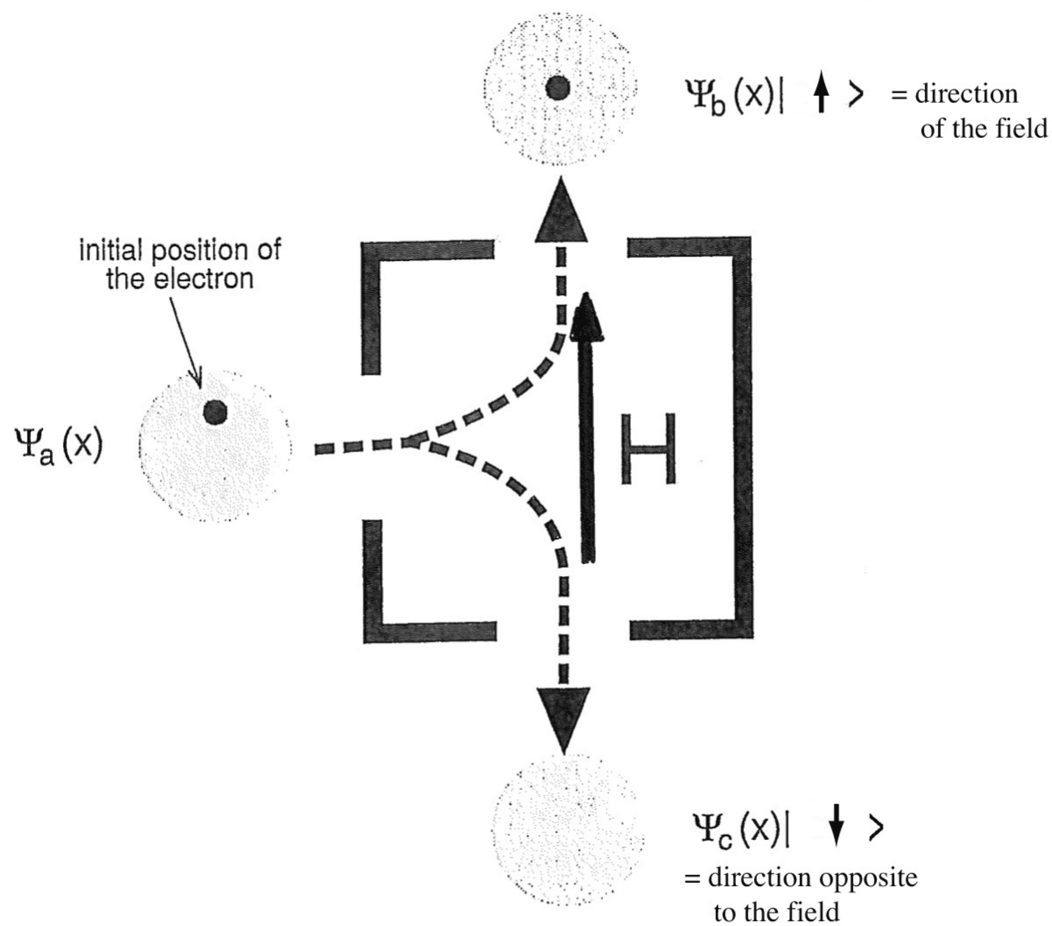
HOW DOES THE THEORY OF DE BROGLIE-BOHM ACCOUNT FOR THE INEXISTENCE OF "SPIN VALUES"?

Consider a Stern-Gerlach apparatus "measuring" spin. Let H be the magnetic field.

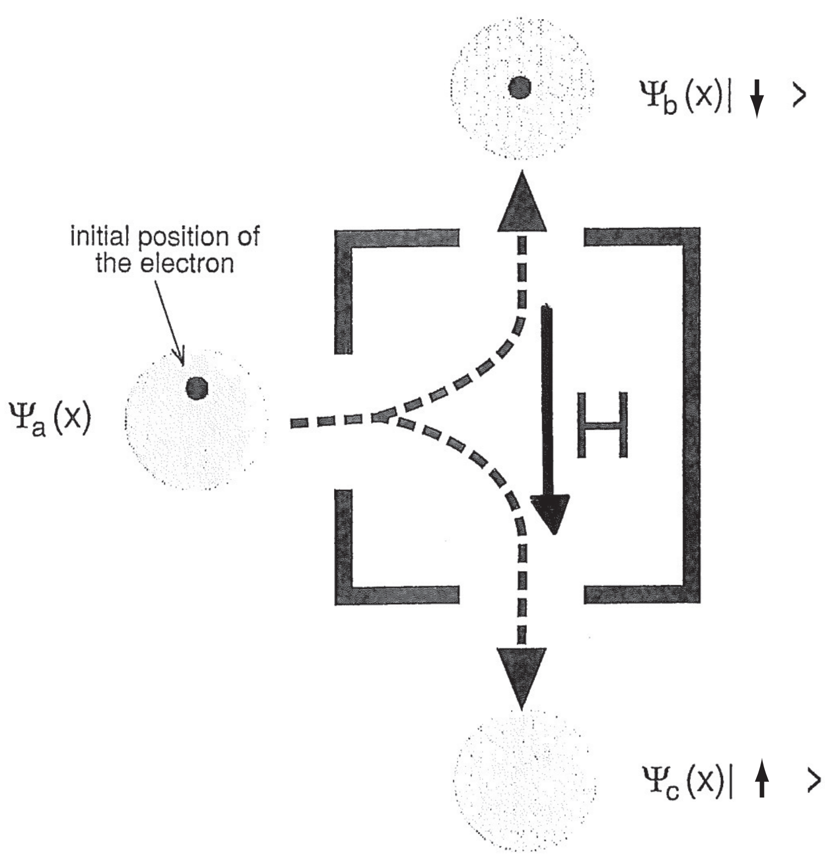


The $|\uparrow>$ part of the state always goes in the direction of the field, and the $|\downarrow>$ part always goes in the opposite direction.

But if the particle is initially in the upper part of the support of the wave function (for a symmetric wave function), it will always go upwards.



Now, repeat the same experiment, but with the direction of the field reversed, and let us assume that the particle starts with exactly the same wave function and the same position as before. The particle whose spin was “up”, will now “have” its spin “down”, although one “measures” exactly the same quantity, with exactly the same initial conditions (for the particle), but with two different arrangements of the apparatus.



Therefore the apparatus does not register something preexisting to the "measurement", but plays an active role. This vindicates the intuition of Bohr and others about the role of the measuring device, but by making it a consequence of the theory and not some philosophical a priori.

CONCLUSION: MAXIMAL CONFUSION IN THE LITERATURE

— Einstein is supposed to have tried to show that QM in incomplete and failed (Bohr replied successfully).

But EPR were posing a dilemma: either QM is incomplete or the world is non local. It is true that they rejected the second half, but neither Bohr nor his followers understood the problem.

— In 1927 de Broglie proposes a non local theory that eliminates any reference to an “observer” . The theory is dismissed (on dubious grounds) and rediscovered in 1952 by Bohm.

— In between (in 1935), von Neumann claims to have shown that no hidden variables can be added to the QM formalism; i.e. QM is "complete". People tend to believe his theorem, without looking at the assumptions. Moreover, a "hidden variable" theory had been proposed before by de Broglie.

— After 1952, Bell understand Bohm's theory but wonders whether one can "do better", i.e. reproduce the QM predictions in a LOCAL theory without observers. He proves in 1964 that this is impossible, by showing that the "hidden variables" (the spin values) that EPR showed were necessary to save locality cannot exist.

— Bell's result is (as he emphasized himself) is an argument **in favor** of Bohm's theory, since it shows that one cannot avoid non locality, which is moreover a rather natural feature in Bohm's theory.

— Bell's result is generally taken to mean that "hidden variable" theories cannot be compatible with the results of QM, including Bohm's theory, which is compatible with those results, which is non local (hence compatible with Bell's results) and explains why one cannot introduce those spin values that Bell shows are impossible.

Can anybody do WORSE?

REMARK BY A SELF-CRITICAL MATHEMATICIAN

The mathematical structure of operators in Hilbert space and unitary transformations is clear enough, as are certain features of the interpretation of this mathematics to give physical assertions, particularly assertions about general scattering experiments. But the larger question here, a systematic elaboration of the world-picture which quantum theory provides, is still unanswered. [...]Here also, the mathematical formalism may be hiding as much as it reveals.

JACOB T. SCHWARTZ, "THE PERNICIOUS INFLUENCE OF MATHEMATICS ON SCIENCE".

MAYBE WE SHOULD TURN TO LITERATURE !

I know that most men, including those at ease with problems of the highest complexity, can seldom accept even the simplest and most obvious truth if it be such as would oblige them to admit the falsity of conclusions which they have delighted in explaining to colleagues, which they have proudly taught to others, and which they have woven, thread by thread, into the fabric of their lives.

TOLSTOY