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A Lagrangian description of entropy solutions of the eikonal equation

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We consider the behaviour as $\varepsilon \to 0^+$ of the following family of functionals introduced by P. Aviles and Y. Giga:
$\mathrm{F}_\varepsilon(u,\Omega) := \int_\Omega \left(\varepsilon|\nabla^2 u|^2 + \frac{1}{\varepsilon}\left|1-|\nabla u|^2\right|^2\right) dx,$ where $\Omega \subset \mathbb{R}^2. Functions with equi-bounded energy as \varepsilon \to 0$ are pre-compact in $L^1(\Omega)$ and all the limits belong to the class of the so called 'entropy solutions' of the eikonal equation $|\nabla u| = 1$ in Ω.

We introduce a Lagrangian description of these solutions and we investigate their fine properties. As a corollary we obtain that if Ω is an ellipse, then minimizers of $F_\varepsilon(\cdot,\Omega)$ in the space $\left\{u \in W^{2,2}(\Omega) : u = 0 \text{ and } \frac{\partial u}{\partial n} = -1 \text{ at } \partial\Omega\right\} converge to \mathrm{u}_*$ $\mathrm{dist}(\cdot,\partial\Omega)$.

Moreover we get a sharp quantitative version of the result in Jabin–Otto–Perthame (2002), stating that the only bounded simply connected domain Ω admitting zero energy states with Dirichlet boundary conditions is the disk.

Part of the work is done in collaboration with Xavier Lamy.

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