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A Lagrangian description of entropy solutions of the eikonal equation

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We consider the behaviour as $\varepsilon\to 0^+$ of the following family of functionals introduced by P. Aviles and Y. Giga:

 $F_{\varepsilon}(u,\Omega) := \int_{\Omega} \left(\varepsilon |\nabla^2 u|^2 + \frac{1}{\varepsilon} \left| 1 - |\nabla u|^2 \right|^2 \right) dx, \quad \text{where } \Omega \subset \mathbb{R}^2. Functions with equi-bounded energy as } \varepsilon \to 0 \text{ are pre-compact in } L^1(\Omega) \text{ and all the limits belong to the class of the so called 'entropy solutions' of the eikonal equation } |\nabla u| = 1 \text{ in } \Omega.$

We introduce a Lagrangian description of these solutions and we investigate their fine properties. As a corollary we obtain that if Ω is an ellipse, then minimizers of $F_{\varepsilon}(\cdot, \Omega)$ in the space $\{u \in W^{2,2}(\Omega) : u = 0 \text{ and } \frac{\partial u}{\partial n} = -1 \text{ at } \partial\Omega\}$ convergetou_{*} dist $(\cdot, \partial\Omega)$.

Moreover we get a sharp quantitative version of the result in Jabin–Otto–Perthame (2002), stating that the only bounded simply connected domain Ω admitting zero energy states with Dirichlet boundary conditions is the disk.

Part of the work is done in collaboration with Xavier Lamy.

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