ABC in Quantum Mechanics

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The *Alphabet* of Quantum Mechanics – or really: *A*gainst *B*abylonian *C*onfusion in Quantum Mechanics

Credits and Contents:

Stimulation and encouragement from *Rutgers gang* & colleagues at ETH (*Graf* and *Hepp*); some motivation from recent experiments, in particular the ones of the *Haroche-Raimond* group; papers by *Bauer & Bernard*, *Maassen & Kümmerer*, and others; joint work with my former PhD student *Baptiste Schubnel*; (some joint efforts with *Ballesteros, Faupin, Fraas, Pickl, Schilling, Schubnel*)

1. *Introduction*
2. *QM systems and projective measurements*
3. *A simple model of a quantum system*
4. *Indirect measurements & "pointer observables"*
5. *Analogy to classical statistical mechanics*
6. *Effective dynamics of pointer observables*
7. *Conclusions*

1. Introduction – Questions to be Addressed

In our courses, we tend to describe quantum-mechanical systems as pairs of a Hilbert space, $\mathcal{H}$, and a propagator, $U(t,s)$, describing time-evolution. Unfortunately, these data encode almost *no invariant structure* (beyond spectral properties of $U(t,s)$) and give the erroneous impression that quantum theory might be deterministic. Thus, among *fundamental problems of quantum theory* are:

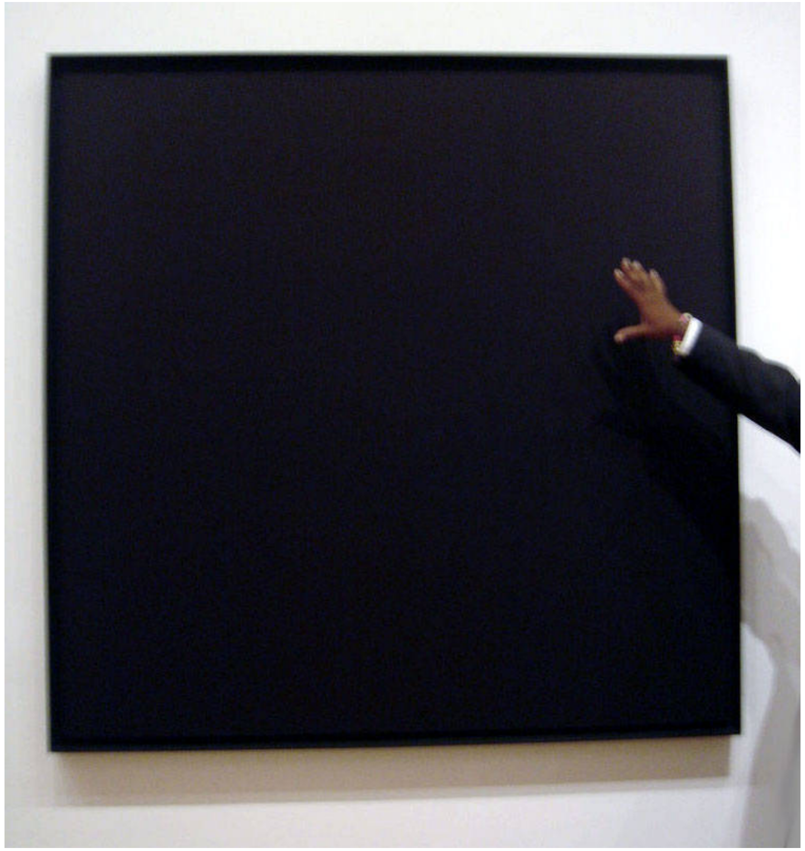
- What do we have to add to the usual formalism of quantum mechanics in order to arrive at a mathematical structure that (through "interpretation") can be given physical meaning, *independently of "observers"*?

Questions, ctd.

- Where does *intrinsic randomness* in quantum mechanics come from, given the deterministic character of the Schrödinger equation? In which way does it differ from classical randomness?
- Do we understand probabilistic phenomena in quantum mechanics, such as *"quantum jumps and tracks"* or *"Quantum Brownian Motion"*, etc.?
- What do we mean by a *"closed system"* in quantum mechanics, and why is this an important notion? How does one prepare a system in a prescribed state?

Etc.

*QM is QM-as-QM and everything else is everything else**



* "The one thing to say about art is that it is one thing. Art is art-as-art and everything else is everything else."

Ad Reinhardt

2. Quantum Systems & projective measurements

A simple-minded definition of quantum-mechanical systems:

A *closed* quantum system, *S*, is characterized by following choices:

(i) $(\mathcal{H}, U(t,s)), \mathbb{R} \ni t, s$ (U = unitary propagator)

(ii) a list, $\mathcal{O}_S = \{a_i\}_{i \in I_S}$, of bounded, selfadjoint operators on $\mathcal{H}$ representing *physical quantities/ potential properties* of *S* that can be measured in *"projective measurements"*; (*S* must be a *closed* system *chosen large enough* for the quantities represented by $a_i,\, i \in I_S$, to be measureable).

QM systems, ctd.

Choose fiducial time, t_0, and define

$$a(t) := U(t_0, t) a U(t, t_0), \qquad a \in \mathcal{O}_S,$$

to be the operator representing the pot. prop. corresp. to $a \in \mathcal{O}_S$ at time t; ➔ list of ops. $\mathcal{O}_S(t)$

Pot. properties, $a(s)$, measureable/observable at times $s \geq t$ generate a **alg.* $\mathcal{E}_{\geq t}$. We set (see also *Appendix*):

$$\mathcal{A}_S := \mathcal{E}_{>-\infty}, \qquad \mathcal{S}_S \text{ (states)}$$

(1)

$$B(\mathcal{H}) \supseteq \mathcal{A}_S \supseteq \mathcal{E}_{\geq t} \supseteq \mathcal{E}_{\geq s} \supseteq \mathcal{O}_S(s), \; s \geq t$$

$$\uparrow$$

$$\neq$$

🡐*Information Loss!*

Projective measurements

Let $\mathcal{O}_S \ni a = a^*$ be a potential property of S that is measured/observed around time t; (i.e., a becomes *"objective"/ "empirical"* around time t). Let $\alpha_1, \alpha_2, \ldots, \alpha_k$ be the eigenvalues of a , and consider the spect. decomposition

$$a(t) = \sum_{j=1}^{k} \alpha_j \Pi_j(t) \;, \qquad (2)$$

with $\{\Pi_j(t)\}_{j=1,\ldots,k}$ the spectral projections. Let ρ be the state of S right before a is measured/observed. Then (as one would expect!)

$$\rho(b) = \sum_{j=1}^{k} \rho(\Pi_j(t) b \Pi_j(t)), \qquad \text{for all } b \in \mathcal{E}_{\geq t} \quad (3)$$

Projective measurements, ctd.

i.e., $\rho|_{\mathcal{E}_{\geq t}}$ is an *incoherent superposition (mixture) of eigenstates of* a . This is what is usually called a

*"projective (*or *von Neumann) measurement".*

Information Loss $\Rightarrow \rho|_{\mathcal{E}_{\geq t}}$ can be a *mixed state* (even if, as a state on $\mathcal{A}_S$, ρ may be a *pure* state).

It is important to note that the choice of time evolution $U(t,s), t, s \in \mathbb{R}$, and of a state, ρ , *determines* which potential property of *S* will first be observed (become empirical) after preparation of ρ!

Details of characterization of "projective measurements" are somewhat subtle; (see *Appendix*).

Projective measurements, ctd.

<u>*Axiom A*</u>

If a is measured around time t then a has a *value* $\in \{\alpha_1, \ldots, \alpha_k\}$ around time t.

The value α_j of a is observed with probability

$$p_j(t) = \rho(\Pi_j(t)) \qquad \text{(“Born Rule”)} \tag{4}$$

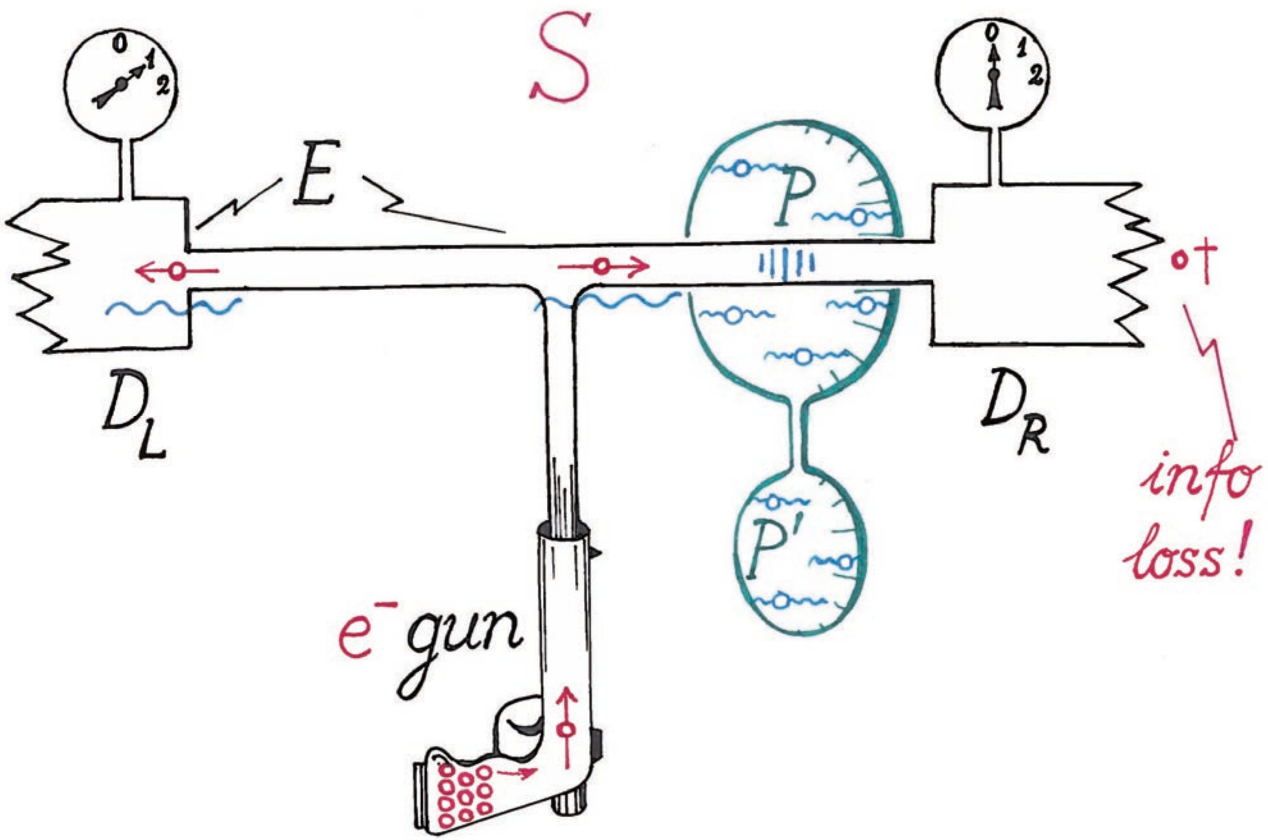
If α_j is observed around time t then the state

$$\rho_j^a(\cdot) := p_j(t)^{-1} \cdot \rho(\Pi_j(t)(\cdot)\Pi_j(t)) \text{ on } \mathcal{E}_{\geq t} \tag{5}$$

should be used for improved predictions of future after time t.

$\Rightarrow$ *Origin of indeterminism (randomness) in QM!*

3. A simple model of a quantum system – theory of indirect (and ND) measurements



Main features of model

$$S = (P \vee P') \vee E$$

At all times, only *one* e$^-$ in "T-channel" $\subset E$.

The only possible projective measnt. in S is to observe whether D_L or D_R has clicked. This measurement is represented by an operator, X, given by

$$X = \mathbf{1}_{\overline{P}} \otimes \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix}_E \qquad (6)$$

Thus, $\mathcal{O}_S = \{X\}$, in this example.

Question: What can one learn about S, in particular about P, by performing long sequences of successive measnts. of X?

Main features of model, ctd.

Up to N e^{--}'s bound by P create "Coulomb blockade" in arm of "T" reaching to D_R, thus discouraging e^- moving inside "T" to be scattered onto D_R. Simple qm calculations yield the probabilities, $p_L(n)$ and $p_R(n)$, for an e^- to be scattered onto D_L, D_R, respectively. Here n is the number of electrons bound by P. Clearly

$$p_L(n) + p_R(n) = 1 \qquad (7)$$

Note that n is an eigenvalue of the electron number operator $\mathcal{N}$ whose eigenvalues correspond to the number of electrons bound by P.

We now imagine that every τ seconds an e^- is injected into "T" and gets scattered onto one of the two detectors, thus resulting in a *measurement* of X. Let $\xi = \pm 1$ denote the eigenvalues of X (corresp. to click of D_L, D_R, resp.)

Main features of model, ctd.

A *measurement protocol* of length k consists of outcomes

$$\underline{\xi}^{(k)} = (\xi_1, \ldots, \xi_k),\ \ k = 1, 2, 3, \ldots \qquad (8)$$

of first k measurements of X. Choosing a state ρ of S enables one to associate a *probability* (or "frequency")

$$\mu_\rho(\xi_1, \ldots, \xi_k) \qquad (9)$$

with each measurement protocol, $\underline{\xi}^{(k)}$, of length k =1,2,3,...
(The unique qm formula for the probabilities (9) has first been found by Schwinger and rediscovered by Wigner,...)
One has that

$$\sum_{\xi_k} \mu_\rho(\xi_1, \ldots, \xi_{k-1}, \xi_k) = \mu_\rho(\xi_1, \ldots, \xi_{k-1})$$

Let Ξ denote the space of all ∞ long measurement protocols. Then μ_ρ defines a probability measure on Ξ.

4. Indirect measurements & pointer observables

We define the *frequency of clicks* of D_L in first k measnts. by

$$\nu_L(\underline{\xi}^{(k)}) := \frac{1}{k}\sharp\{j \in \{1, ..., k\} | \xi_j = 1\} \qquad (10)$$

and $\nu_L(\underline{\xi})$ the limit of $\nu_L(\underline{\xi}^{(k)})$, as $k \to \infty$. We expect:

$$\nu_L(\underline{\xi}) = p_L(n), \text{ where } n \text{ is an eigenvalue} \qquad (11)$$

of the number operator $\mathcal{N}$ introduced after (7).

Important observation:

There is a close analogy between *S*, with (7) through (11), and the *classical stat. mech. of Spin Chains (SC)*:

time of *S* $\leftrightarrow$ *1D space* of SC; $\mu_\rho \leftrightarrow$ Gibbs state of SC

Indirect measurements, ctd.

Limit, as (discrete) time $k \to \infty \leftrightarrow$ *TD limit* of SC

Appealing to *"equivalence of ensembles"* in TD limit, we expect that the *fluctuations* of $\nu_L(\underline{\xi}^{(k)})$ around one of the possible limiting values $p_L(n)$ tend to 0, as $k \to \infty$.

Precise statement

We define

$$\Xi_n(k;\underline{\epsilon}) := \{\underline{\xi} \in \Xi \mid |\nu_L(\underline{\xi}^{(k)}) - p_L(n)| < \epsilon_k\}$$

and (12)

$$\Xi(k;\underline{\epsilon}) := \cup_{n=1}^{N} \Xi_n(k;\underline{\epsilon}) \subset \Xi$$

where $\epsilon_k \searrow 0$, as $k \to \infty$.

Indirect measurements, ctd.

Theorem

Hyp. *(ND measnt.)*: Operator $\mathcal{N}(t) = \mathcal{N}$ const. in time t;
& suppose that $\Delta := min_{n_1 \neq n_2} | p_L(n_1) - p_L(n_2) | > 0$.
Then:

(A) If k is so large that $\epsilon_k < \Delta/2$ then the sets

$$\Xi_1(k, \underline{\epsilon}), \ldots, \Xi_N(k, \underline{\epsilon})$$

are all disjoint from one another; and

(B) $$\mu_\rho(\Xi(k; \underline{\epsilon})^c) = 1 - \mu_\rho(\Xi(k; \underline{\epsilon})) < \delta_k,$$

with $\delta_k \searrow 0$, as $k \to \infty$.
The sequences $\underline{\epsilon}$ and $\underline{\delta}$ can "usually" be chosen to be independent of the state ρ .

General insight

Think of a more general quantum system, S, with $\mathcal{O}_S = \{X\}$.

5.Analogy to classical statistical mechanics

Imagine that many successive projective measurements of X at times $t_1 < t_2 < \cdots < t_k$ are made, with outcomes $\underline{\xi}^{(k)} = (\xi_1, \ldots, \xi_k)$, where $\xi_j \in \mathrm{spec}(X)$ is the value of an an obs. X measured at time t_j, j = 1,…, k, k = 1,2,3,… .

Given any state ρ of S, one may predict the *frequency*, $\mu_\rho(\xi_1, \ldots, \xi_k)$, of the measnt. protocol $\underline{\xi}^{(k)} = (\xi_1, \ldots, \xi_k)$.

Let $\Xi_S \subset \Xi$ be the smallest set of arbitrarily long measnt. prot. s.t. $\mu_\rho(\Xi_S^c) = 0$, for all states, ρ, of S.

We define $\mathcal{O}_{S,\infty}$ to be the algebra of functions on Ξ_S *measureable at* ∞, which are also called *"pointer obs."*.

(If S is autonomous and there is complete decoherence then $\mathcal{O}_{S,\infty}$ consists of *all fus. invariant under right-shift*!)

Analogy to class. Stat. mech., ctd.

An example of a pointer observable is $\nu_L(\underline{\xi})$; (see Eq. (10)). The projections (characteristic functions) in the algebra $\mathcal{O}_{S,\infty}$ describe *events* in the system *S* detected with the help of very many successive measurements of *X.*

All states ρ / probability measures μ_ρ can be decomposed into a convex combination of mutually singular states/ prob. measures indexed by points in the spectrum of $\mathcal{O}_{S,\infty}$ ($= \mathcal{E}_\infty := \bigcap_t \mathcal{E}_{\geq t}$).

In our simple model, $\mathcal{O}_{S,\infty}$ consists of all functions on {1,…,N}, which is the spectrum of the number operator $\mathcal{N}$.

Many successive measurements of *X* (detector clicks) provide information about number of e^- bound by dot *P.*

6. Effective dynamics of pointer observables

Most indirect measurements are *not* non-demolition measurements. Usually $\mathcal{O}_{S,\infty}$ is empty. However, there are functions, $\vec{\nu}_k$, on Ξ_S depending on measurement protocols

$$\underline{\xi}^{(m,k)} = (\xi_{mk+1}, \ldots, \xi_{(m+1)k}), \;\; m = 0, 1, 2, \ldots$$

of length k, which approximate pointer observables – example $\nu_L(\underline{\xi}^{(m,k)})$ - whose level sets provide a decomposition of Ξ_S into disjoint subsets,

$$\Xi_{\sigma(m)}(m, k; \underline{\epsilon}), \;\; \sigma(m) \in \Sigma, \qquad (13)$$

with the property that the μ_ρ - measure of the complement of their union, $\bigcup_{\sigma \in \Sigma} \Xi_\sigma(m, k; \underline{\epsilon})$, is tiny.

Effective dynamics, ctd.

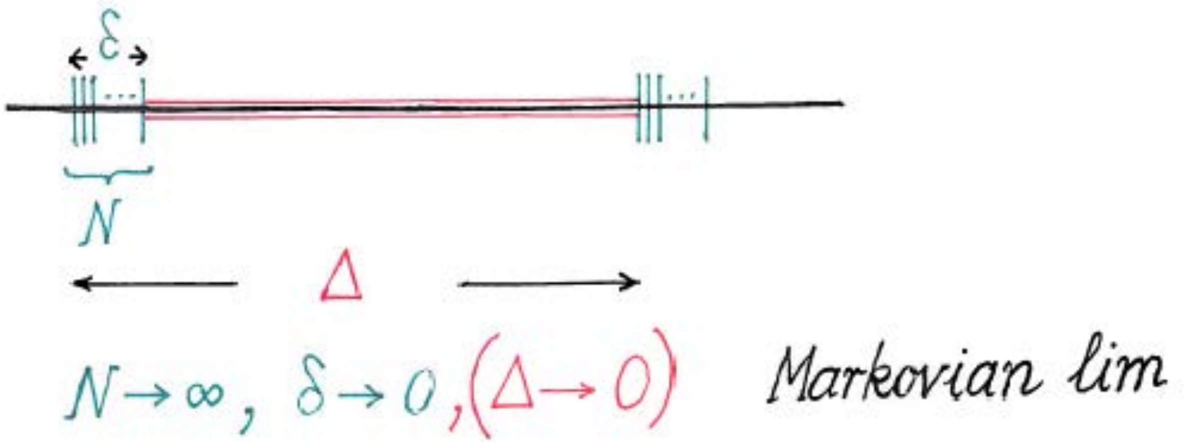
A measurement protocol of length *Mk, M* < ∞ not too large, then determines a *trajectory* (or *"history"*)

$$\{\sigma(m)\}_{m=1,2,3,...} \tag{14}$$

of *events*, $\sigma(m)$ $m = 1,2,...,M$.

Important problem

We would like to determine the effective stochastic dynamics that determines the *frequencies of event histories*. Here is a very simple model, where this can be done!



Effective dynamics, ctd.

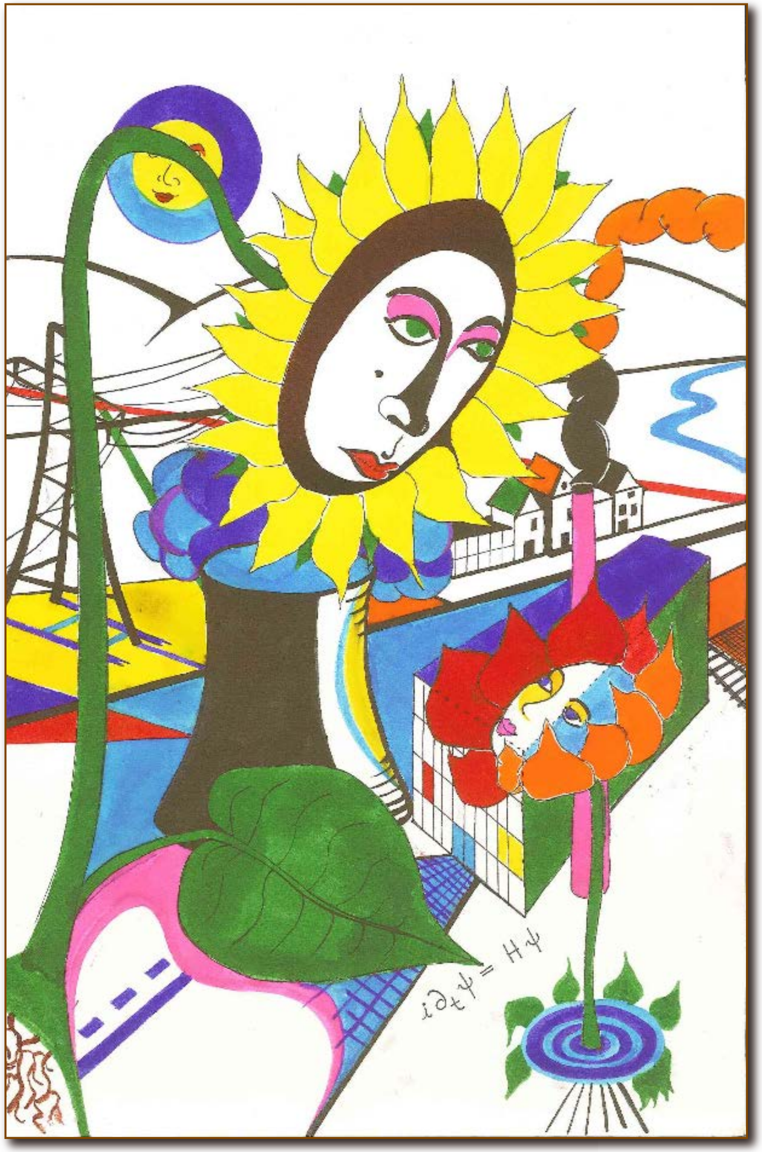
In this model, the state tends to purify after $N \gg 1$ projective measurements of X within a time interval of length δ and then evolves *unitarily* during a time interval of length $\Delta - \delta$. In the limiting regime, where first N tends to ∞ and then δ to 0, a *Markov chain* with transition function

$$p(n, n') := |[exp(-i\Delta H)]_{n,n'}|^2 \qquad (15)$$

describes the *effective dynamics of events* consisting of a jump process on the spectrum of the number operator $\mathcal{N}$.

There is a more interesting limiting regime that can be analyzed with the help of Kurtz' stochastic Trotter product formula. But we don't have time to describe it here.

7. Conclusions



"In all my films, I have been faithful to these suspension points in the conclusions. Besides, I have never written the word 'END' on the screen."
(Federico Fellini)

"Everyone wants to understand art (physics). Why don't we try to understand the song of a bird? Why do we love the night, the flowers, everything around us, without trying to understand them? But in the case of a painting (result in physics), people think they have to understand." (Pablo Picasso)

Thank you for listening!

Appendix to Sect. 2 (Quantum Systems…)

Recall definitions of $a(t)$, $\mathcal{O}_S$ and $\mathcal{O}_S(t)$. Potential properties measureable/obs. at times $\geq t$ generate *W*-alg.*

$$\mathcal{E}_{\geq t} := \langle \sum \prod_i a_i(t_i) | a_i \in \mathcal{O}_S, t_i \geq t \rangle^- \qquad \text{(A1)}$$

$$\mathcal{A}_S := \mathcal{E}_{>-\infty}, \qquad \mathcal{S}_S \text{ (states)}$$

(A2)

$$B(\mathcal{H}) \supseteq \mathcal{A}_S \supseteq \mathcal{E}_{\geq t} \supseteq \mathcal{E}_{\geq s} \supseteq \mathcal{O}_S(s), \; s \geq t$$

$\neq$ ← *Information Loss!*

Appendix, ctd.

Define

$$\tau_s(a(t)) := a(s+t) \quad , \qquad \text{(A3)}$$

so that

$$\tau_s : \mathcal{E}_{\geq t} \to \mathcal{E}_{\geq (t+s)}$$

τ_s is a *endom; τ_s *not* a *autom $\Leftrightarrow$ *information loss*

$\Rightarrow$ *entanglement!*

Some fundamental questions to be answered:

(1) What is meant by a "measurement" of $a \in \mathcal{O}_S$? Around which time does it take place ? A measurement of a ought to result in " a having a value", i.e., become an "empirical/objective property" of *S*

$\Leftrightarrow$ state on $\mathcal{E}_{\geq t} \simeq$ incoherent mixture of eigenstates of $a(t)$,

for some time *t*.

Appendix, ctd.

(2) Given a state of *S*, does *QM* *predict* which $a \in \mathcal{O}_S$ will be measured first; what does *QM* predict about the outcome of measnt. of a ? In which way is *QM* intrinsically *indeterministic*? Why does a meant. a of have a *random outcome*?

Projective measurements

Consider

$$\mathcal{O}_S \ni a = a^* \quad \text{with eigenvalues} \quad \alpha_1, \alpha_2, \ldots, \alpha_k,$$

$$a(t) = \sum_{j=1}^{k} \alpha_j \Pi_j(t) \qquad \text{(A4)}$$

"Measurement/observation" of a around time *t*, or

Appendix, ctd.

$\Leftrightarrow a$ is an *"empirical/objective property"* of *S* around time t :

$$\rho(b) = \sum_{j=1}^{k} \rho(\Pi_j(t) b \Pi_j(t)), \qquad \text{for all } b \in \mathcal{E}_{\geq t} \quad \text{(A5)}$$

where ρ is the state of *S* right before measnt. of a , i.e.,

$$\rho = \rho_t := \rho|_{\mathcal{E}_{\geq t}} \qquad \text{(A6)}$$

Information Loss $\Rightarrow \rho = \rho_t$ is usually a mixed state on $\mathcal{E}_{\geq t}$ even if the initial state of *S* has been pure, as a state on $\mathcal{A}_S$!

Appendix, ctd.

Suppose, for simplicity, that $\mathcal{E}_{\geq t}$ is isomorphic to some $B(\mathcal{H}_t)$, (i.e., $\mathcal{E}_{\geq t}$ is of type I_∞). Then

$$\text{Eq. (5)} \quad \Leftrightarrow \quad [a(t), P_t] = 0, \qquad \text{(A7)}$$

where P_t is the density matrix on $\mathcal{E}_{\geq t}$ corresp. to ρ_t

Definition

$a \in \mathcal{O}_S$ is measured/observed around time $t \Leftrightarrow a$ is an *"empirical/objective prop."* of *S* around time t iff

$$a(t)|_{RangeP_t} \approx_t F(P_t) \cdot z, \text{ for some bd. fu. } F, \qquad \text{(A8)}$$

and some z in the center of $\mathcal{E}_{\geq t}$.

(More generally, $a(t)$ belongs to the *"center of the centralizer of the state* ρ_t*"*; use of **Tomita-Takesaki th**. !)

Appendix, ctd.

Axiom A

If a is measured (i.e., an empirical/objective prop. of *S*) around time t then a has a *value* $\in \{\alpha_1, \ldots, \alpha_k\}$ around time t.

The value α_j of a is observed w. probability

$$p_j(t) = \rho(\Pi_j(t))$$

If α_j is observed around time t then the state

$$\rho_j^a(\cdot) := p_j(t)^{-1} \cdot \rho(\Pi_j(t)(\cdot)\Pi_j(t)) \text{ on } \mathcal{E}_{\geq t} \qquad \text{(A9)}$$

should be used for improved predictions of future after time t.

Appendix, ctd.: Summary

(1) Given the *initial state* of the system *S*, *time evolution*, $\{U(t,s)\}$, *determines* which pot. prop. $a \in \mathcal{O}_S$ will first become empirical (objective, measureable), and around which time.

(2) Measnt. of a_2 is *independent* of an *earlier* measnt. of a_1 iff a_2 becomes empirical/objective *after* time of measnt. of a_1, no matter what the outcome of measnt. of a_1 was, i.e., *for all states* $\rho_j^{a_1}(\cdot)$ *, j= 1, … , k,* with $\rho_j^{a_1}(\cdot)$ as in (8).

$\Rightarrow$ *Decoherence, "consistent histories".*

(3) Time of measurement*:* Time, t_*, of observation of a det. by minimizing the fu. $\| a(t)|_{Range P_t} - F(P_t) \cdot z \|$, for some fu. *F* and some *z.*

(4) General theory of repeated measurements ("POVM's"), etc.: Another time!

My Manifesto

I propose that, at all decent institutions of higher education, *one or two days per semester* will be declared to be

Days of Reflection and of Protest!

During these days, we will not teach or attend committee meetings, and there won't be any exercise classes. Instead, we will discuss some of the serious problems threatening our civilization, draft declarations and reach out to the media, with the aim to make it clear to all circles wielding power that we no longer accept – (to mention some examples among others):

My Manifesto, ctd.

- That internal tensions and conflicts in countries, such as the *Ukraine*, are “solved” by armed conflicts rather than by political dialogue and compromise;
- that innocent people are slaughtered in ugly civil wars and by terrorist activities, such as those in Syria and Iraq;
- that countries threaten other countries with warfare;
- that weapons are sold to (clans) in countries plagued by civil war or other forms of unrest and conflict;
- that religions are abused for purposes of power and suppression of people;
- that the dignity and the rights of women are abused and offended in the name of religion;

My Manifesto, ctd.

- that people are harassed or killed because of their race or faith;
- that nothing is done against the perversions of 21st Century Capitalism;
- that the resources of Planet Earth continue to be looted shamelessly.

These are *but some examples of numerous problems* threatening the survival of humankind in peace and dignity. –

Where is the *"Peace Movement"*, where are movements such as *"Occupy Wall Street"*, *"Survivre et Vivre"*? What is the *"Club of Rome"* doing? Why are the media silent about the activities of these and other groups? Why do academics not have a strong voice in political debates, anymore?

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Students and Academics raise your voices, arise!