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Persistent order in CFTs

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This presentation is based on the works done in collaboration with

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SciPost Phys. 12 (2022) 181, e-Print: 2111.02474 [hep-th]
Phys.Rev.Lett. 128 (2022) 1, 011601, e-Print:2106.09723 [hep-th]
Phys.Rev.Lett. 125 (2020) 13, 131603
Phys.Rev.D. 102 (2020) 6, 065014, e-Print: 2005.03676 [hep-th]

A wide perception:

While global symmetries can be broken at $T=0$, all the symmetries are restored in a thermal state for sufficiently high T.

Is it actually true in a UV complete, unitary, local, relativistic QFT?

Landau & Lifschitz “Statistical Physics Part 1”, Chapter XIV

“In the great majority of the known instances of phase transitions, the more symmetrical phase corresponds to higher temperatures and the less symmetrical one to lower temperatures. In particular, a transition of the second kind from an ordered to a disordered state always occurs with increasing temperature. This is not a law of thermodynamics, however, and exceptions are therefore possible.”

Landau & Lifschitz “Statistical Physics Part 1”, Chapter XIV

“One exception, for example, is the “lower Curie point” of Rochelle salt, below which the crystal is orthorhombic, but above which it is Monoclinic”

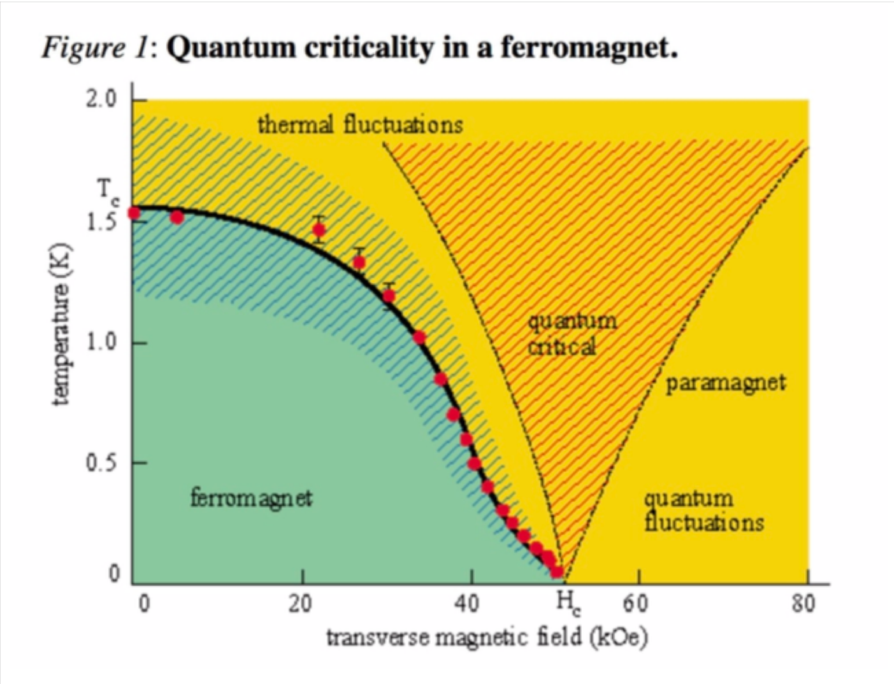
Rochelle salt is sodium potassium tartrate [$KNaC_4H_4O_6 \cdot 4H_2O$], and has Curie temperatures at –18°C and 24°C. Between these temperatures it has a lower crystal symmetry than at lower T.

- In QFT Weinberg in 74' constructed a UV **incomplete** (3+1)D model which exhibits symmetry non-restoration to all orders in the weak coupling.

- These examples (Rochelle salt and Weinberg's construction) have higher symmetry at T=0. They reveal symmetry breaking for some intermediate temperature only.

Are there UV complete, unitary, local, relativistic QFTs which exhibit symmetry non-restoration at all temperatures (persistent symmetry breaking)?

- To answer this question it is natural to consider CFTs. Such theories are UV complete and represent the UV end of the RG flow in the asymptotically safe QFTs. The question is therefore clear: consider a CFT in d+1 dimensions and turn on some T. Can symmetry breaking take place?



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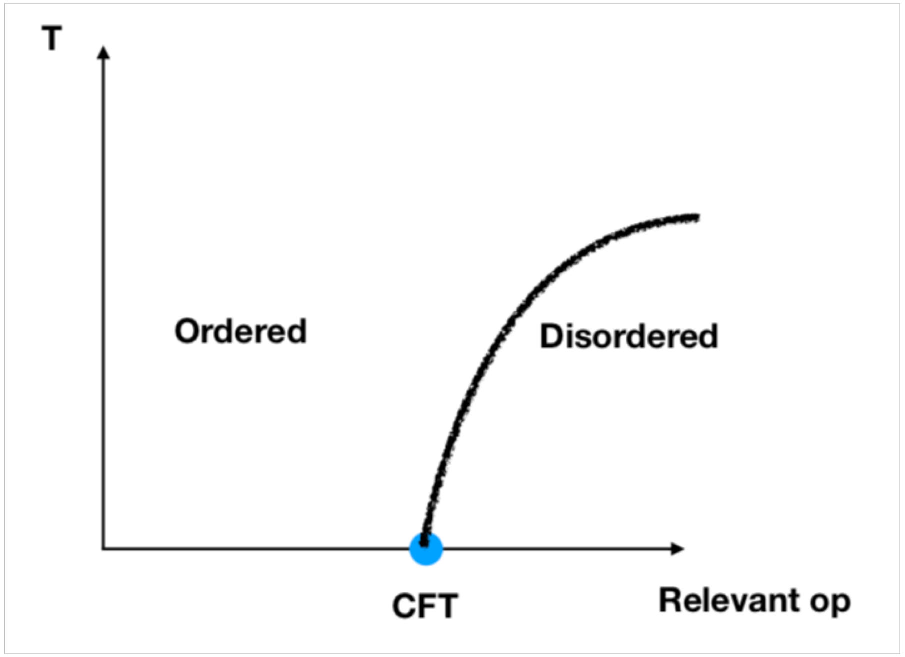

Ising ferromagnet LiHoF4 (Bitko et. al.)

- There are related constructions in AdS/CFT (e.g., see recent papers by A. Buchel on hairy BH), but in these constructions, one gets

✓ symmetric phase is preferred

✓ symmetry broken phase is subdominant in the canonical ensemble, but it is perturbatively stable at linear and nonlinear orders.

We build a class of nonlocal CFTs, which have some of its internal symmetries broken at arbitrary finite temperature.

The model:

- Start from the gaussian theory (GFF) in d Euclidean dimensions

$$S_0 = \mathcal{N}_\phi \int d^d x_1 \int d^d x_2 \frac{\vec{\phi}(x_1)\cdot\vec{\phi}(x_2)}{|x_1 - x_2|^{d+\eta_\phi}} + \mathcal{N}_\sigma \int d^d x_1 \int d^d x_2 \frac{\sigma(x_1)\sigma(x_2)}{|x_1 - x_2|^{d+\eta_\sigma}}$$

$$\Delta_\phi = \frac{d-\eta_\phi}{2}\,,\ \Delta_\sigma = \frac{d-\eta_\sigma}{2}$$

$$\eta_\phi = \tfrac{d+\epsilon_1}{2} \text{ and } \eta_\sigma = \tfrac{d+\epsilon_3}{2} \text{ with } \epsilon_{1,3} \ll 1,$$

- For this choice of the scaling dimensions there are three Quartic operators which become weakly relevant

$$\mathcal{O}_1 = (\phi^2)^2\ , \mathcal{O}_2 = \phi^2\sigma^2\ , \mathcal{O}_3 = \sigma^4$$

- Consider the following deformation of the gaussian theory

$$S = S_0 + \sum_{i=1}^{3} \frac{g_i \mu^{\epsilon_i}}{N} \int d^d x\, \mathcal{O}_i(x)$$

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$$\Delta_\phi = \frac{d - \eta_\phi}{2} \, , \ \Delta_\sigma = \frac{d - \eta_\sigma}{2}$$

$$\eta_\phi = \tfrac{d+\epsilon_1}{2} \text{ and } \eta_\sigma = \tfrac{d+\epsilon_3}{2} \text{ with } \epsilon_{1,3} \ll 1,$$

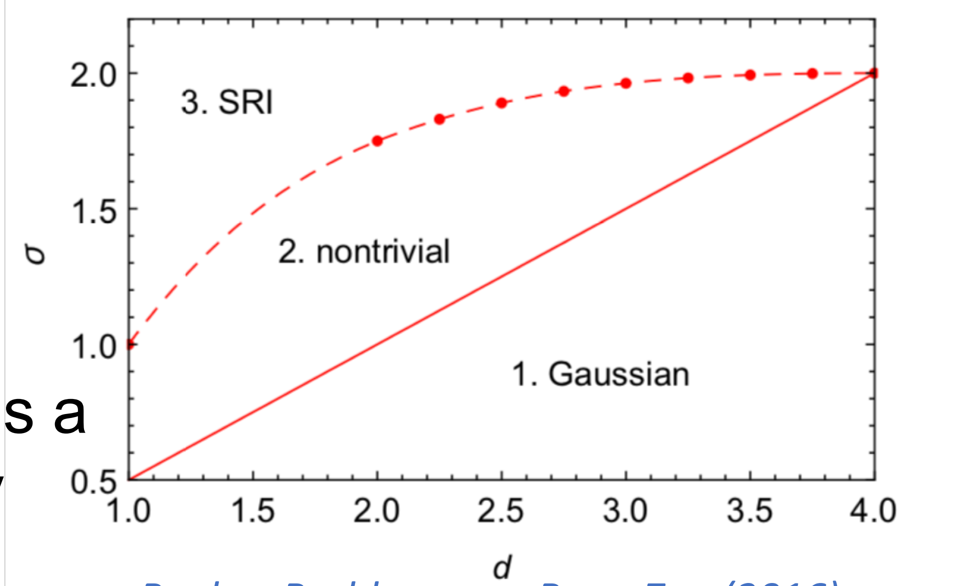
It has $O(N) \times Z2$ with $\vec{\phi}$ transforming in the vector representation of the $O(N)$. In 3+1 this model has only Gaussian fixed point – trivial. Below d=3+1 it has interacting Wilson-Fisher fixed points - these are CFTs of interest. The focus is on d=2+1=3 dimensions.

Motivation for deformed GFF?

AdS/CFT, Long Range Ising model

- Just like SRI, the LRI has a 2nd order phase transition at a critical temperature.

$$H_{\mathrm{LRI}} = -J \sum_{i,j} s_i s_j / r_{ij}^{d+\sigma}$$



Paulos, Rychkov, van Rees, Zan (2016)

- For $\sigma < d/2$ the critical point is a gaussian theory described by a nonlocal action

$$S_0 = \int \mathrm{d}^d x\, \mathrm{d}^d y\, \phi(x)\phi(y)/|x-y|^{d+\sigma}$$

$$\Delta_\phi = (d-\sigma)/2$$

- For $d/2 < \sigma < \sigma_*$ the critical theory can be obtained by perturbing the above action by a local quartic interaction

What are the critical points of the model?

$$S = S_0 + \sum_{i=1}^{3} \frac{g_i \mu^{\epsilon_i}}{N} \int d^d x \, \mathcal{O}_i(x)$$

$$\mathcal{O}_1 = (\phi^2)^2 \, , \mathcal{O}_2 = \phi^2 \sigma^2 \, , \mathcal{O}_3 = \sigma^4$$

- The quartic interactions induce an RG flow of the form

$$\mu\frac{dg_i}{d\mu} = -\epsilon_i g_i + \frac{\pi^{d/2}}{N\Gamma\left(\frac{d}{2}\right)}\sum_{j,k} C^i_{jk} g_j g_k$$

$$\mathcal{O}_i(0)\mathcal{O}_j(x) \supset \frac{C^k_{ij}\mathcal{O}_k(0)}{|x|^{\Delta_i+\Delta_j-\Delta_k}}$$

- The critical couplings satisfy

$$\begin{aligned}
\epsilon_1 g_1 &= \frac{\pi^{d/2}}{N\Gamma\left(\frac{d}{2}\right)}\left(C^1_{11} g_1^2 + C^1_{22} g_2^2\right) \ , \\
\epsilon_2 g_2 &= \frac{\pi^{d/2}}{N\Gamma\left(\frac{d}{2}\right)}\left(2\, C^2_{12}\, g_1 g_2 + C^2_{22} g_2^2 + 2\, C^2_{23} g_2 g_3\right) \ , \\
\epsilon_3 g_3 &= \frac{\pi^{d/2}}{N\Gamma\left(\frac{d}{2}\right)}\left(C^3_{22}\, g_2^2 + C^3_{33}\, g_3^2\right) \ .
\end{aligned}$$

- As long as the f.p. equations are satisfied the potential is always bounded from below, see Halperin 19’

- There are many solutions to the above system of quadratic equations. Some of them are trivial or not interesting or complex.

- Given a fixed point one can calculate finite T effects by putting the model on a thermal cylinder $S^1 \times \mathbb{R}^{d-1}$

- Calculate the effective potential for the zero mode to understand the symmetry pattern of the critical model

$$f(\phi,\sigma;\beta) = \mathcal{M}_\phi(\beta)\phi^2 + \mathcal{M}_\sigma(\beta)\sigma^2 + \frac{g_1\mu^{\epsilon_1}}{N}(\phi^2)^2 + \frac{g_2\mu^{\epsilon_2}}{N}\phi^2\sigma^2 + \frac{g_3\mu^{\epsilon_3}}{N}\sigma^4 + \mathcal{O}(\epsilon_i^2)\ ,$$

where

$$\mathcal{M}_\phi(\beta) = 4g_1\mu^{\epsilon_1}\left(1+\frac{2}{N}\right)\frac{\zeta(2\Delta_\phi)}{\beta^{2\Delta_\phi}} + 2\frac{g_2\mu^{\epsilon_2}}{N}\frac{\zeta(2\Delta_\sigma)}{\beta^{2\Delta_\sigma}}\ ,$$

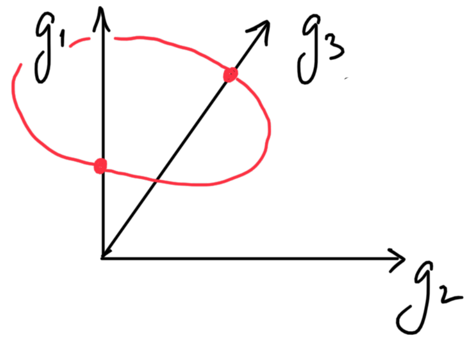
$$\mathcal{M}_\sigma(\beta) = 2g_2\mu^{\epsilon_2}\frac{\zeta(2\Delta_\phi)}{\beta^{2\Delta_\phi}} + 12\,\frac{g_3\mu^{\epsilon_3}}{N}\frac{\zeta(2\Delta_\sigma)}{\beta^{2\Delta_\sigma}}\ .$$

- For N>10 the coupling $g_2 < 0$, for N>17 the quadratic terms are not positive definite $\mathcal{M}_\sigma < 0$, the symmetry breaks !!

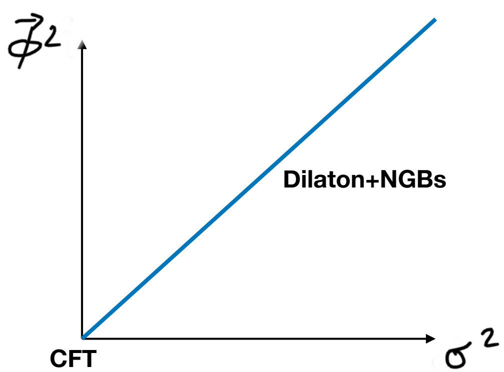
$$O(N)\times\mathbb{Z}_2 \quad\rightarrow\quad O(N)$$

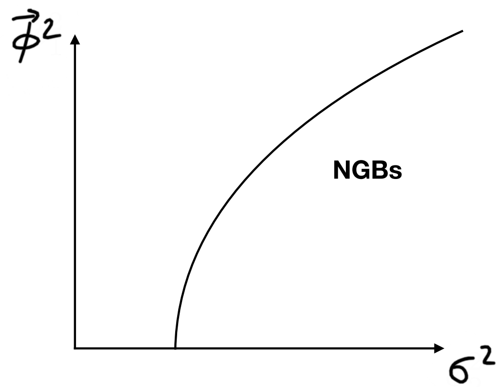
Higher order perturbative corrections cannot restore the symmetry, because multiloop quadratic terms are suppressed in epsilon, whereas terms with higher powers of the fields are subdominant in the vicinity of origin.

- Large-N analysis can be done. One finds a line of f.p.



- For each f.p. on the conformal manifold there is a moduli space of vacua (hyperbola at finite T) etc.





- At 1/N all these disappear, but persistent symmetry breaking remains.

Are there local models of the same kind?

- CHMW theorem protects continuous symmetries in 3D. However, there is a candidate for discrete symmetry breaking in 4-epsilon dimensions *Chai, Chaudhuri, Choi, Komargodski, Rabinovici, MS (2020)*

$$H = \frac{1}{2}\partial\vec{\phi}\,\partial\vec{\phi} + \frac{1}{2}\partial\sigma\partial\sigma + \frac{g_1}{4N}\sigma^4 + \frac{g_2}{4N}\left(\vec{\phi}^2\right)^2 + \frac{g_3}{2N}\sigma^2\left(\vec{\phi}\right)^2$$

- O(1) symmetry breaks in this model for sufficiently high T, but it is hard to prove that symmetry breaking pattern survives in 3D.

Conclusions

- We provided examples of CFTs, including 2+1, where a global $O(m)\times O(N-m)$ symmetry is broken at ***any*** finite T.

- The large N limit (with m/N fixed) of this models exhibits a line of WF fixed points, and for each fixed point there is a moduli space of vacua (straight line at T=0 is deformed to a hyperbola at finite temperature)

- We considered both large-N limit and epsilon expansion and found full agreement. At finite N, however, conformal manifold and moduli space of vacua disappear.

- The **local** examples live in a fractional space-time dimension, so are there similar constructions in 3+1? For infinite N there is an example in 3+1, finite N?
  Chaudhuri, Choi, Rabinovici (2020) Chaudhuri, Rabinovici (2021)

- In 2+1 dimensions continuous symmetry cannot be broken in a local model, but what about discrete symmetry breaking? We propose a candidate.

- What about adding fermions, gauge fields etc. in 2+1?
  Thermally induced Quantum Hall effect?

The End

Thank You!