

Vortex sheet solutions for the Ginzburg–Landau system in cylinders

Tuesday, July 5, 2022 2:00 PM (1 hour)

We consider the Ginzburg-Landau energy E_ϵ for \mathbb{R}^M -valued maps defined in a cylinder $B^N \times (0, 1)^n$ satisfying the degree-one vortex boundary condition on $\partial B^N \times (0, 1)^n$ in dimensions $M \geq N \geq 2$ and $n \geq 1$. The aim is to study the radial symmetry of global minimizers of this variational problem. We prove the following: if $N \geq 7$, then for every $\epsilon > 0$, there exists a unique global minimizer which is given by the non-escaping radially symmetric vortex sheet solution $u_\epsilon(x, z) = (f_\epsilon(|x|) \frac{x}{|x|}, 0_{\mathbb{R}^{M-N}})$, $\forall x \in B^N$ that is invariant in $z \in (0, 1)^n$. If $2 \leq N \leq 6$ and $M \geq N + 1$, the following dichotomy occurs between escaping and non-escaping solutions: there exists $\epsilon_N > 0$ such that

- if $\epsilon \in (0, \epsilon_N)$, then every global minimizer is an escaping radially symmetric vortex sheet solution of the form $R\tilde{u}_\epsilon$ where $\tilde{u}_\epsilon(x, z) = (\tilde{f}_\epsilon(|x|) \frac{x}{|x|}, 0_{\mathbb{R}^{M-N-1}}, g_\epsilon(|x|))$ is invariant in z -direction with $g_\epsilon > 0$ in $(0, 1)$ and $R \in O(M)$ is an orthogonal transformation keeping invariant the space $\mathbb{R}^N \times \{0_{\mathbb{R}^{M-N}}\}$;
- if $\epsilon \geq \epsilon_N$, then the non-escaping radially symmetric vortex sheet solution $u_\epsilon(x, z) = (f_\epsilon(|x|) \frac{x}{|x|}, 0_{\mathbb{R}^{M-N}})$, $\forall x \in B^N, z \in (0, 1)^n$ is the unique global minimizer; moreover, there are no bounded escaping solutions in this case.

We also discuss the problem of vortex sheet \mathbb{S}^{M-1} -valued harmonic maps.

Presenter: IGNAT, Radu (Université Toulouse III - Paul Sabatier)