Q = "quantum"   C = "classical"

Universität Bielefeld

QUANTUM PHYSICS : OUTLINE OF A PROGRAMME

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- QM incredibly succesfull ⇒ Standard QM clearly works pragmatically
  ⇒ Justification FAPP (for all practical purpose) of the Quantum Rules starting from v. Neumann's epistemic principle (1932)

- A "Unified Algebraic Framework" valid just as well for C- and Q- Physics (Segal Heisenberg, Jordan, v. Neumann, Segal, Haag-Kastler, Araki
  Algebra of observables $\mathcal{A}$
  States (preparation procedure) $\varphi : \mathcal{A} \to \mathbb{C}$ $\varphi(\mathbb{1}) = 1$ linear, positive $\varphi(A^*A) \geq 0$, continuous
  Dynamic Automorphism $\alpha_t : \mathcal{A} \to$

  Physical system ⟺ $(\mathcal{A}, \varphi, \alpha_t)$

- Connections to experiments
  Experiments more and more sophisticated
  History   Non-demolition experiments

  $A = A^* \Rightarrow \varphi(A)$ real   Expectation (A in state φ)

  To be computed   Prob $[A, \varphi \mid \varphi(A) \in I \subset \mathbb{R}]$

  History in the sense of the history books   Prob [History] = Prob $[P_n P_{n-1} \dots P_1]$

- C^*-Algebra $\mathcal{A}$ v. Neumann algebra $\mathcal{M} = \mathcal{M}'' \subset \mathcal{B}($

  Algebraic Probability Theory $(\mathcal{M}, \varphi)$
  φ faithful normal state ≈ non-commutative measure

  GNS $\pi_\varphi(\mathcal{A})$ in $\mathcal{B}(\mathcal{H}_\varphi)$ ⇒ recovering the Hilbert space
  $\langle \cdot , \cdot \rangle_{\mathcal{H}_\varphi} = \varphi(A^*B)$

  $\mathcal{M}$ commutative ⟺ Kolmogorov probability theory (1932)

  The 2 logics of science $\mathcal{M}$ non commutative   "Quantum" Probability Theory

- From Q to C : Decoherence (emergent phenomenon : multiscale environment, time)

  S + E closed system

  $$\mathcal{H}_{S+E} = \mathcal{H}_S \otimes \mathcal{H}_E$$
  $$\mathcal{N} = \mathcal{M} \otimes \mathcal{M}_E$$

  $\alpha_t : \mathcal{N} \to \mathcal{N}$ automorphism

  ω_E Reference state of E

  $i : \mathcal{M} \to \mathcal{N}$ embedding $i(A) = A \otimes \mathbb{1}_E$

  $E_{\omega_E} : \mathcal{N} \to \mathcal{M}$ conditional expectation

  $$T_t = E_{\omega_E} \circ \alpha_t \circ i$$

  $\rho \otimes \omega_E$ state on $\mathcal{N}$

  $$T_t(\rho) = (E_{\omega_E} \circ \alpha_t \circ i)(\rho \otimes \omega_E)$$

  T_t is completely positive as combination of 3 CP-maps

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Mathematical definition of decoherence

B.O. RMP 15 217–243 (2003)

Splitting of $\mathcal{M} = \mathcal{M}_1 \oplus \mathcal{M}_2$

$\mathcal{M}_1$ ↖ v. N. subalgebra of $\mathcal{M}$

$A \in \mathcal{M} \qquad A = A_1 \oplus A_2$

$\mathcal{M}_1 \approx$ effective observables, decoherence free part, can be detected in practice

$\beta_t \equiv \alpha_t \restriction \mathcal{M}_1$ is an automorphism

$\mathcal{M}_2 \approx$ not detectable by measurements for $t \to \infty$

$$\lim_{t \to \infty} \varphi(\alpha_t(A_2)) = 0$$

- Automorphism $\qquad \alpha_t : \mathcal{M} \otimes \mathcal{M}_E$

$\qquad \Big\downarrow \; E_{\omega_E}$: conditional expectation ~ averaging over $\mathcal{M}_E$

- CP – map $\qquad T_t : \mathcal{M} \to \mathcal{M}$

$\qquad \Big\downarrow \; \lim_{t \to \infty}$ asymptotic decoherence (Hepp, ..)

- Automorphism FAPP $\qquad \beta_t = T_t \restriction_{\mathcal{M}_1}$

Decoherence is nothing else as the application of the standard formalism of Q – Physics

Scenarios of Decoherence

Attractors of the Dynamics

$$\mathcal{Z}(\mathcal{M}) = \mathcal{M} \cap \mathcal{M}' = \mathbb{C}\mathbb{1} \Leftrightarrow S \text{ genuine (purely) Q – system}$$

- Pointer states $\qquad \mathcal{M}_1$ commutative $\approx L^\infty(\Omega)$

  $\{\beta_t\}_{t \in \mathbb{R}}$ trivial $\quad \beta_t = \mathbb{1}$

- SSR $\qquad \mathcal{M}_1$ non commutative

  $\mathcal{Z}(\mathcal{M}_1)$ non trivial $\Leftrightarrow$ superselection observables

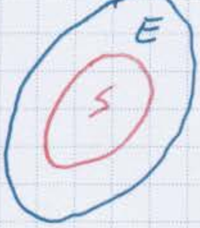
- From Q to q $\qquad \mathcal{M}_1$ non-commutative and

  $\mathcal{Z}(\mathcal{M}_1) = \mathbb{C}\mathbb{1}$

- Classical system $\qquad \mathcal{M}_1$ commutative

- Ergodicity $\qquad \mathcal{M}_1 = \mathbb{C}\mathbb{1} \qquad \mathcal{M}_1$ is trivial

The notion of Decoherence in the algebraic framework



Closed system $S \times E$ described by

- $\mathcal{N}$ von Neumann algebra
- continuous group of automorphism $\{\beta_t\}_{t \in \mathbb{R}}$
- S described by a v. N. subalgebra $\mathcal{M} \subseteq \mathcal{N}$
- $\exists$ a normal conditional expectation $E : \mathcal{N} \to \mathcal{M}$

$\Rightarrow$ Define the reduced dynamics as follows

$$T_t(x) = E \circ \beta_t(x) \qquad x \in \mathcal{M} \quad t \geq 0$$

This is the Heisenberg picture time evolution for an observer with experimental capabilities limited to $\mathcal{M}$.

T_t is in general irreversible (S is an open system)

Mathematical properties of T_t

1. $\{T_t\}_{t \geq 0}$ family of CP normal linear maps on $\mathcal{M}$
2. $T_t(\mathbb{1}) = \mathbb{1} \quad \forall t \geq 0 \qquad \|T_t x\| \leq \|x\|$
3. $t \to T_t(x)$ is (weak) continuous
4. In general $\{T_t\}_{t \geq 0}$ is not Markovian.

Remark In some physically relevant situations $\exists$ good approximation to describe T_t by a semigroup $\tilde{T}_t$

$\tilde{T}_t \Rightarrow$ Q dynamical semi group

Models

$$\mathcal{N} = \underset{S}{\mathcal{M}} \otimes \underset{E}{\mathcal{M}_0} \quad \text{acting on} \quad \underset{S}{\mathcal{H}} \otimes \underset{E}{\mathcal{H}_0}$$

$$\beta_t(x) = e^{itH} x \, e^{-itH}$$

$$H = H_1 \otimes \mathbb{1} + \mathbb{1} \otimes H_2 + H_{int}$$

ω reference state of E

$$\varphi \otimes \omega(x) = \varphi(E_\omega(x)) \quad x \in \mathcal{N}$$

φ state on $\mathcal{M}$

Time evolution

$$T_t(\varphi) = tr_2 \left[e^{-itH} (\varphi \otimes \omega) \, e^{itH} \right]$$

φ normal state on $\mathcal{M}$

tr_2 partial trace with respect to the degrees of freedom of E

T_t not reversible $\Rightarrow$ new phenomena can appear

- approach to equilibrium

Physical problems as motivation $\Rightarrow$ ... problems

Definition of Decoherence

The reduced dynamics $\{T_t\}_{t\geq 0}$ displays decoherence if

- $\exists$ a T_t invariant v. Neumann subalgebra $\mathcal{M}_1$ of $\mathcal{M}$ and a group of automorphisms $\{\alpha_t\}_{t\geq 0}$ on $\mathcal{M}_1$ such that
$$T_t\restriction_{\mathcal{M}_1} = \alpha_t \quad \text{for } t \geq 0$$ The effective dynamics
- $\exists$ a T_t invariant Banach subspace $\mathcal{M}_2$ of $\mathcal{M}$ such that
$$\mathcal{M} = \mathcal{M}_1 \oplus \mathcal{M}_2$$
with $$\lim_{t\to\infty} T_t(x) = 0 \quad \forall x \in \mathcal{M}_2$$

We require that $\mathcal{M}_1$ is a maximal v. N. subalgebra of $\mathcal{M}$ (not contained in any larger von Neumann subalgebra on which $\{T_t\}_{t\geq 0}$ extends to an automorphism)

$\mathcal{M}_1$ is called the algebra of effective observables

Physical interpretation

$x \in \mathcal{M} \qquad x = x_1 \oplus x_2 \qquad x_1 \in \mathcal{M}_1 \quad x_2 \in \mathcal{M}_2$

$$\lim_{t\to\infty} \varphi(T_t(x_2)) = 0 \quad \Rightarrow \mathcal{M}_2 = 0 \text{ FAPP}$$

After sufficiently long time S behaves effectively (FAPP) like a closed system described by $\mathcal{M}_1$ and $\{\alpha_t\}_{t\geq 0}$

Question: Is $\mathcal{M}_1$ unique?

Yes if T_t is a QDS

In certain cases the splitting $\mathcal{M} = \mathcal{M}_1 \oplus \mathcal{M}_2$ is always given by a conditional expectation

Proposition: $\{T_t\}_{t\geq 0}$ displays decoherence and $T_t\restriction_{\mathcal{M}_1} = \mathbb{1}_{\mathcal{M}_1}$ then there exists a normal conditional expectation E from $\mathcal{M}$ into $\mathcal{M}_1$

Problem: Given T_t under which conditions decoherence appears?

- Markovian case CMP 239 241–255 (2003) L O

Theorem: Let $\{T_t\}_{t\geq 0}$ be a continuous one parameter SG with a faithful normal state ω such that
$$\omega \circ T_t = \omega \quad \forall t \geq 0$$
and assume that

1. T_t $t\geq 0$ is strongly positive and unital
2. $\{\sigma_t^\omega\}_{t\in\mathbb{R}}$ the modular group corresponding to then $[T_t, \sigma_s^\omega] = 0 \quad \forall s \in \mathbb{R}$ and $t \geq 0$

Then $\{T_t\}_{t\geq 0}$ displays decoherence and $\exists$ a normal conditional expectation $E: \mathcal{M} \to \mathcal{M}_1$ such that $[T_t, E] = 0$ and $\omega \circ E = \omega$

Références sur la décohérence Wroclaw - Bielefeld 5. II. 2015

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