Conference on Calculus of Variations in Lille - 3rd edition - July 4-6 2022

Report of Contributions

Contribution ID: **2** Type: **not specified**

Rigidity results for measurable sets

Monday, July 4, 2022 9:30 AM (1 hour)

Let $\Omega \subset \mathbb{R}^d$ be a set with finite Lebesgue measure such that, for a fixed radius $r > 0$, the Lebesgue measure of $\Omega \cap B_r(x)$ is equal to a positive constant when x varies in the essential boundary of Ω. We prove that Ω is a ball (or a finite union of equal balls) provided it satisfies a nondegeneracy condition, which holds in particular for any set of diameter larger than r which is either open and connected, or of finite perimeter and indecomposable. This is a joint work with Ilaria Fragalà.

Authors: BUCUR, Dorin (Université Savoie Mont Blanc); FRAGALÀ, Ilaria

Presenter: BUCUR, Dorin (Université Savoie Mont Blanc)

Contribution ID: **3** Type: **not specified**

Curvature functionals for surfaces with fractional regularity

Monday, July 4, 2022 11:00 AM (30 minutes)

The curvature functionals (such as the Willmore functional) are usually defined under $W^{2,2}$ regularity assumptions on the given surface. We will explain how this assumption could be relaxed to the fractional Sobolev setting $W^{1+s,2/s}$ for $s > 1/2$, and will discuss the related problematics of a fractional variational plate model in nonlinear elasticity.

Author: PAKZAD, Reza (University of Pittsburgh)

Presenter: PAKZAD, Reza (University of Pittsburgh)

Contribution ID: **4** Type: **not specified**

From local energy bounds to dimensional estimates in a reduced model for type-I superconductors

Monday, July 4, 2022 2:00 PM (1 hour)

In the limit of vanishing but moderate external magnetic field, we derived a few years ago together with S. Conti, F. Otto and S. Serfaty a branched transport problem from the full Ginzburg–Landau model. In this regime, the irrigated measure is the Lebesgue measure and, at least in a simplified 2d setting, it is possible to prove that the minimizer is a self-similar branching tree. In the regime of even smaller magnetic fields, a similar limit problem is expected but this time the irrigation of the Lebesgue measure is not imposed as a hard constraint but rather as a penalization. While an explicit computation of the minimizers seems here out of reach, I will present some ongoing project with G. De Philippis and B. Ruffini relating local energy bounds to dimensional estimates for the irrigated measure.

Authors: RUFFINI, Berardo; DE PHILIPPIS, Guido; GOLDMAN, Michael (Université Paris Cité)

Presenter: GOLDMAN, Michael (Université Paris Cité)

Contribution ID: **5** Type: **not specified**

Dividing a set in half

Monday, July 4, 2022 11:30 AM (1 hour)

In this talk I will consider the following problem of isoperimetric type:

Given a set E in $\mathbb{R}^d$ with finite volume, is it possible to find an hyperplane P that splits E in two parts with equal volume, and such that the area of the cut (that is, the intersection of P and E) is of the expected order, namely $(vol(E))^{1-1/d}$?

We can show that the answer is positive if the dimension d is 3 or higher, but, somewhat surprisingly, our proof breaks down completely in dimension $d = 2$, and we do not know what happens in this case.
(However we know that the answer is positive even for $d = 2$ if we allow cuts that are not exactly planar, but close to planar.)

This is a work in progress with Alan Chang (Princeton University).

Authors: CHANG, Alan; ALBERTI, Giovanni (Università di Pisa)

Presenter: ALBERTI, Giovanni (Università di Pisa)

Contribution ID: **6** Type: **not specified**

Tamped functions: A rearrangement in dimension 1

Monday, July 4, 2022 3:00 PM (30 minutes)

We define a new rearrangement, called rearrangement by tamping, for non-negative measurable functions defined on $\mathbb{R}_+$. This rearrangement has many properties in common with the well-known Schwarz non-increasing rearrangement such as the Pólya–Szegő inequality.
Contrary to the Schwarz rearrangement, the tamping also preserves the homogeneous Dirichlet boundary condition of a function. This presentation aims at presenting the construction of the rearrangement by tamping (with an algorithmic approach) and some recent developments around this idea.

Author: GODARD-CADILLAC, Ludovic (Nantes Université)

Presenter: GODARD-CADILLAC, Ludovic (Nantes Université)

Contribution ID: **7** Type: **not specified**

Exploiting convex duality in the calculus of variations

Wednesday, July 6, 2022 3:00 PM (30 minutes)

I will recall the classical theory of convex duality and explain how this can be used to obtain regularity statements in the study of minimisers of the problem
$\min_{u\in W^{1,p}(\Omega)} \int_\Omega F(x,\mathrm{D}u)\mathrm{d}x. In particular, I will comment on recent results obtained in collaboration with Cristiana de$
$valued integrands \mathrm{F} satisfying no upper growth condition as well as concerning integrands \mathrm{F} satisfying controlled duali$

Authors: DE FILIPPIS, Cristiana; KRISTENSEN, Jan; KOCH, Lukas (MPI for MiS, Leipzig)

Presenter: KOCH, Lukas (MPI for MiS, Leipzig)

Contribution ID: **8** Type: **not specified**

Controlling nonconvexity and nonlinearity in gradient flows: two methods and two model problems

Monday, July 4, 2022 4:30 PM (1 hour)

Together with Felix Otto, Richard Schubert, and other collaborators, we have developed two different energy-based methods to capture convergence rates and metastability of gradient flows. We will present the methods and their application to the two model problems that drove their development: the 1-d Cahn–Hilliard equation and the Mullins–Sekerka evolution. Both methods can be viewed as quantifying "how nonconvex"or "how nonlinear"a problem can be while still retaining the optimal convergence rates, i.e., the rates for the convex or linear problem. Our focus is on fairly large (ill-prepared) initial data.

Authors: OTTO, Felix (Max Planck Institute for Mathematics in the Sciences); WESTDICKENBERG, Maria G. (RWTH Aachen University); SCHUBERT, Richard

Presenter: WESTDICKENBERG, Maria G. (RWTH Aachen University)

Contribution ID: **9** Type: **not specified**

Derivation of surface tension of grain boundaries in polycrystals

Wednesday, July 6, 2022 9:00 AM (1 hour)

Inspired by a recent result of Lauteri and Luckhaus, we derive, via Gamma convergence, a surface tension model for polycrystals in dimension two. The starting point is a semi-discrete model accounting for the possibility of having crystal defects. The presence of defects is modelled by incompatible strain fields with quantised curl. In the limit as the lattice spacing tends to zero we obtain an energy for grain boundaries that depends on the relative angle of the orientations of the two neighbouring grains. The energy density is defined through an asymptotic cell problem formula. By means of the bounds obtained by Lauteri and Luckhaus we also show that the energy density exhibits a logarithmic behaviour for small angle grain boundaries in agreement with the classical Read and Shockley formula.
The talk is based on a paper in preparation in collaboration with Emanuele Spadaro.

Authors: GARRONI, Adriana (Sapienza University of Rome); SPADARO, Emanuele

Presenter: GARRONI, Adriana (Sapienza University of Rome)

Contribution ID: **10** Type: **not specified**

Discrete approximation of the Griffith functional by adaptative finite elements

Wednesday, July 6, 2022 10:00 AM (30 minutes)

This joint work with Jean-François Babadjian is devoted to showing a discrete adaptative finite element approximation result for the isotropic two-dimensional Griffith energy arising in fracture mechanics. The problem is addressed in the geometric measure theoretic framework of generalized special functions of bounded deformation which corresponds to the natural energy space for this functional. It is proved to be approximated in the sense of Γ-convergence by a sequence of integral functionals defined on continuous piecewise affine functions. The main feature of this result is that the mesh is part of the unknown of the problem, and it gives enough flexibility to recover isotropic surface energies.

Authors: BABADJIAN, Jean-François; BONHOMME, Élise (Université Paris-Saclay)

Presenter: BONHOMME, Élise (Université Paris-Saclay)

Contribution ID: **11** Type: **not specified**

A free discontinuity problem in fluid mechanics

Monday, July 4, 2022 4:00 PM (30 minutes)

We consider an incompressible Stokes fluid contained in a box B that flows around an obstacle $K \subset B$ with a Navier boundary condition on ∂K. I will present existence and partial regularity results for the minimization of the drag of K among all obstacles of given volume.

Author: NAHON, Mickaël (Université Savoie Mont Blanc)

Presenter: NAHON, Mickaël (Université Savoie Mont Blanc)

Contribution ID: **12** Type: **not specified**

Frobenius theorem for weak submanifolds

Tuesday, July 5, 2022 11:30 AM (1 hour)

The question of producing a foliation of the n-dimensional Euclidean space with k-dimensional submanifolds which are tangent to a prescribed k-dimensional simple vectorfield is part of the celebrated Frobenius theorem: a decomposition in smooth submanifolds tangent to a given vectorfield is feasible (and then the vectorfield itself is said to be integrable) if and only if the vectorfield is involutive. In this seminar I will summarize the results obtained in collaboration with G. Alberti, A. Merlo and E. Stepanov when the smooth submanifolds are replaced by weaker objects, such as integral or normal currents or even contact sets with "some"boundary regularity. I will also provide Lusin-type counterexamples to the Frobenius property for rectifiable currents. Finally, I will try to highlight the connection between involutivity/integrability à la Frobenius and Carnot–Carathéodory spaces and how to apply our techniques in this framework.

Authors: MERLO, Andrea; MASSACCESI, Annalisa (University of Padova); ALBERTI, Giovanni (Università di Pisa)

Presenter: MASSACCESI, Annalisa (University of Padova)

Contribution ID: **13** Type: **not specified**

Vortex sheet solutions for the Ginzburg–Landau system in cylinders

Tuesday, July 5, 2022 2:00 PM (1 hour)

We consider the Ginzburg-Landau energy E_ϵ for $\mathbb{R}^M$-valued maps defined in a cylinder $B^N \times (0,1)^n$ satisfying the degree-one vortex boundary condition on $\partial B^N \times (0,1)^n$ in dimensions $M \geq N \geq 2$ and $n \geq 1$. The aim is to study the radial symmetry of global minimizers of this variational problem. We prove the following: if $N \geq 7$, then for every $\epsilon > 0$, there exists a unique global minimizer which is given by the non-escaping radially symmetric vortex sheet solution $u_\epsilon(x,z) = (f_\epsilon(|x|)\frac{x}{|x|}, 0_{\mathbb{R}^{M-N}}), \forall x \in B^N$ that is invariant in $z \in (0,1)^n$. If $2 \leq N \leq 6$ and $M \geq N+1$, the following dichotomy occurs between escaping and non-escaping solutions: there exists $\epsilon_N > 0$ such that

• if $\epsilon \in (0, \epsilon_N)$, then every global minimizer is an escaping radially symmetric vortex sheet solution of the form $R\tilde{u}_\epsilon$ where $\tilde{u}_\epsilon(x,z) = (\tilde{f}_\epsilon(|x|)\frac{x}{|x|}, 0_{\mathbb{R}^{M-N-1}}, g_\epsilon(|x|))$ is invariant in z-direction with $g_\epsilon > 0$ in $(0,1)$ and $R \in O(M)$ is an orthogonal transformation keeping invariant the space $\mathbb{R}^N \times \{0_{\mathbb{R}^{M-N}}\}$;

• if $\epsilon \geq \epsilon_N$, then the non-escaping radially symmetric vortex sheet solution $u_\epsilon(x,z) = (f_\epsilon(|x|)\frac{x}{|x|}, 0_{\mathbb{R}^{M-N}})$, $\forall x \in B^N, z \in (0,1)^n$ is the unique global minimizer; moreover, there are no bounded escaping solutions in this case.

We also discuss the problem of vortex sheet $\mathbb{S}^{M-1}$-valued harmonic maps.

Presenter: IGNAT, Radu (Université Toulouse III - Paul Sabatier)

Contribution ID: **14** Type: **not specified**

Fractional Allen–Cahn systems with multi-well potential and nonlocal minimal partitions

Tuesday, July 5, 2022 3:00 PM (30 minutes)

The aim of this talk is to present results on the asymptotic analysis of a fractional version of the vectorial Allen–Cahn equation with multiple-well in arbitrary dimension. In contrast to usual Allen–Cahn equations, the Laplace operator is replaced by the fractional Laplacian as defined in Fourier space. Our results concern the singular limit $\varepsilon \to 0$ and show that arbitrary solutions with uniformly bounded energy converge both in the energetic and geometric sense to nonlocal minimal partitions in Ω. The notion of nonlocal minimal partition corresponds to the stationary version of the nonlocal minimizing clusters introduced by M. Colombo & F. Maggi (2017) and A. Cesaroni & M. Novaga (2020), and generalizing the nonlocal minimal surfaces of L. Caffarelli, J.M. Roquejoffre, & O. Savin (2010).

Author: GABARD, Thomas (Université Paris-Est Créteil)

Presenter: GABARD, Thomas (Université Paris-Est Créteil)

Contribution ID: **15** Type: **not specified**

A Γ-convergence result for non-self dual U(1)-Yang–Mills–Higgs energies of Ginzburg–Landau type

Tuesday, July 5, 2022 4:00 PM (30 minutes)

Let $E \to M$ be a Hermitian complex line bundle with structure group $\mathrm{U}(1)$ over a closed smooth orientable connected Riemannian manifold M. Fix a smooth metric connection D_0 on E and consider, for $\varepsilon > 0$, the non-self dual $\mathrm{U}(1)$-Yang–Mills–Higgs energies

$$G_\varepsilon(u_\varepsilon, A_\varepsilon) := \int_M \frac{1}{2}|\mathrm{D}_{A_\varepsilon} u_\varepsilon|^2 + \frac{1}{4\varepsilon^2}\left(1 - |u_\varepsilon|^2\right)^2 + \frac{1}{2}|F_{A_\varepsilon}|^2 \,\mathrm{vol}_g,$$

where $(u, A) \in W^{1,2}(M, E) \times W^{1,2}(M, \mathrm{T}^*M)$, $\mathrm{D}_A := \mathrm{D}_0 - iA$, and F_A denotes the curvature form of D_A. The functionals G_ε arise as natural generalisation of the usual Ginzburg–Landau energy on domains of $\mathbb{R}^n$.

The aim of the talk is to illustrate the following Γ-convergence result, obtained in collaboration with G. Canevari and G. Orlandi (Università di Verona): as $\varepsilon \to 0$, the rescaled functionals $\frac{G_\varepsilon}{|\log \varepsilon|}$ Γ-converge, in the flat topology of Jacobians, to (π times) the codimension two area functional.

Author: DIPASQUALE, Federico Luigi (University of Verona)

Presenter: DIPASQUALE, Federico Luigi (University of Verona)

Contribution ID: **16** Type: **not specified**

A Lagrangian description of entropy solutions of the eikonal equation

Tuesday, July 5, 2022 4:30 PM (1 hour)

We consider the behaviour as $\varepsilon \to 0^+$ of the following family of functionals introduced by P. Aviles and Y. Giga:
$\mathrm{F}_\varepsilon(u,\Omega) := \int_\Omega \left(\varepsilon|\nabla^2 u|^2 + \frac{1}{\varepsilon}\left|1-|\nabla u|^2\right|^2\right)dx,$ where $\Omega \subset \mathbb{R}^2 . Functions with equi-bounded energy as \varepsilon \to$ 0 are pre-compact in $L^1(\Omega)$ and all the limits belong to the class of the so called 'entropy solutions' of the eikonal equation $|\nabla u| = 1$ in Ω.

We introduce a Lagrangian description of these solutions and we investigate their fine properties. As a corollary we obtain that if Ω is an ellipse, then minimizers of $F_\varepsilon(\cdot,\Omega)$ in the space $\left\{u \in W^{2,2}(\Omega) : u = 0 \text{ and } \frac{\partial u}{\partial n} = -1 \text{ at } \partial\Omega\right\} converge to \mathrm{u}_* := \mathrm{dist}(\cdot,\partial\Omega)$.

Moreover we get a sharp quantitative version of the result in Jabin–Otto–Perthame (2002), stating that the only bounded simply connected domain Ω admitting zero energy states with Dirichlet boundary conditions is the disk.

Part of the work is done in collaboration with Xavier Lamy.

Authors: MARCONI, Elio (EPFL); LAMY, Xavier (Université Toulouse III - Paul Sabatier)

Presenter: MARCONI, Elio (EPFL)

Contribution ID: **17** Type: **not specified**

Variational models with data driven regularisation for inverse problems

Tuesday, July 5, 2022 9:00 AM (1 hour)

Inverse problems are about the reconstruction of an unknown physical quantity from indirect measurements. Most inverse problems of interest are ill-posed and require appropriate mathematical treatment for recovering meaningful solutions. Variational regularization is one of the main mechanisms to turn inverse problems into well-posed ones by adding prior information about the unknown quantity to the problem, often in the form of assumed regularity of solutions. Classically, such regularization approaches are handcrafted. Examples include Tikhonov regularization, the total variation and several sparsity-promoting regularizers such as the L1 norm of Wavelet coefficients of the solution. While such handcrafted approaches deliver mathematically and computationally robust solutions to inverse problems, providing a universal approach to their solution, they are also limited by our ability to model solution properties and to realise these regularization approaches computationally. Recently, a new paradigm has been introduced to the regularization of inverse problems, which derives regularization approaches for inverse problems in a data driven way. Here, regularization is not mathematically modelled in the classical sense, but modelled by highly over-parametrised models, typically deep neural networks, that are adapted to the inverse problems at hand by appropriately selected (and usually plenty of) training data. In this talk, I will review some machine learning based regularization techniques, present some work on unsupervised and deeply learned convex regularisers and their application to image reconstruction from tomographic and blurred measurements, and finish by discussing some open mathematical problems.

Author: SCHÖNLIEB, Carola-Bibiane (University of Cambridge)

Presenter: SCHÖNLIEB, Carola-Bibiane (University of Cambridge)

Contribution ID: **18** Type: **not specified**

Strong convergence results for total variation regularized inverse problems in a low noise regime

Tuesday, July 5, 2022 11:00 AM (30 minutes)

We consider an imaging inverse problem which consists in recovering a “simple”function from a set of noisy linear measurements. Our approach is variationnal: we produce an approximation of the unknown function by solving a least squares problem with a total variation regularization term. Our aim is to prove this approximation converges to the unknown function in a low noise regime. Specifically, we are interested in a convergence of “geometric”type: convergence of the level sets, of the number of non-trivial level sets, etc. This result is closely related to stability questions for solutions of the prescribed curvature problem. This is a joint work with Vincent Duval and Yohann De Castro.

Authors: PETIT, Romain (Université Paris Dauphine); DUVAL, Vincent; DE CASTRO, Yohann

Presenter: PETIT, Romain (Université Paris Dauphine)

Contribution ID: **19** Type: **not specified**

Isoperimetric problems on periodic lattices

Wednesday, July 6, 2022 11:00 AM (1 hour)

Motivated by the crystallization issue, we focus on the minimization of Heitman–Radin potential energies for configurations of N particles in a periodic lattice, and in particular on the connection with anisotropic isoperimetric problems in the suitably rescaled limit as $N \to \infty$. Besides identifying the asymptotic Wulff shapes through Gamma-convergence, we obtain fluctuation estimates for quasiminimizers that include the well-known $N^{3/4}$ conjecture for minimizers in planar lattices. Our technique combines the sharp quantitative Wulff inequality with a notion of quantitative closeness between discrete and continuum problems. These results have been obtained in collaborations with Marco Cicalese and Leonard Kreutz.

Authors: LEONARDI, Gian Paolo (University of Trento); KREUTZ, Leonard; CICALESE, Marco

Presenter: LEONARDI, Gian Paolo (University of Trento)

Contribution ID: **20** Type: **not specified**

Smooth Bilevel Programming for Sparse Regularization

Wednesday, July 6, 2022 1:30 PM (1 hour)

Iteratively reweighted least square (IRLS) is a popular approach to solve sparsity-enforcing regression problems in machine learning. State of the art approaches are more efficient but typically rely on specific coordinate pruning schemes. In this work, we show how a surprisingly simple reparametrization of IRLS, coupled with a bilevel resolution (instead of an alternating scheme) is able to achieve top performances on a wide range of sparsity (such as Lasso, group Lasso and trace norm regularizations), regularization strength (including hard constraints), and design matrices (ranging from correlated designs to differential operators). Similarly to IRLS, our method only involves linear systems resolutions, but in sharp contrast, corresponds to the minimization of a smooth function. Despite being non-convex, we show that there is no spurious minima and that saddle points are "ridable", so that there always exists a descent direction. We thus advocate for the use of a BFGS quasi-Newton solver, which makes our approach simple, robust and efficient. At the end of the talk, I will discuss the associated gradient flows as well as the connection with Hessian geometry and mirror descent. This is a joint work with Clarice Poon (Bath Univ.). The corresponding article is available: https://arxiv.org/abs/2106.01429. A python notebook introducing the method is available at this address: https://nbviewer.org/github/gpeyre/numerical-tours/blob/master/python/optim_7_noncvx_pro.ipynb

Authors: POON, Clarice; PEYRÉ, Gabriel (École Normale Supérieure)

Presenter: PEYRÉ, Gabriel (École Normale Supérieure)

Contribution ID: **21** Type: **not specified**

Uniform Convergence Rates for Lipschitz Learning Down to Graph Connectivity

Tuesday, July 5, 2022 10:00 AM (30 minutes)

Discrete to continuum convergence results for graph-based learning have seen an increased interest in the last years. In particular, the connections between discrete machine learning and continuum partial differential equations or variational problems, lead to new insights and better algorithms.

This talk considers Lipschitz learning —which is the limit of p-Laplacian learning for p to infinity — and introduces new proof strategies for the discrete to continuum limit. Our framework provides a convergence result in a sparse graph regime and additionally yields convergence rates. Employing a homogenized non-local operator with a much larger bandwidth allows us to extend uniform convergence rates to any graph length scale strictly above graph connectivity. We will sketch the ideas of the proof and indicate how the approach may be used in other problems, like spectral convergence of the graph Laplacian.

Author: ROITH, Tim (Friedrich-Alexander-Universität Erlangen-Nürnberg)

Presenter: ROITH, Tim (Friedrich-Alexander-Universität Erlangen-Nürnberg)

Contribution ID: **22** Type: **not specified**

A variational approach to data-driven problems in fluid mechanics

Wednesday, July 6, 2022 2:30 PM (30 minutes)

In this talk, we discuss a data-driven approach to viscous fluid mechanics. Typically, in order to describe the behaviour of fluids, two different kinds of modelling assumptions are used. On the one hand, there are first principles like the balance of forces or the incompressibility condition. On the other hand there are material specific constitutive laws that describe the relation between the strain and the viscous stress of the fluid. Combining both, one obtains the partial differential equations of fluid mechanics like the Stokes or Navier–Stokes equations. The constitutive laws are obtained by fitting a law from a certain class (for example linear, power law, etc.) to experimental data. This leads to modelling errors.

Instead of using a constitutive relation, we introduce a data-driven formulation that has previously been examined in the context of solid mechanics and directly draws on material data. This leads to a variational solution concept, that incorporates differential constraints coming from first principles and produces fields that are optimal in terms of closeness to the data. In order to derive this formulation we recast the differential constraints of fluid mechanics in the language of constant-rank differential operators. We show a Γ-convergence result for the functionals arising in the data-driven fluid mechanical problem which implies that the method is well-adapted to the convergence of experimental data through increasing experimental accuracy.
Furthermore, we will see that the data-driven solutions are consistent with PDE solutions if the data are given by a constitutive law and discuss advantages of this new solution concept.

Authors: LIENSTROMBERG, Christina (Universität Stuttgart); SCHUBERT, Richard (University of Bonn); SCHIFFER, Stefan (Bonn University)

Presenter: SCHUBERT, Richard (University of Bonn)

Contribution ID: **23** Type: **not specified**

Convergence rate of general entropic optimal transport costs

Wednesday, July 6, 2022 4:00 PM (1 hour)

Entropic optimal transport (EOT) has received a lot of attention in recent years because it is related to efficient solvers. In this talk, I will address the rate of convergence of the value to the optimal transport cost as the noise parameter vanishes. This is a joint work with Paul Pegon and Luca Tamanini.

Authors: CARLIER, Guillaume (Université Paris Dauphine); TAMANINI, Luca; PEGON, Paul (Université Paris Dauphine)

Presenter: CARLIER, Guillaume (Université Paris Dauphine)