Polynomial preserving processes and discrete-tenor interest rate models

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Based on joint work with K. Glau and M. Keller-Ressel

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Introduction and motivation

- Developing interest rate models that ensure on the one side nonnegative interest rates and/or spreads, and on the other side analytical pricing of *both* caplets and swaptions and enough flexibility for calibration, is a challenging problem.

Recall:

- Caplet with strike K and maturity T_k, settled in arrears:

$$\mathrm{Cpl}_t^k = B(t, T_{k+1})\delta_k \mathbb{E}_{\mathbb{P}_{T_{k+1}}}\left[(L(T_k, T_k) - K)^+ \,|\mathcal{F}_t\right]$$

- Swaption with swap rate S and exercise date T_k – option to enter an interest rate swap:

$$\begin{aligned}\mathrm{Swp}_t &= B(t, T_k)\mathbb{E}_{\mathbb{P}_{T_k}}\left[\left(\sum_{i=k+1}^{n} \delta_i L(T_k, T_i) B(T_k, T_i) - \sum_{i=k+1}^{n} \delta_i S B(T_k, T_i)\right)^+ \middle| \mathcal{F}_t\right] \\ &= B(t, T_k)\mathbb{E}_{\mathbb{P}_{T_k}}\left[\left(1 - \sum_{i=k+1}^{n} c_i B(T_k, T_i)\right)^+ \middle| \mathcal{F}_t\right],\end{aligned}$$

where $c_i = \delta_i S$, for $k + 1 \leq i < n$ and $c_n = 1 + \delta_n S$.

Interest rate models based on polynomial preserving processes

Polynomial preserving processes seem to be very suitable to tackle these issues...

Seminal paper introducing rational interest rate models:

- B. Flesaker and L.P. Hughston (1996). *Positive interest*

Some references from recent literature:

- S. Cheng and M. Tehranchi (2014). *Polynomial models for interest rates and stochastic volatility*
- D. Filipović, M. Larsson and A. Trolle (2014). *Linear-rational term structure models*
- S. Crépey, A. Macrina, T.M. Nguyen and D. Skovmand (2014). *Rational multi-curve models with counterparty-risk valuation adjustments*

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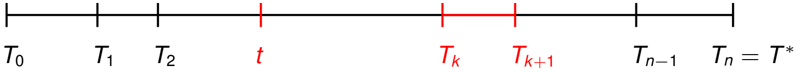
In this paper: we work in the discrete-tenor setup and make use of the polynomial property already in the model construction to ensure the main theoretical and practical modeling requirements

$\rightarrow$ we shall limit the presentation for simplicity to a single curve only

Introduction - main ingredients

Discrete tenor structure: $0 = T_0 < T_1 < \ldots < T_n = T^*$, with $\delta_k = T_{k+1} - T_k$, for all k



- zero coupon bond with maturity T_k: $B(t, T_k)$
- forward price process: $F(t, T_k, T_{k+1}) = \frac{B(t, T_k)}{B(t, T_{k+1})}$
- forward Libor rate for the interval $[T_k, T_{k+1}]$: $L(t, T_k)$

Master relation

$$1 + \delta_k L(t, T_k) = F(t, T_k, T_{k+1})$$

Forward measures

- forward martingale measure with numeraire $B(\cdot, T_k)$: $\mathbb{P}_{T_k}$
- Density process for the change between two forward measures

$$\left.\frac{\mathrm{d}\mathbb{P}_{T_k}}{\mathrm{d}\mathbb{P}_{T_{k+1}}}\right|_{\mathcal{F}_t} = \frac{B(0, T_{k+1})}{B(0, T_k)} \frac{B(t, T_k)}{B(t, T_{k+1})} = \frac{F(t, T_k, T_{k+1})}{F(0, T_k, T_{k+1})}$$

- No arbitrage:

$$\frac{B(\cdot, T_j)}{B(\cdot, T_k)} \in \mathcal{M}(\mathbb{P}_{T_k}), \forall j, k \quad \Leftrightarrow \quad \frac{B(\cdot, T_{k-1})}{B(\cdot, T_k)} \in \mathcal{M}(\mathbb{P}_{T_k}), \forall k$$

Martingale condition

$$F(\cdot, T_{k-1}, T_k), \quad L(\cdot, T_{k-1}) \in \mathcal{M}(\mathbb{P}_{T_k}).$$

Main modeling requirements

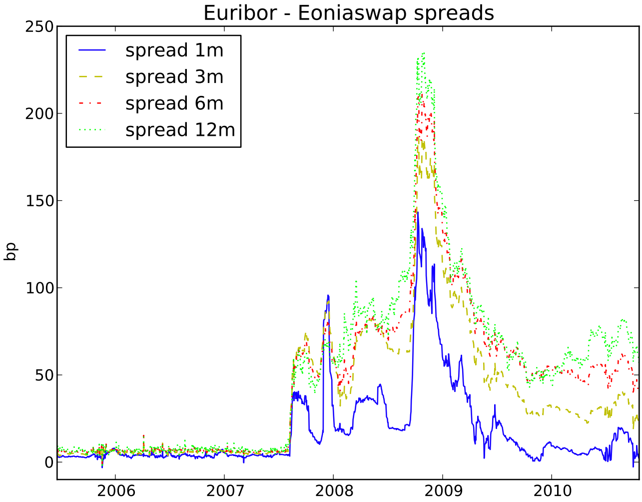
1. Libor rates should be non-negative: $L(t, T_k) \geq 0$, for all t, k
2. The model should be arbitrage-free: $L(\cdot, T_k)$ are $\mathbb{P}_{T_{k+1}}$-martingales
3. The model should be analytically tractable: closed or semi-closed formulas for most liquid derivatives (caps and swaptions) or efficient and accurate approximations
4. The model should be flexible, i.e. provide good calibration

Post-crisis modeling: Libor rates depend on the tenor and also differ from the discounting rates

$\Longrightarrow$ various other rates have to be modeled in addition to (1), or equivalently their spreads

$\Longrightarrow$ the rates can become negative, whereas the spreads still always remain positive in the current market situation

New interest rate models – multiple curve setup



Modeling of the forward price processes

The *forward price process* with respect to the terminal tenor date:

$$F(t, T_k, T_n) := \frac{B(t, T_k)}{B(t, T_n)}, \qquad t \in [0, T_k],$$

for all $1 \leq k \leq n$.

The modeling requirements now become:

1. For all $k = 1, \dots, n-1$ and all $t \in [0, T_k]$

$$1 \leq F(t, T_{k+1}, T_n) \leq F(t, T_k, T_n)$$

2. The forward prices $F(\cdot, T_k, T_n)$ should be $\mathbb{P}_{T_n}$-martingales
3. Tractability
4. Calibration (flexibility)

Comparison of some existing approaches

- Libor market model and extensions (Sandmann et al., Brace et al., Musiela and Rutkowski, Jamshidian, Eberlein and Özkan, Joshi, Andersen et al., Rebonato, Schoenmakers et al.)

  $$L(t, T_k) = L(0, T_k) \exp X_t^k,$$

  where X^k are semimartingales

- Forward price models (Musiela and Rutkowski, Eberlein et al.)

  $$F(t, T_k, T_{k+1}) = F(0, T_k, T_{k+1}) \exp X_t^k,$$

  where X^k are semimartingales

- Affine Libor model (Keller-Ressel et al., Da Fonseca et al.)

  $$F(t, T_k, T_n) = \mathbb{E}_{\mathbb{P}_{T_n}}[e^{\langle u_k, X_{T_n} \rangle} | \mathcal{F}_t] = e^{\phi_{T_n - t}(u_k) + \langle \psi_{T_n - t}(u_k), X_t \rangle}$$

  where X is an affine process

Additive construction of forward price models

Instead of modeling directly the forward prices, we model the forward price spreads:

$$S(t, T_k, T_n) := F(t, T_k, T_n) - F(t, T_{k+1}, T_n)$$

for all $k = 1, \ldots, n-1$.

Then, requirements (1) and (2) become

($\mathbb{S}$) *The forward price spreads $S(\cdot, T_k, T_n)$ are $\mathbb{P}_{T_n}$-martingales and*

$$S(t, T_k, T_n) \geq 0$$

for all $k = 1, \ldots, n$ and all $t \in [0, T_k]$.

The forward prices are sums of the forward price spreads:

$$F(t, T_k, T_n) = \sum_{j=k}^{n} S(t, T_j, T_n)$$

with

$$S(t, T_j, T_n) = \begin{cases} \frac{B(t,T_j) - B(t,T_{j+1})}{B(t,T_n)} & \text{for } j < n, \\ 1 & \text{for } j = n. \end{cases}$$

Additive construction of forward price models

Expressed in terms of bond prices, we have the following decomposition:

$$\frac{B(t,T_k)}{B(t,T_n)} = \underbrace{\frac{B(t,T_k) - B(t,T_{k+1})}{B(t,T_n)}}_{\geq 0} + \cdots + \underbrace{\frac{B(t,T_{n-1}) - B(t,T_n)}{B(t,T_n)}}_{\geq 0} + 1$$

and each summand is a $\mathbb{P}_{T_n}$-martingale.

Additive construction of forward price models

To specify the model, we set

$$S(t, T_j, T_n) := S(0, T_j, T_n) N_t^j$$

where the initial values $S(0, T_j, T_n)$ are market data and $(N^j)_{1 \leq j \leq n-1}$ nonnegative $\mathbb{P}_{T_n}$-martingales starting at 1.

Furthermore, set

$$N_t^j := \frac{\mathbb{E}_{\mathbb{P}_{T_n}}[f^j(Y^j_{T_n}) | \mathcal{F}_t]}{\mathbb{E}_{\mathbb{P}_{T_n}}[f^j(Y^j_{T_n})]},$$

where f^j are nonnegative functions and Y^j are semimartingales such that the conditional expectation above is analytically tractable.

Our choice: polynomial functions and polynomial preserving processes

Caplets and swaptions – 2

Proposition

The price of the caplet at time $t \leq T_k$ is given by

$$Cpl_t^k = B(t, T_n)\mathbb{E}_{\mathbb{P}_{T_n}}\left[\left(\sum_{j=k}^{n} \mu_j N_{T_k}^j\right)^+ \Big| \mathcal{F}_t\right]$$

where $\mu_k := S(0, T_k, T_n)$ and $\mu_j := -\delta_k K S(0, T_j, T_n)$, for $j > k$.

Proposition

The price of the swaption at time $t \leq T_k$ is given by

$$Swp_t = B(t, T_n)\mathbb{E}_{\mathbb{P}_{T_n}}\left[\left(\sum_{j=k}^{n} \eta_j N_{T_k}^j\right)^+ \Big| \mathcal{F}_t\right],$$

where $\eta_k := S(0, T_k, T_n)$ and $\eta_j := (1 - \sum_{i=k+1}^{j} c_i) S(0, T_j, T_n)$, for $j > k$.

Polynomial preserving processes

- Let X be a time-homogeneous Markov process and a semimartingale on the state space $E \subset \mathbb{R}^d$, relative to some filtration $(\mathcal{F}_t)_{t\geq 0}$
- Transition semigroup
$$P_t f(x) := \int_E f(y) p_t(x, \mathrm{d}y),$$
where $(p_t)_{t\geq 0}$ is the transition kernel of X.
- Then
$$\mathbb{E}_x[f(X_{t+s})|\mathcal{F}_s] = \mathbb{E}_{X_s}[f(X_t)] = P_t f(X_s)$$
- Denote by $\mathcal{P}_m$ the vector space of polynomials on E up to degree $m \geq 0$:
$$\mathcal{P}_m = \left\{ x \mapsto \sum_{|\mathrm{k}|=0}^{m} \alpha_{\mathrm{k}} x^{\mathrm{k}}, x \in E, \alpha_{\mathrm{k}} \in \mathbb{R} \right\}$$

Polynomial preserving processes

Cuchiero, Keller-Ressel and Teichmann (2012), Filipović, Gourier, Mancini, Trolle (2012, 2013)

- The process X is *m-polynomial preserving* (m-PP) if for all $k \leq m$, $P_t(\mathcal{P}_k) \subset \mathcal{P}_k$.

Or equivalently:

- the generator $\mathcal{A}$ of X is m-polynomial preserving: $\mathcal{A}(\mathcal{P}_k) \subset \mathcal{P}_k$, for all $k \leq m$.
- for every $k \leq m$, there exists a linear map A on $\mathcal{P}_k$ such that

$$P_t\big|_{\mathcal{P}_k} = e^{tA}$$

If $B = \{e_1, \ldots, e_M\}$ denotes a basis of $\mathcal{P}_k$, then $A = (A_{ij})_{i,j=1,\ldots,M}$ is obtained from $\mathcal{A}e_i = \sum_{j=1}^{M} A_{ij} e_j$ and

$$P_t f = (\alpha_1, \ldots, \alpha_M) e^{tA} (e_1, \ldots, e_M)^\top,$$

for any $f = \sum_{i=1}^{M} \alpha_i e_i \in \mathcal{P}_k$

Polynomial preserving processes

- Hence: the expected value of any polynomial of (X_t) is again a polynomial in the initial value $X_0 = x$

- Moments of X_t can be computed explicitly and easily without knowing the probability distribution or characteristic function of X_t:

$$\mathbb{E}_x[(X_t)^k] = (0, \dots, 0, 1, 0, \dots, 0)e^{tA}(x^0, x^1, \dots, x^m)^\top,$$

  where we assumed $d = 1$ for simplicity.

- The only task is to compute the matrix exponential e^{tA}

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- The only task is to compute the matrix exponential e^{tA}

Applications in finance: explicit formulas for polynomial claims, approximations for other claims, variance reduction techniques for Monte Carlo computations in pricing and hedging

Polynomial preserving processes

- Semimartingale characteristics (B, C, ν) of X is necessarily of the form:

$$B_t^i = \int_0^t b^i(X_s)\mathrm{d}s$$
$$C_t^{ij} + \int_0^t \int_{\mathbb{R}^d} y_i y_j \, \nu(\mathrm{d}s, \mathrm{d}y) = \int_0^t a^{ij}(X_s)\mathrm{d}s,$$

  where $b^i \in \mathcal{P}_1$ and $a^{ij} \in \mathcal{P}_2$

- Examples of polynomial preserving processes: affine processes (with finite moments), exponential Lévy processes, quadratic term structure models
- Pearson diffusions (Forman and Sørensen (2010))

$$\mathrm{d}X_t = -b(X_t - \theta)\mathrm{d}t + \sqrt{a + a_1 X_t + a_2 X_t^2}\mathrm{d}W_t, \quad X_0 = 0,$$

  for $b > 0$ and a, a_1, a_2 such that the square root is well-defined

Polynomial specification for the additive model

- $X = (X_t)_{t \geq 0}$ an m-polynomial preserving process on $E \subset \mathbb{R}^d$ and $X_0 = x_0 \in E$
- $x \mapsto p_j(x)$, $j = 1, \ldots, n-1$, be a family of nonnegative polynomial functions of degree m.
- the $\mathbb{P}_{T_n}$-martingales N^j, $j = 1, \ldots, n-1$, are defined as follows:

$$N_t^j := \frac{\mathbb{E}_{\mathbb{P}_{T_n}}[p_j(X_{T_n}) | \mathcal{F}_t]}{\mathbb{E}_{\mathbb{P}_{T_n}}[p_j(X_{T_n})]} = \frac{P_{T_n - t} p_j(X_t)}{P_{T_n} p_j(x_0)}.$$

- $P_{T_n - t} p_j(X_t)$ is a polynomial of degree m in X_t, hence each N_t^j is a polynomial in X_t

Caplet and swaption pricing

Revisiting the formulas for the time-0 price of the caplet and the swaption:

Caplet Pricing Formula

$$\mathrm{Cpl}_0^k = B(0, T_n)\mathbb{E}_{\mathbb{P}_{T_n}}\left[\left(\sum_{|i|=0}^{m} a_i \left(X_{T_k}\right)^i\right)^+\right],$$

where the coefficients a_i are explicitly determined by the infinitesimal generator $\mathcal{A}$ of the process X.

Swaption Pricing Formula

$$\mathrm{Swp}_0 = B(0, T_n)\mathbb{E}_{\mathbb{P}_{T_n}}\left[\left(\sum_{|i|=0}^{m} b_i \left(X_{T_k}\right)^i\right)^+\right],$$

with the coefficients b_i explicitly determined by the infinitesimal generator $\mathcal{A}$ of the process X.

Tractable examples

- quadratic OU Gaussian model (OU Gaussian process as driver combined with quadratic functions)
- quadratic OU Lévy model
- linear model (a positive PP-process as driver and linear functions), e.g. linear CIR model

Note:

- in the first two cases, polynomial functions of higher (even) degree can be used as well
- if the assumption on the positivity of the interest rates (and/or spreads) is relaxed, in the first two cases, the linear function can be applied, as well as, any other polynomial of odd degree. In the third case, there is no need to take a positive process

Examples: Quadratic Lévy case

- Let (X_t) with values in $\mathbb{R}^d$ be given by

$$\mathrm{d}X_t = (b + VX_t)\mathrm{d}t + \mathrm{d}L_t,$$

where L is a Lévy process with the triplet $(0, a, \nu)$.

- The generator $\mathcal{A}$ of X is

$$\mathcal{A}f(x) = b^\top \nabla f(x) + x^\top V \nabla f(x) + \frac{1}{2}\sum_{j,k=1}^{d} a_{jk} \frac{\partial^2 f(x)}{\partial x_j \partial x_k}$$
$$+ \int_{\mathbb{R}^d} \left(f(x+y) - f(x) - \sum_{j=1}^{d} \frac{\partial f(x)}{\partial x_j} h(y)_j \right) \nu(\mathrm{d}y)$$

$\Longrightarrow$ X is a polynomial preserving process

- Assume $V = \mathrm{diag}\,(v_1, \ldots, v_d)$ and consider the subspace of $\mathcal{P}_2$ with basis

$$B = \{1, x_1, \ldots, x_d, x_1^2, \ldots, x_d^2\}$$

Examples: Quadratic Lévy case

Then

$$\mathcal{A}(1) = 0$$
$$\mathcal{A}(x_i) = b_i + x_i v_i + \int_{\mathbb{R}^d} (y_i - h(y)_i)\nu(\mathrm{d}y)$$
$$\mathcal{A}(x_i^2) = 2x_i\Big(b_i + \underbrace{\int_{\mathbb{R}^d} (y_i - h(y)_i)\nu(\mathrm{d}y)}_{=:\gamma_i}\Big) + 2x_i^2 v_i + a_{ii} + \underbrace{\int_{\mathbb{R}^d} y_i^2 \nu(\mathrm{d}y)}_{=:\xi_i}$$

and

$$A := \mathcal{A}\big|_B = \begin{bmatrix} 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ \gamma_1 + b_1 & v_1 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \gamma_d + b_d & 0 & \dots & v_d & 0 & \dots & 0 \\ \xi_1 + a_{11} & 2(\gamma_1 + b_1) & \dots & 0 & 2v_1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \xi_d + a_{dd} & 0 & \dots & 2(\gamma_d + b_d) & 0 & \dots & 2v_d \end{bmatrix}$$

Examples: Quadratic Lévy case

- Particular case: $V = 0 \Longrightarrow X$ is a Lévy process and A is nilpotent, i.e. $A^n = 0$, for every $n \geq n_0$
- We have

$$P_t f(x) = (\alpha_1, \dots, \alpha_{2d+1}) e^{tA} (1, x_1, \dots, x_d, x_1^2, \dots, x_d^2)^\top,$$

  where (α_i) are the coefficients of $f(x)$ in terms of the basis $B = \{1, x_1, \dots, x_d, x_1^2, \dots, x_d^2\}$

- Therefore,

$$P_{T_n - t} f(X_t) = \sum_{i=1}^{d} C_{2,i}(T_n - t)(X_t^i)^2 + \sum_{i=1}^{d} C_{1,i}(T_n - t) X_t^i + C_0(T_n - t)$$

  and the coefficients $C_{2,i}(T_n - t)$, $C_{1,i}(T_n - t)$ and $C_0(T_n - t)$ are explicitly given as linear combinations of elements from the matrix $C(T_n - t) := e^{(T_n - t)A}$

Examples: Quadratic Lévy case

- Now in order to obtain the price of all considered interest rate derivatives an expectation of the following form must be computed

$$\pi_0 := \mathbb{E}_{\mathbb{P}_{T_n}}\left[\left(X_{T_k}^\top u X_{T_k} + v^\top X_{T_k} + w\right)^+\right],$$

for some $u \in \mathbb{R}^{d\times d}$, $v \in \mathbb{R}^d$ and $w \in \mathbb{R}$.

- Then we proceed using the Fourier transform methods:

$$\pi_0 = \frac{1}{2\pi}\int_{\mathbb{R}} \hat{g}(iR + v) M_{h(X_{T_k})}(R - iv)\mathrm{d}v,$$

where $M_{h(X_{T_k})}$ is the moment generating function of $h(X_{T_k})$ and $\hat{g}$ is the Fourier transform of the payoff function g.

Examples: Quadratic Lévy case

We have two possible choices:

(1) Setting

$$g(x) = x^+, \qquad g : \mathbb{R} \to \mathbb{R}$$
$$h(y) = y^\top u y + v^\top y + w, \quad h : \mathbb{R}^d \to \mathbb{R}$$

$\Longrightarrow$ Fourier transform of g is easy, mgf $M_{h(X_{T_k})}$ has to be computed

(2) Setting

$$g(x) = (x^\top u x + v^\top x + w)^+, \quad g : \mathbb{R}^d \to \mathbb{R}$$
$$h(y) = y, \qquad h : \mathbb{R} \to \mathbb{R}$$

$\Longrightarrow$ mgf $M_{h(X_{T_k})}$ is given in closed form for typical Lévy processes used in finance, but the Fourier transform of g can be numerically demanding to compute (finding zeros and dimension of integration)

$\Longrightarrow$ If $d = 1$, (2) is more convenient; if $d \geq 2$, (1) is preferred

Quadratic Lévy case: Transform Formula

To apply Fourier pricing we need a new result on computing the mgf of quadratic forms of Lévy processes.

Condition A

The characteristic exponent ψ of X can be extended to an analytic function on a domain $D \subset \mathbb{C}^d$ which contains the set

$$\Theta = \left\{ u \in \mathbb{C}^d : \arg u_i \in \left(\frac{\pi}{4}, \frac{3\pi}{4}\right) \cup \left(\frac{5\pi}{4}, \frac{7\pi}{4}\right) \right\}.$$

Moreover, the extended function ψ satisfies the growth bound

$$\limsup_{r \to \infty} \frac{\Re(\psi(yre^{i\theta}))}{r^2} \leq 0 \quad \text{for all} \quad y \in \mathbb{R}^d,\ \theta \in \left(\tfrac{\pi}{4}, \tfrac{3\pi}{4}\right) .$$

This is a multivariate version of a similar condition from Keller-Ressel, Muhle-Karbe (2013).

Quadratic Lévy case: Transform Formula

Proposition (Transform Formula for Quadratic Lévy Processes)

Let X_t be an $\mathbb{R}^d$-valued Lévy process with characteristic exponent $\psi(u)$ that satisfies Condition A and let

$$Q_t = X_t^\top \Sigma X_t + X_t^\top \mu.$$

Then the Fourier-Laplace transform of Q_T is given by

$$\mathbb{E}\left[e^{-uQ_T}\right] = \exp\left(\frac{u}{4}\mu^\top \Sigma^{-1}\mu\right)\mathbb{E}\left[\exp\left(\frac{i}{2}\sqrt{2u}\mu^\top \Sigma^{-1} Z\right)\exp\left(T\psi\left(i\sqrt{2u}Z\right)\right)\right]$$

for all $u \in \mathbb{C}$ with $\Re u \in (0, \mathcal{C})$, where the expectation on the right hand side is taken with respect to the d-dimensional normal random variable Z with zero mean and covariance Σ.
$\mathcal{C}$ is a constant depending on μ and the order of finite exponential moments of X.

Remark: for the quadratic Gaussian case, Filipović et al. (2013) provide the expression for the moment generating function

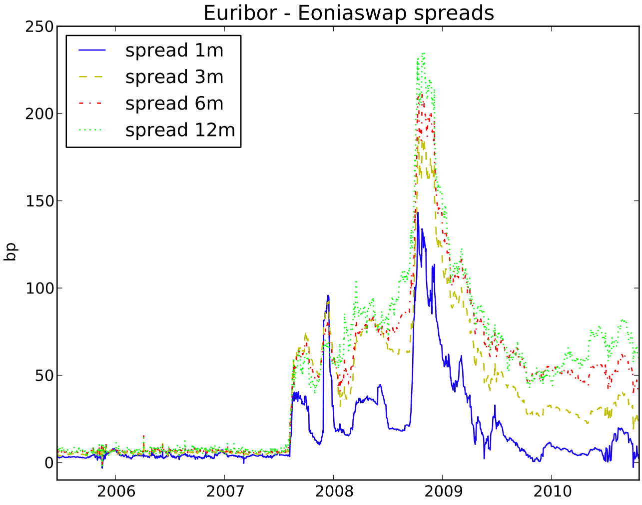
Concluding remarks

- Additive construction of discrete-tenor interest rate models based on polynomial preserving process
- Efficient pricing of caplets and swaptions using Fourier methods (no approximations needed)
- Tractable specification based on quadratic Lévy Process
- New transform formula for quadratic forms of Lévy processes
- Straightforward extension to post-crisis multiple curve framework
- Work in progress: calibration and study of implied volatilities in the model
- Based on:

K. Glau, Z.G. and M. Keller-Ressel (2015): Construction of Libor models from polynomial preserving processes. Working paper.

Thank you for your attention

New interest rate models – multiple curve setup



New interest rate models – multiple curve setup

- In addition to the family (N_t^j), model another family of nonnegative $\mathbb{P}_{T_n}$-martingales $(\tilde{N}_t^j)$, $j = 1, \dots, n-1$
- The OIS-forward prices are modeled in terms of (N_t^j), which do not have to necessarily positive given the current market conditions

$$F^{OIS}(t, T_k, T_n) = \sum_{j=k}^{n} \beta_j N_t^j$$

- The Libor rates are modeled as rational functions of (N_t^j) and $(\tilde{N}_t^j)$
- The prices of caplets and swaptions are again of the same additive form

$$\mathbb{E}_{\mathbb{P}_{T_n}}\left[\left(\sum_{j=k}^{n} a_j N_t^j + \sum_{i=k}^{n} \tilde{a}_i \tilde{N}_t^i\right)^+\right]$$

- Polynomial specification produces an equally tractable and flexible multiple curve model as in the classical single curve case