PROBABILISTIC WELL-POSEDNESS FOR THE SCHRÖDINGER EQUATION POSED FOR THE GRUSHIN LAPLACIAN

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ABSTRACT. In this talk we study the local well-posedness of the equation

$$i\partial_t u + \Delta_G u = |u|^2 u\,,$$

where $\Delta_G = \partial_x^2 + x^2\partial_y^2$ is the *Grushin Laplacian* and $u(t) : \mathbb{R}^2 \to \mathbb{C}$ is the solution, to be constructed with initial data $u(0) = u_0 \in H^s_G(\mathbb{R}^d)$ (the adapted Grushin-Sobolev spaces). From a deterministic perspective, the best local well-posedness theory is in $\mathcal{C}^\cdot([0,T), H^{\frac{3}{2}^+}_G)$ and the proof only uses the Sobolev embedding.

Our main goal is to provide a *probabilistic* construction of local solutions for initial data $u_0 \in H^s_G$ where $s < 3/2$. This is achieved using linear and bilinear random estimates.

In the first part of the talk I will introduce the random initial data which we will consider. Then I will explain why randomisation helps to lower the well-posedness threshold: this is a general argument in the study of dispersive equations with random initial data. Then I will explain how bilinear random estimates relate to our probabilistic well-posedness problem, which we will prove if time permits.

This talk is based on a joint work with Louise Gassot [GL21].

REFERENCES

[GL21] Louise Gassot and Mickaël Latocca. Probabilistic local well-posedness for the schrödinger equation posed for the grushin laplacian. *arXiv:2103.03560*, 2021.

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