Renormalization of Horava gravity

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Plan

Horava gravity:

1) Renormalizability and regular gauges: projectable models

2) BRST structure of renormalization and background formalism

3) Asymptotic freedom in (2+1)-dimensional model

4) Method of universal functional traces in background field formalism

5) Beta functions and RG fixed points in (3+1)-dimensions

D. Blas, M. Herrero-Valea, S. Sibiryakov C. Steinwachs & A.B.,
PRD 93, 064022 (2016), arXiv:1512.02250; arXiv:1705.03480;
PRL 119, 211301 (2017), arXiv:1706.06809;

M. Herrero-Valea, S. Sibiryakov & A.B., PRD100 (2019) 026012;

A.Kurov, S.Sibiryakov & A.B., Phys.Rev.D 105 (2022) 4, 044009
 arXiv: 2110.14688

Saving unitarity in renormalizable QG

Einstein GR

$$S_{EH} = \frac{M_P^2}{2} \int dt d^d x \, R$$

nonrenormalizable

$$\Longrightarrow \frac{M_P^2}{2} \int dt d^d x \, \left(h_{ij} \Box h_{ij} + h^2 \Box h + \dots \right)$$

Higher derivative gravity

$$\int \left(M_P^2 R + R_{\mu\nu} R^{\mu\nu} + R^2 \right)$$

Stelle (1977)

$$\Longrightarrow \int \left(M_P^2 h_{ij} \Box h_{ij} + h_{ij} \Box^2 h_{ij} + \dots \right)$$

dominates at $k \gg M_P$

The theory is renormalizable and asymptotically free !

Fradkin, Tseytlin (1981)
Avramidy & A.B. (1985)

But has ghost poles $\Longrightarrow$ no unitary interpretation

$$\int \underbrace{dt\, d^d x}\left(\dot{h}_{ij}\dot{h}_{ij} - h_{ij}(-\Delta)^z h_{ij} + \dots\right)$$

$$\propto b^{-(z+d)}$$
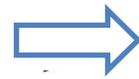
$$\Longrightarrow \qquad h_{ij} \mapsto b^{(d-z)/2} h_{ij}$$

$$\mathbf{x} \mapsto b^{-1}\mathbf{x}\,, \qquad t \mapsto b^{-z}t$$

Critical theory in $z = d$

LI is necessarily broken. We want to preserve as many symmetries, as possible

$$x^i \mapsto \tilde{x}^i(\mathbf{x}, t) \qquad \Longrightarrow \qquad \gamma_{ij} \quad N^i, \qquad i = 1, \dots, d$$

$$t \mapsto \tilde{t}(t) \qquad \Longrightarrow \qquad N$$

Foliation preserving diffeomorphisms

$$x^i \mapsto \tilde{x}^i(\mathbf{x}, t) \ , \quad t \mapsto \tilde{t}(t)$$

ADM metric decomposition

$$ds^2 = N^2 dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt) \ , \quad i, j = 1, \ldots, d$$

space dimensionality

Anisotropic scaling transformations and scaling dimensions

$$x^i \to \lambda^{-1} x^i, \quad t \to \lambda^{-z} t, \quad N^i \to \lambda^{z-1} N^i, \quad \gamma_{ij} \to \gamma_{ij},$$

$$[x] = -1, \ [t] = -z, \ [N^i] = z - 1, \ [\gamma_{ij}] = 0, \quad [K_{ij}] = z.$$

extrinsic curvature

$$K_{ij} = \frac{1}{2N}(\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i)$$

``*Projectable*" theory $N = \text{const} = 1$

kinetic term -- unitarity

Horava gravity action

$$S = \frac{1}{2G} \int dt\, d^d x \sqrt{\gamma} N \left(K_{ij} K^{ij} - \lambda K^2 - \mathcal{V}(\gamma) \right)$$

$$K_{ij} = \frac{1}{2N}(\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i)$$

Potential term

$$\mathcal{V}(\gamma) = 2\Lambda - \eta R + \mu_1 R^2 + \mu_2 R_{ij} R^{ij} + \nu_1 R^3 + \nu_2 R R_{ij} R^{ij}$$

$$+ \nu_3 R^i_j R^j_k R^k_i + \nu_4 \nabla_i R \nabla^i R + \nu_5 \nabla_i R_{jk} \nabla^i R^{jk} + \ldots$$

Many more versions: extra structures in non-projectable theory, reduction of structures for detailed balance case . . .

$$N \neq \text{const}, \quad a_i = \nabla_i \ln N, \ldots$$

$$d + 1 = 4$$ *DoF: tt-graviton and scalar*

Unitarity domain (no ghosts) $\dfrac{1 - \lambda}{1 - 3\lambda} > 0$

$$\omega_{tt}^2 = \eta k^2 + \mu_2 k^4 + \nu_5 k^6 \, ,$$

$$\omega_s^2 = \frac{1 - \lambda}{1 - 3\lambda}\Big(-\eta k^2 + (8\mu_1 + 3\mu_2)k^4 + (8\nu_4 + 3\nu_5)k^6 \Big)$$

tachyon in IR

Divergences power counting

Deg of div $\int \dfrac{d^{d+1}p}{\left(p^2\right)^N} = d + 1 - 2N =$ physical dimensionality
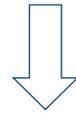
$$p = (\omega, \mathbf{k}), \ \ p^2 \to \omega^2 + \mathbf{k}^{2z}$$

Deg of div $\int \dfrac{d\omega \, d^d k}{\left(\omega^2 + \mathbf{k}^{2z}\right)^N} = z + d - 2zN =$ scaling dimensionality

physical dimensionality $\neq$ scaling dimensionality

$$z = d$$ *critical value*

$$\mathcal{V}^{(d=2)}(\gamma) = 2\Lambda - \eta R + \mu_1 R^2$$

$$\mathcal{V}^{(d=3)}(\gamma) = 2\Lambda - \eta R + \mu_1 R^2 + \mu_2 R_{ij}^2 + O(R^3, R\nabla^2 R)$$

Things are not so simple: power counting is not enough:

$$\int \prod_{l=1}^{L} d^{d+1}k^{(l)} \, \mathcal{F}(k) \, \prod_{m=1}^{M} \frac{1}{\left(P^{(m)}(k)\right)^2} \quad \Rightarrow$$

$$\int \prod_{l=1}^{L} d\omega^{(l)} d^d k^{(l)} \, \mathcal{F}(\omega, \mathbf{k}) \, \prod_{m=1}^{M} \frac{1}{A_m \left(\Omega^{(m)}(\omega)\right)^2 + B_m \left(\mathbf{K}^{(m)}(\mathbf{k})\right)^{2z}}$$

Generalization of BPHZ renormalization theory (subtraction of subdivergences)
works only *for* $A_m > 0$ *and* $B_m > 0$

depends on gauge fixing

Invention of **regular gauges** *for* **projectable** *HG*

$$F^\mu \equiv \partial_\nu h^{\nu\mu} + \ldots \Rightarrow F^i = \dot{N}^i + c\partial_j \Delta^{d-1} h^{ji} + \ldots$$

extra derivatives to have homogeneity in scaling

$$[F^i] = 2d - 1 \Rightarrow \mathcal{O}_{ij} = \Delta^{2-d}(\Delta\delta_{ij} + \xi\partial_i\partial_j)^{-1}$$

Gauge fixing term $\quad S_{\mathsf{gf}} = \frac{\sigma}{2G} \int dt \, d^d x \, \sqrt{\gamma} \, F_i \mathcal{O}^{ij} F_j$

σ, ξ **free gauge fixing parameters**

Projectable HG is renormalizable in any *d*

Gauge invariance of counterterms

Background covariant gauge conditions + BRST structure of renormalization

Background field method:

$$\gamma_{ij} = \bar{\gamma}_{ij} + h_{ij}, \ \ \partial_i \to \bar{\nabla}_i, \ \ \mathcal{O}^{-1\,ij} = \bar{\Delta}^{d-1}\,\bar{\gamma}^{ij} + \xi\bar{\nabla}^i\Delta^{d-2}\bar{\nabla}^j, \ ...$$

DeWitt, Tyutin, Voronov, Stelle, Batalin, Vilkovisky, Slavnov, Arefieva, Abbott…
Barnich, Henneaux, Grassi, Anselmi,…

Blas, Herrero-Valea, Sibiryakov, Steinwachs & A.B.
arXiv: 1705.03480, JHEP07(2018)035

Background field extension of the BRST operator + inclusion of generating functional sources into it and the asso_ciared gauge fermion

1. **BRST structure of renormalization via decoupling of the background field**
2. **No power counting or use of field dimensionalities**
3. **Extension to Lorentz symmetry violating theories**
4. **Extension to (nonrenormalizable) effective field theories**

Extended BRS operator and gauge fermion $\quad Q \to Q_{\text{ext}}, \quad \Psi \to \Psi_{\text{ext}}[\Phi, J]$

$$e^{-W/\hbar} = \int D\Phi \, e^{-(S + Q\Psi + J\Phi)/\hbar} \implies e^{-W/\hbar} = \int D\Phi \, e^{-(S + Q_{\text{ext}}\Psi_{\text{ext}})/\hbar}$$

Gauge independence on shell $\quad \boxed{\delta_\Psi W[J] \Big|_{J=0} = 0}$

physical gauge invariant local counterterm

Renormalization: $\quad S[\varphi] \to S[\varphi] + \triangle_\infty S[\varphi]$

$$\Psi_{\text{ext}}[\Phi] \to \Psi_{\text{ext}}[\Phi] + \triangle_\infty \Psi_{\text{ext}}[\Phi]$$

local counterterm to gauge fermion (irrelevant)

Asymptotic freedom in (2+1)-dimensions

$$S = \frac{1}{2G} \int dt\, d^2x\, N\sqrt{\gamma}\, \left(K_{ij}K^{ij} - \lambda K^2 + \mu R^2 \right)$$

Off-shell extension is not unique:

$$\Gamma_{1-\mathsf{loop}} \to \Gamma_{1-\mathsf{loop}} + \int dt\, d^dx\, \Omega_{ij}\frac{\delta S}{\delta \gamma_{ij}}$$

Essential coupling constants: $\lambda,\quad \mathcal{G} \equiv \dfrac{G}{\sqrt{\mu}}$

$$S_{\text{gf}} = \frac{\sigma}{2G} \int dt\, d^2x\, \sqrt{\gamma}\, F_i\, \mathcal{O}^{ij} F_i$$

$$F_i = \partial_t n_i + \frac{1}{2\sigma} \mathcal{O}_{ij}^{-1} (\nabla^k h_k^{\,j} - \lambda \nabla^j h)$$

$$\mathcal{O}^{ij} = -[\gamma_{ij}\Delta + \xi \nabla_i \nabla_j]^{-1}$$

**background covariant
gauge-fixing term**
σ, ξ **– free gauge parameters**

Mathematica package xAct

**D. Brizuela, J. M. Martin-Garcia, and G. A. Mena
Marugan, Gen. Rel. Grav. 41, 2415 (2009),
arXiv:0807.0824**

$$\beta_\lambda = \frac{15 - 14\lambda}{64\pi} \sqrt{\frac{1 - 2\lambda}{1 - \lambda}}\, \mathcal{G}$$

$$\beta_{\mathcal{G}} = -\frac{(16 - 33\lambda + 18\lambda^2)}{64\pi(1-\lambda)^2} \sqrt{\frac{1-\lambda}{1-2\lambda}}\, \mathcal{G}^2$$

Powerful check – gauge independence of essential couplings

Check in conformal gauge $h_{ij} = e^{2\phi}\gamma_{ij}$

$$\beta_\mu = \frac{2 - 7\lambda + 6\lambda^2}{32\pi(1-\lambda)^{3/2}\sqrt{1-2\lambda}} G\sqrt{\mu}$$

$$\beta_G = -\frac{6\lambda - 7}{32\pi\sqrt{(1-2\lambda)(1-\lambda)}} \frac{G^2}{\mu}$$

$\Rightarrow \beta_{\mathcal{G}}$
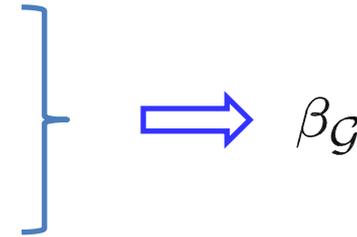
Compare to regular ``relativistic'' gauge

$$\beta_\mu = \frac{30 - 73\lambda + 42\lambda^2}{32\pi(1-\lambda)^{3/2}\sqrt{1-2\lambda}} G\sqrt{\mu}$$

$$\beta_G = -\frac{30\lambda - 23}{32\pi\sqrt{(1-2\lambda)(1-\lambda)}} \frac{G^2}{\mu}$$

$\Rightarrow \beta_{\mathcal{G}}$
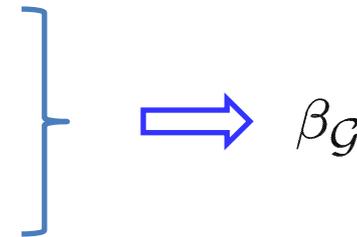
same

Renormalization flows:
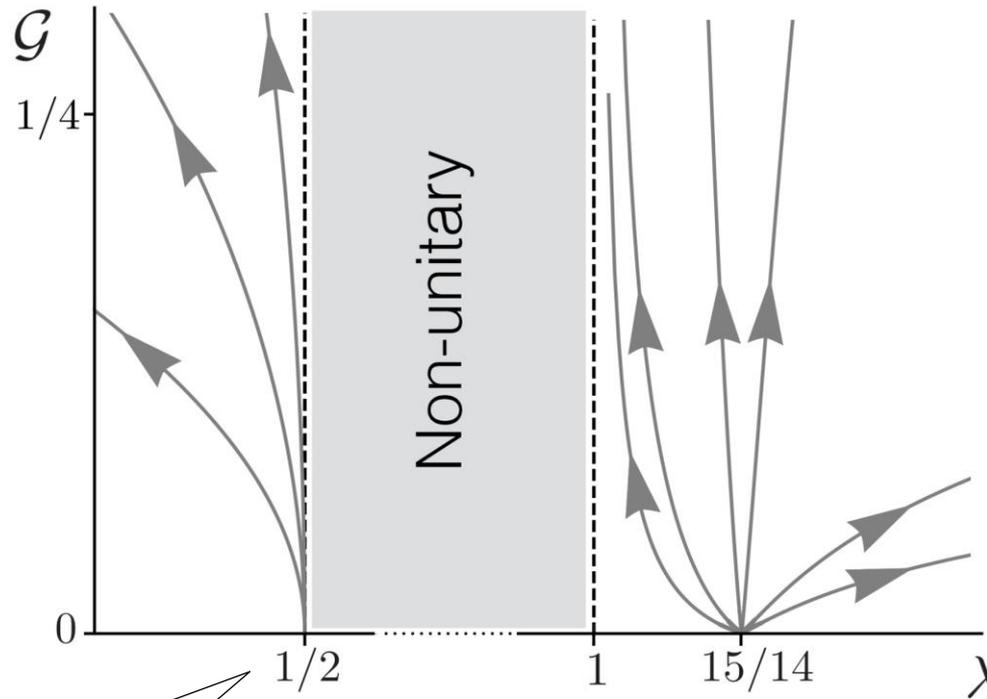
strongly coupled fixed point

$$\mathcal{G} \to \tilde{\mathcal{G}} = \frac{G}{\sqrt{1 - 2\lambda}}$$

$$\beta_{\tilde{\mathcal{G}}} = -\frac{(1 - 2\lambda)^2}{64\pi(1 - \lambda)^{3/2}} \tilde{\mathcal{G}}^2$$

AF UV fixed point

Towards RG flows of (3+1)-dimensional Horava gravity

$$S = \frac{1}{2G} \int dt\, d^d x \sqrt{\gamma} \Big(K_{ij} K^{ij} - \lambda K^2 - \mathcal{V}(\gamma) \Big)$$

$$\mathcal{V}(\gamma) = \nu_1 R^3 + \nu_2 R R_{ij} R^{ij} + \nu_3 R^i_j R^j_k R^k_i + \nu_4 \nabla_i R \nabla^i R + \nu_5 \nabla_i R_{jk} \nabla^i R^{jk} + \dots$$

$$\beta_G, \qquad \beta_\lambda$$

$$\alpha \equiv \frac{1}{\nu_5} \frac{(1-\lambda)(8\nu_4 + 3\nu_5)}{1 - 3\lambda}$$

$$\lambda(\alpha) = \frac{9 + 7\sqrt{\alpha} - 2\alpha + 2\sqrt{10(\alpha + \alpha^{3/2})}}{3(3 + \sqrt{\alpha} - 2\alpha)}$$

The lines $\lambda = \lambda(\alpha)$ correspond to potential location of fixed points of the full RG flow. The region $\lambda \in [1/3,1]$ is excluded by the requirement of unitarity.
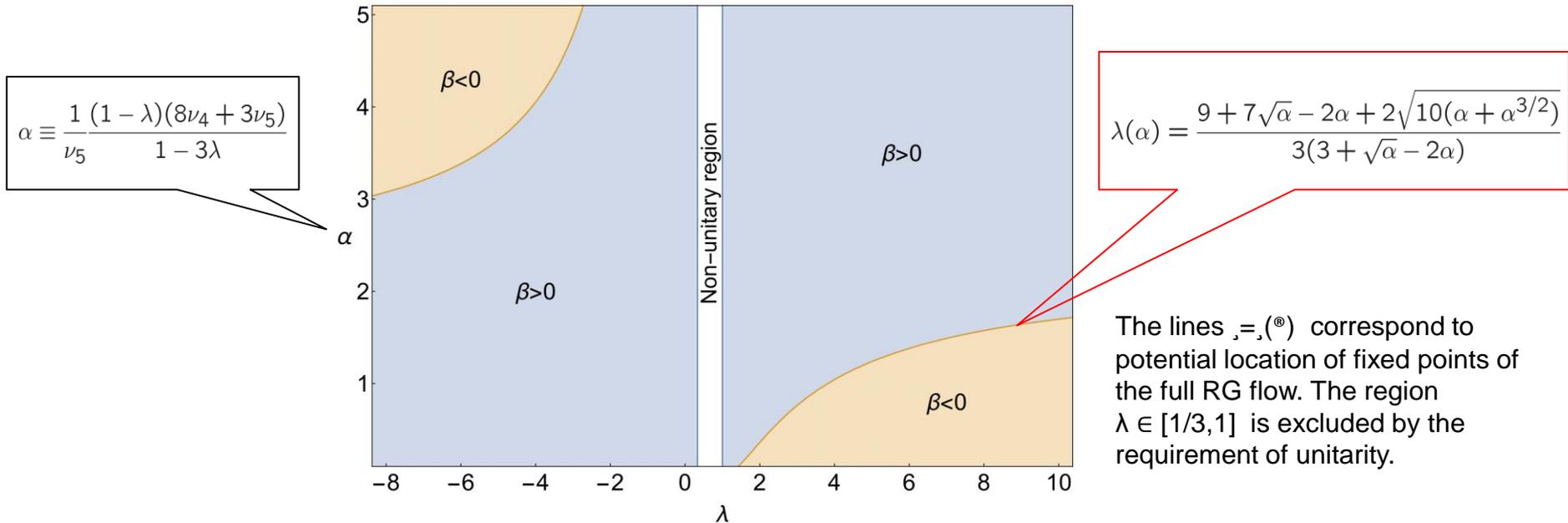
Background field method

One-loop effective action

$$\Gamma_{\text{one−loop}} = \frac{1}{2}\text{Tr}_4 \ln \hat{F}(\nabla) = -\frac{1}{2}\int_0^\infty \frac{ds}{s} e^{-s\hat{F}(\nabla)}$$

Action Hessian $\hat{F}(\nabla) = F^A_B(\nabla)$ **acting in the space of fields** $\varphi = \varphi^A(x)$

$$\hat{F}(\nabla) = \Box + \hat{P} - \frac{\hat{1}}{6} R, \qquad \Box = g^{\mu\nu}\nabla_\mu\nabla_\nu$$

$$[\nabla_\mu, \nabla_\nu] V^\lambda = R^\lambda{}_{\rho\mu\nu} V^\rho, \quad [\nabla_\mu, \nabla_\nu] \phi = \hat{R}_{\mu\nu}\phi, \quad [\nabla_\mu, \nabla_\nu] \hat{X} = [\hat{R}_{\mu\nu}, \hat{X}]$$

Heat kernel (Schwinger-DeWitt) expansion

$$e^{-s\hat{F}(\nabla)}\delta(x,y) = \frac{\mathcal{D}^{1/2}(x,y)}{(4\pi s)^{d/2}} g^{1/2}(y) e^{-\frac{\sigma(x,y)}{2s}} \sum_{n=0}^\infty s^n \hat{a}_n(x,y)$$

Schwinger-DeWitt (Gilkey-Seely) coefficients

$$\hat{a}_0\Big|_{y=x} = \hat{1}, \quad \hat{a}_1\Big|_{y=x} = \hat{P},$$

$$\hat{a}_2\Big|_{y=x} = \frac{1}{180}\left(R^2_{\alpha\beta\gamma\delta} - R^2_{\mu\nu} + \Box R\right)\hat{1} + \frac{1}{12}\hat{R}^2_{\mu\nu} + \frac{1}{2}\hat{P}^2 + \frac{1}{6}\Box\hat{P}, ...$$

One-loop divergences

$$\Gamma^{\text{div}}_{\text{one−loop}} = -\frac{1}{32\pi^2\varepsilon}\int dx\, g^{1/2}\text{tr}\,\hat{a}_2(x,x), \quad \varepsilon = 2 - \frac{d}{2} \to 0$$

Extension to non-minimal and higher-derivative operators

*The method of **universal functional traces** (I. Jack and H. Osborn (1984),*
G.A. Vilkovisky & A.B., Phys. Rept. 119 (1985) 1)

$$\text{Tr} \ln \left(\square^N + P(\nabla) \right) = N \, \text{Tr} \ln \square + \text{Tr} \ln \left(1 + P(\nabla) \frac{1}{\square^N} \right)$$

$$= N \, \text{Tr} \ln \square + \text{Tr} \, P(\nabla) \frac{1}{\square^N} + \cdots$$

$$\Gamma^{\text{div}} = \sum_{m,n} \int d^4x \, \mathcal{R}_n^{\mu_1 \cdots \mu_m} \nabla_{\mu_1} ... \nabla_{\mu_m} \frac{\widehat{1}}{\square^n} \delta(x,y) \Big|_{y=x}^{\text{div}}$$

universal functional traces

$$\nabla ... \nabla \frac{\widehat{1}}{\square^n} \delta(x,y) \Big|_{y=x}^{\text{div}} = \frac{(-1)^n}{\Gamma(n)} \nabla ... \nabla \int_0^\infty ds \, s^{\alpha-1} \, e^{s\square} \, \widehat{\delta}(x,y) \Big|_{y=x}^{\text{div}}$$

Schwinger-DeWitt expansion

Horava gravity:

$$\varphi^A(x) = h_{ij}(x), n^i(x) \ + \ \text{FP ghosts}$$

Static 3-metric background with generic 3-metric $\bar{\gamma}_{ij}(\mathbf{x})$

$$\gamma_{ij}(x) = \bar{\gamma}_{ij}(\mathbf{x}) + h_{ij}(\tau, \mathbf{x}), \quad N^i(x) = 0 + n^i(\tau, \mathbf{x})$$

Hessian structure $\qquad \hat{F}(\nabla) = -\hat{1}\,\partial_\tau^2 + \hat{\mathbb{F}}(\nabla_{\mathbf{x}})$

Space parts of metric and vector (shifts and ghosts) operators: $\qquad \hat{\mathbb{F}} = \mathbb{F}^A_{\ B} = \left\{ \mathbb{F}_{ij}^{\ kl}, \mathbb{F}^k_{\ i} \right\} \sim \nabla^6 + ...$

Example – for the ghost operator in σ, ξ *-family of gauges:*

$$\mathbb{F}^i_{\ j}(\nabla) = -\frac{1}{2\sigma}\delta^i_j\Delta^3 - \frac{1}{2\sigma}\Delta^2\nabla_j\nabla^i - \frac{\xi}{2\sigma}\nabla^i\Delta\nabla^k\nabla_j\nabla_k$$

$$-\frac{\xi}{2\sigma}\nabla^i\Delta\nabla_j\Delta + \frac{\lambda}{\sigma}\Delta^2\nabla^i\nabla_j + \frac{\lambda\xi}{\sigma}\nabla^i\Delta^2\nabla_j, \quad \Delta = \gamma^{ij}\nabla_i\nabla_j$$

*Dimensional reduction method on a **static** background with generic 3-metric*

$$\mathsf{Tr}_4 \ln(-\partial_\tau^2 + \mathbb{F}) = -\int_0^\infty \frac{ds}{s} \mathsf{Tr}_4 e^{-s(-\partial_\tau^2 + \mathbb{F})}$$

$$= \int d\tau\, d^3x \int \frac{ds}{s} \mathsf{tr}\, e^{-s(-\partial_\tau^2 + \mathbb{F})} \delta(\tau - \tau')\, \delta(\mathbf{x} - \mathbf{x}') \Big|_{\tau = \tau',\, \mathbf{x} = \mathbf{x}'}$$

$$= -\int d\tau\, \mathsf{Tr}_3 \sqrt{\mathbb{F}}$$

How to proceed with the square root of the 6-th order differential operator?

$$\mathbb{F} = \sum_{a=0}^6 \mathcal{R}_{(a)} \sum_{6 \geq 2k \geq a} \alpha_{a,k} \nabla_1 \dots \nabla_{2k-a} (-\Delta)^{3-k}, \quad \mathcal{R}_{(a)} = O\left(\frac{1}{l^a}\right)$$

Pseudodifferential *operator – infinite series in curvature invariants* $\mathcal{R}_{(a)}$

$$\sqrt{\mathbb{F}} = \sum_{a=0}^\infty \mathcal{R}_{(a)} \sum_{k \geq a/2}^{K_a} \tilde{\alpha}_{a,k} \nabla_1 \dots \nabla_{2k-a} \frac{1}{(-\Delta)^{k-3/2}}$$

How to find the coefficients $\tilde{\alpha}_{a,k}$?

$$\sqrt{\mathbb{F}} = \mathbb{Q}^{(0)} + \mathbb{X}$$

principal symbol of $\mathbb{F} \equiv \mathbb{F}(\nabla)\Big|_{\nabla \to p,\, \mathcal{R} \to 0}$

$$\mathbb{Q}^{(0)} = \left(\textit{principal symbol of } \mathbb{F}\right)^{1/2}\Big|_{p \to \nabla}$$

Solving by iterations the linear equation for $\mathbb{X}$ as expansion in the curvature

$$\mathbb{Q}^{(0)}\mathbb{X} + \mathbb{X}\mathbb{Q}^{(0)} = \mathbb{F} - \left(\mathbb{Q}^{(0)}\right)^2 - \mathbb{X}^2 \propto \mathcal{R} \sim [\nabla, \nabla]$$

$$\text{Tr}_3\sqrt{\mathbb{F}}\Big|^{\text{div}} = \sum_{a=2}^{6}\sum_{k}\tilde{\alpha}_{a,k}\int d^3x\,\mathcal{R}_{(a)}(\mathbf{x})\nabla_1...\nabla_{2k-a}\frac{1}{(-\triangle)^{k-3/2}}\delta(\mathbf{x},\mathbf{x}')\Big|^{\text{div}}_{\mathbf{x}=\mathbf{x}'}$$

Divergences of universal functional traces

$$\nabla...\nabla\frac{\hat{1}}{(-\triangle)^{\alpha}}\,\delta(x,y)\Big|^{\text{div}}_{y=x} = \frac{1}{\Gamma(\alpha)}\nabla...\nabla\int_0^{\infty}ds\,s^{\alpha-1}\,e^{s\triangle}\,\hat{\delta}(x,y)\Big|^{\text{div}}_{y=x}$$

↑

Schwinger-DeWitt expansion

Examples:

$$g^{ij}(-\triangle)^{1/2}\delta_{ij}{}^{kl}(x,y)\Big|^{\text{div}}_{y=x} = -\frac{1}{16\pi^2\varepsilon}\sqrt{g}\,g^{kl}\frac{1}{30}\left(\frac{1}{2}R_{ij}^2+\frac{1}{4}R^2+\triangle R\right)$$

$$\int d^3x\,\delta_{kl}{}^{ij}(-\triangle)^{3/2}\delta_{ij}{}^{kl}(x,y)\Big|^{\text{div}}_{y=x} = \frac{3}{32\pi^2\varepsilon}\int d^3x\,\sqrt{g}\,\delta_{kl}{}^{ij}\,a_{3ij}{}^{kl}(x,x)$$

$$= \frac{3}{32\pi^2\varepsilon}\int d^3x\,\sqrt{g}\left(\frac{31}{45}R_j^i R_k^j R_i^k - \frac{233}{210}R_{ij}^2 R + \frac{673}{2520}R^3 + \frac{5}{84}R\triangle R - \frac{67}{420}R_{ij}\triangle R^{ij}\right)$$

Results for beta functions of (3+1)-dimensional Horava gravity

$$S = \frac{1}{2G} \int dt\, d^d x \sqrt{\gamma} N \left(K_{ij} K^{ij} - \lambda K^2 - \mathcal{V}(\gamma) \right)$$

$$\mathcal{V}(\gamma) = \nu_1 R^3 + \nu_2 R R_{ij} R^{ij} + \nu_3 R^i_j R^j_k R^k_i + \nu_4 \nabla_i R \nabla^i R + \nu_5 \nabla_i R_{jk} \nabla^i R^{jk}$$

Six essential coupling constants $\quad \mathcal{G}, \ \lambda$ and $\ \chi = (u_s, v_1, v_2, v_3)$

$$\mathcal{G} = \frac{G}{\sqrt{\nu_5}}, \quad \lambda, \quad u_s = \sqrt{\frac{(1-\lambda)(8\nu_4 + 3\nu_5)}{(1-3\lambda)\nu_5}}, \quad v_a = \frac{\nu_a}{\nu_5}, \quad a = 1,2,3$$

$$\beta_{\mathcal{G}} = \frac{\mathcal{G}^2}{26880\pi^2(1-\lambda)^2(1-3\lambda)^2(1+u_s)^3 u_s^3} \sum_{n=0}^{7} u_s^n \, \mathcal{P}_n^{\mathcal{G}}[l, v_1, v_2, v_3]$$

$$\beta_{\lambda} = \frac{\mathcal{G}}{120\pi^2(1-\lambda)(1+u_s)u_s} \left[27(1-\lambda)^2 + 3u_s(11-3\lambda)(1-\lambda) - 2u_s^2(1-3\lambda)^2 \right]$$

$$\beta_{\chi} = \frac{A_{\chi}\,\mathcal{G}}{26880\pi^2(1-\lambda)^3(1-3\lambda)^3(1+u_s)^3 u_s^5} \sum_{n=0}^{9} u_s^n \, \mathcal{P}_n^{\chi}[l, v_1, v_2, v_3]$$

$$A_{u_s} = u_s(1-\lambda), \quad A_{v_1} = 1, \quad A_{v_2} = A_{v_3} = 2$$

$\mathcal{P}_n^{\chi}[l, v_1, v_2, v_3,]$ *are polynomials in* λ *and* v_a,

Use of Mathematica package xAct

Example (one of the longest ones):

$$
\begin{aligned}
\mathcal{P}_5^{v_1} =& -2(1-\lambda)^2(1-3\lambda)\Big\{168v_2^3(51\lambda^3 - 149\lambda^2 + 125\lambda - 27) - 108v_3^3(9\lambda^3 + 9\lambda^2 \\
& -25\lambda + 7) - 4v_2^2(1-\lambda)\big[18v_3(117\lambda^2 - 366\lambda + 109) - 284\lambda^2 - 7265\lambda + 5425\big] \\
& +40320v_1^2(1-\lambda)^2(\lambda + 1) - 9v_3^2(3467\lambda^3 - 8839\lambda^2 + 6237\lambda - 865) \\
& +v_1\big[64v_2^2(1-\lambda)^2(1717\lambda - 581) - 16v_2(1-\lambda)\big(3v_3(2741\lambda^2 - 3690\lambda + 949) \\
& +25940\lambda^2 - 40662\lambda + 12022\big) + 27v_3^2(961\lambda^3 - 2395\lambda^2 + 1835\lambda - 401) \\
& +6v_3(52267\lambda^3 - 148963\lambda^2 + 129881\lambda - 33185) - 288353\lambda^3 + 542255\lambda^2 \\
& -333355\lambda + 83485\big] - 2v_2\big[162v_3^2(3\lambda^3 + 35\lambda^2 - 51\lambda + 13) + 24v_3(1265\lambda^3 \\
& -2191\lambda^2 + 691\lambda + 235) + 30971\lambda^3 - 40323\lambda^2 + 13167\lambda - 4451\big] - 12v_3(6551\lambda^3 \\
& -11593\lambda^2 + 6124\lambda - 1112) + 109519\lambda^3 - 252396\lambda^2 + 177357\lambda - 34396\Big\}
\end{aligned}
$$

Check of the results: independence of essential beta functions on the choice of gauge (σ, ξ - family of gauge conditions) and spectral sum method in dimensional and zeta-functional regularization.

Discussion: detailed balance and asymptotic freedom

Special (not fully fixed) point: $\{v^*\} : v_1 = 1/2, \quad v_2 = -5/2, \quad v_3 = 3$

$$\beta_{v_a}\Big|_{\{v^*\},\, u_s \to 0} = 0 \,,\, a = 1, 2, 3 \,, \qquad \beta_{u_s}\Big|_{\{v^*\},\, u_s \to 0}, \beta_{\mathcal{G}}\Big|_{\{v^*\},\, u_s \to 0} \text{ are regular}$$

Detailed balance version of HG

$$S_{v^*, u_s \to 0} = \frac{1}{2G} \int d\tau \, d^3x \, \sqrt{\gamma} \, (K_{ij} K^{ij} - \lambda K^2 + \nu_5 \, C^{ij} C_{ij})$$
$$= \frac{2}{G} \int d\tau \, d^3x \, \sqrt{\gamma} \, (K_{ij} + \sqrt{\nu_5} C_{ij}) \, \mathbb{G}^{ij,kl} \, (K_{kl} + \sqrt{\nu_5} C_{kl}),$$

Cotton tensor

$$C^{ij} = \varepsilon^{ikl} \nabla_k \left(R_l^j - \frac{1}{4} R \delta_l^j \right) = \varepsilon^{kl(i} \nabla_k R_l^{j)}$$

Connection to gravitational Chern-Simons theory

$$C^{ij} = -\frac{1}{\sqrt{g}} \frac{\delta W_{\mathrm{CS}}[g]}{\delta g_{ij}(x)}, \quad W_{\mathrm{CS}}[g] = \frac{1}{2} \int d^3x \, \epsilon^{ijk} \left(\Gamma_{il}^m \partial_j \Gamma_{km}^l + \frac{2}{3} \Gamma_{il}^n \Gamma_{jm}^l \Gamma_{kn}^m \right)$$

Covariant Langevin equation – stochastic quantization of CS theory

$$K_{ij} - \frac{\sqrt{\nu_5}}{\sqrt{g}} \frac{\delta W_{\mathrm{CS}}[g]}{\delta g_{ij}(x)} = J^{ij}$$

$$\mathcal{G} \to 0 \qquad \textit{asymptotic freedom}$$

$$\mathcal{G} \to \infty \qquad \textit{Landau pole}$$

Fixed points equations:

$$\beta_\lambda/\mathcal{G} = 0 \,,$$

$$\beta_\chi/\mathcal{G} = 0 \,, \qquad \chi = u_s, v_1, v_2, v_3$$

λ	u_s	v_1	v_2	v_3	$\beta_\mathcal{G}/\mathcal{G}^2$	AF?	UV attractive along λ?
0.1787	60.57	-928.4	-6.206	-1.711	-0.1416	yes	no
0.2773	390.6	-19.88	-12.45	2.341	-0.2180	yes	no
0.3288	54533	3.798×10^8	-48.66	4.736	-0.8484	yes	no
0.3289	57317	-4.125×10^8	-49.17	4.734	-0.8784	yes	no

Special limit: $\lambda \to \infty$ *(non-relativistic gravity vs Perelman-Ricci flow, A. Frenkel, P. Horava and S. Randall, 2011.1914; cosmology implication, A.E. Gumrukcuoglu, S. Mukohyama, 1104.2087)*

u_s	v_1	v_2	v_3	$\beta_{\mathcal{G}}/\mathcal{G}^2$	asymptotically free?	UV attractive along λ?
0.01950	0.4994	-2.498	2.999	-0.2004	yes	no
0.04180	-0.01237	-0.4204	1.321	-1.144	yes	no
0.05530	-0.2266	0.4136	0.7177	-1.079	yes	no
12.28	-215.1	-6.007	-2.210	-0.1267	yes	yes
21.60	-17.22	-11.43	1.855	-0.1936	yes	yes
440.4	-13566	-2.467	2.967	0.05822	no	yes
571.9	-9.401	13.50	-18.25	-0.07454	yes	yes
950.6	-61.35	11.86	3.064	0.4237	no	yes

Conclusions

Renormalization of Horava-Lifshitz gravity

*Salvation of unitarity in **local renormalizable QG via LI violation***

BPHZ renormalization and "regularity" of propagators

Gauge invariance of UV counterterms

Asymptotic freedom in (2+1)-dimensional theory

Method of universal functional traces

Beta functions of (3+1)-dimensional theory and fixed points candidates for AF

THANK YOU!