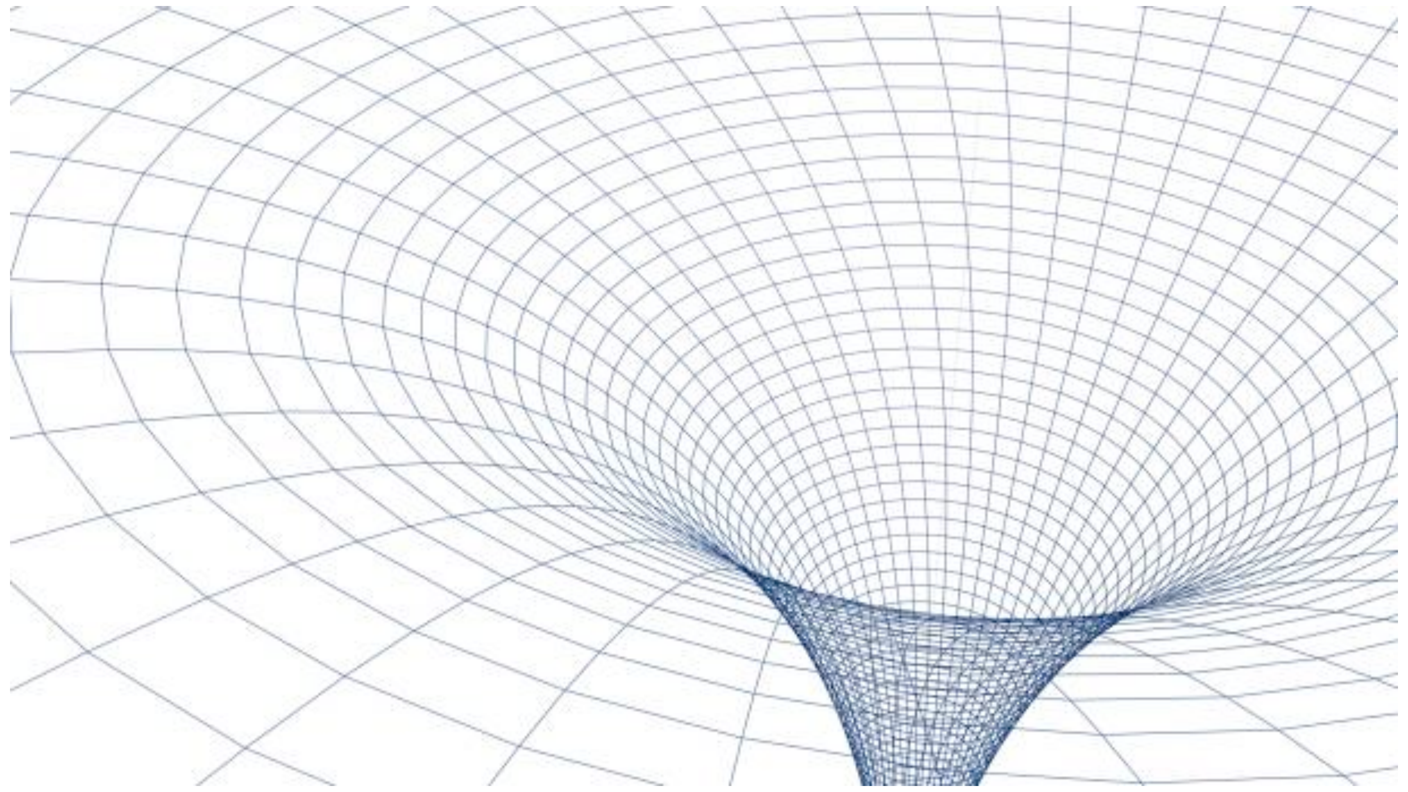
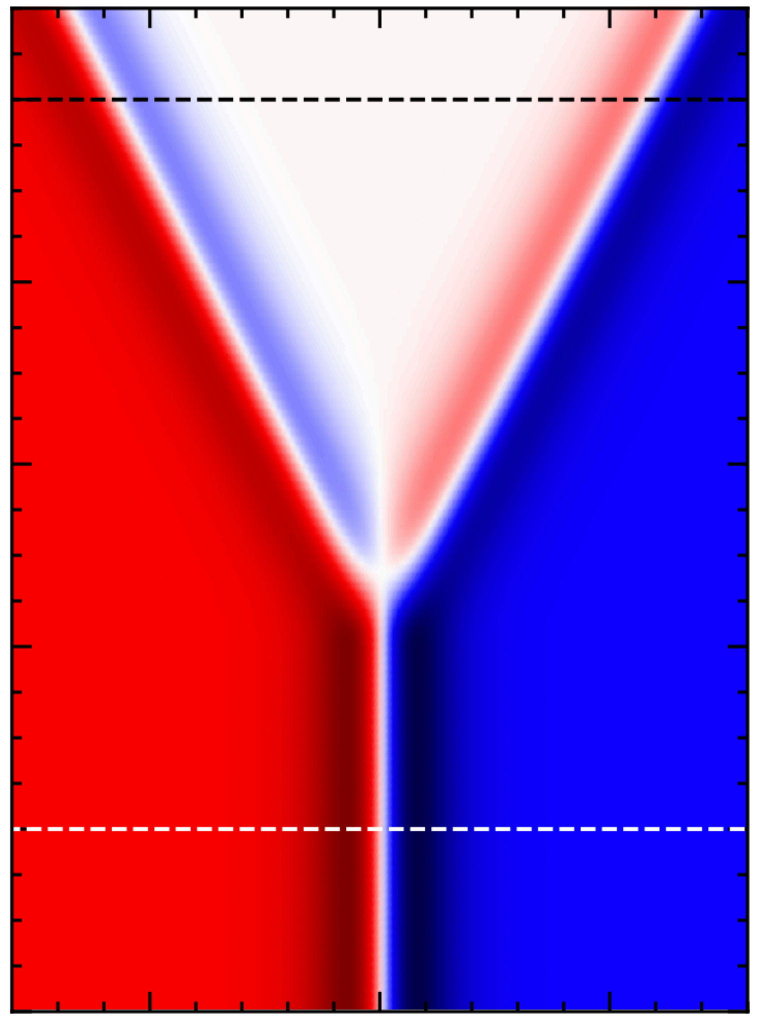
Anomalous Luttinger relation for energy transport: From black hole's atmosphere to thermal quenches





Baptiste Bermond

Collab.

David Carpentier (ENS de Lyon)
Adolfo Grushin (Néel Institute, Grenoble)
Maxim Chernodub (CNRS, LMPT, Tours)

Lab PHYS ENS de LYON

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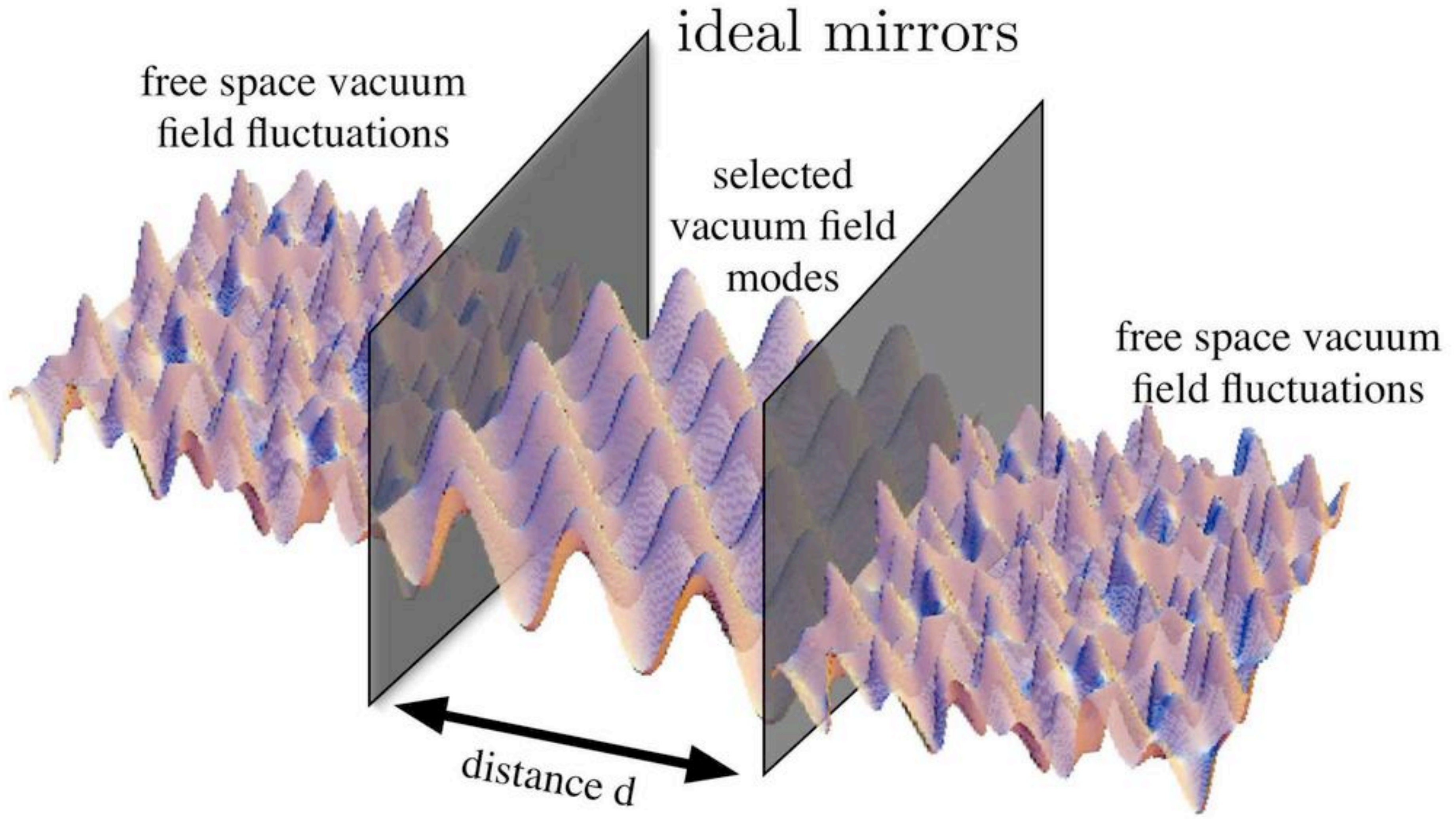
When geometry affects quantum fluctuations

- **Geometrical confinement** can change **the vacuum properties**

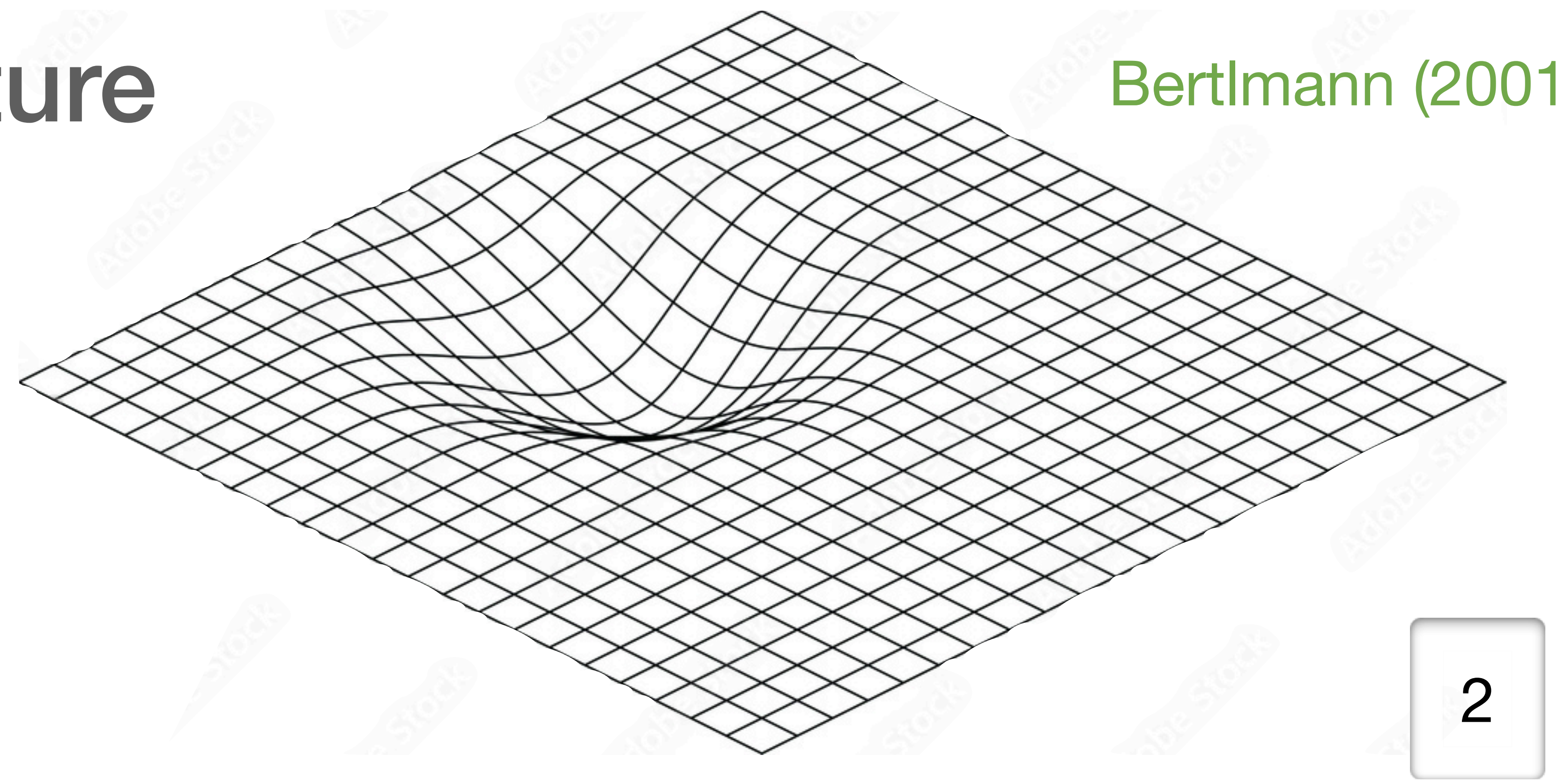
⇨ Casimir effect



Casimir (1948)
Ruser (2007)

- **Here: anomalous fluctuations** induced by **spacetime curvature**

⇨ Gravitational anomalies



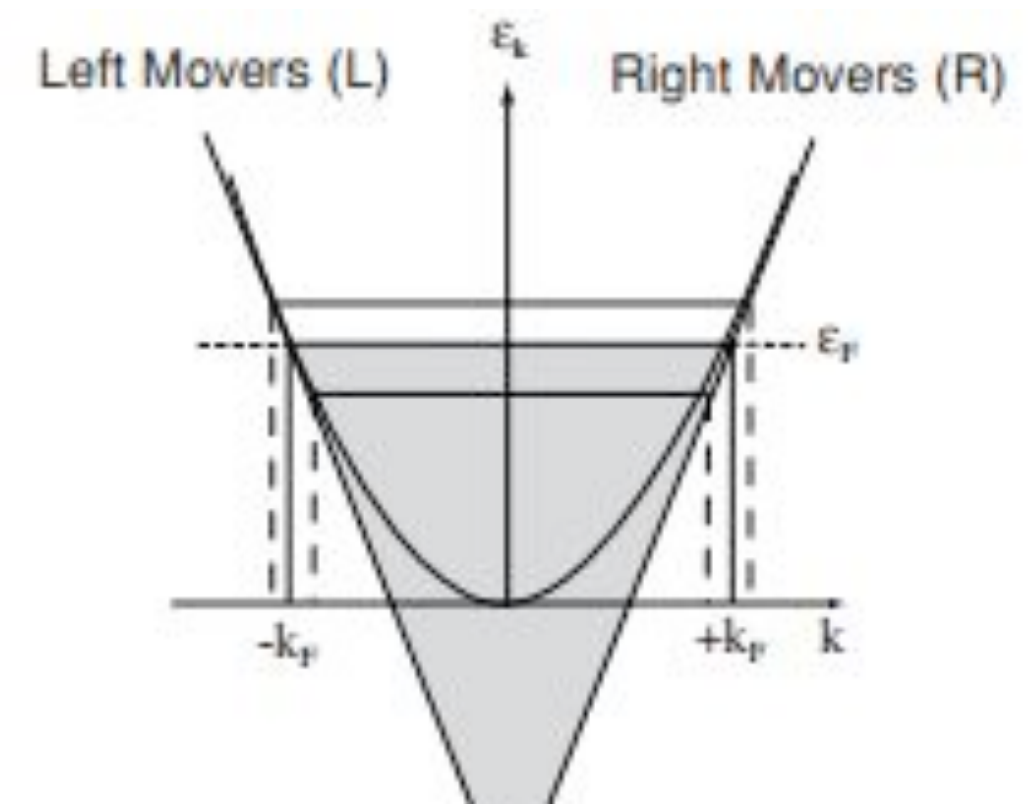
Bertlmann (2001)

Relativistic materials: playground for exotic relativistic quantum effects

- Linear dispersion relation

D=1+1

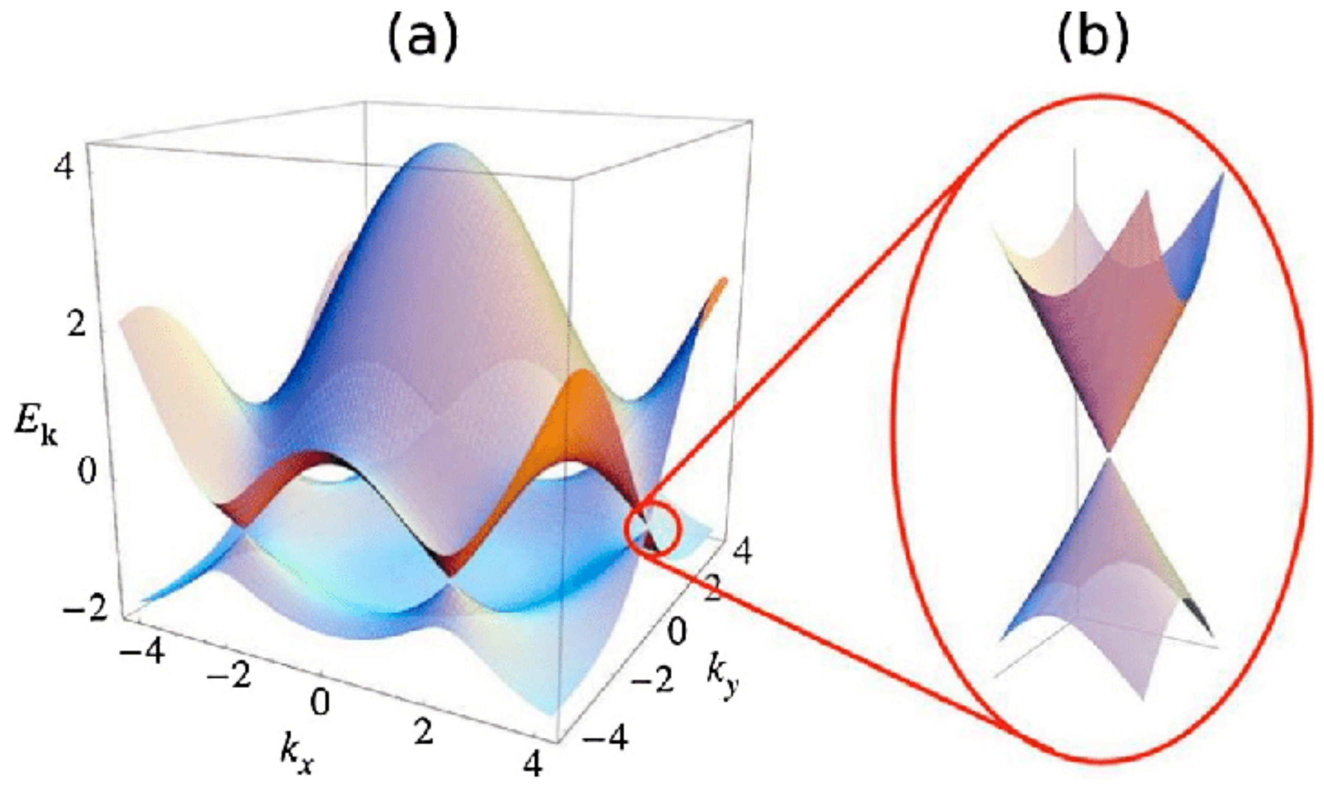
Luttinger liquid



Schonhammer (2003)

D=2+1

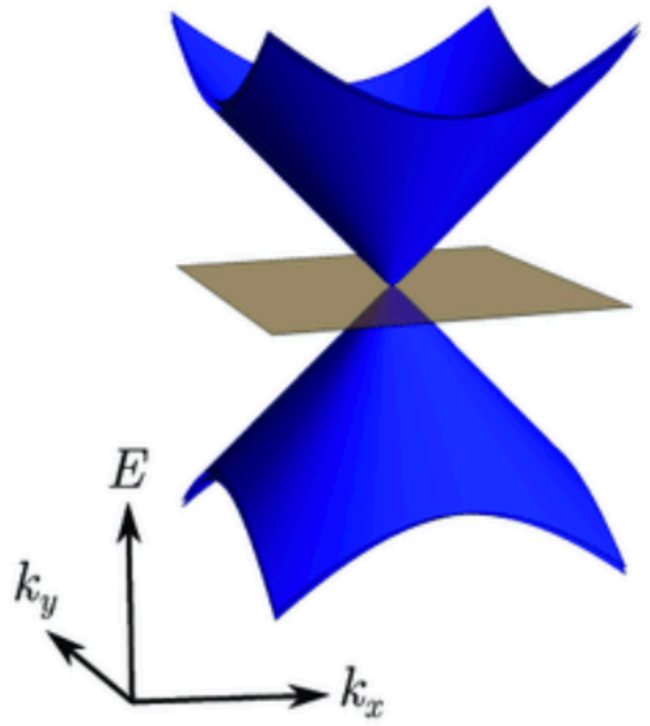
Graphene



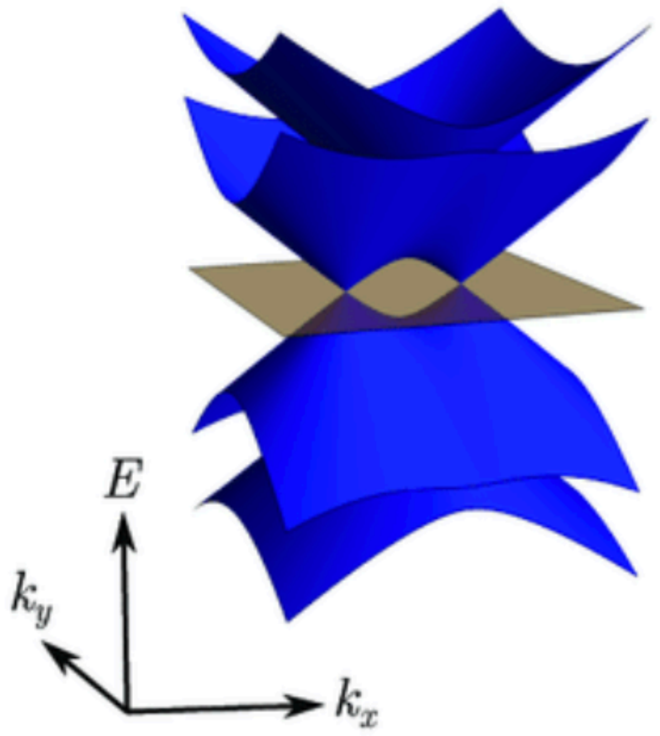
Zhao et al. (2013)

D=3+1

Dirac semimetals



Weyl semimetals



Alexander Lau (2018)

⇨ **Ideal platforms to test** sizable consequences of **gravitational anomalies**

Objectives:

- Revisit relation **Temperature ↔ curved spacetime** in presence of **gravitational anomalies**
- **Measurable effects** : black holes and **condensed matter** ?

Plan:

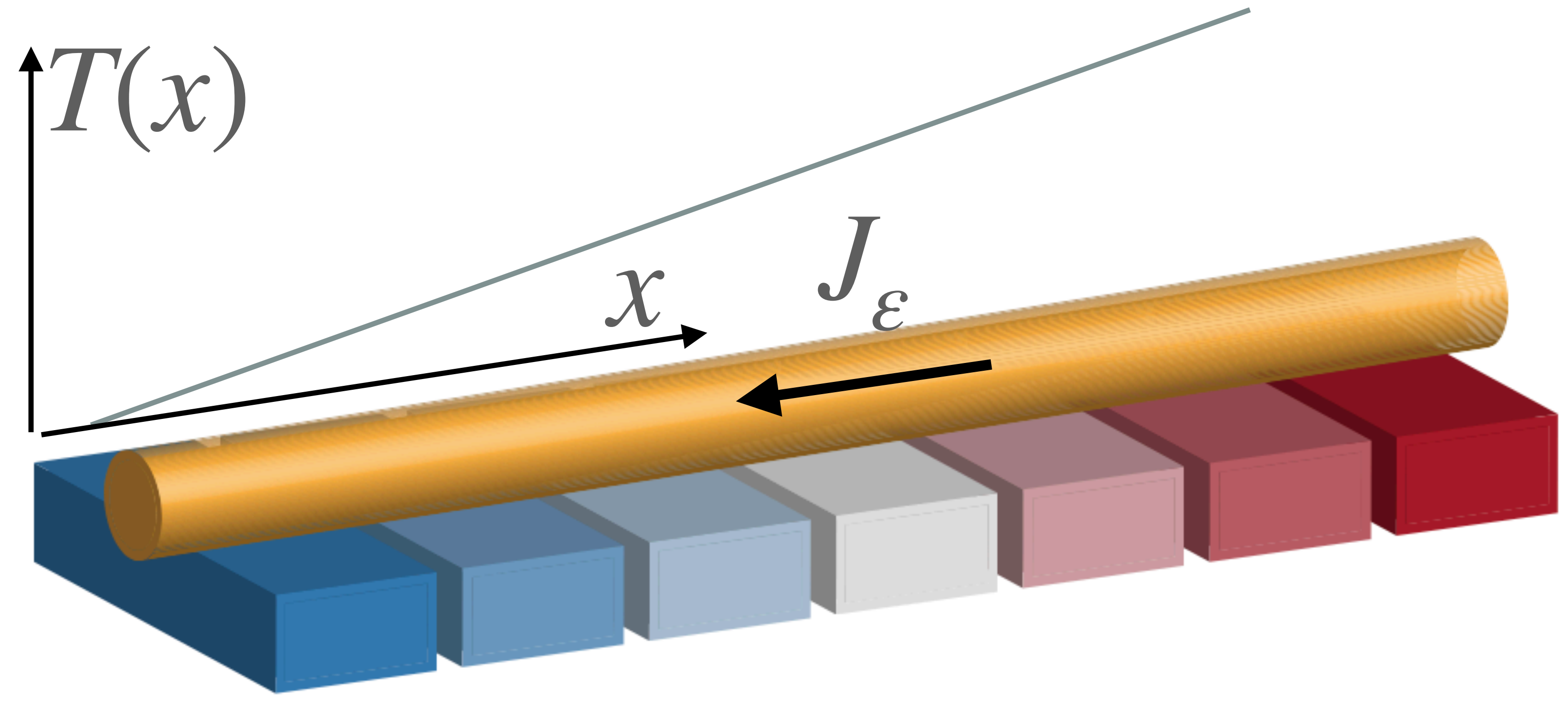
1. From Tolman-Ehrenfest theorem to the Luttinger trick
2. Gravitational corrections to Tolman-Ehrenfest theorem
3. Physical consequences

Thermal Response theory

(Linear) transport theory

- Charge and energy transport:

$$J = \sigma\, E \qquad\qquad J_\epsilon = \kappa\, \nabla T$$



Charge Transport

- Transported quantity: electric charge Q
- Coupling potential: ϕ $\quad (-\nabla\phi = E\,)$

$\Rightarrow$ $\delta H = Q\,\delta\phi$

Energy Transport

- Transported quantity: energy density ε
- What is the potential ?

Luttinger Trick (1964)

- Solution provided by **Luttinger** in 1964

PHYSICAL REVIEW VOLUME 135, NUMBER 6A 14 SEPTEMBER 1964

Theory of Thermal Transport Coefficients*

J. M. LUTTINGER
Department of Physics, Columbia University, New York, New York
(Received 20 April 1964)

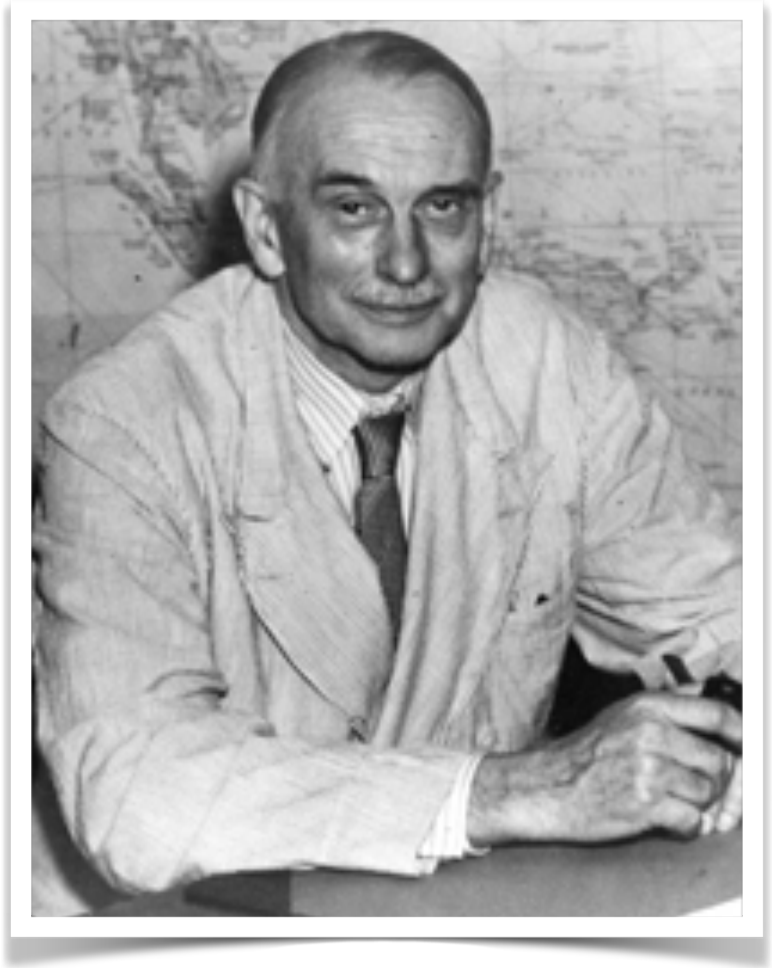
J.M.Luttinger

Energy Transport

- **Transported quantity:** energy density ε
- **Coupling potential:** gravitational potential ϕ_G , $-\nabla \phi_G = \dfrac{\nabla T}{T}$

$$\Rightarrow \quad \delta H = \varepsilon\, \delta\phi_G$$

… based on the Tolman-Ehrenfest theorem



R.C. Tolman

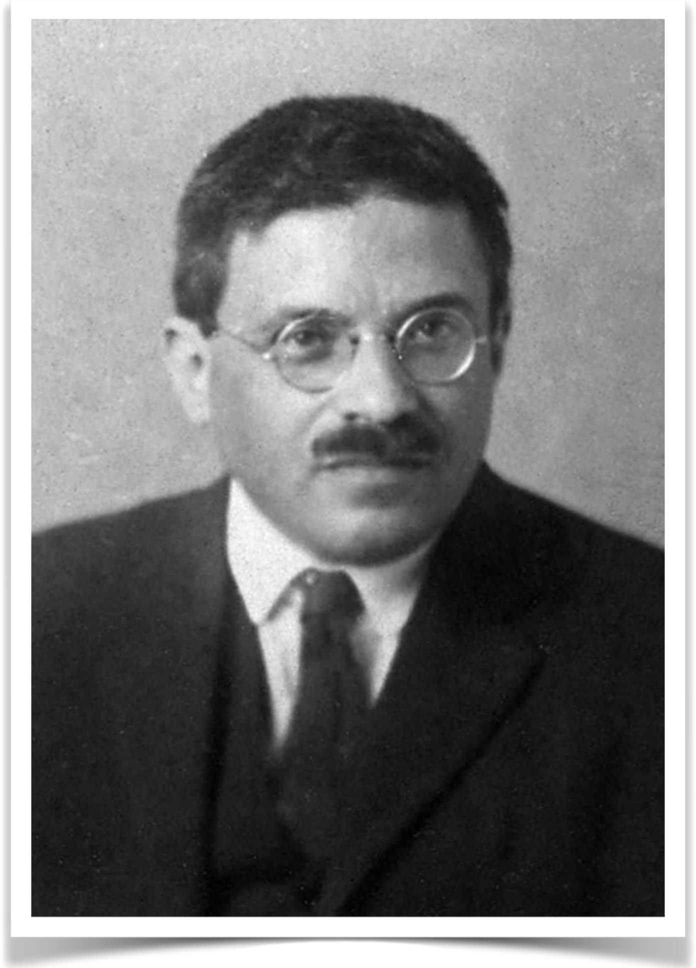
DECEMBER 15, 1930 PHYSICAL REVIEW VOLUME 36

TEMPERATURE EQUILIBRIUM IN A STATIC GRAVITATIONAL FIELD

By Richard C. Tolman and Paul Ehrenfest

Norman Bridge Laboratory of Physics, Pasadena, California

(Received October 27, 1930)



P. Ehrenfest

- Thermal equilibrium in a curved spacetime

Curved spacetime

$$ds^2 = g_{00}(x) v_F^2 \, dt^2 - dx^2$$

⇨

Inhomogeneous temperature

$$T(x) = T_0 \sqrt{\frac{g_{00}(x_0)}{g_{00}(x)}}\,, \quad T_0 = T\left(x_0\right)$$

… based on the Tolman-Ehrenfest theorem

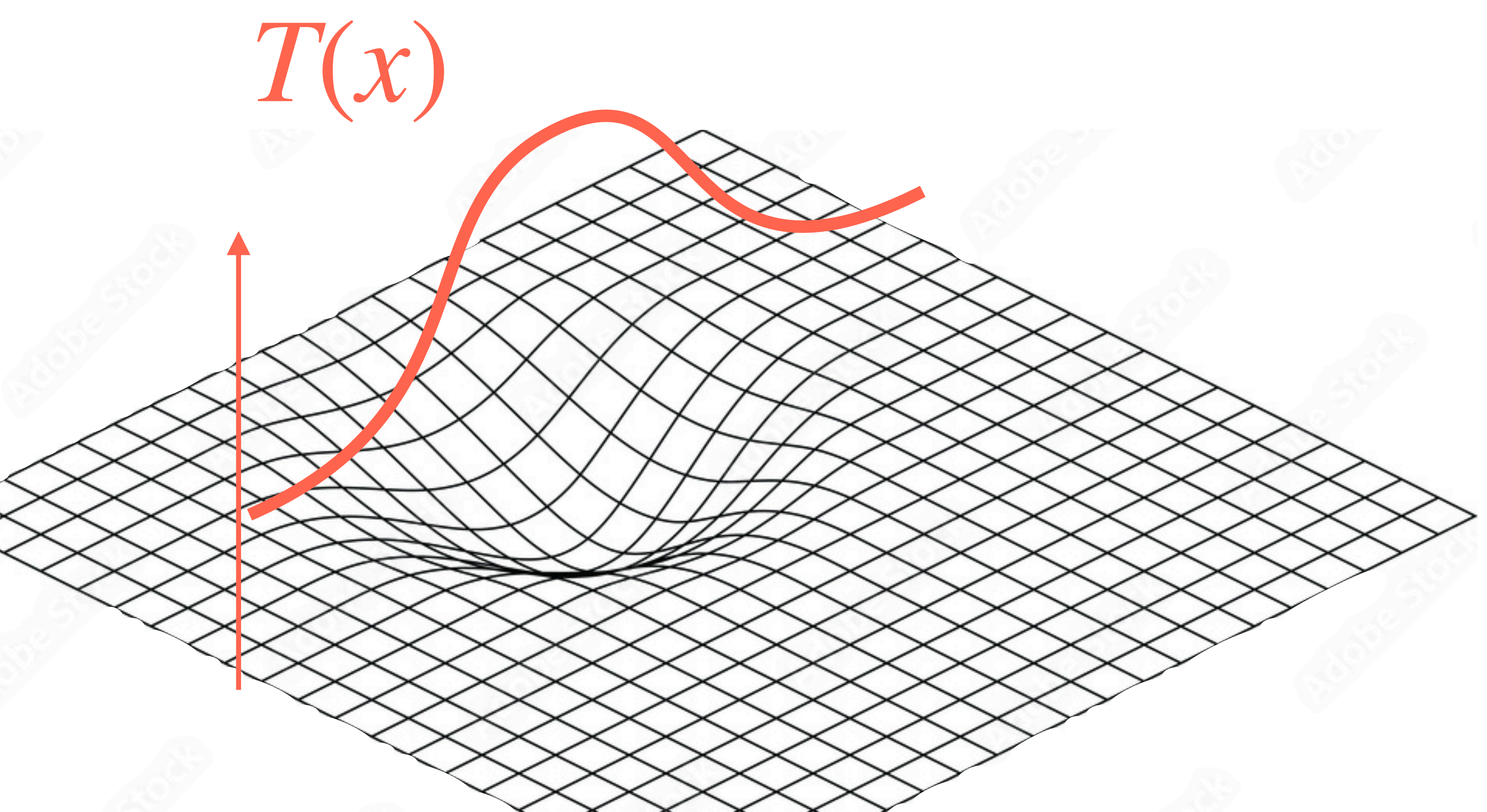
Tolman-Ehrenfest

Inhomogeneous $T(x)$

Curved Spacetime

$$ds^2 = \left(\frac{T_0}{T(x)}\right)^2 v_F^2\, dt^2 - dx^2$$

Equilibrium, $J_\varepsilon = 0$



… based on the Tolman-Ehrenfest theorem

Tolman-Ehrenfest

Inhomogeneous $T(x)$

Curved Spacetime

$$ds^2 = \left(\frac{T_0}{T(x)}\right)^2 v_F^2\, dt^2 - dx^2$$

Equilibrium, $J_\varepsilon = 0$

$=$

Inhomogeneous $T(x)$

Flat Spacetime

$$ds^2 = v_F^2\, dt^2 - dx^2$$

Out of Equilibrium, $J_\varepsilon \neq 0$

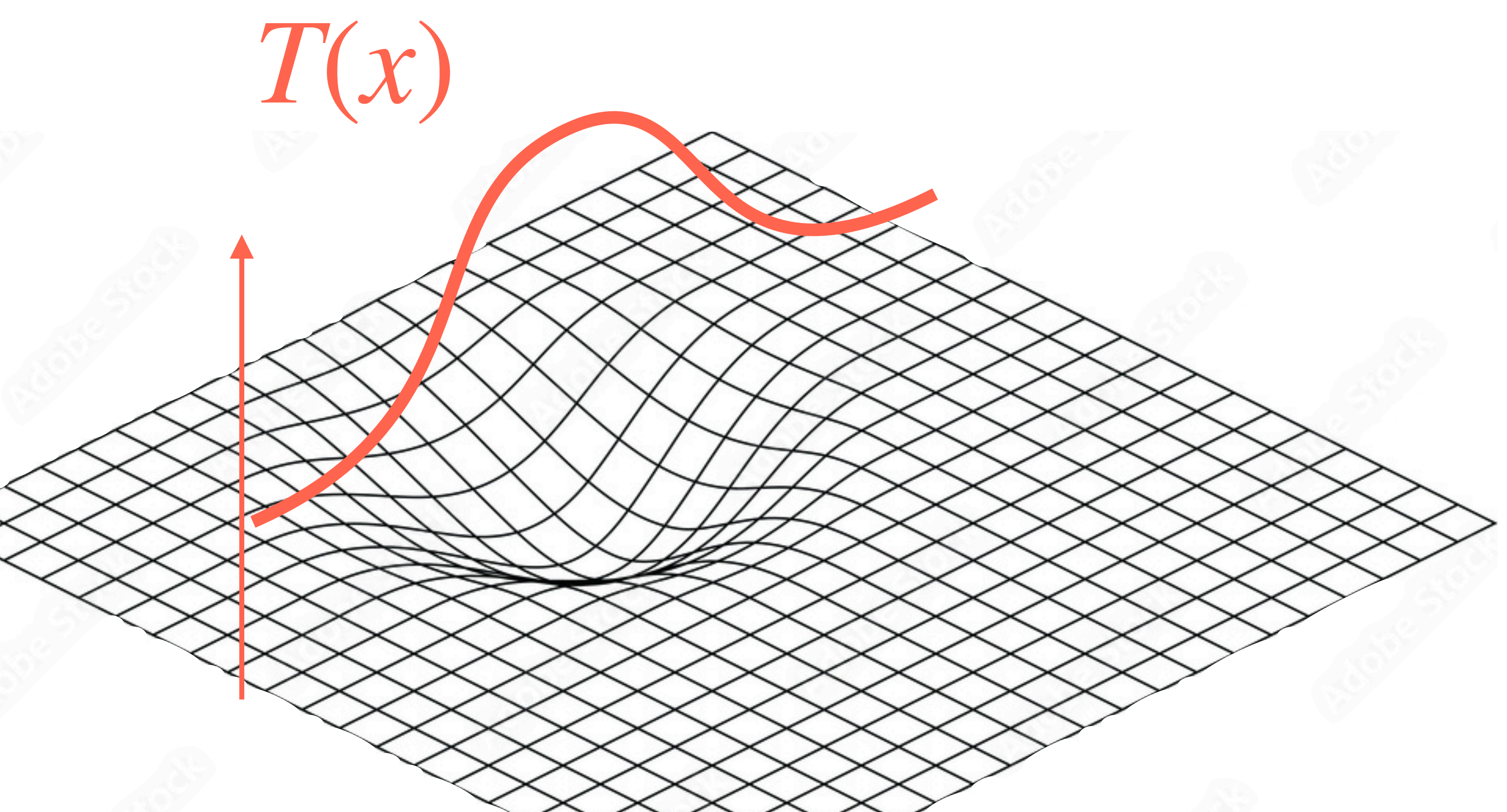
$+$

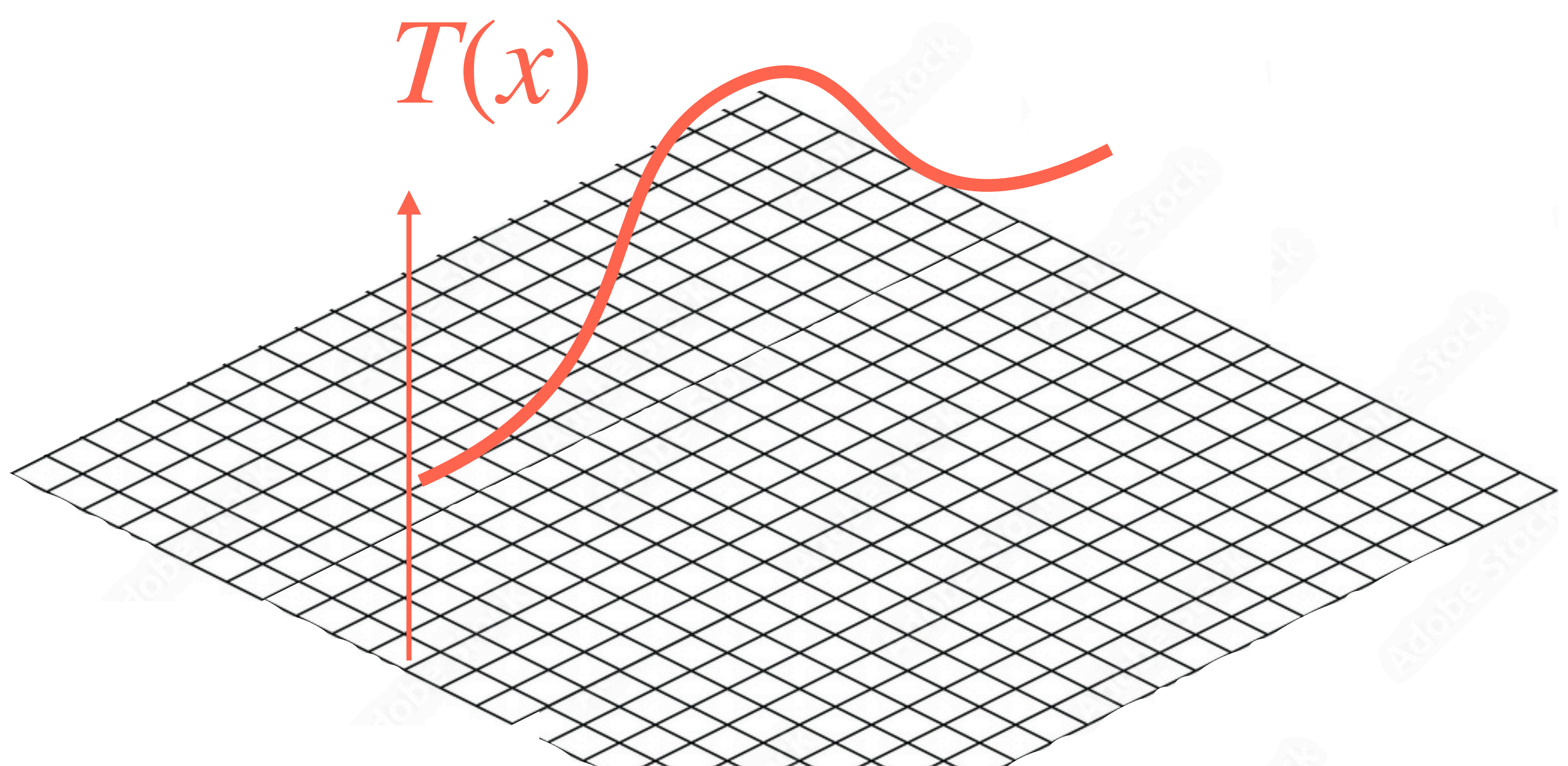
Homogeneous T_0

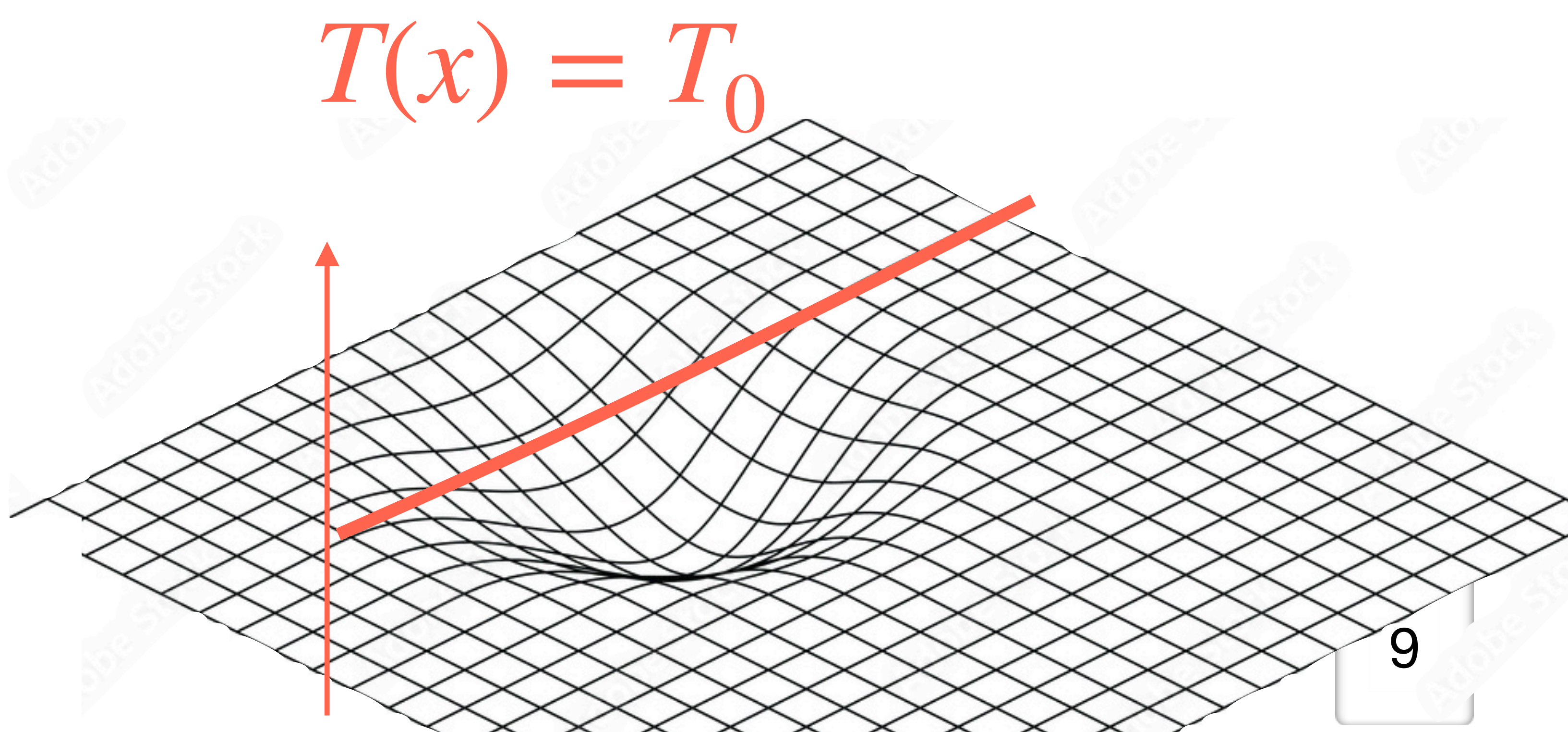
Curved Spacetime

$$ds^2 = \left(\frac{T_0}{T(x)}\right)^2 v_F^2\, dt^2 - dx^2$$

Out of Equilibrium, $-J_\varepsilon$







… based on the Tolman-Ehrenfest theorem

Luttinger equivalence

Inhomogeneous $T(x)$

Flat Spacetime

$$ds^2 = v_F^2\, dt^2 - dx^2$$

Out of Equilibrium, $J_\varepsilon \neq 0$

$\Leftrightarrow$

Homogeneous T_0

Curved Spacetime

$$ds^2 = \left(\frac{T_0}{T(x)}\right)^2 v_F^2\, dt^2 - dx^2$$

Out of Equilibrium, $-J_\varepsilon$

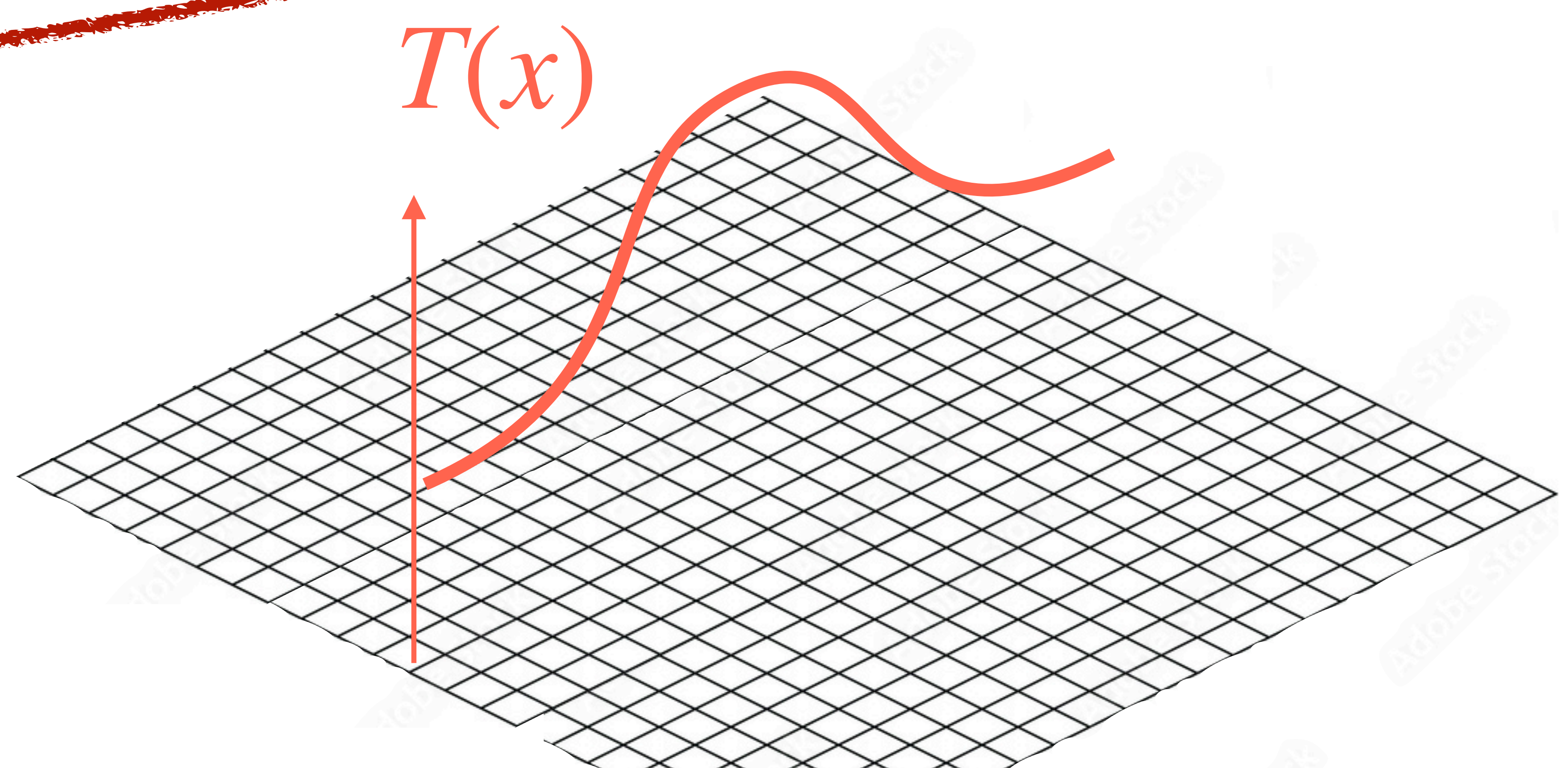
Luttinger trick

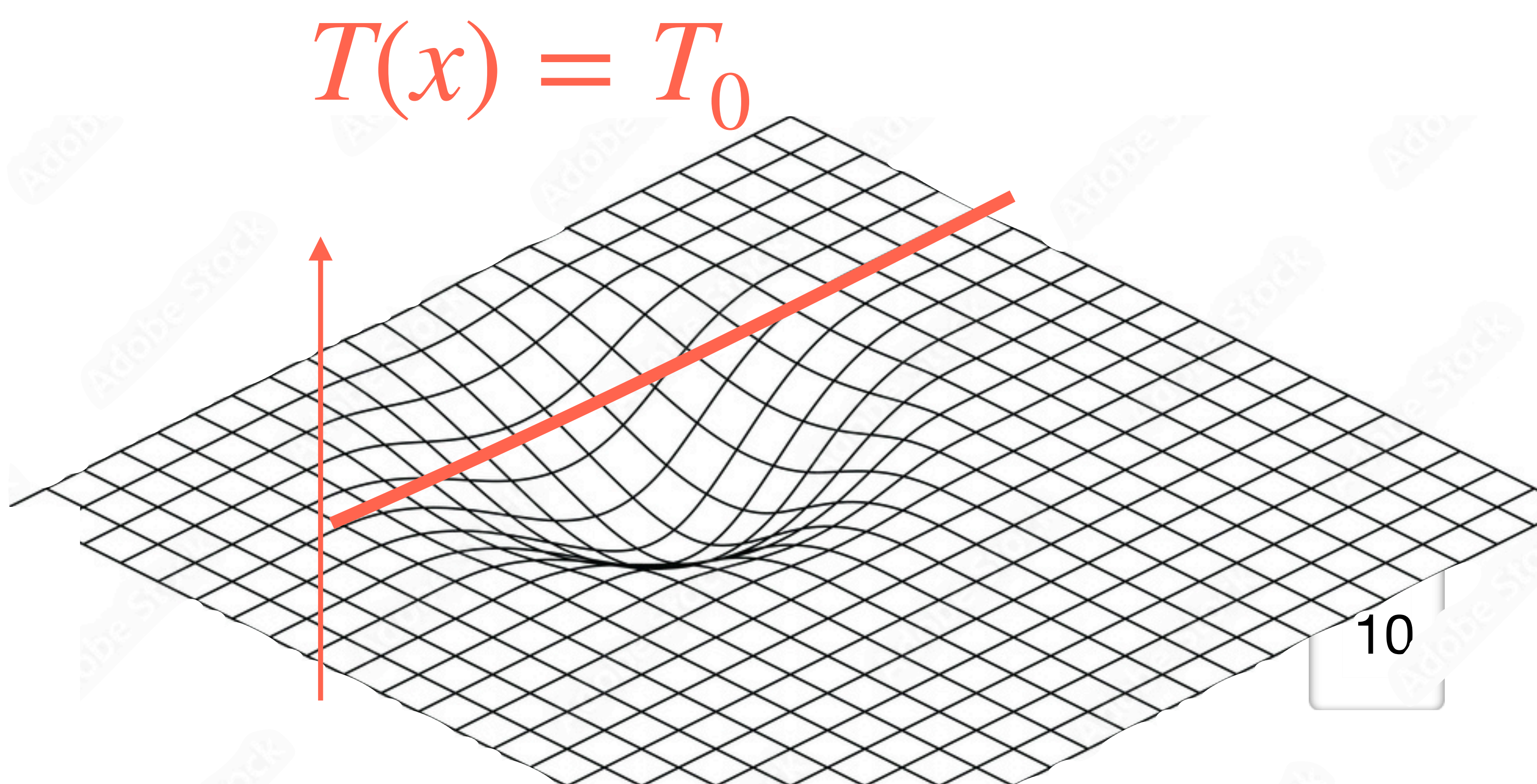
$$ds^2 = e^{2\phi(x)} v_F^2\, dt^2 - dx^2$$

with grav. potential

$$e^{\phi(x)} = \frac{T_0}{T(x)}$$

$$\partial_x \phi(x) = -\frac{\partial_x T(x)}{T(x)}$$





Probing an inhomogeneous temperature

How to probe the temperature profile in an inhomogeneous system?

Probing an inhomogeneous $T(x)$ via Stefan-Boltzmann Law

J. Stefan

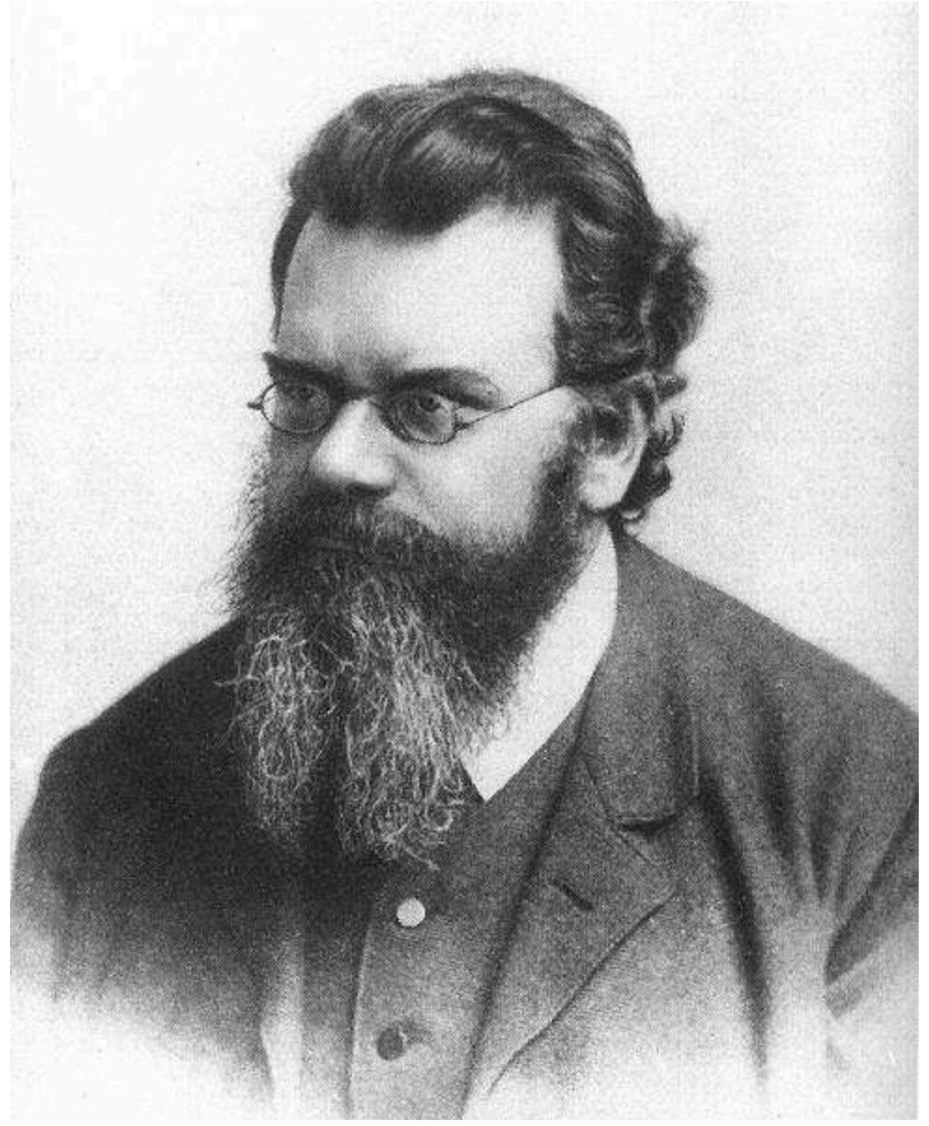
Define T through local energy density ϵ

Stefan Law (1879): Black body radiation

$\varepsilon \propto T^4$ in $d = 3$

J. Stefan (1879)

Probing an inhomogeneous $T(x)$ via Stefan-Boltzmann Law



L. Boltzmann

Define T through local energy density ϵ

Boltzmann derivation (1884):

L. Boltzmann (1884)

- Thermodynamics

$$\varepsilon dV = TdS - PdV \qquad \left.\frac{dS}{dV}\right|_T = \left.\frac{dP}{dT}\right|_V \qquad \varepsilon = T\frac{dP}{dT} - P$$

- Conformal symmetry in d+1Dimensions: $\varepsilon = d \, . \, P$

⇨

$\varepsilon \propto T^{d+1}$ and $P \propto T^{d+1}$

In d=1, $\varepsilon = P = \gamma_{1D} T^2, \ \gamma_{1D} = \dfrac{\pi k_B^2}{6 v_F}$

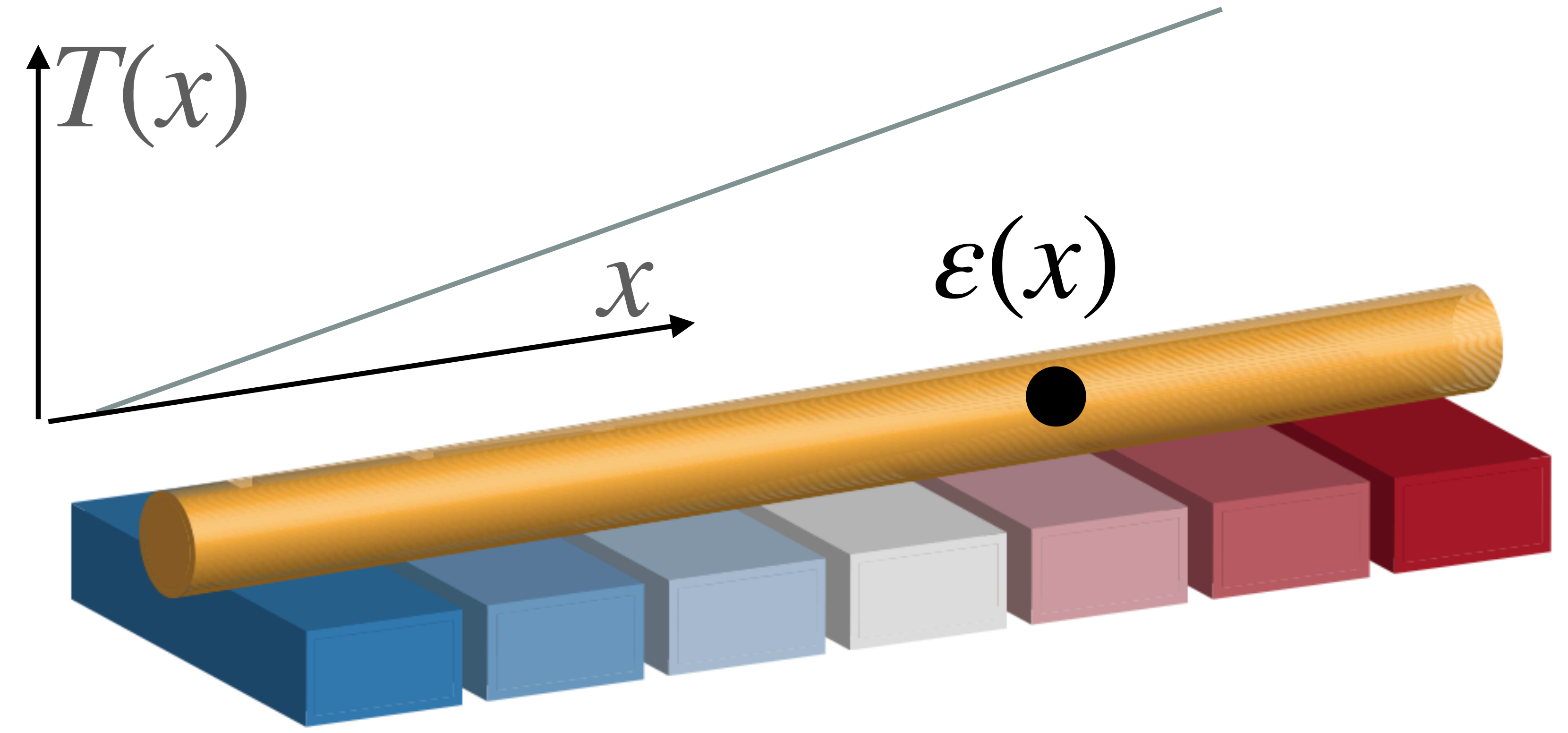
Probing an inhomogeneous $T(x)$ via Stefan-Boltzmann Law

Blöte et al. (1986)
Cardy (2010)

Generalized Stefan-Boltzmann law for interacting theories in D=1+1

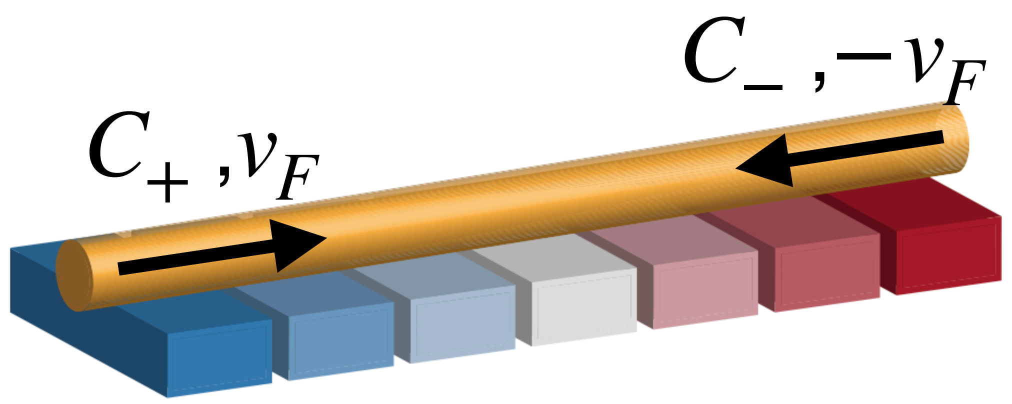
$$\varepsilon = P = \frac{C_+ + C_-}{2}\gamma_{1D}T^2, \qquad \gamma_{1D} = \frac{\pi k_B^2}{6\hbar v_F}$$

$C_w = (C_+ + C_-)/2$
Central charge of the theory



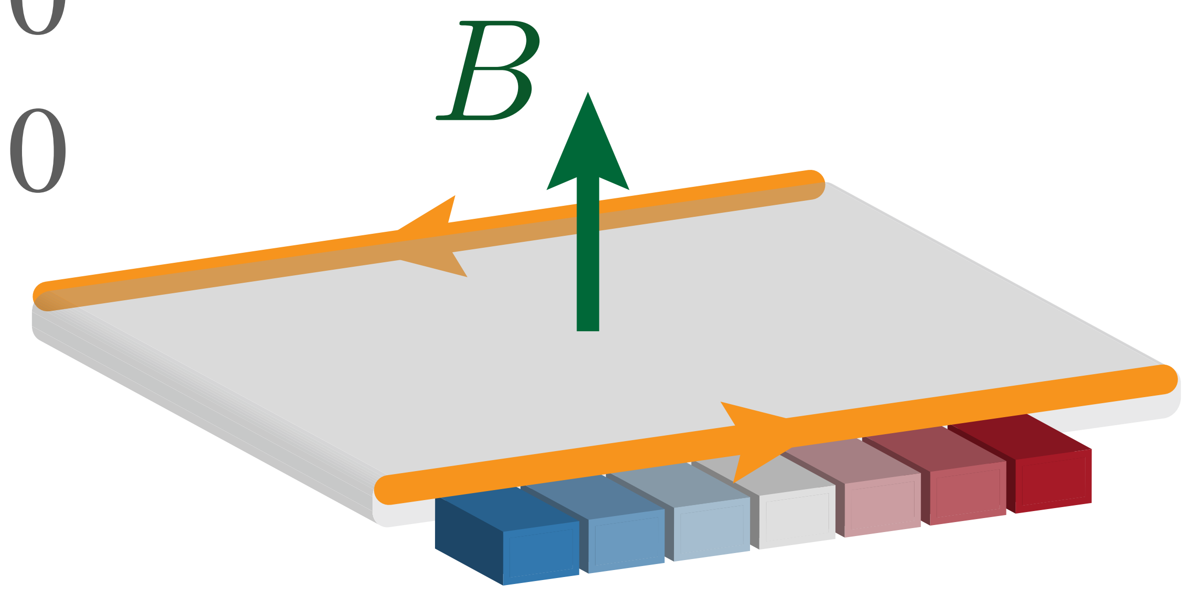
Examples :

- Interacting Quantum wires (Luttinger liquids) $C_+ = C_-$



- Chiral edge of a quantum Hall system

$C_+ \neq 0$
$C_- = 0$

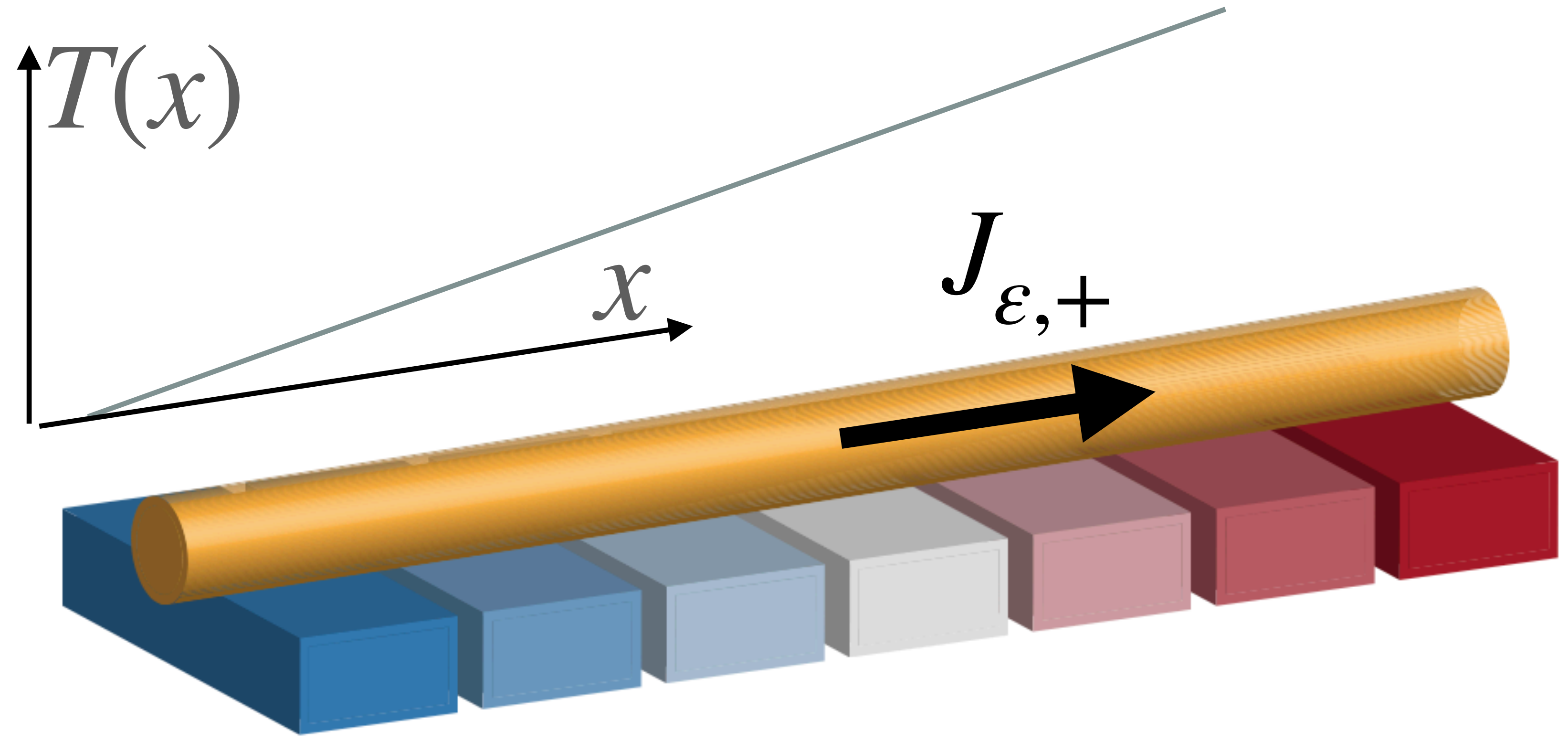


Probing an inhomogeneous $T(x)$ via energy current

- Focusing on **right movers**

$$\varepsilon = P = C_+ \gamma_{1D} T^2$$

$$J_\varepsilon = v_F\, \varepsilon = C_+ \frac{\pi}{6\hbar} k_B^2 T^2 \qquad \left(\Pi = \frac{1}{v_F}\varepsilon\right)$$



(Chiral) Energy currents : alternative probe for T^2

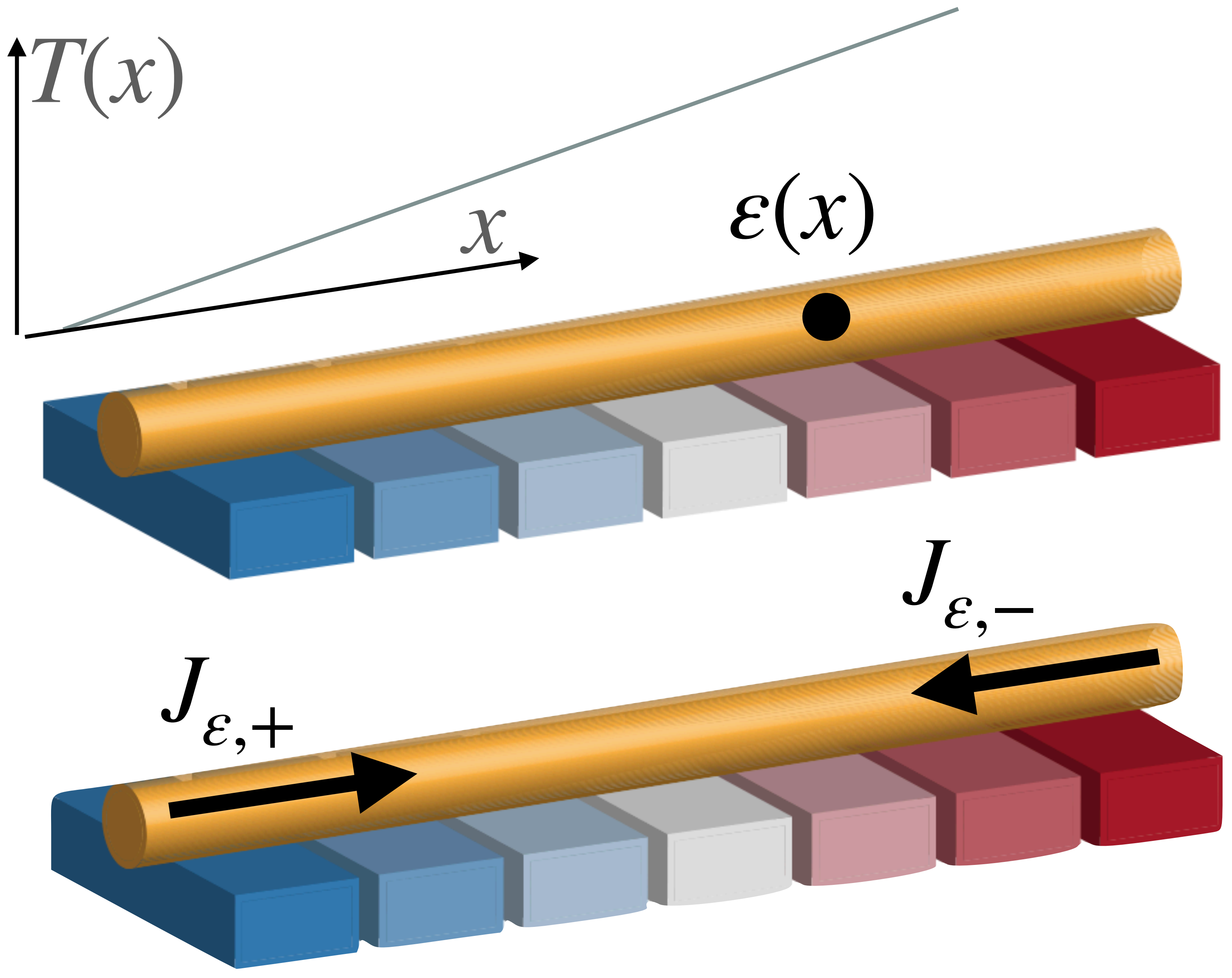
Probing an inhomogeneous $T(x)$

via Stefan-Boltzman law (density)

$$P = \varepsilon = \frac{1}{2}\overbrace{(C_+ + C_-)}^{C_w}\frac{\pi k_B^2}{6\hbar v_F}T^2$$

via transport (current)

$$J_\varepsilon = v_F^2\Pi = \overbrace{(C_+ - C_-)}^{C_g}\frac{\pi}{6\hbar}k_B^2 T^2$$



Probing an inhomogeneous $T(x)$

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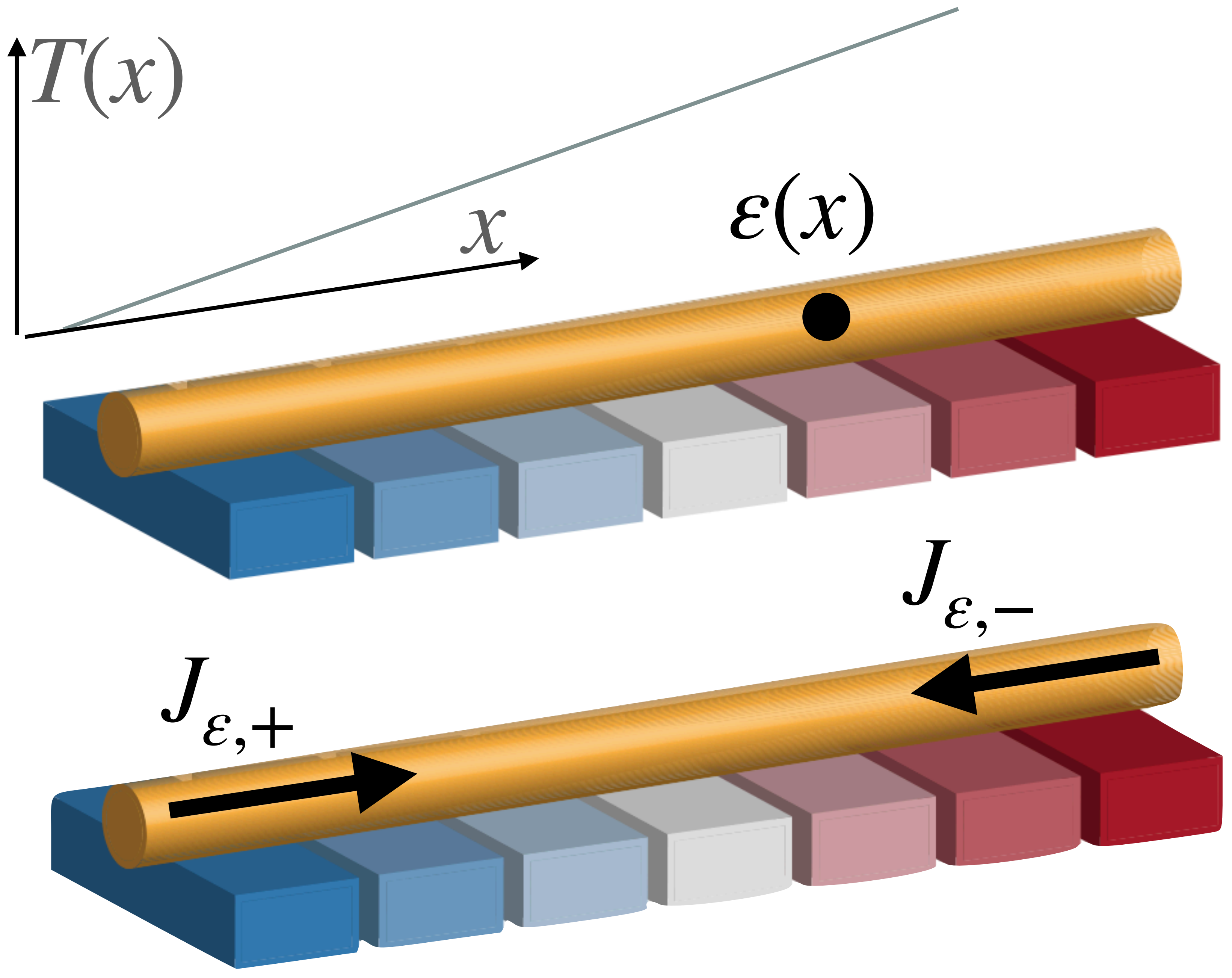


Convenient object: Stress-energy tensor

- In flat space, $\mathcal{T}^\mu_{\ \nu} = \begin{pmatrix} \varepsilon & \frac{1}{v_F}J_\varepsilon \\ -v_F\Pi & -P \end{pmatrix}$

Inhomogeneous temperature in curved D=1+1 spacetime

Static spacetimes in 1+1 dimensions

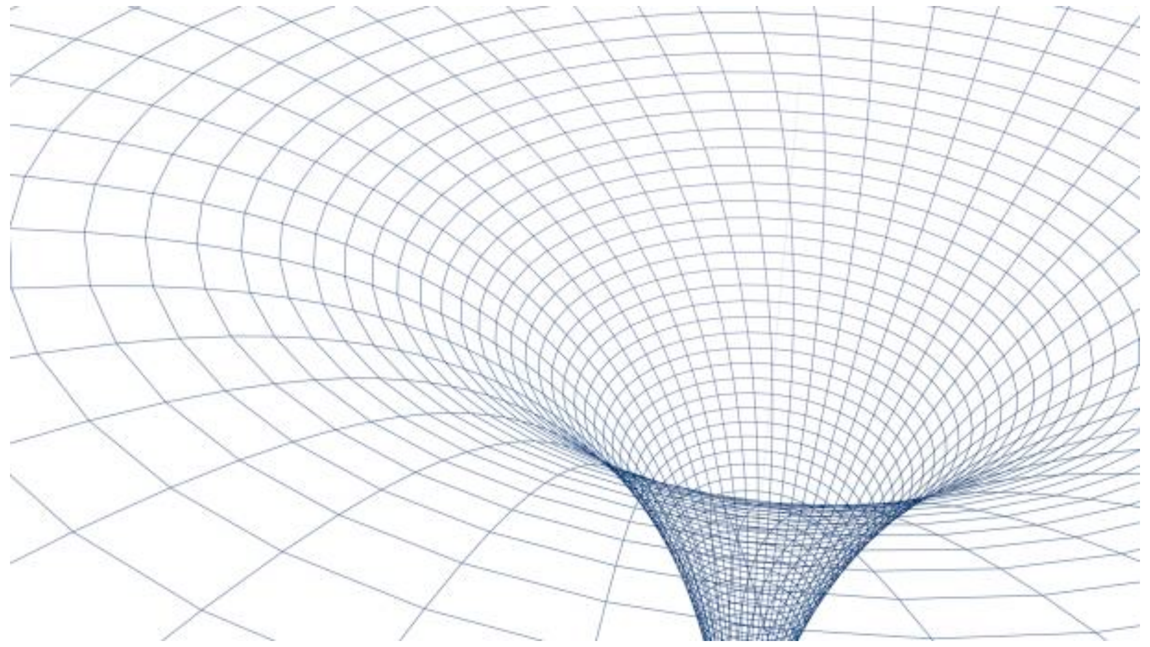
$$g_{\mu\nu} = \begin{pmatrix} f_1(x) & 0 \\ 0 & -f_2(x) \end{pmatrix}$$

$$ds^2 = f_1(x)\, v_F^2\, dt^2 - f_2(x)\, dx^2$$

Examples :

- Schwarzschild Black-hole:

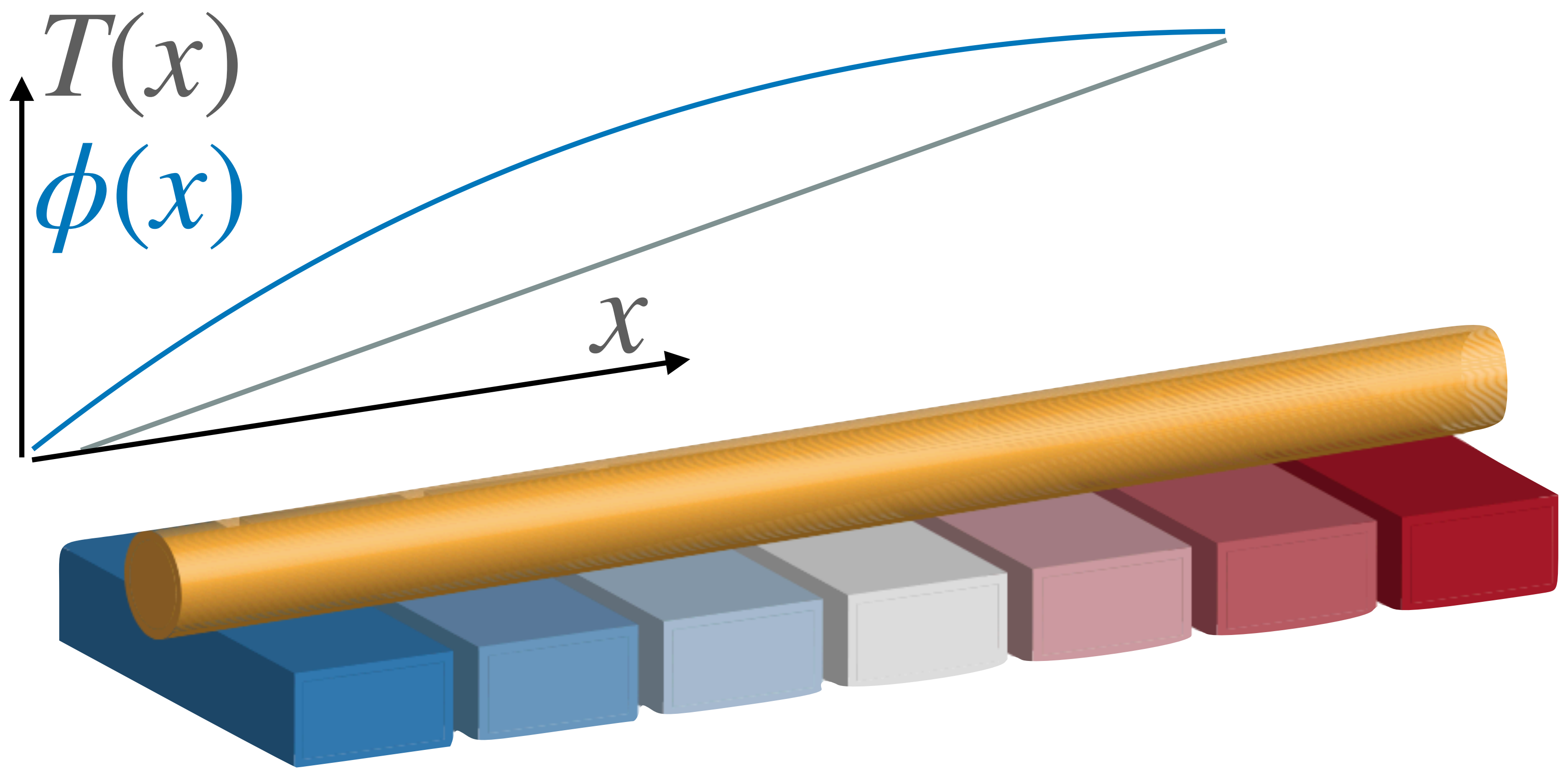
$$f_1(x) = 1/f_2(x) = 1 - \frac{x_H}{x}$$



- Condensed matter: Luttinger metric

$$f_1(x) = e^{2\phi(x)}$$
$$f_2(x) = 1$$

$$\partial_x \phi(x) = -\frac{\partial_x T(x)}{T(x)}$$



Inhomogeneous temperature in curved D=1+1 spacetime

Tolman (1930)
Tolman et al. (1930)

- Conformal/Weyl symmetry

$$\mathcal{T}^{\mu}_{\ \mu} = 0 \quad \longleftrightarrow \quad \varepsilon = P$$

- Diffeomorphism invariance

$$\nabla_{\mu}\mathcal{T}^{\mu\nu} = 0 \quad \longleftrightarrow \quad \sqrt{f_1 f_2}\,\partial_t \varepsilon + \partial_x \left(f_1 J_\varepsilon\right) = 0$$

(+ Momentum conservation)

- Lorentz invariance

$$\mathcal{T}^{\mu\nu} - \mathcal{T}^{\nu\mu} = 0 \quad \longleftrightarrow \quad v_F^2\,\Pi = J_\varepsilon$$

Inhomogeneous temperature in curved D=1+1 spacetime

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(+ Momentum conservation)

- Lorentz invariance

$$\mathcal{T}^{\mu\nu} - \mathcal{T}^{\nu\mu} = 0 \quad \longleftrightarrow \quad v_F^2\,\Pi = J_\varepsilon$$

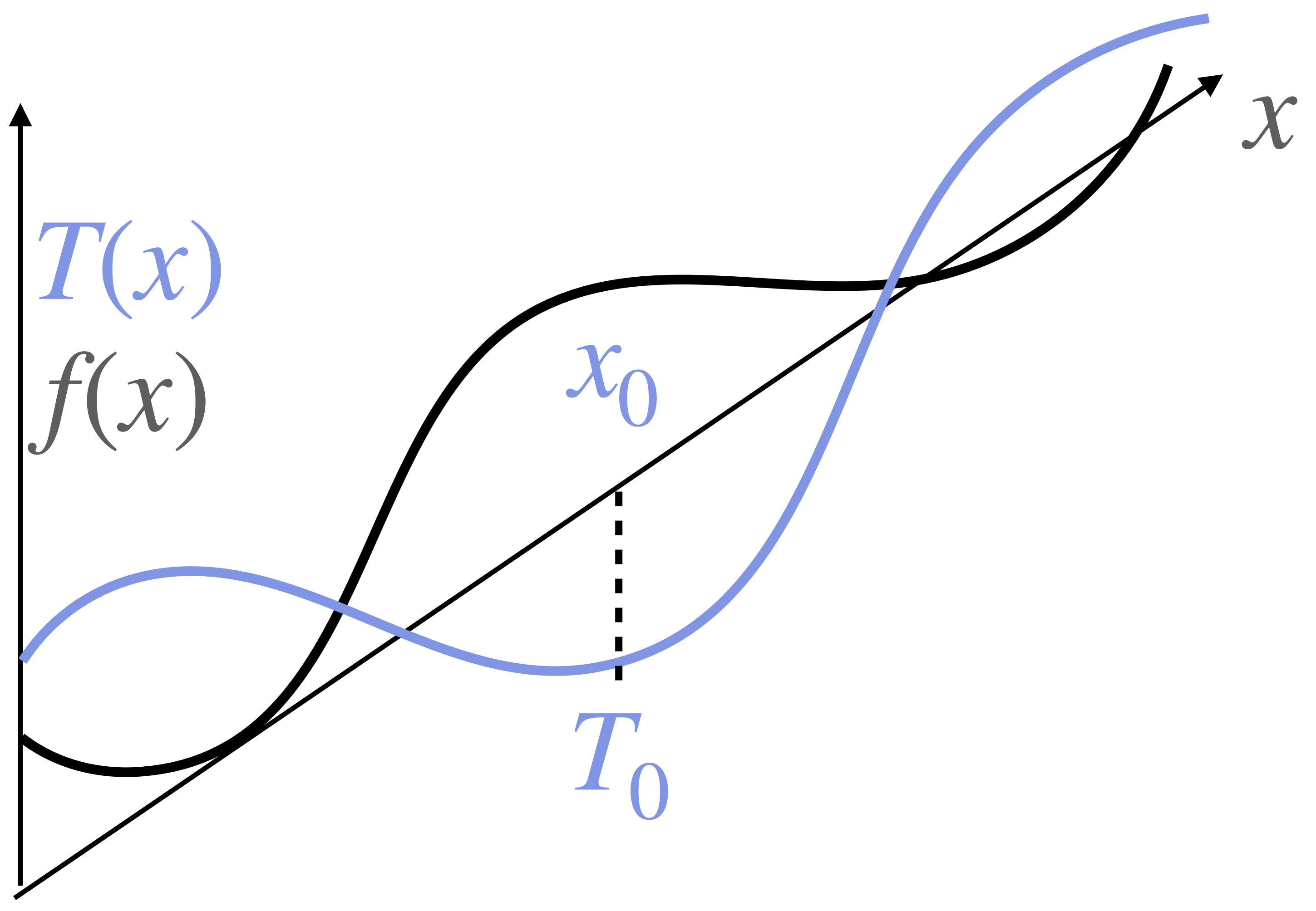
Stationary solution (equilibrium)

$$\varepsilon = P = C_w\,\gamma_{1D}\,T_0^2\,\frac{f_1(x_0)}{f_1(x)}$$

$$J_\varepsilon = v_F^2\Pi = C_g\,v_F\,\gamma_{1D}\,T_0^2\,\frac{f_1(x_0)}{f_1(x)}$$

Inhomogeneous temperature in curved D=1+1 spacetime

Tolman (1930)
Tolman et al. (1930)



$$\varepsilon = P$$

$$\sqrt{f_1 f_2}\, \partial_t \varepsilon + \partial_x \left(f_1\, J_\varepsilon \right) = 0$$

(+ Momentum conservation)

$$v_F^2\, \Pi = J_\varepsilon$$

Tolman-Ehrenfest temperature

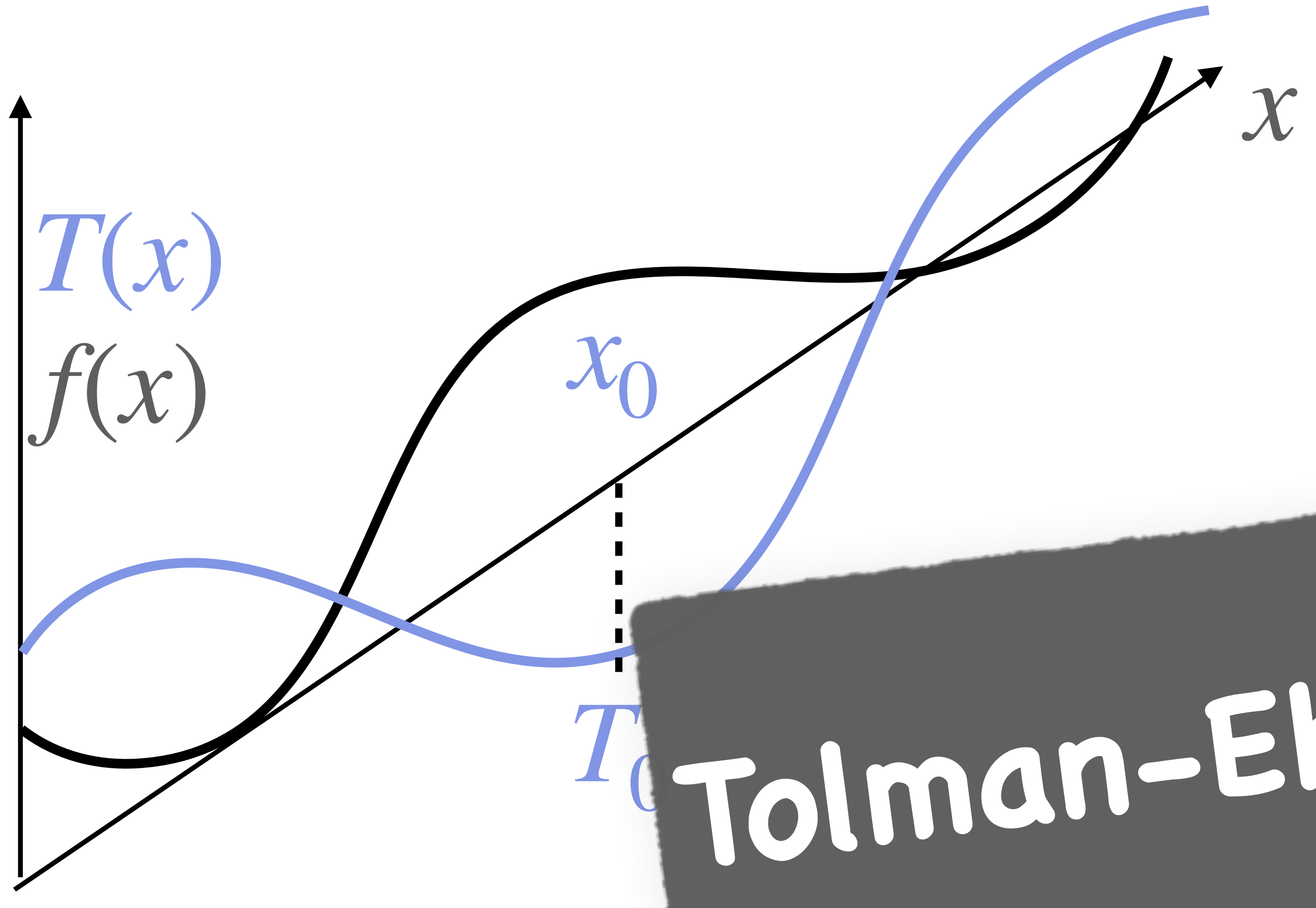
$$T_{TE}(x) = T_0 \sqrt{\frac{f_1(x_0)}{f_1(X)}}$$

Stationary solution (equilibrium)

$$\begin{cases} \varepsilon = P = C_w\, \gamma_{1D}\, T_0^2\, \dfrac{f_1(x_0)}{f_1(x)} \\ J_\varepsilon = v_F^2 \Pi = C_g\, v_F\, \gamma_{1D}\, T_0^2\, \dfrac{f_1(x_0)}{f_1(x)} \end{cases}$$

Inhomogeneous temperature in curved D=1+1 spacetime

Tolman (1930)
Tolman et al. (1930)



$$\varepsilon = P$$

$$\partial_t \varepsilon + \partial_x \left(f_1 J_\varepsilon\right) = 0$$

ntum conservation)

$$v_F \Pi = J_\varepsilon$$

Tolman-Ehrenfest temperature and gravitational anomalies ?

Tolman-Ehrenfest temperature

$$T_{TE}(x) = T_0 \sqrt{\frac{f_1(x_0)}{f_1(X)}}$$

Stationary solution (equilibrium)

$$\left\{ \begin{aligned} \varepsilon &= P = C_w \gamma_{1D} T_0^2 \frac{f_1(x_0)}{f_1(x)} \\ J_\varepsilon &= v_F^2 \Pi = C_g v_F \gamma_{1D} T_0^2 \frac{f_1(x_0)}{f_1(x)} \end{aligned} \right.$$

Plan:

1. From Tolman-Ehrenfest theorem to the Luttinger trick

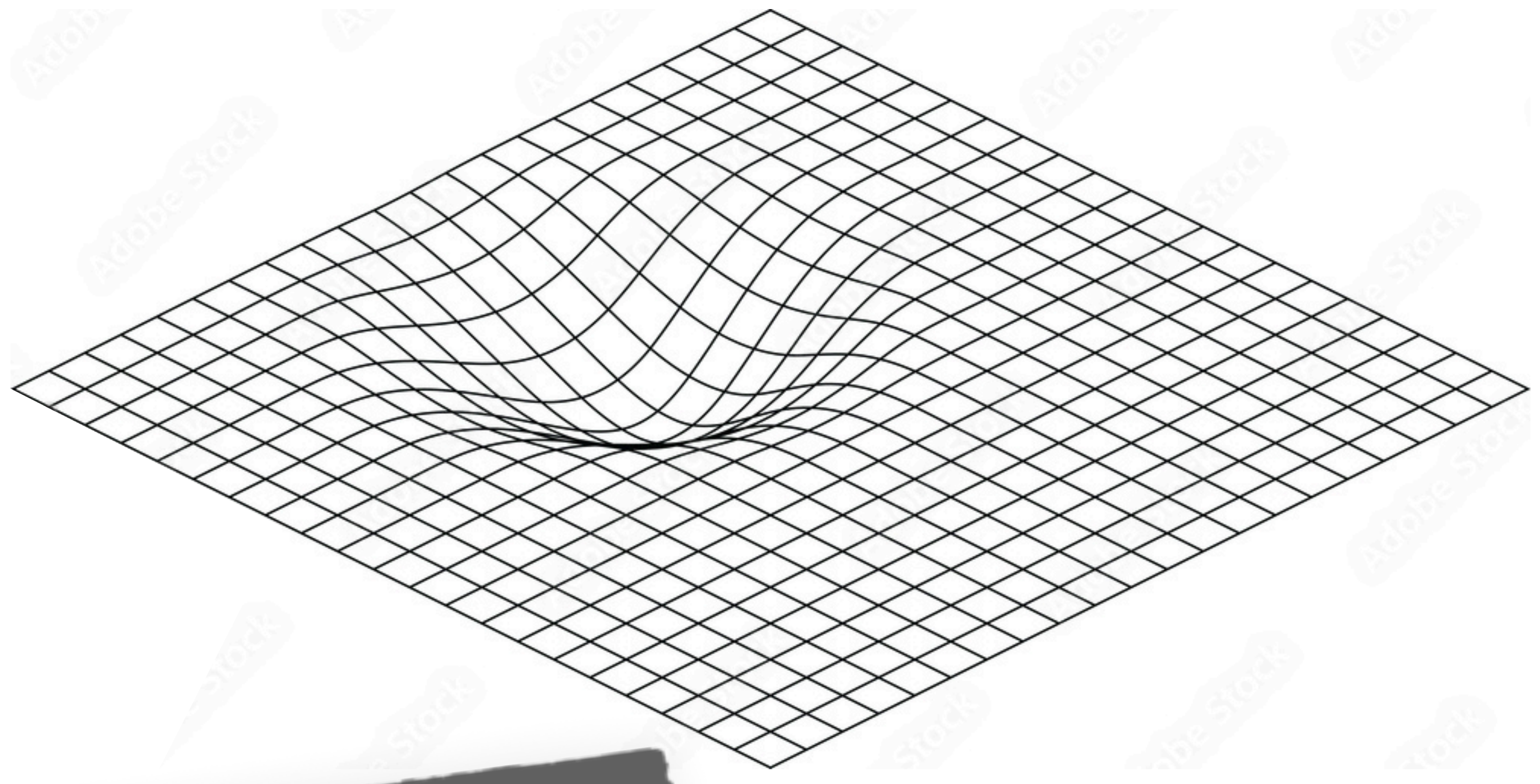
2. Gravitational corrections to Tolman-Ehrenfest theorem

3. Physical consequences

Anomalous quantum fluctuations and local temperature

- **anomalous fluctuations** induced by **spacetime curvature**

⇨ Gravitational anomalies



Effect of anomalous fluctuations on the Luttinger/Tolman-Ehrenfest relation

$$T \leftrightarrow \phi_G \ ?$$

Anomaly ?

Noether Theorem

- Continuous symmetry of Hamiltonian
- Conserved current
  $$\partial_\mu J^\mu = 0$$
- Example:
  $U(1)$ symmetry and charge conservation

Anomaly

- Symmetry of Hamiltonian broken by quantum fluctuations
- Conservation spoiled by quantum fluctuations
  $$\partial_\mu J^\mu \neq 0$$
- Example:
  Chiral anomaly
  Adler, Bell, Jackiw (1969)

Gravitational anomalies

Bertlmann (2001)

Classical

- Conformal/Weyl symmetry

$$\mathcal{T}^{\mu}_{\mu} = 0$$

- Diffeomorphism invariance

$$\nabla_{\mu}\mathcal{T}^{\mu}_{\nu} = 0$$

- Lorentz invariance

$$\mathcal{T}^{\mu\nu} = \mathcal{T}^{\nu\mu}$$

Quantum

- Conformal **anomaly**

$$\mathcal{T}^{\mu}_{\mu} = C_w \frac{\hbar v_F}{48\pi}\mathcal{R}$$

- Einstein **anomaly**

$$\nabla_{\mu}\mathcal{T}^{\mu\nu} = C_g \frac{\hbar v_F}{96\pi}\frac{1}{\sqrt{-det(g)}}\epsilon^{\nu\mu}\nabla_{\mu}\mathcal{R}$$

- Lorentz **anomaly**

$$\mathcal{T}^{\mu\nu} - \mathcal{T}^{\nu\mu} = C_g \frac{\hbar v_F}{48\pi}\frac{1}{\sqrt{-det(g)}}\epsilon^{\mu\nu}\mathcal{R}$$

Modified Tolman-Ehrenfest temperature

- Conformal anomaly

$$\mathcal{T}^{\mu}_{\mu} = C_w \frac{\hbar v_F}{48\pi}\mathcal{R}$$

- Einstein anomaly

$$\nabla_\mu \mathcal{T}^{\mu\nu} = C_g \frac{\hbar v_F}{96\pi}\frac{1}{\sqrt{-det(g)}}\epsilon^{\nu\mu}\nabla_\mu \mathcal{R}$$

- Lorentz Symmetry

$$\mathcal{T}^{\mu\nu} - \mathcal{T}^{\nu\mu} = 0$$

$$\varepsilon = P + C_w \frac{\hbar v_F}{48\pi}\mathcal{R}$$

$$\sqrt{f_1 f_2}\,\partial_t \varepsilon + \partial_x \left(f_1 J_\varepsilon\right) = C_g \frac{\hbar v_F}{96\pi} f_1 \partial_x \mathcal{R}$$

(+Momentum conservation)

$$v_F^2 \Pi = J_\varepsilon$$

Modified Tolman-Ehrenfest temperature

- Conformal anomaly

$$\mathcal{T}^{\mu}_{\mu} = C_w \frac{\hbar v_F}{48\pi} \mathcal{R}$$

$\longleftrightarrow$ $\varepsilon = P + C_w \frac{\hbar v_F}{48\pi} \mathcal{R}$

- Einstein anomaly

$$\nabla_\mu \mathcal{T}^{\mu\nu} = C_g \frac{\hbar v_F}{96\pi} \frac{1}{\sqrt{-det(g)}} \epsilon^{\nu\mu} \nabla_\mu \mathcal{R}$$

$\longleftrightarrow$ $\sqrt{f_1 f_2}\, \partial_t \varepsilon + \partial_x \left(f_1 J_\varepsilon \right) = C_g \frac{\hbar v_F}{96\pi} f_1 \partial_x \mathcal{R}$

(+Momentum conservation)

- Lorentz Symmetry

$$\mathcal{T}^{\mu\nu} - \mathcal{T}^{\nu\mu} = 0$$

$\longleftrightarrow$ $v_F^2 \Pi = J_\varepsilon$

➡ **Stationary solution** (equilibrium) $\mathcal{T}^{\mu}{}_{\nu} = \left(\mathcal{T}_{cl}\right)^{\mu}{}_{\nu} + \left(\mathcal{T}_{q}\right)^{\mu}{}_{\nu}$

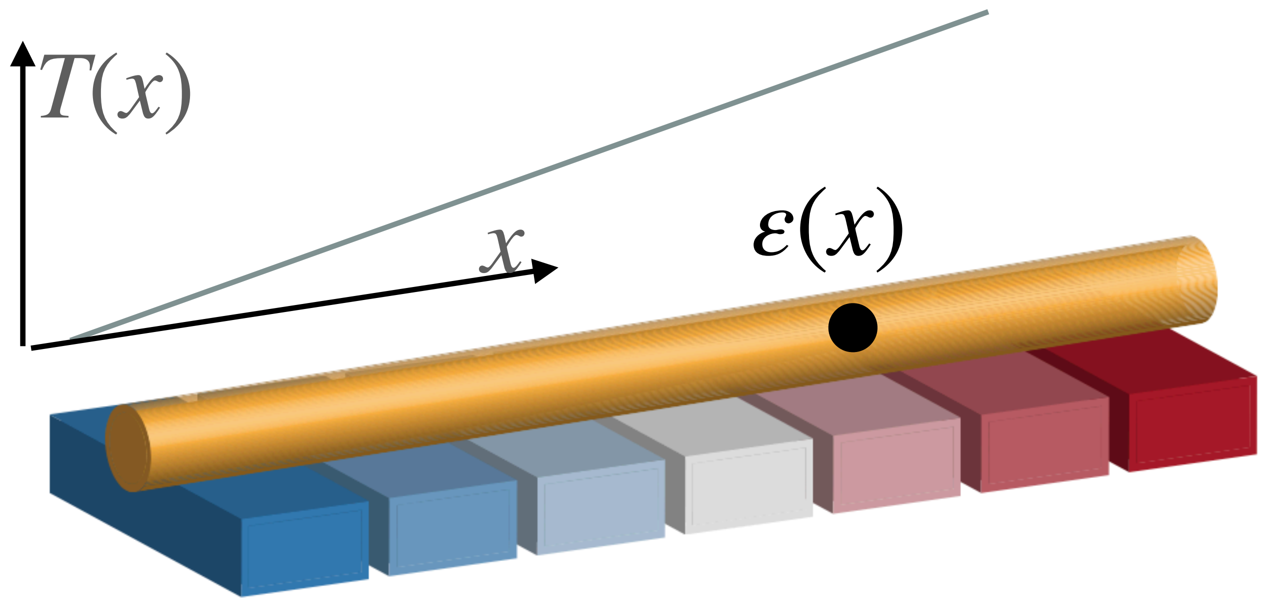
Classical (Tolman-Ehrenfest) — **Quantum corrections**

Modified equilibrium temperature ?

via Stefan-Boltzman law (density)

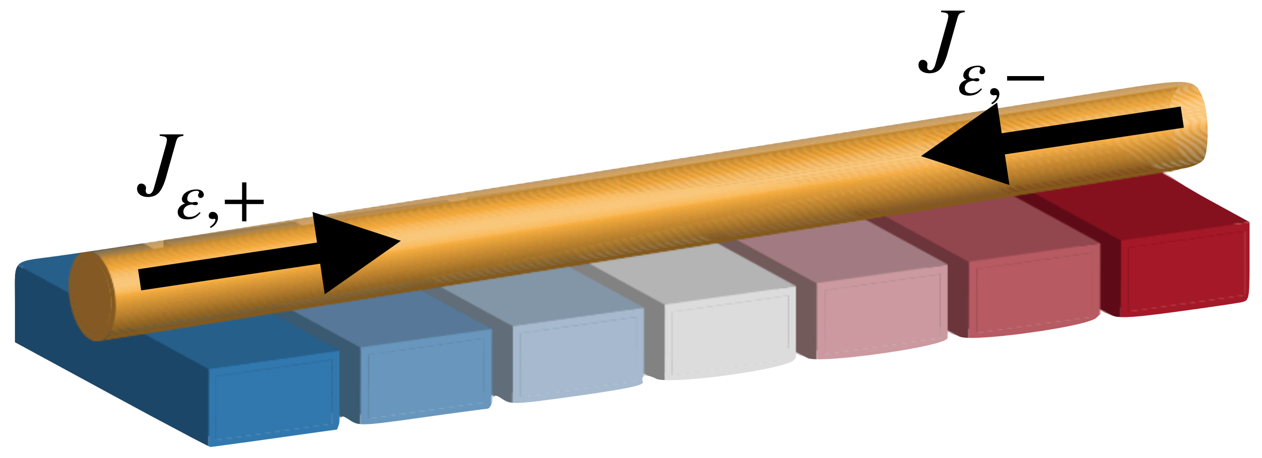
$$\varepsilon + P = C_w \gamma_{1D} T_m^2$$

$$\varepsilon - P = C_w \, \epsilon_q^{(1)}$$



via transport (current)

$$J_\varepsilon = v_F^2 \Pi = C_g \, v_F \, \gamma_{1D} \, T_m^2$$



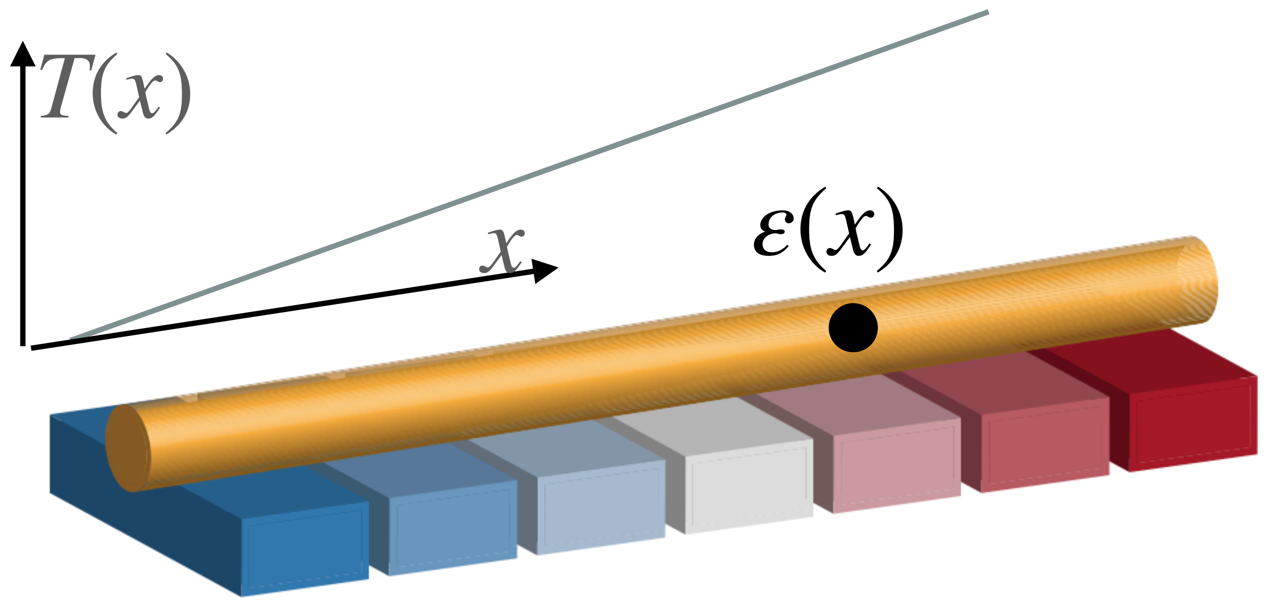
Modified equilibrium temperature ?

via Stefan-Boltzman law (density)

trace anomaly

$$\varepsilon + P = C_w \gamma_{1D} T_m^2$$

$$\varepsilon - P = C_w \, \epsilon_q^{(1)}$$



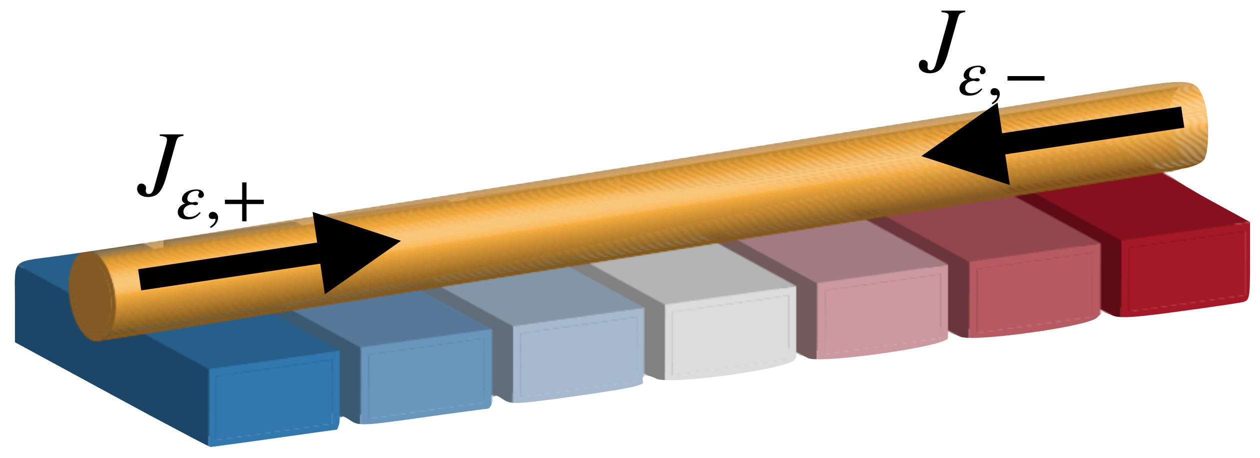
Modified Tolman-Ehrenfest temp.

$$\gamma_{1D} T_m^2(x) = \gamma_{1D} \, T_{TE}^2(x) + \epsilon_q^{(2)}(x)$$

via transport (current)

Einstein anomaly

$$J_\varepsilon = v_F^2 \Pi = C_g \, v_F \, \gamma_{1D} \, T_m^2$$



2 new energy scales: $\epsilon_q^{(1)} = \frac{\hbar v_F}{48\pi} \mathscr{R}$; $\epsilon_q^{(2)} = \frac{\hbar v_F}{48\pi} \left(\mathscr{R} - \frac{1}{f_1(x)} \int_0^x \mathscr{R} \partial_x f_1 \right)$

Plan:

1. From Tolman-Ehrenfest theorem to the Luttinger trick

2. Gravitational corrections to Tolman-Ehrenfest theorem

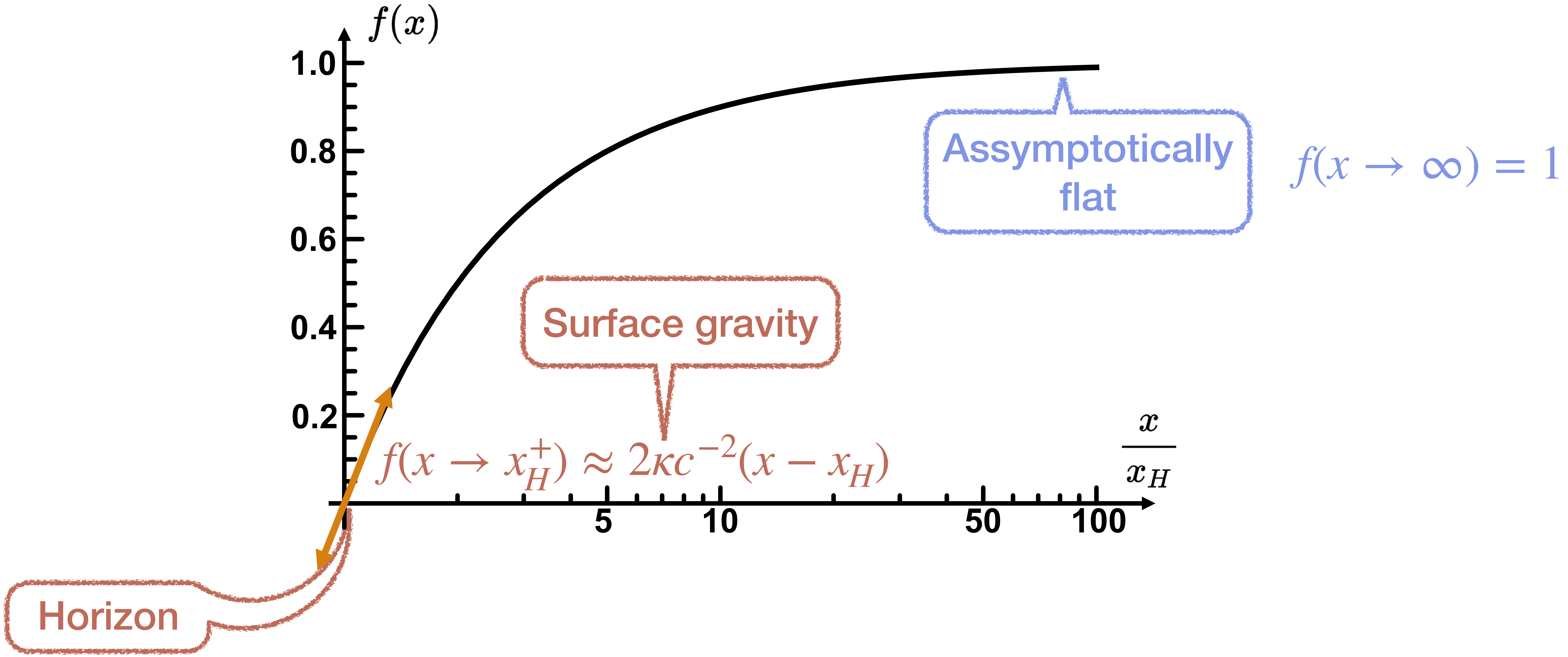
3. Physical consequences

1. Hawking radiation and Black-hole atmosphere

Black hole metric

$$g_{\mu\nu} = \begin{pmatrix} f(x) & 0 \\ 0 & -1/f(x) \end{pmatrix}$$

- Schwarzschild Black holes $f(x) = 1 - \frac{x_H}{x}$
- Evanescent CGHS black holes $f(x) = 1 - e^{-\alpha(x - x_H)}$

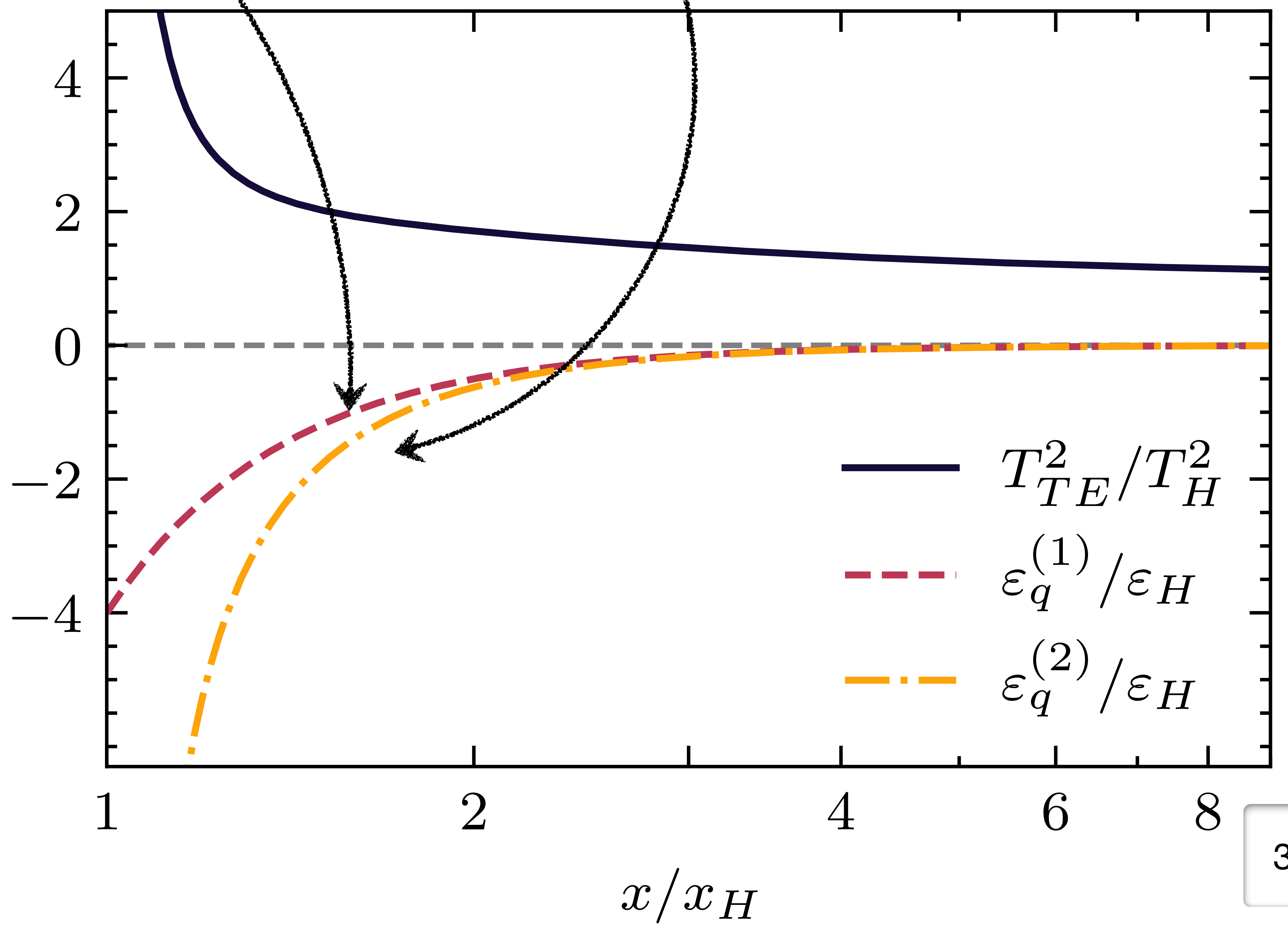


1. Hawking radiation and Black-hole atmosphere

Christensen et al. (1977)
Robinson et al. (2005)
M.Eune et al. (2017)

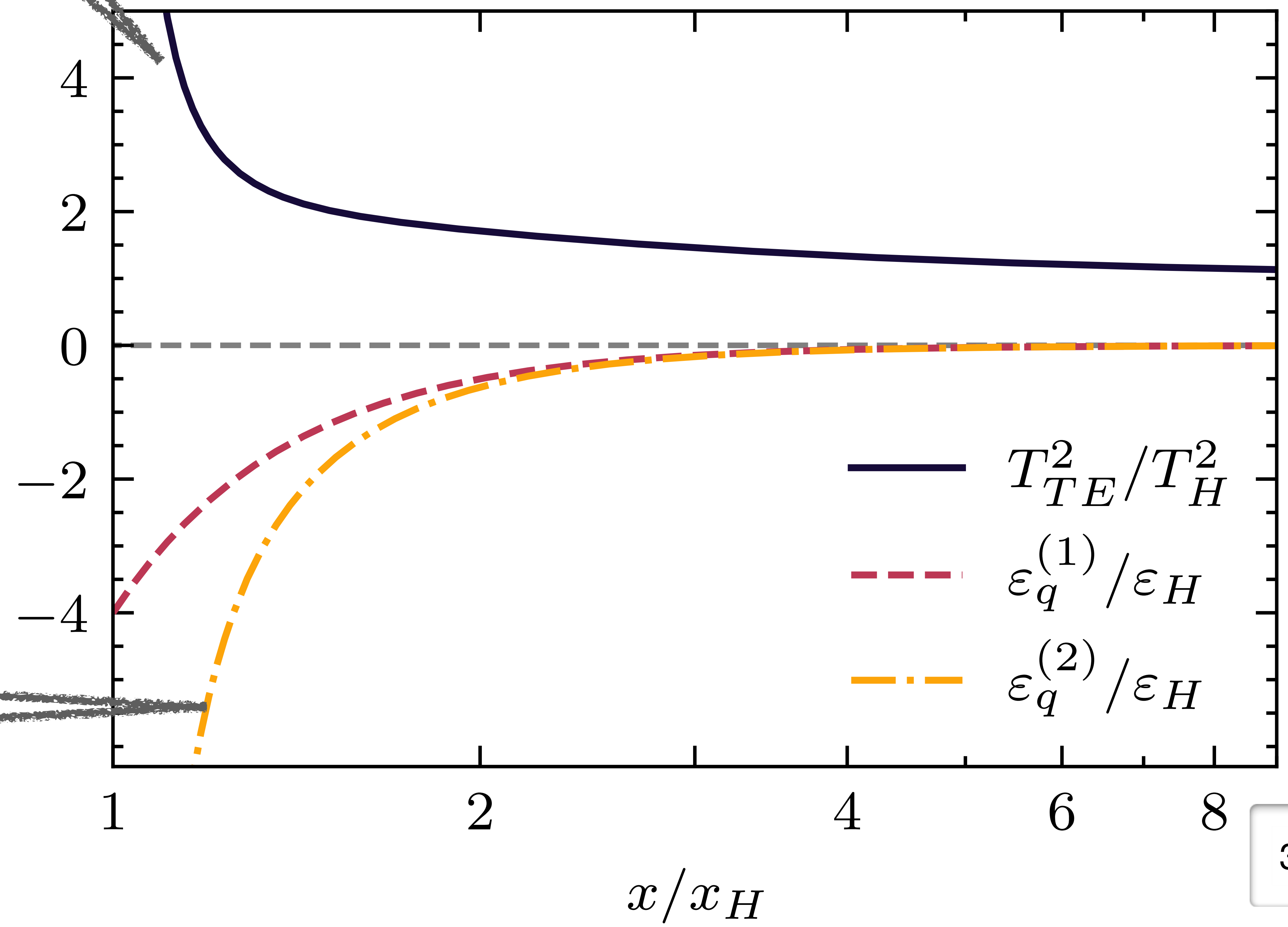
2 new scales: large close to the horizon

$$\epsilon_q^{(1)} = \frac{\hbar c}{48\pi} f'' \qquad \epsilon_q^{(2)} = \frac{\hbar c}{48\pi} \left[f'' - \frac{(f')^2}{2f} \right]$$



1. Hawking radiation and Black-hole atmosphere

$T_{TE}^2 = \frac{T_H^2}{f}$ diverges at the horizon



$$\epsilon_q^{(2)} = \frac{\hbar c}{48\pi}\left[f'' - \frac{(f')^2}{2f}\right]$$

diverges at the horizon

1. Hawking radiation and Black-hole atmosphere

$T_{TE}^2 = \frac{T_H^2}{f}$ diverges at the horizon

Close to the horizon

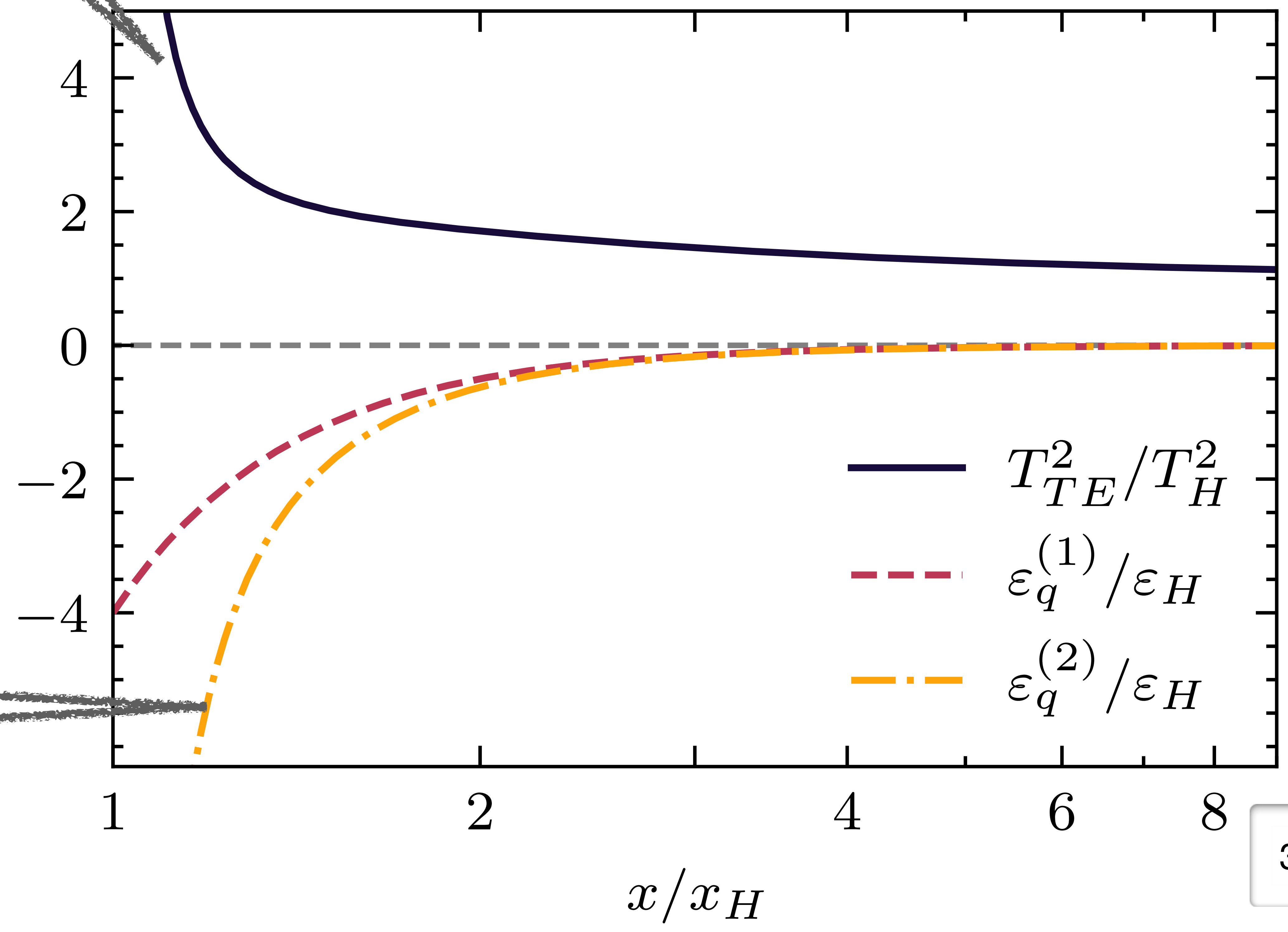
$$\gamma_{1D} T_m^2(x) \approx \gamma_{1D}\frac{T_H^2}{f(x)} - \frac{\hbar c}{48\pi}\frac{f'(x_H)^2}{2f(x)}$$

Cancelling the divergence imposes

$$T_H = \frac{\hbar}{4\pi c} f'(x_H)$$

$$\epsilon_q^{(2)} = \frac{\hbar c}{48\pi}\left[f'' - \frac{(f')^2}{2f}\right]$$

diverges at the horizon



1. Hawking radiation and Black-hole atmosphere

Giddings (2016)

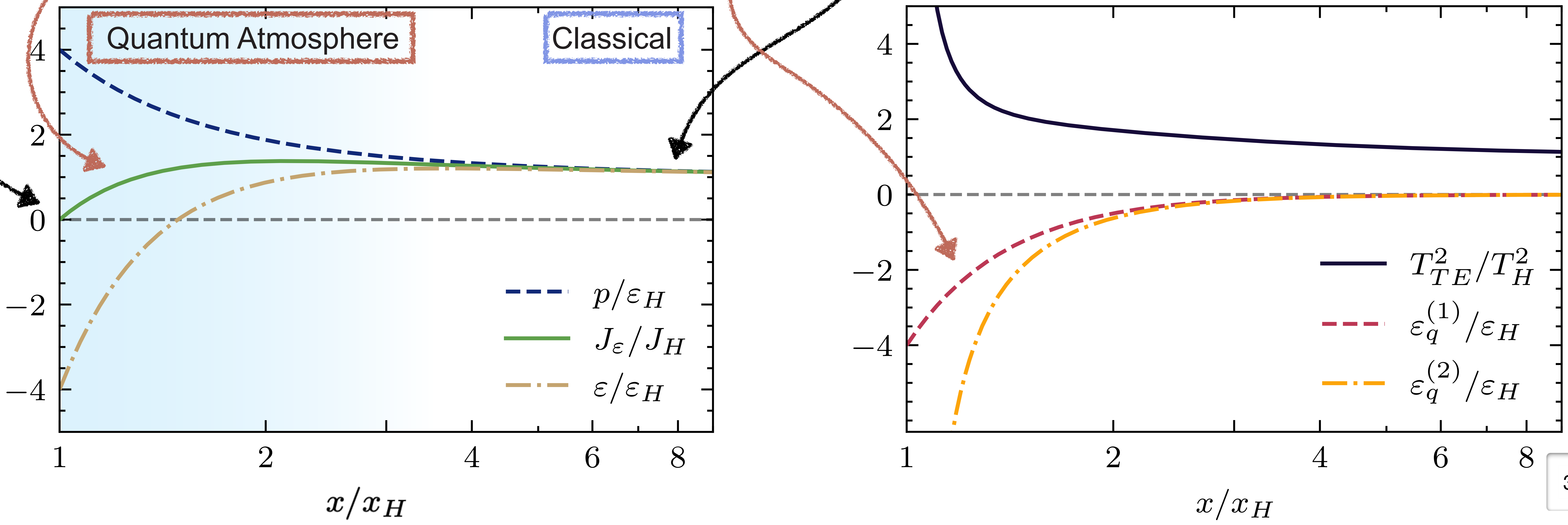
Outgoing energy current:

$J_\varepsilon \left(x \to x_H\right) = 0$: **Nothing exits the black-hole**

Asymptotic outgoing energy current:

$J_{\varepsilon,+} (x \to \infty) = \frac{\pi}{12\hbar} T_H^2$: **Hawking Radiation**

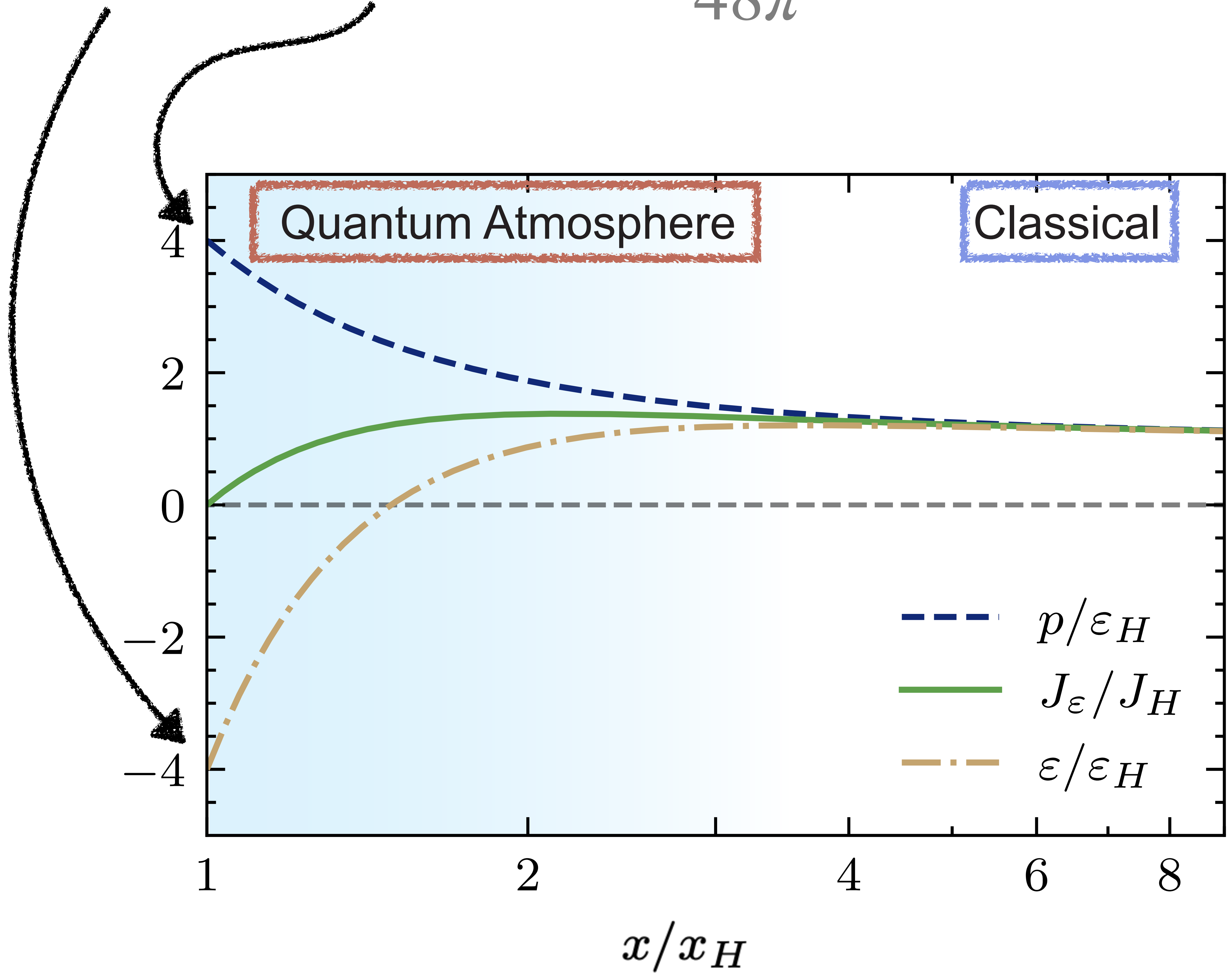
Quantum atmosphere:
strong anomalous quantum fluctuations

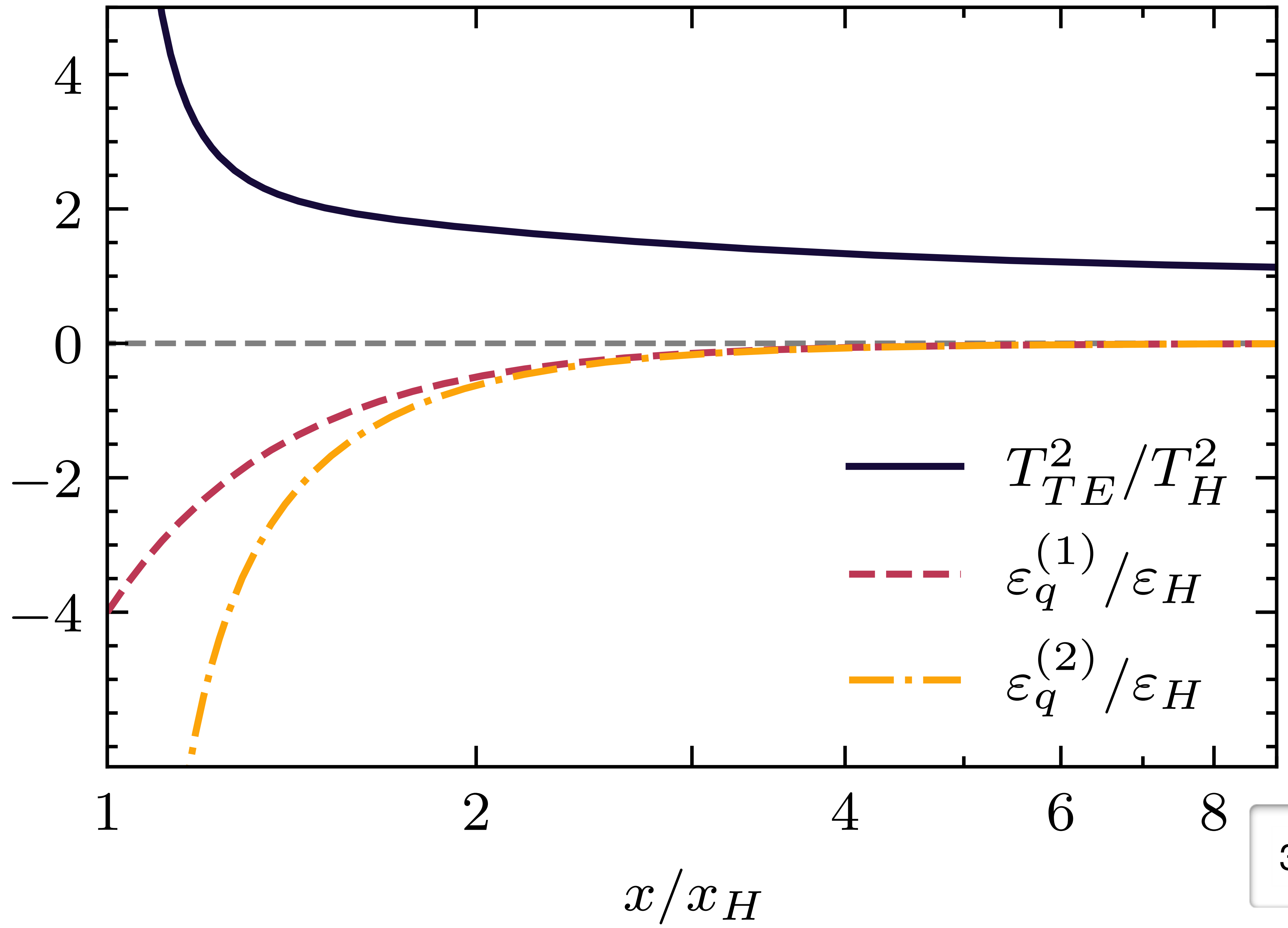


1. Hawking radiation and Black-hole atmosphere

In the close atmosphere (quantum troposphere)

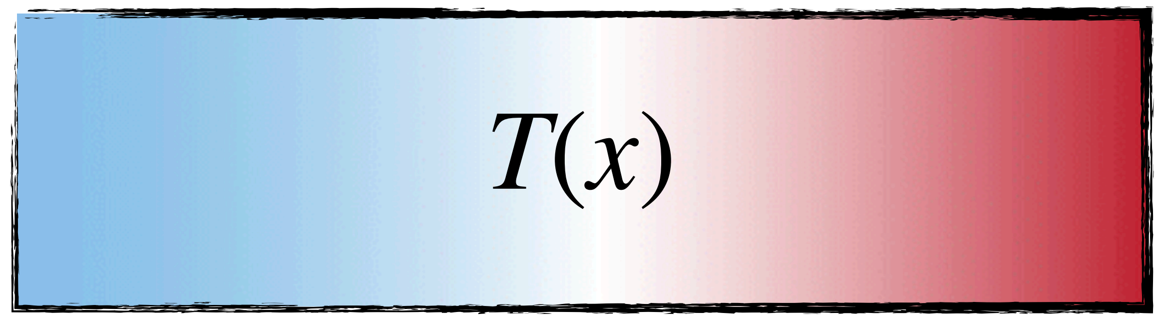
$$\varepsilon\left(x_H\right) = -P\left(x_H\right) = \frac{\hbar c}{48\pi} f''(x_H)$$: **Analog of the Casimir effect**



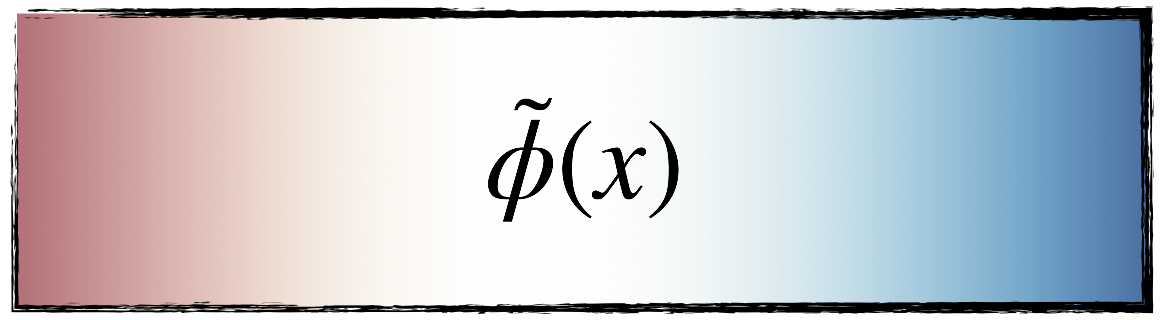


2. Generalized Luttinger trick and response theory

$$g_{\mu\nu} = \begin{pmatrix} e^{2\phi(x)} & 0 \\ 0 & -1 \end{pmatrix}$$



Modified equivalent gravitational potential



In **presence** of gravitational anomalies

Corrected Luttinger relation

$$\frac{\partial_x T(x)}{T(x)} = -\partial_x \tilde{\phi}(x) + \lambda_T^2 \left[\partial_x \tilde{\phi}(x) \partial_x^2 \tilde{\phi}(x) + \frac{1}{2} \partial_x^3 \tilde{\phi}(x) \right]$$

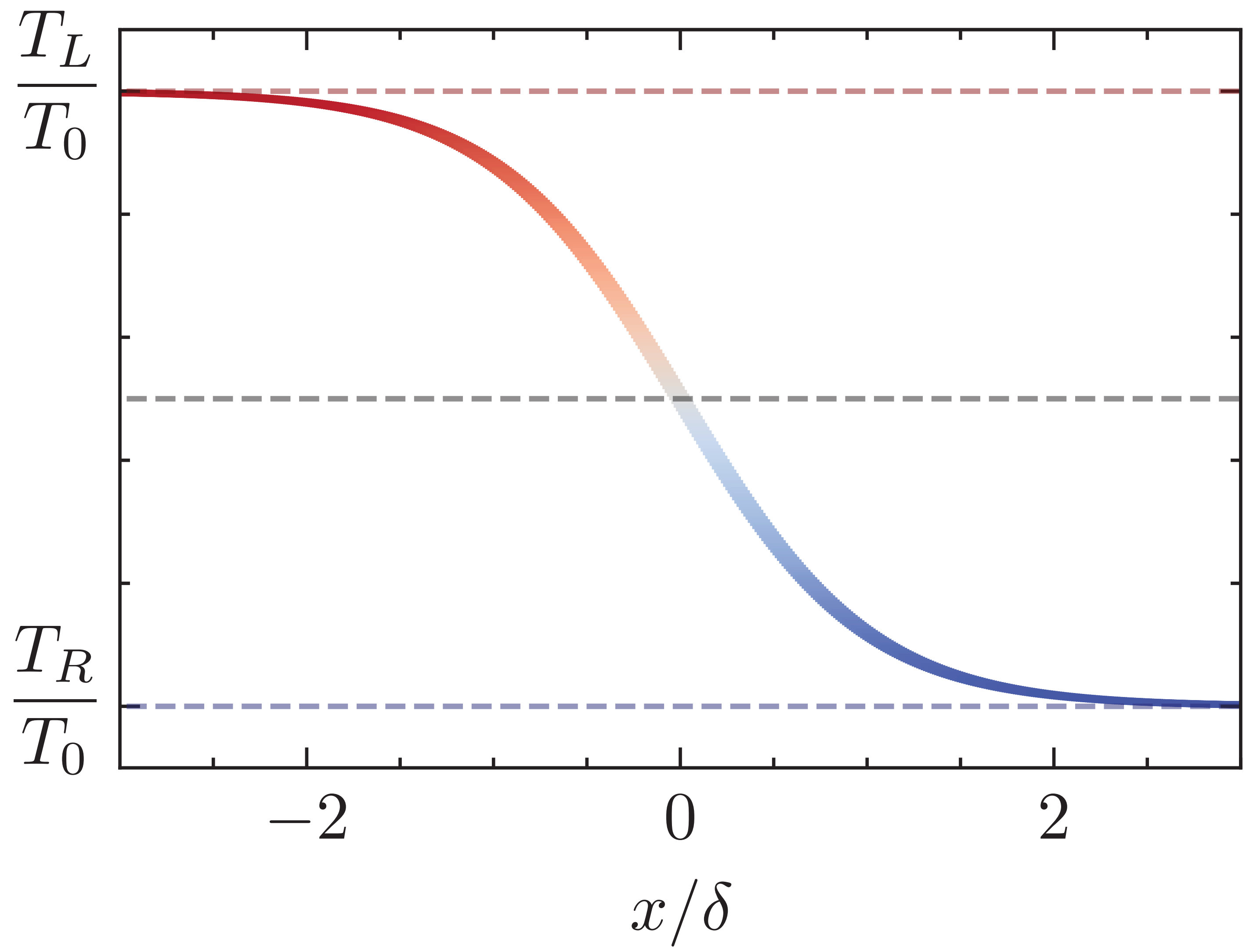
with $\lambda_T = \dfrac{\hbar v_F}{2\pi k_B T(x)}$

⇨ Correction of **non-linear** response $\quad J_\varepsilon(x) = J_\varepsilon^0(x) + \delta J_\varepsilon(x)$

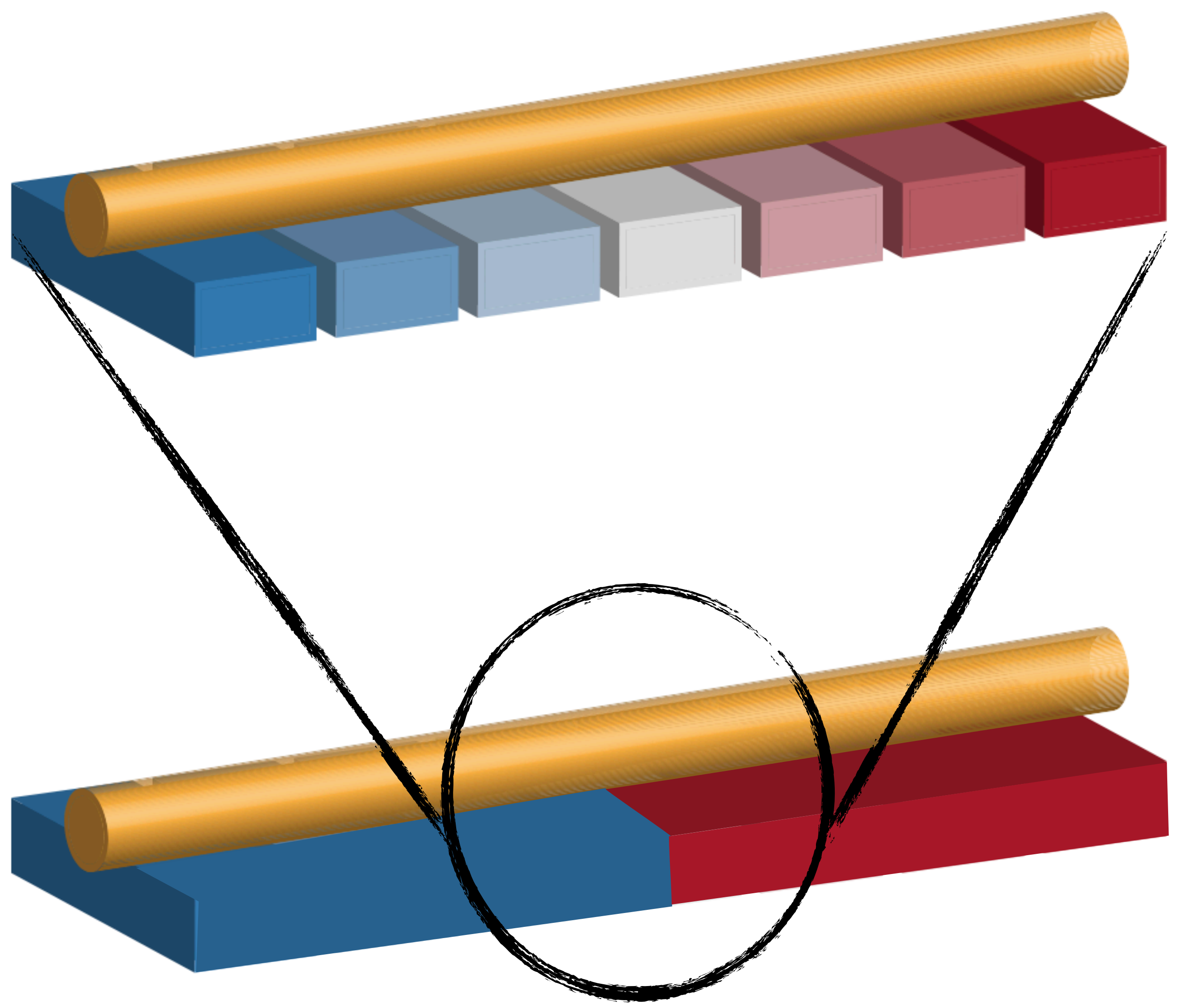
Classical ballistic energy current $J_\varepsilon^0(x) = \dfrac{\pi}{6} k_B^2 T^2(x)$

Quantum corrections
$\delta J_\varepsilon(x) \propto \lambda_{T_0}^2 (\nabla T)^2$

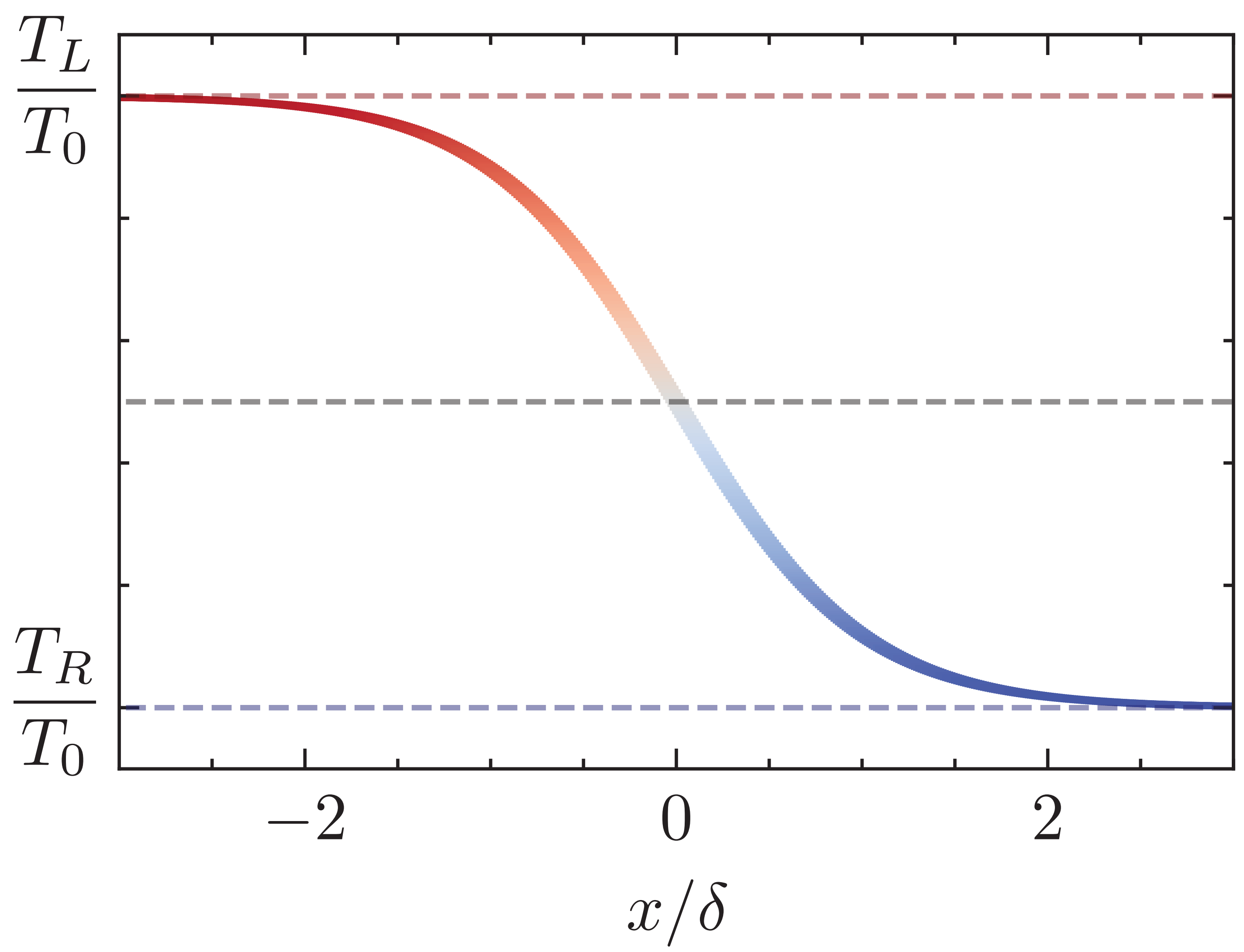
3. Out-of-equilibrium states by an inhomogeneous T

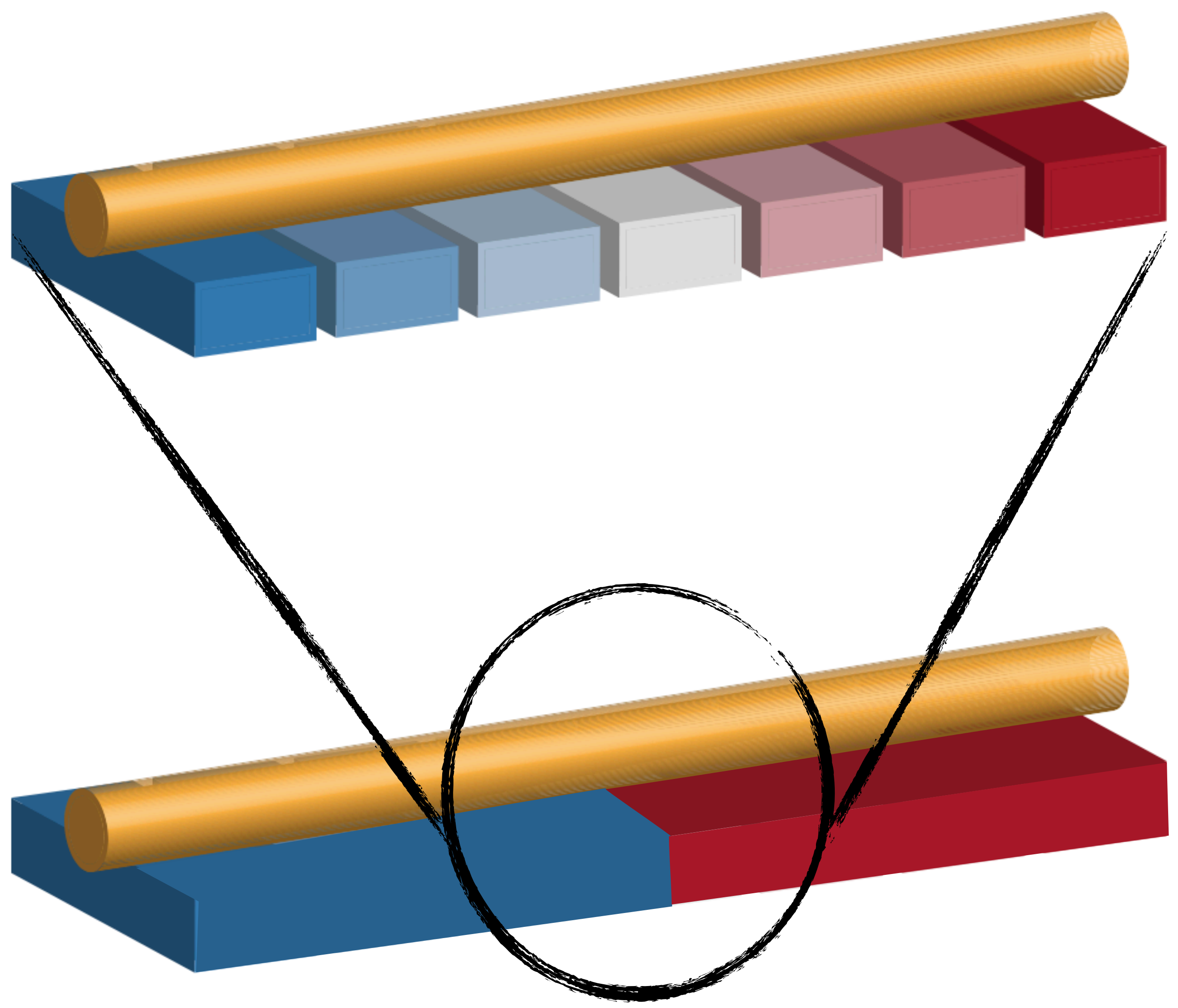


Imposing externally a temperature profile $T(x)$:



3. Out-of-equilibrium states by an inhomogeneous T





Imposing externally a temperature profile $T(x)$:
Modified Gibbs measure

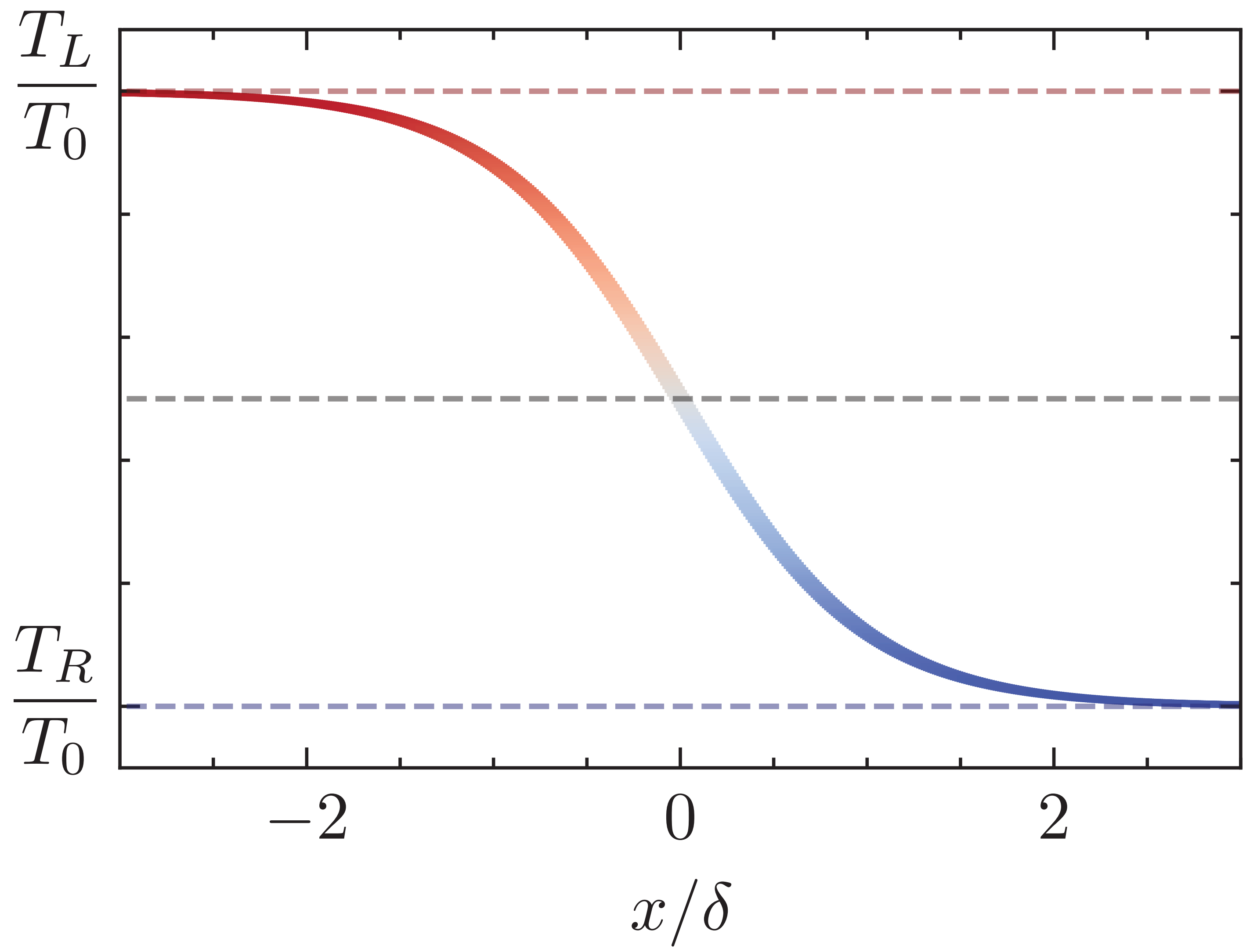
$$\rho = \frac{e^{-\beta H}}{Tr(e^{-\beta H})}$$

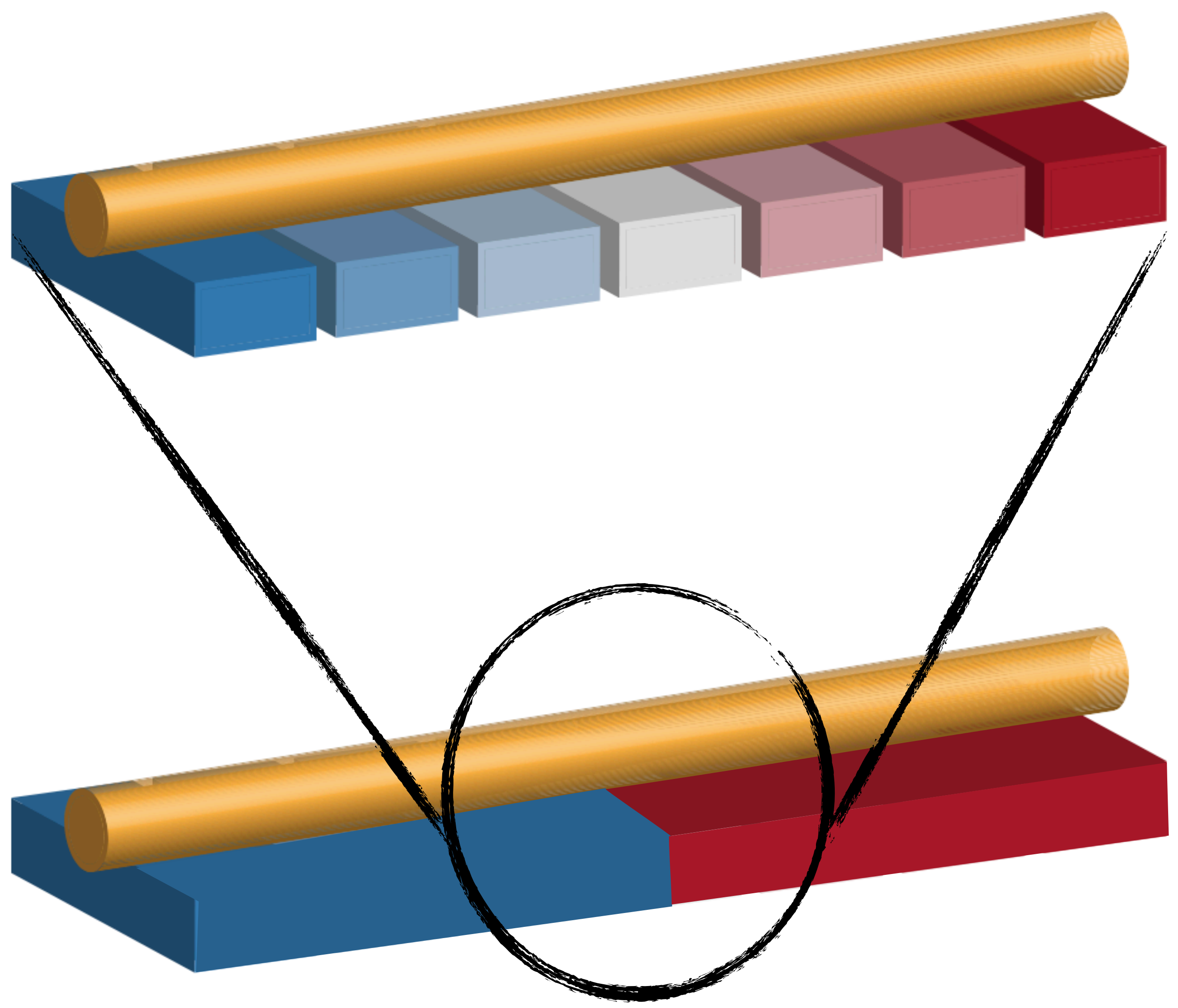
with $\beta H = \int dx \frac{1}{k_B T(x)} h(x)$

Inhomogeneous $T(x)$

Flat Space

3. Out-of-equilibrium states by an inhomogeneous T





Imposing externally a temperature profile $T(x)$:
Modified Gibbs measure

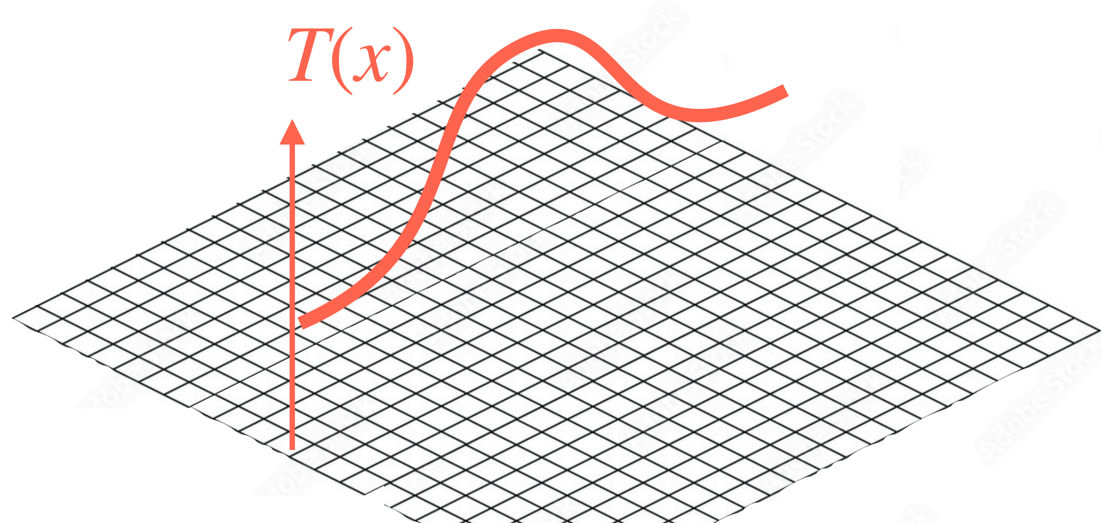
$$\rho = \frac{e^{-\beta H}}{Tr(e^{-\beta H})}$$

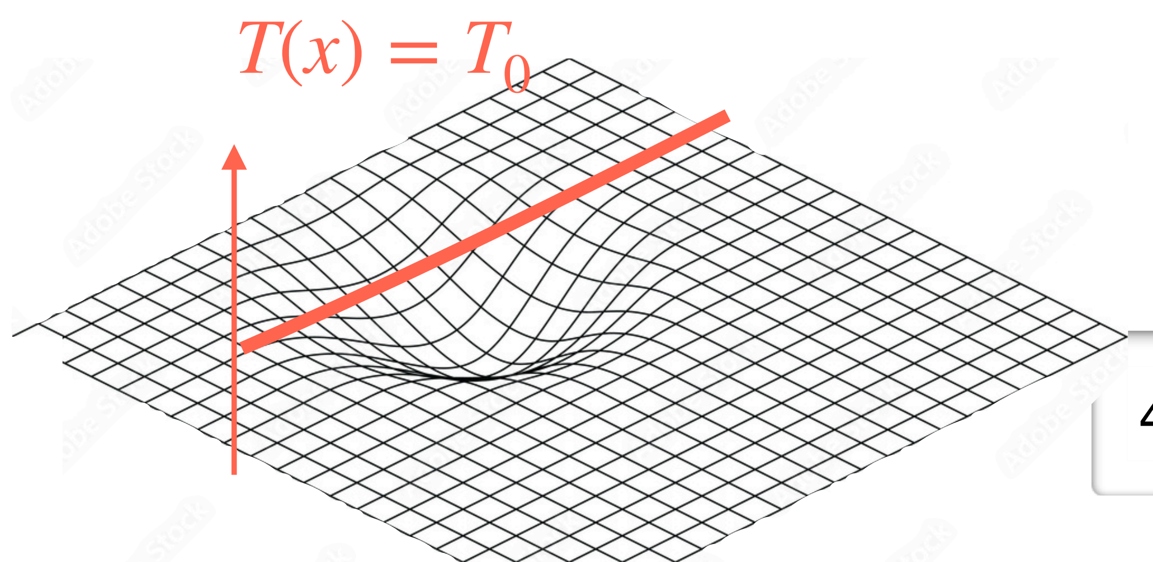
with $\beta H = \int dx \frac{1}{k_B T(x)} h(x) = \frac{1}{k_B T_0} \int dx \frac{T_0}{T(x)} h(x)$

Inhomogeneous $T(x)$ — **Flat** Space ⇔ **Homogeneous** T_0 — **Curved** space

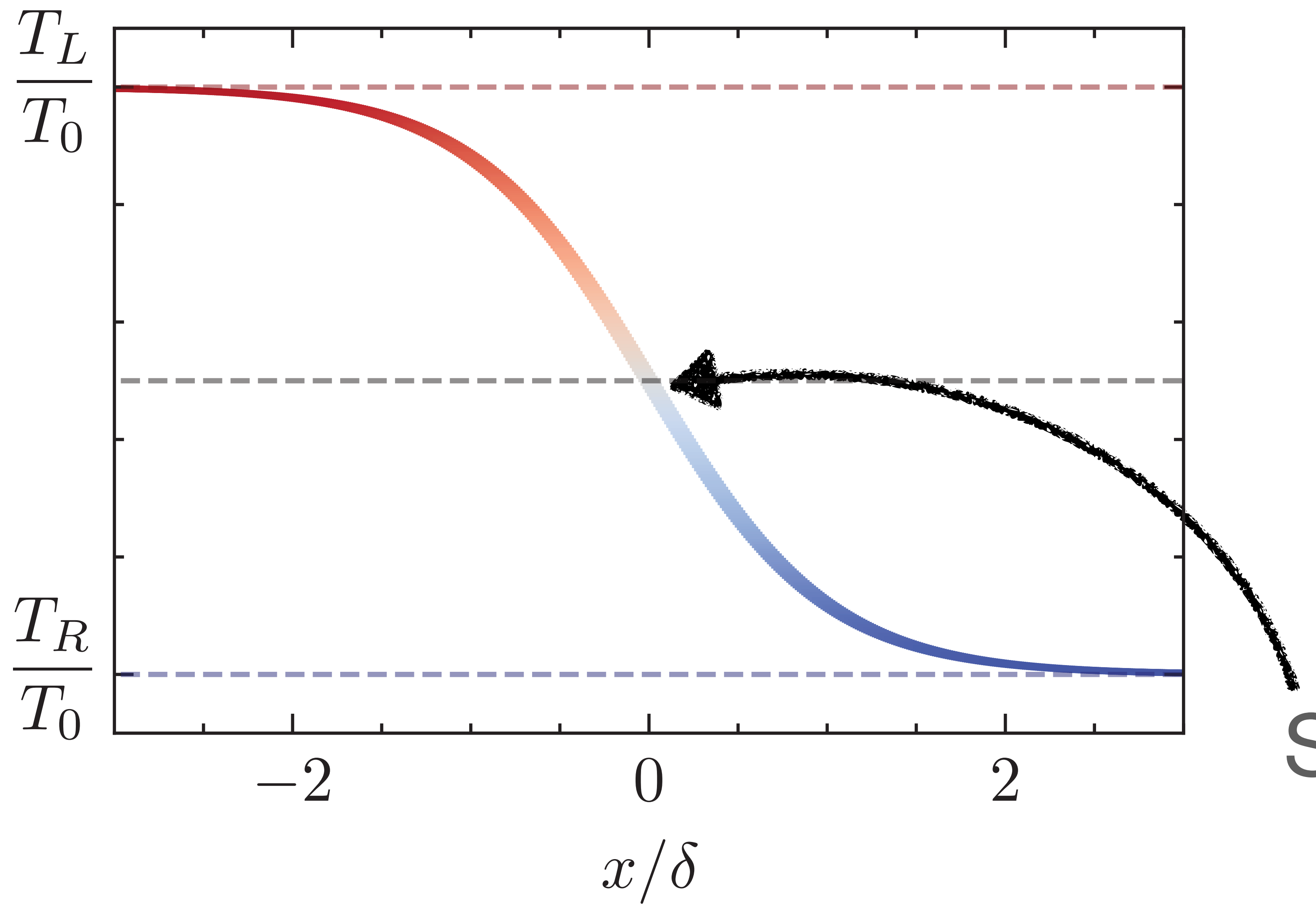
$$g_{\mu\nu} = \begin{pmatrix} \left(\frac{T_0}{T(x)}\right)^2 & 0 \\ 0 & -1 \end{pmatrix}$$

Extended Luttinger equivalence





3. Out-of-equilibrium states by an inhomogeneous T



Sizable at the temperature jump

2 new energy scales

$$\epsilon_q^{(1)} = \frac{\hbar v_F}{24\pi}\left[-\frac{\partial_x^2 T}{T} + 2\left(\frac{\partial_x T}{T}\right)^2\right]$$

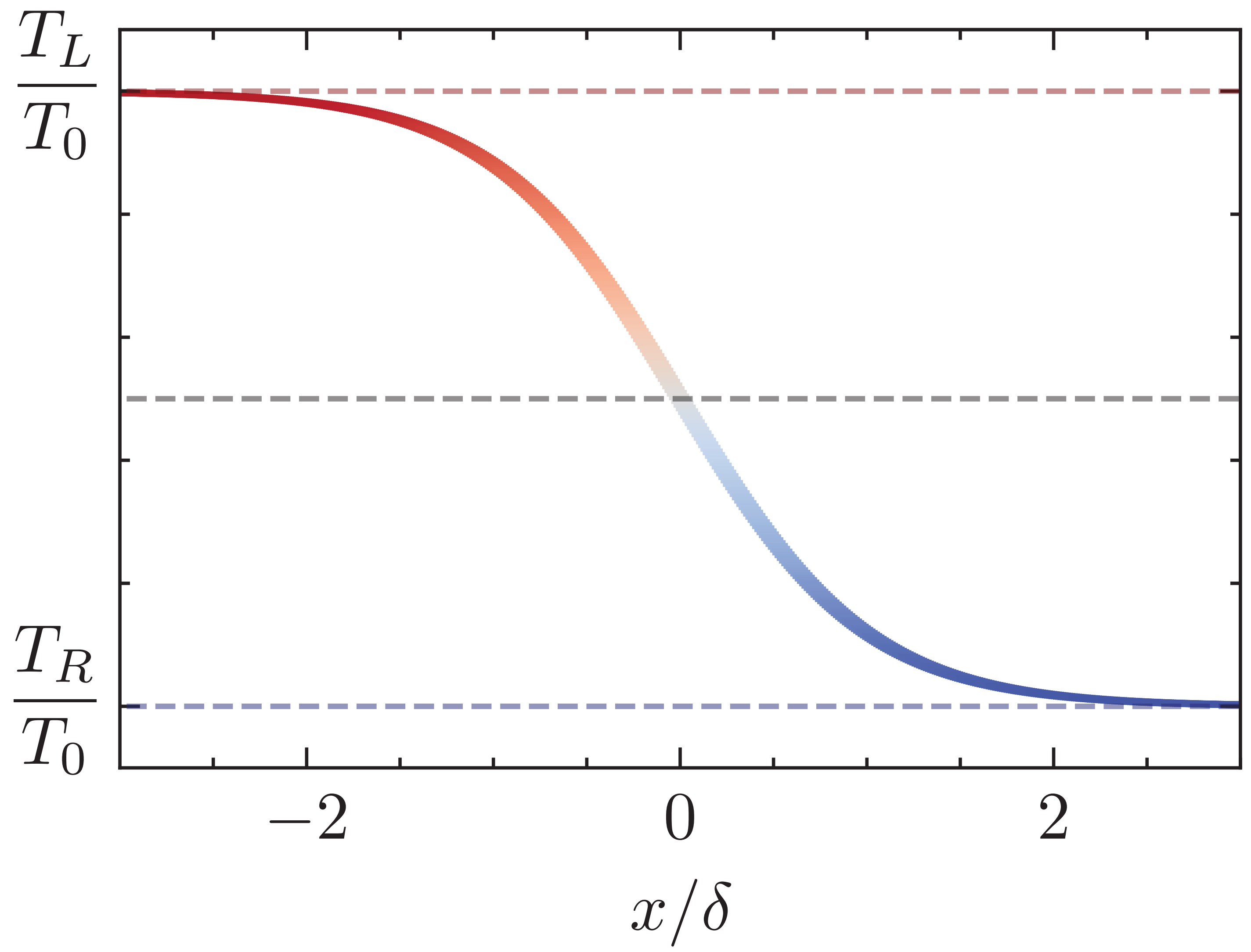
$$\epsilon_q^{(2)} = \frac{\hbar v_F}{24\pi}\left[-\frac{\partial_x^2 T}{T} + \left(\frac{\partial_x T}{T}\right)^2\right]$$

Characteristic length scales for T variations

$$l_T^{(1)} = \frac{T}{\partial_x T} \qquad l_T^{(2)} = \sqrt{\left|\frac{T}{\partial_x^2 T}\right|}$$

→ compare with thermal length $\lambda_T = \frac{\hbar v_F}{2\pi k_B T}$

3. Out-of-equilibrium states by an inhomogeneous T



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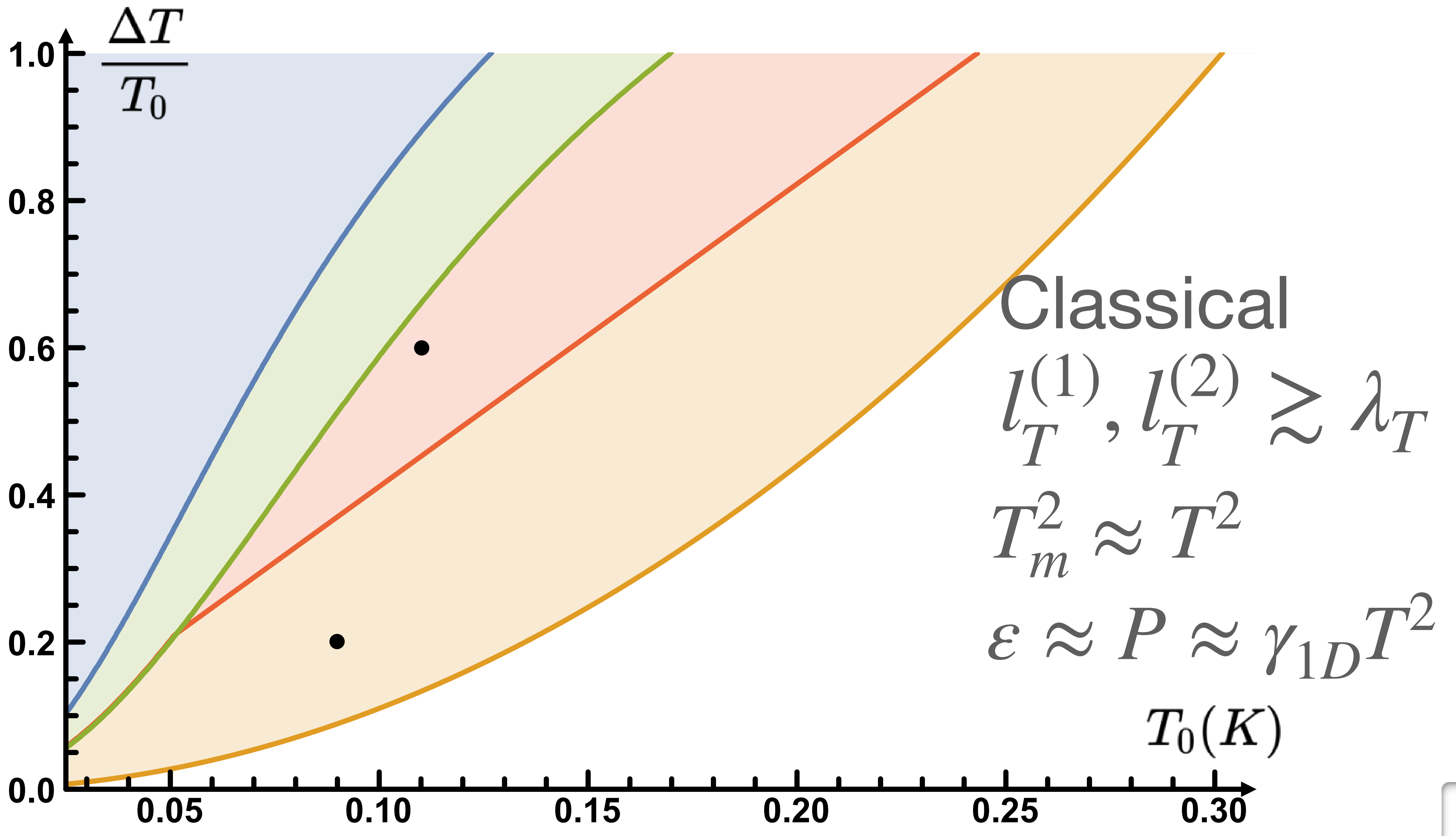
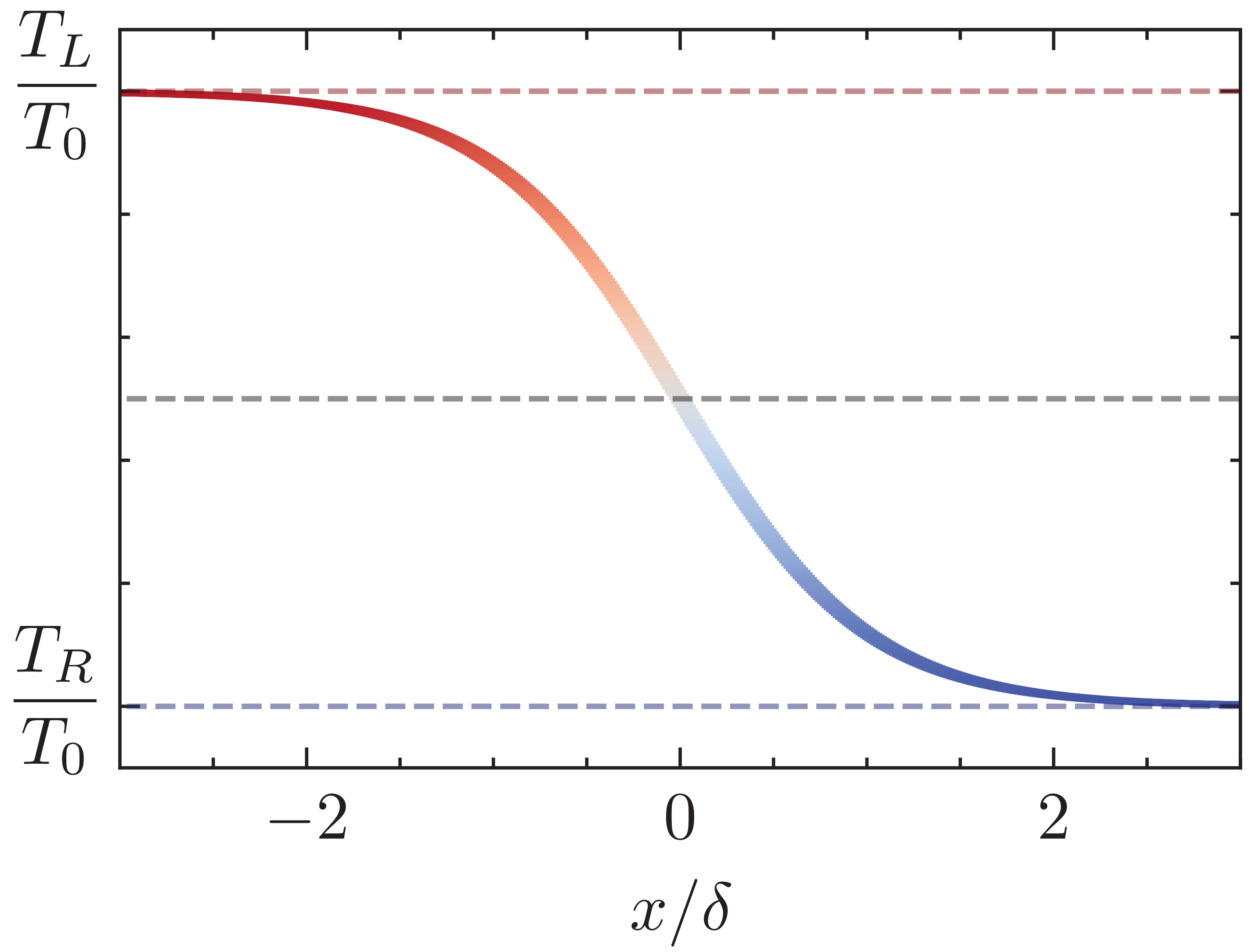
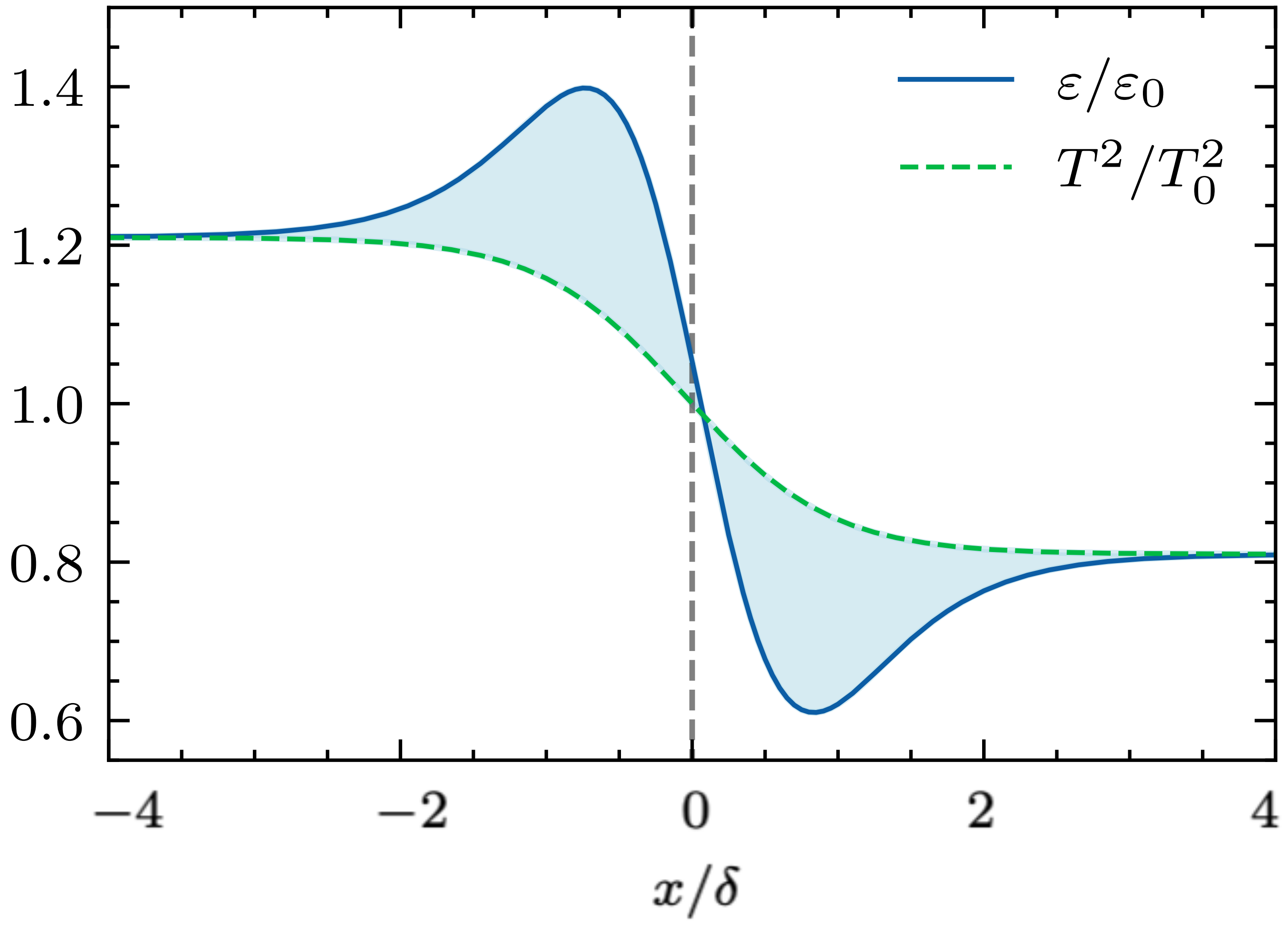
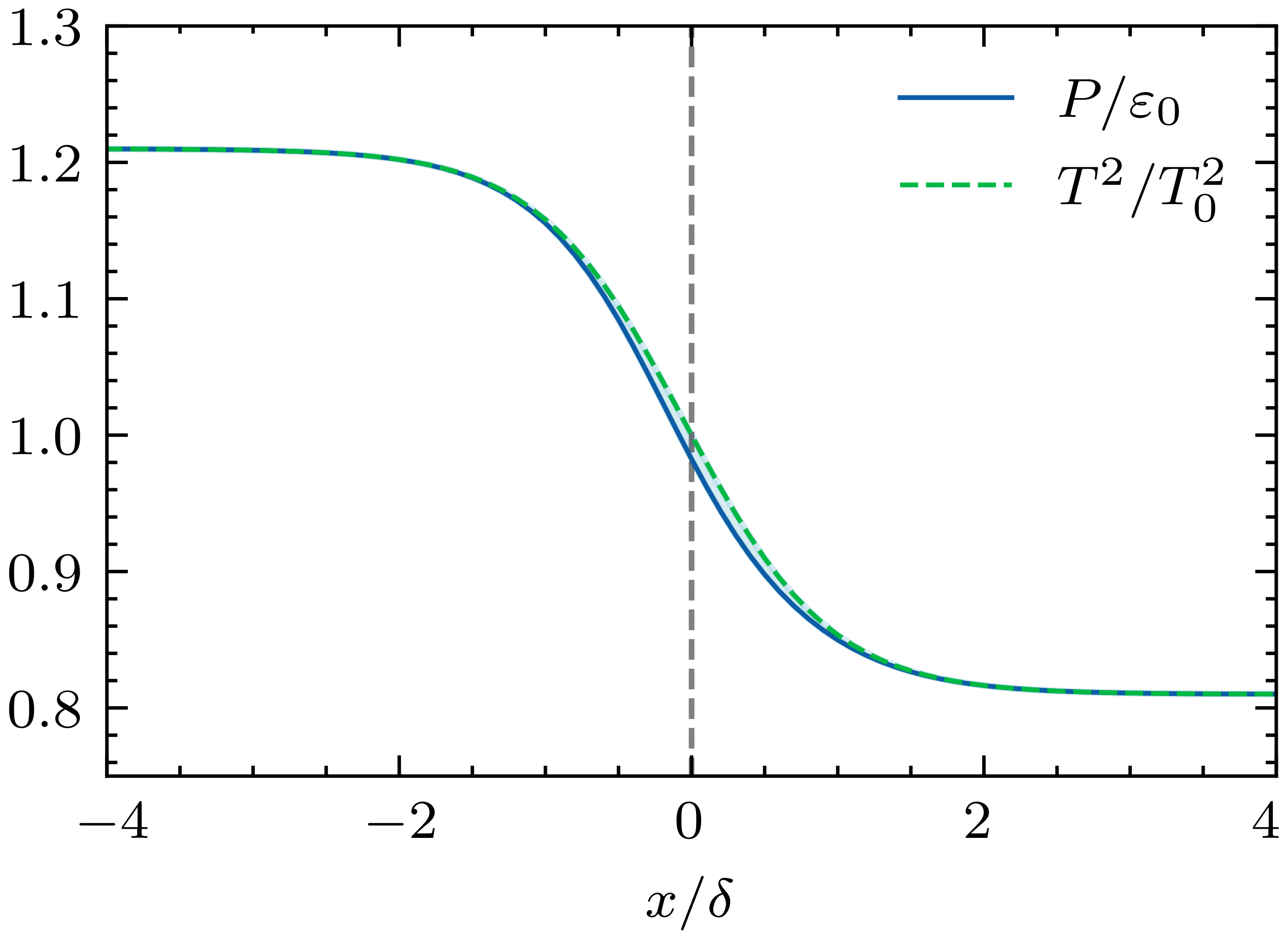


3. Out-of-equilibrium states by an inhomogeneous T







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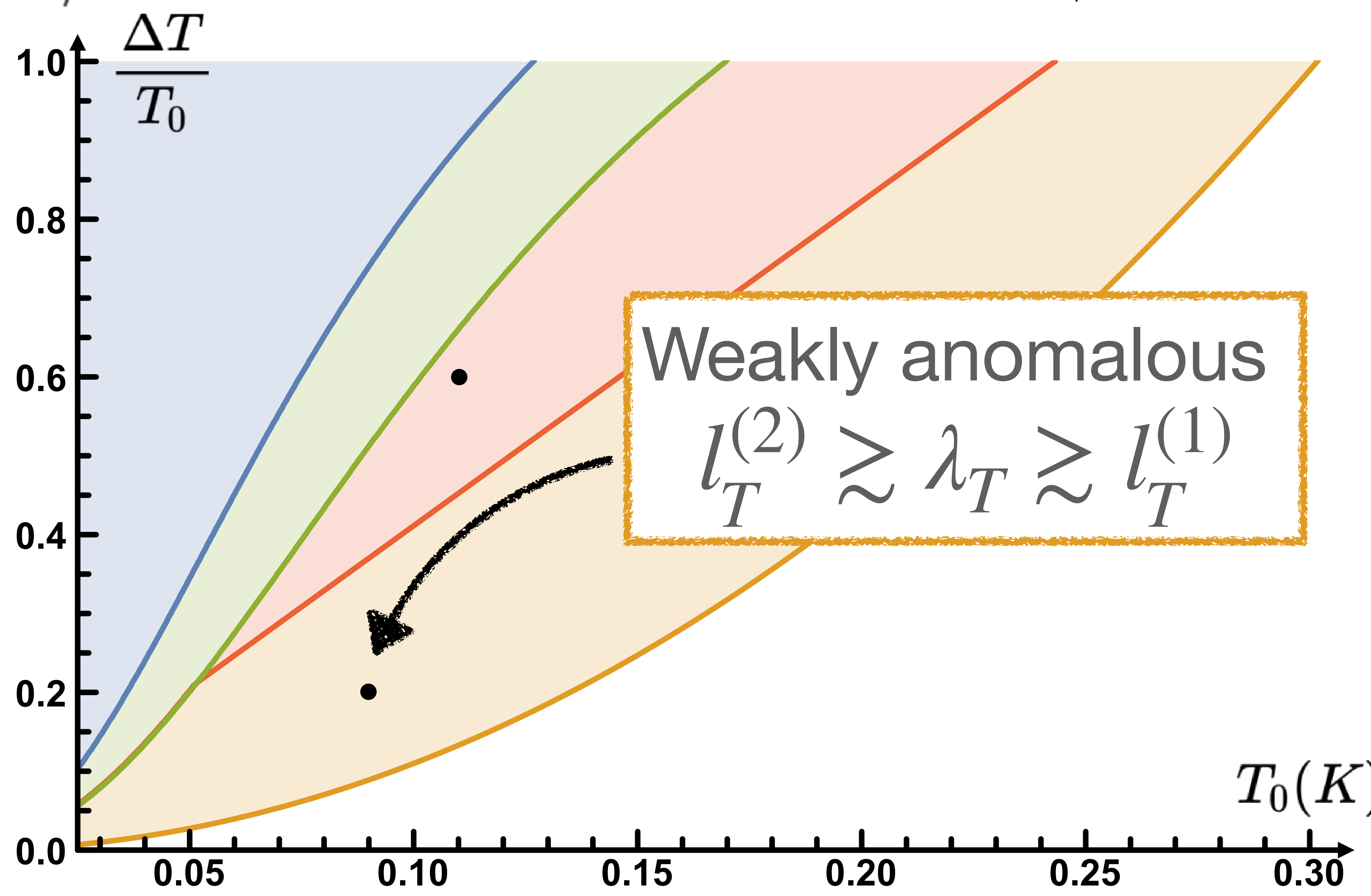
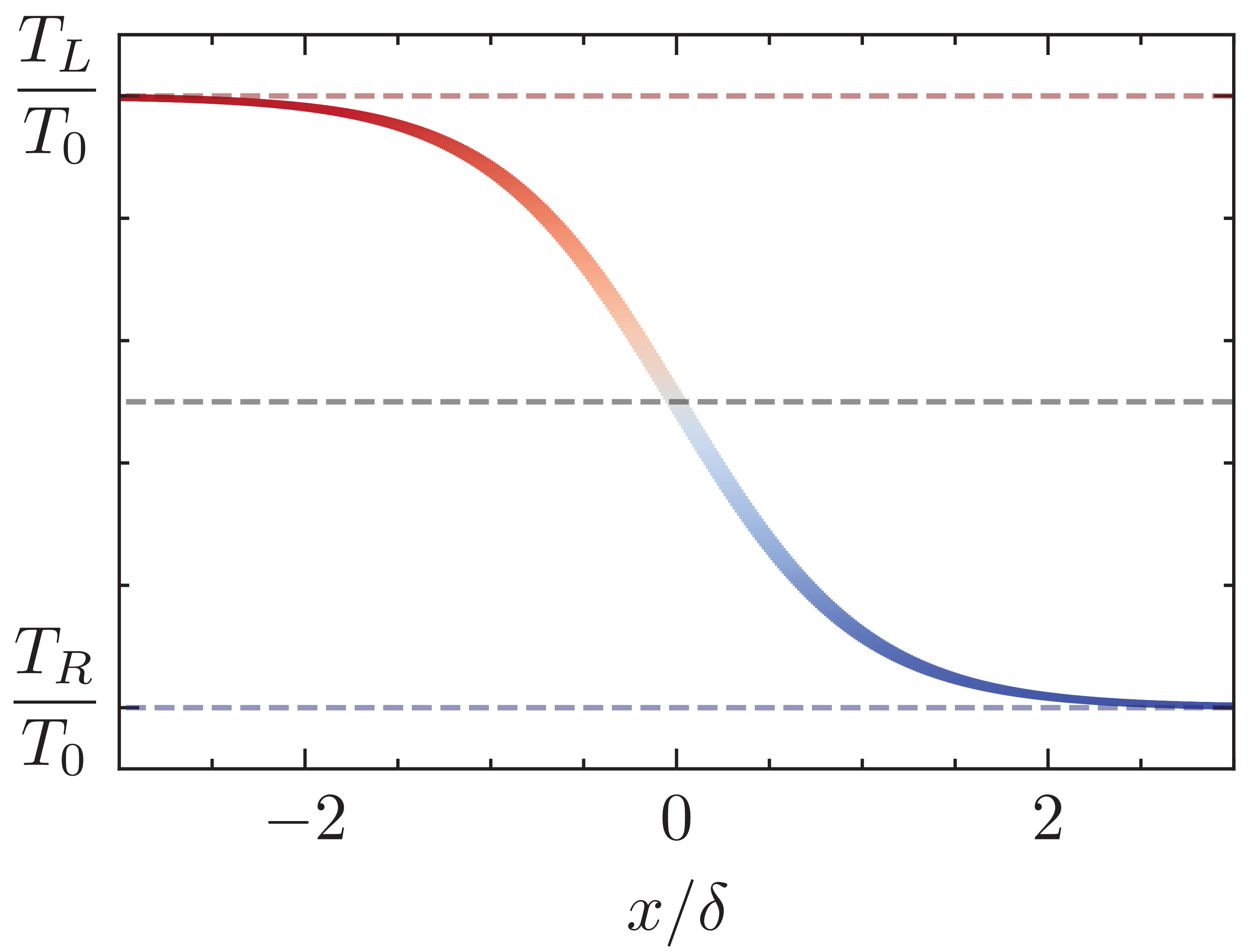


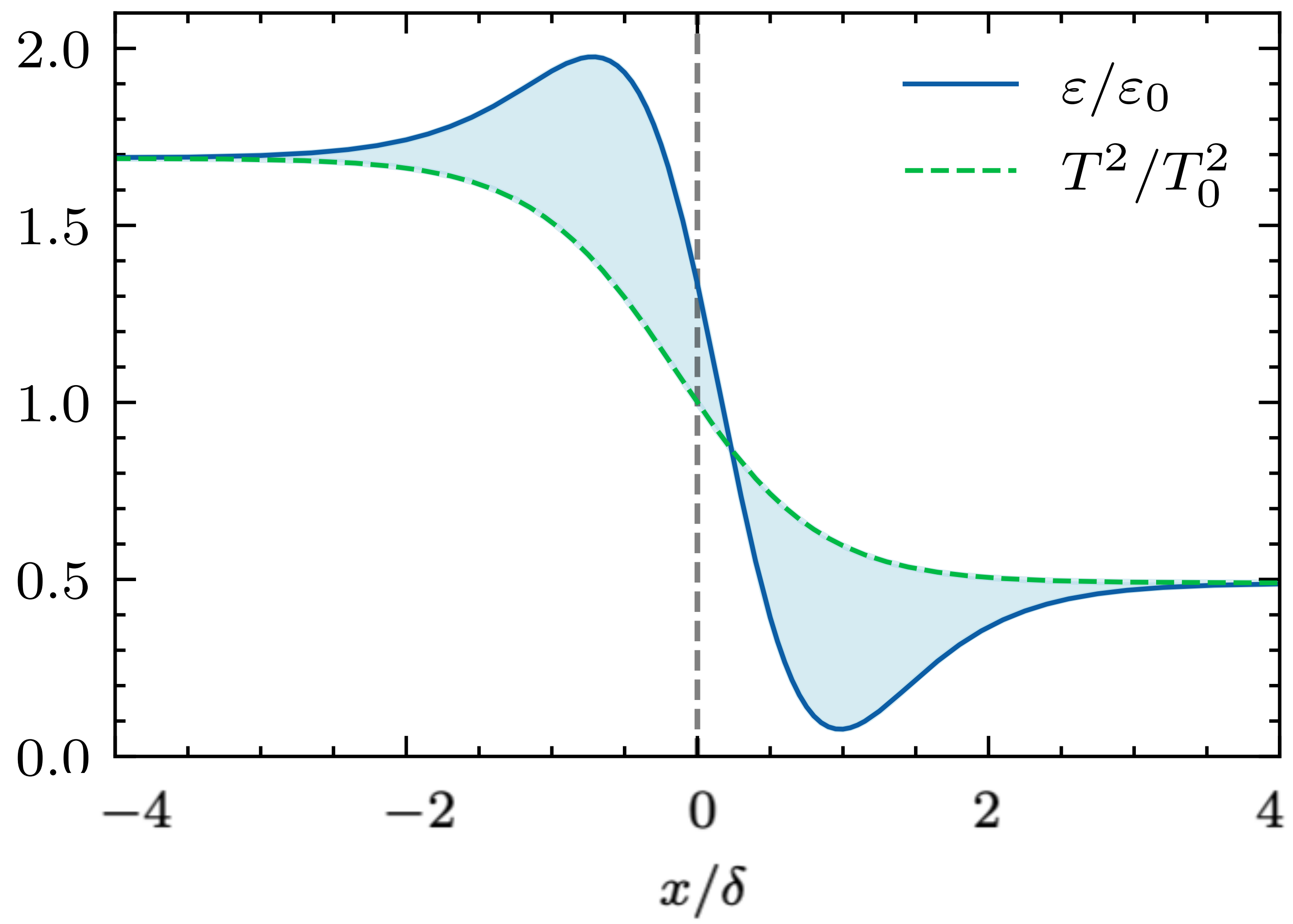
$T_m^2 \neq T^2$

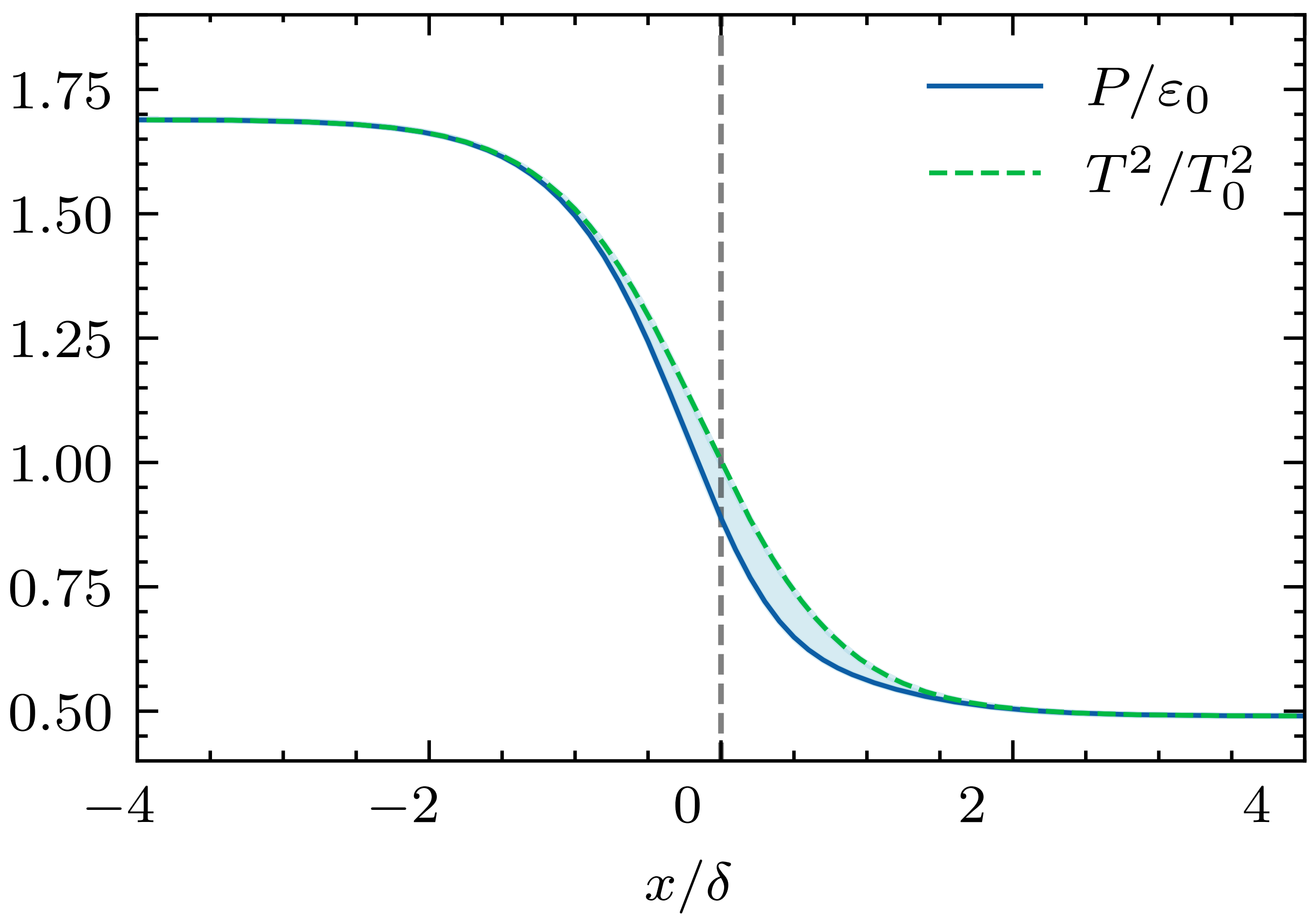
$\varepsilon \neq \gamma_{1D} T^2$

$P \approx \gamma_{1D} T^2$

3. Out-of-equilibrium states by an inhomogeneous T







Characteristic length scales for T variations

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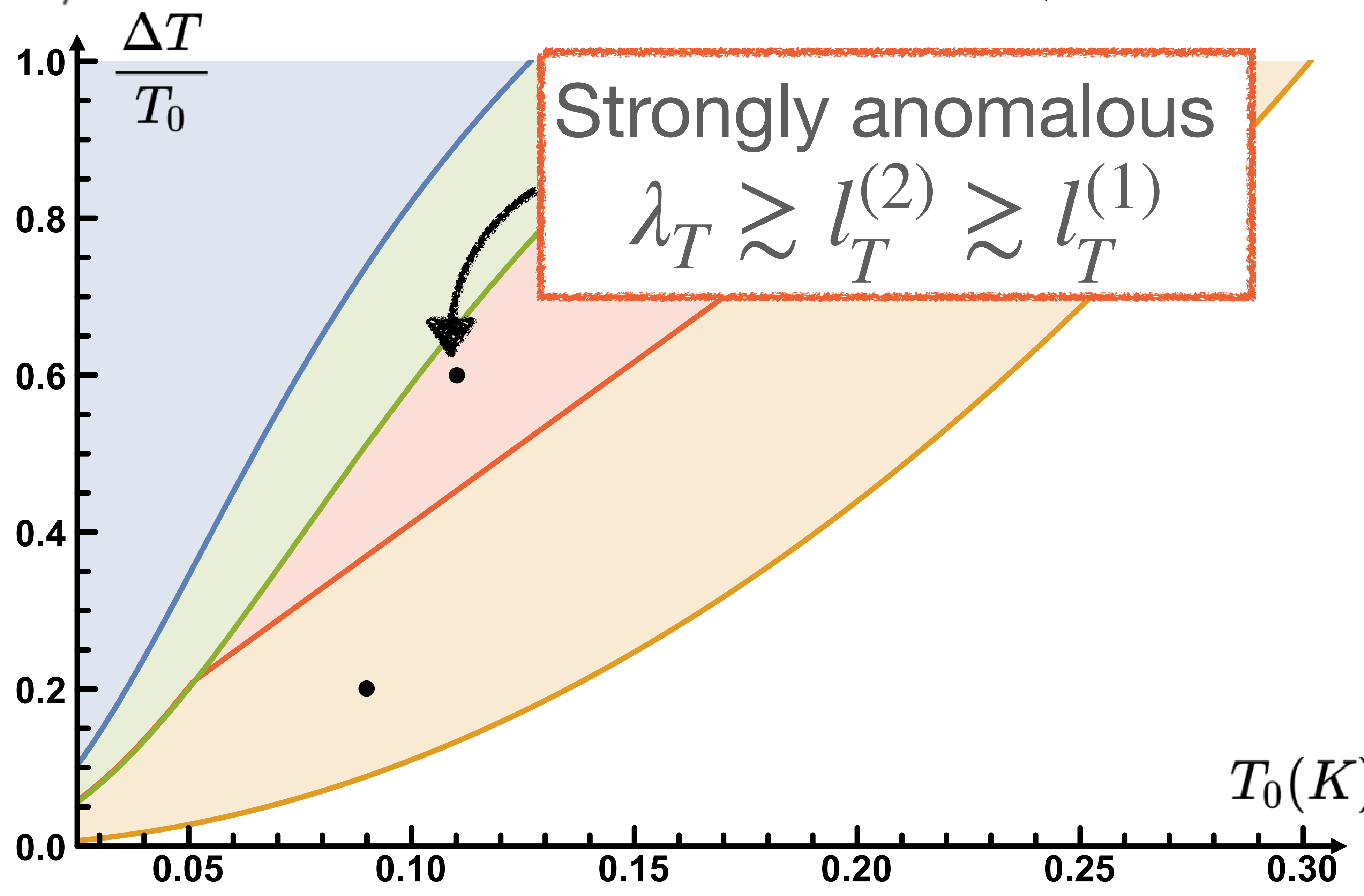

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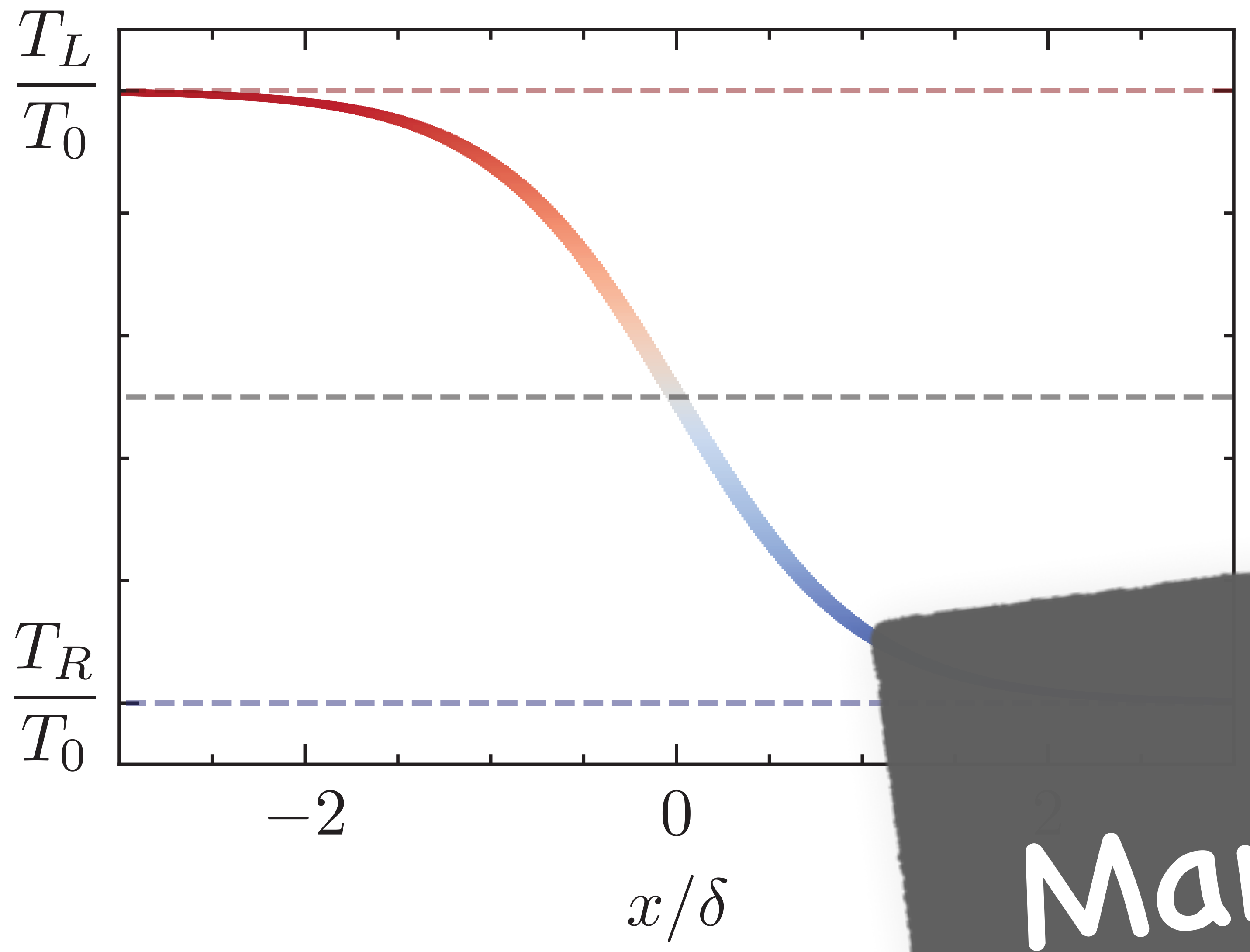

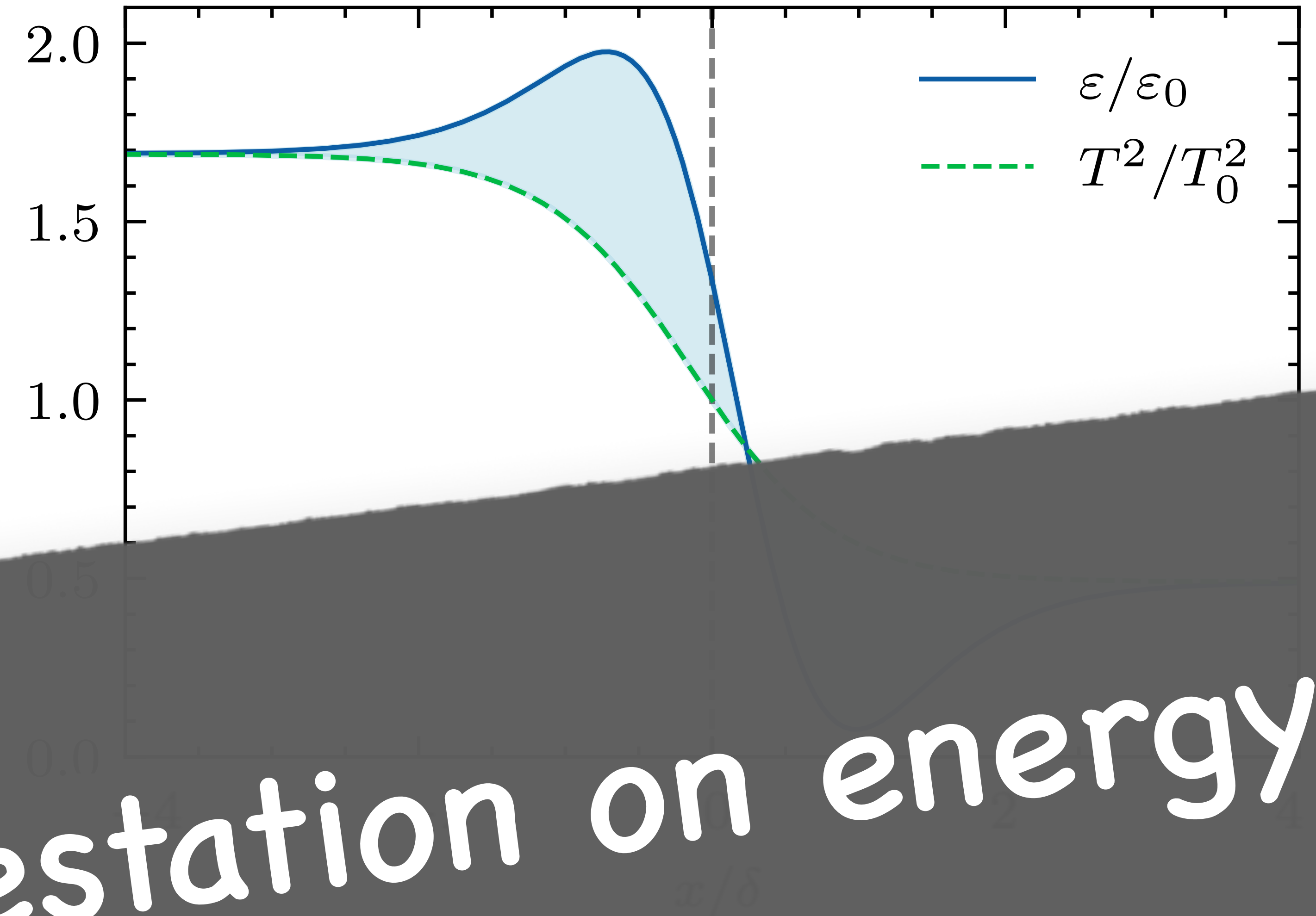

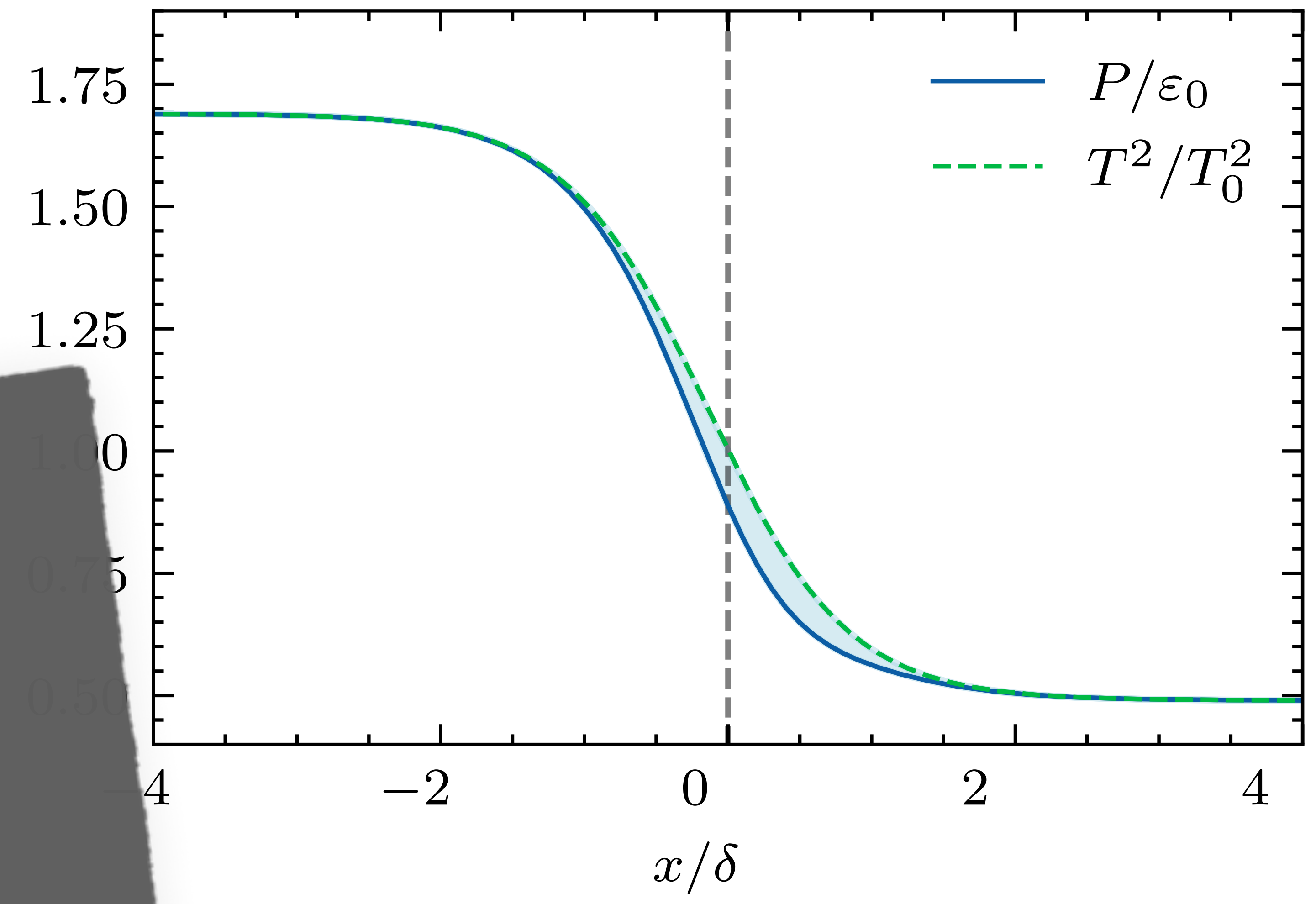

Manifestation on energy currents?

Characteristic length scales

$$l_T^{(1)} = \frac{T}{\partial_x T} \qquad l_T^{(2)} = \sqrt{\left|\frac{T}{\partial_x^2 T}\right|}$$

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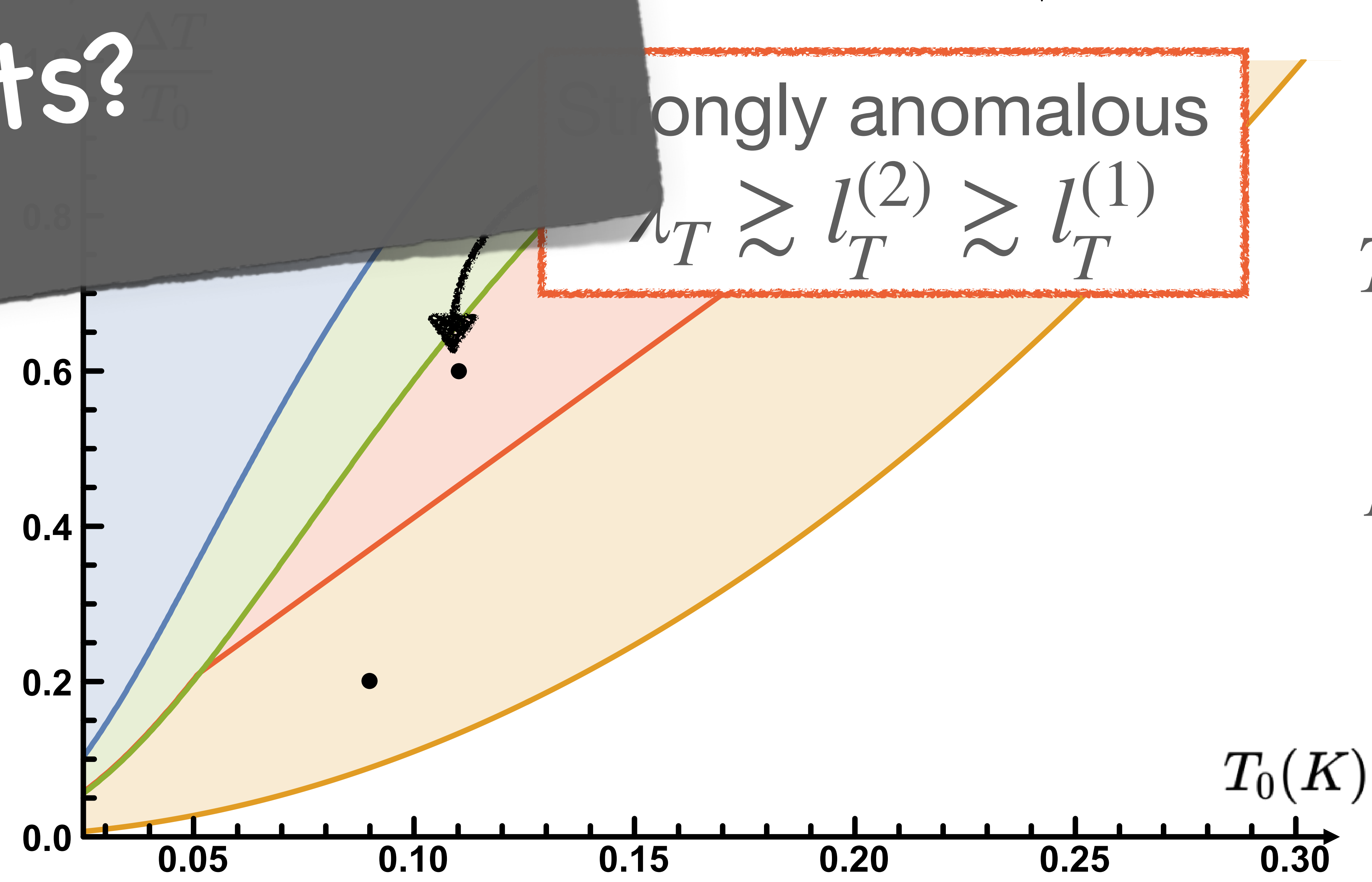
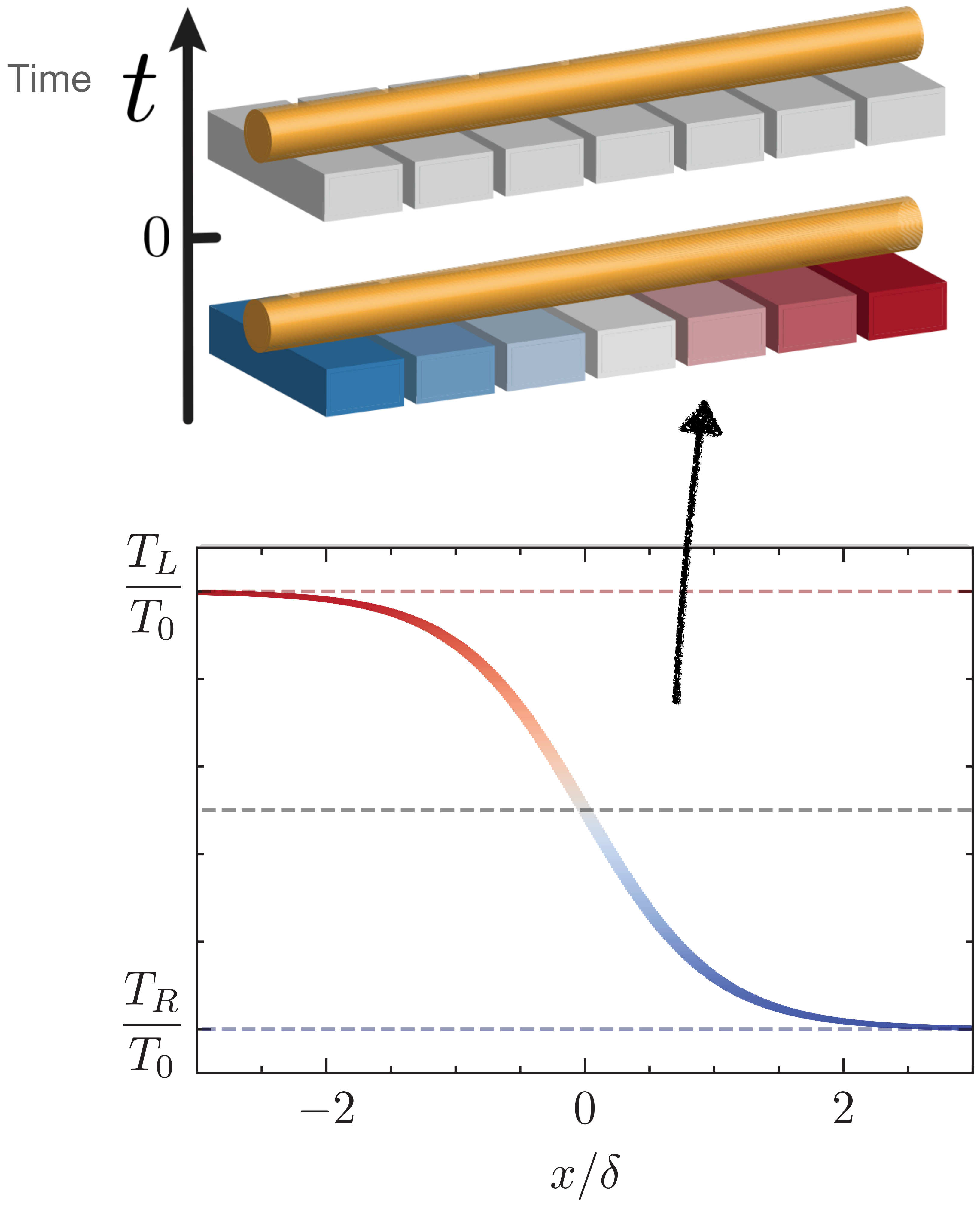


$T_m^2 \neq T^2$
$\varepsilon \neq \gamma_{1D} T^2$
$P \neq \gamma_{1D} T^2$

3. Out-of-equilibrium: quench dynamics

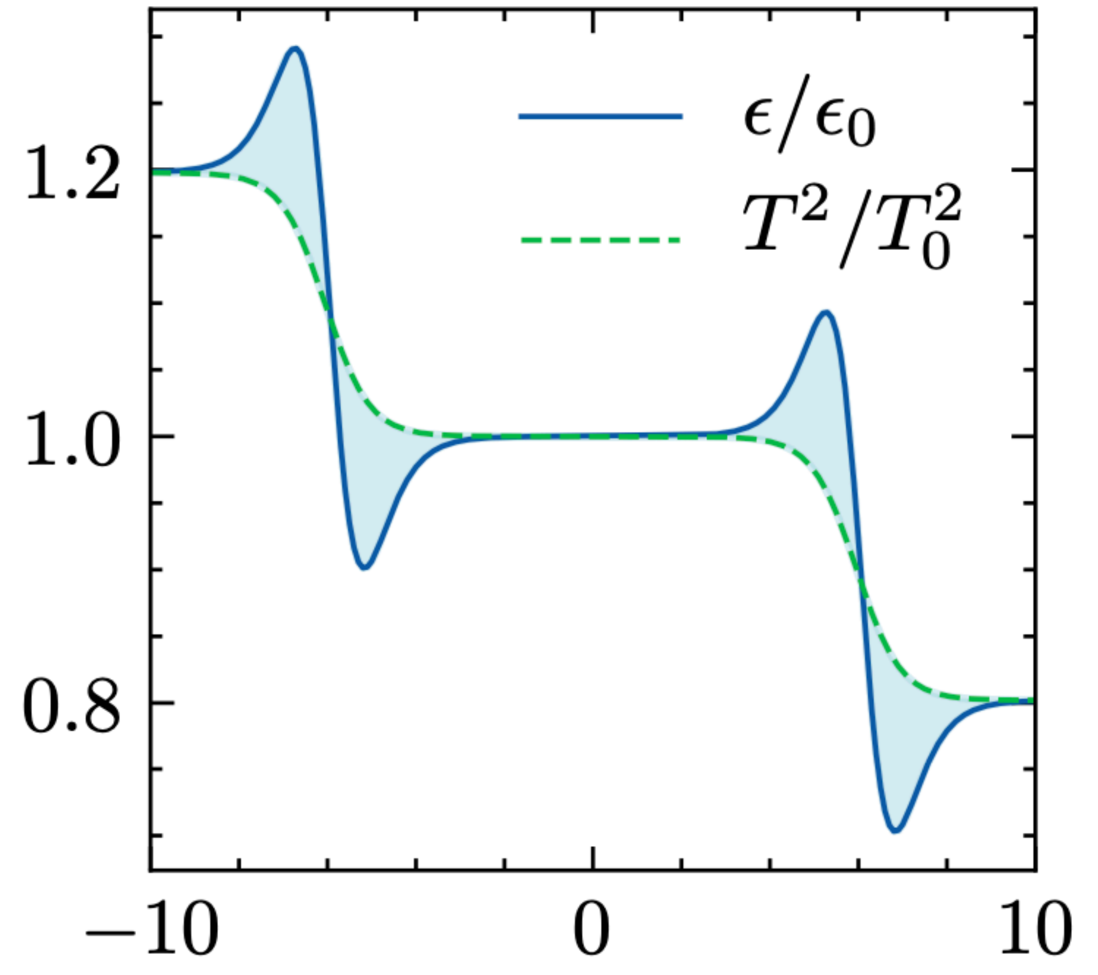
- t<0: previous out-of-equilibrium state via external $T(x)$
- t=0: release external T
- Amounts to realize a **quench of spacetime** following (generalized) Luttinger equivalence

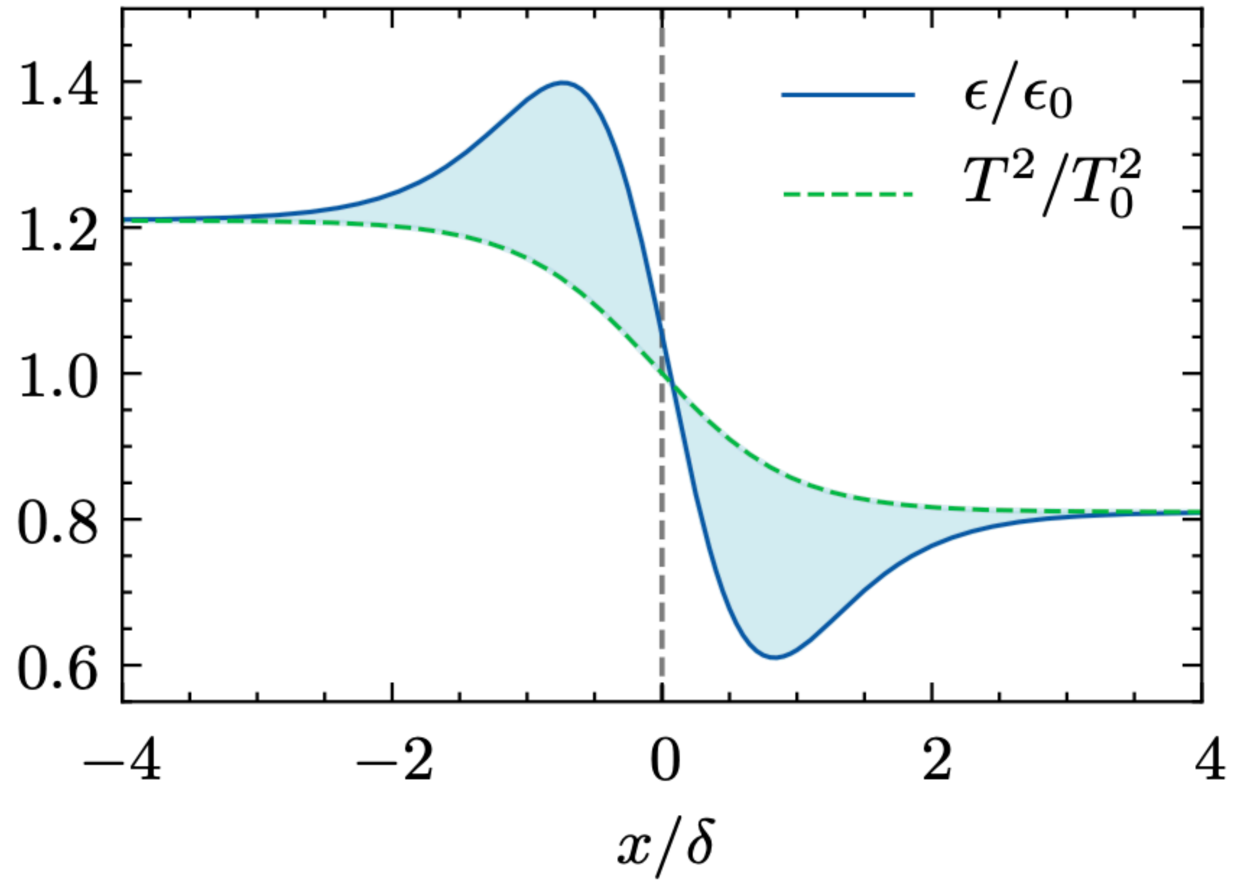


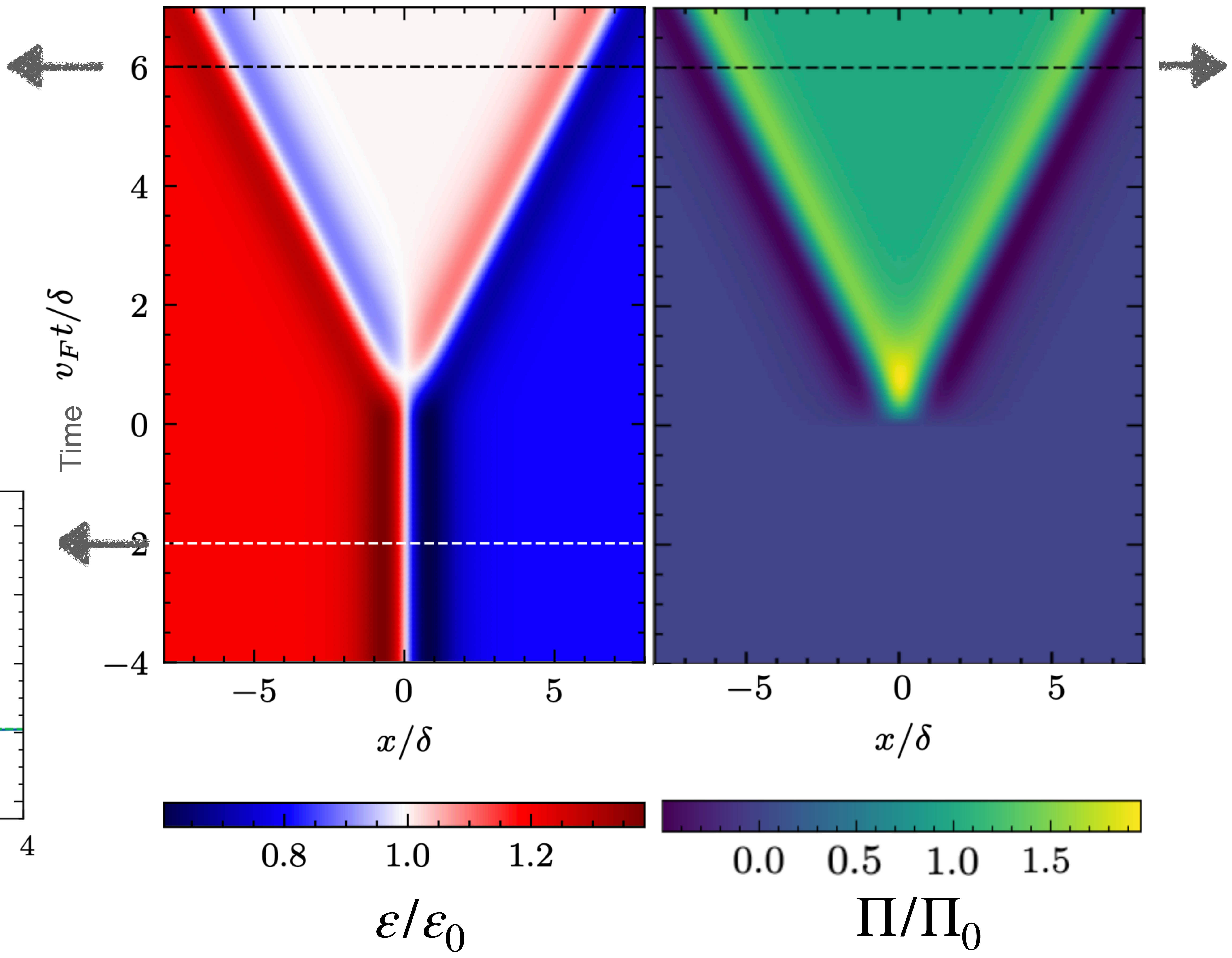
3. Out-of-equilibrium: quench dynamics

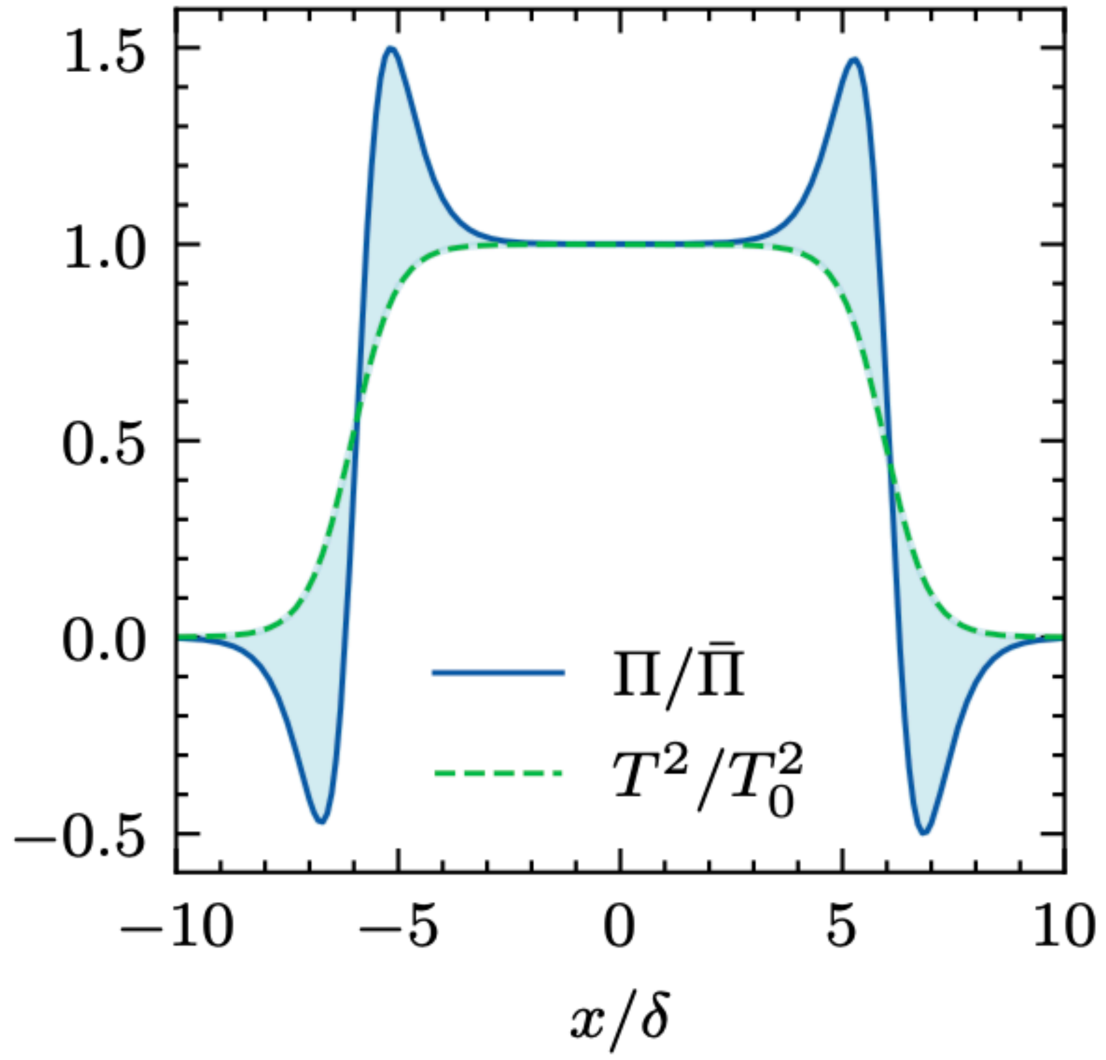
Langmann et al. (2017)
Gawedzki et al.(2018)
Karrasch et al. (2013)

2 « heat waves » of **energy** and momentum propagating at a velocity $\pm v_F$









Consistent with numerical simulation (XXZ chain), perturbative analysis and CFT computations

Langmann et al. (2017)
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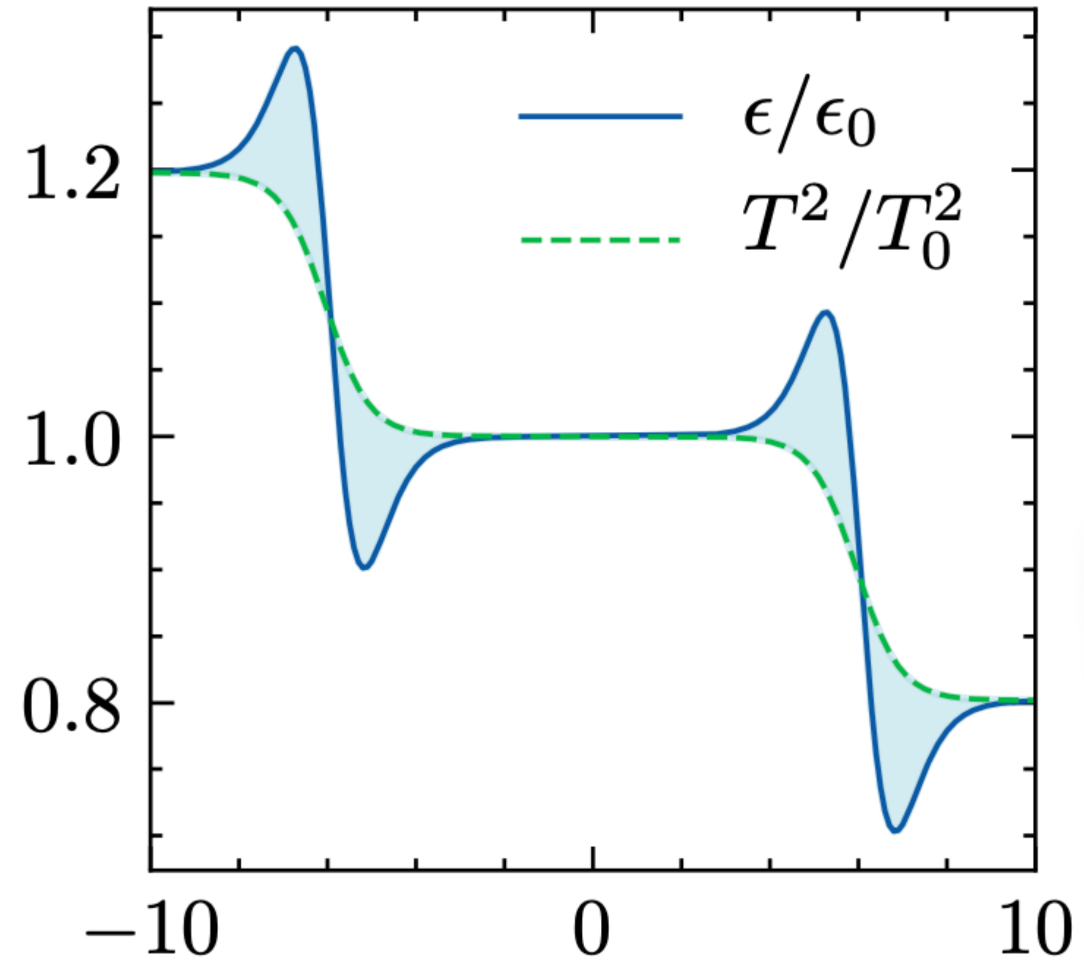
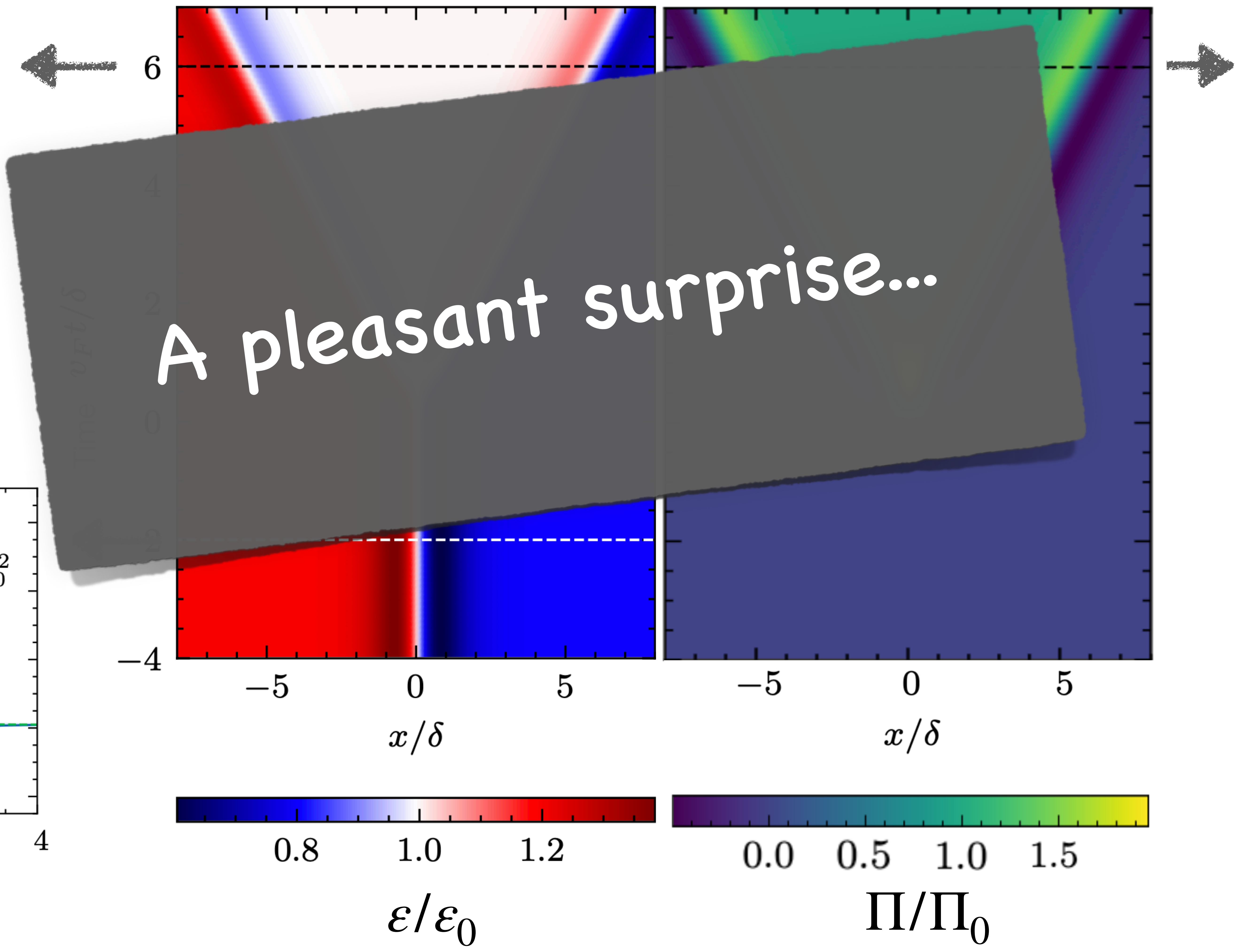
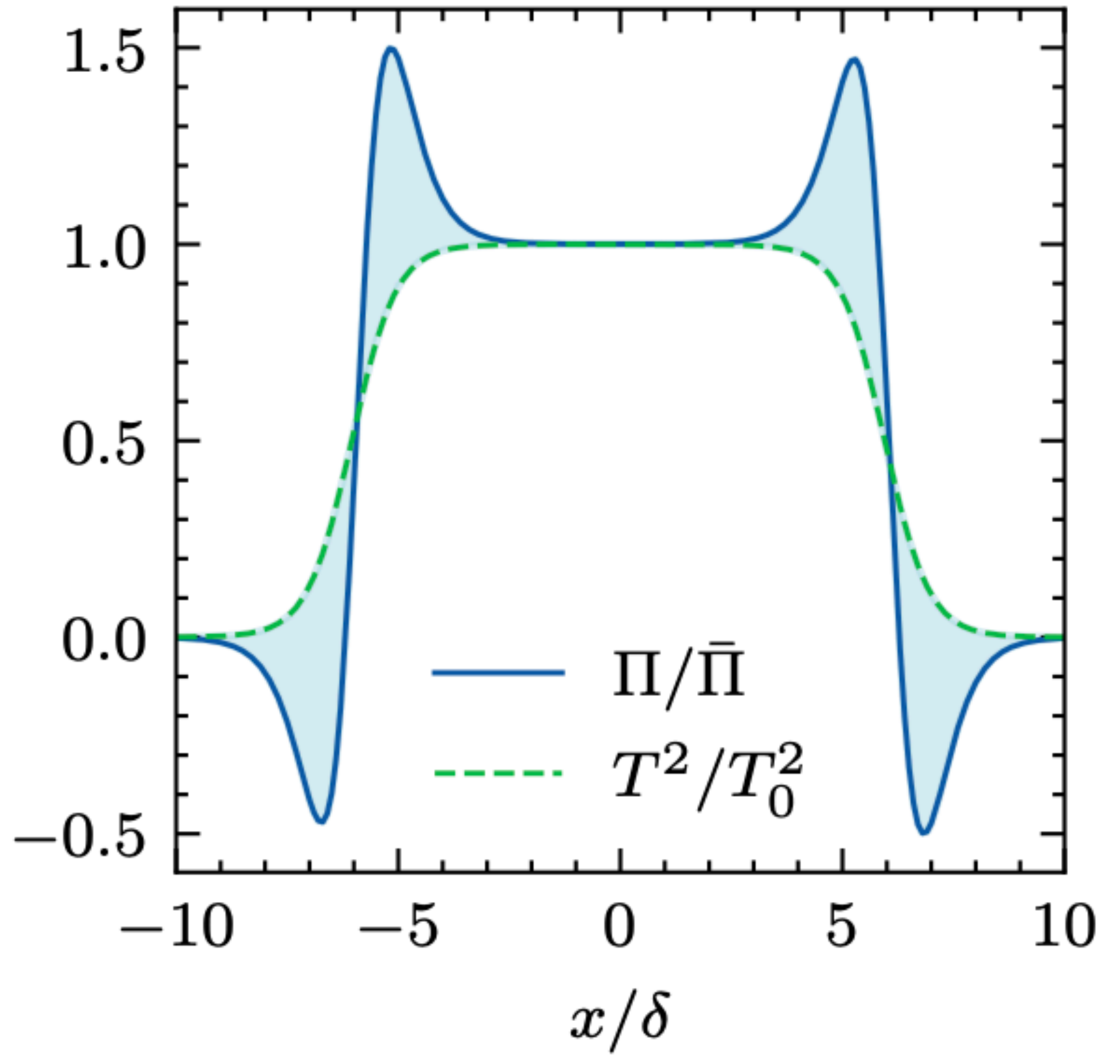
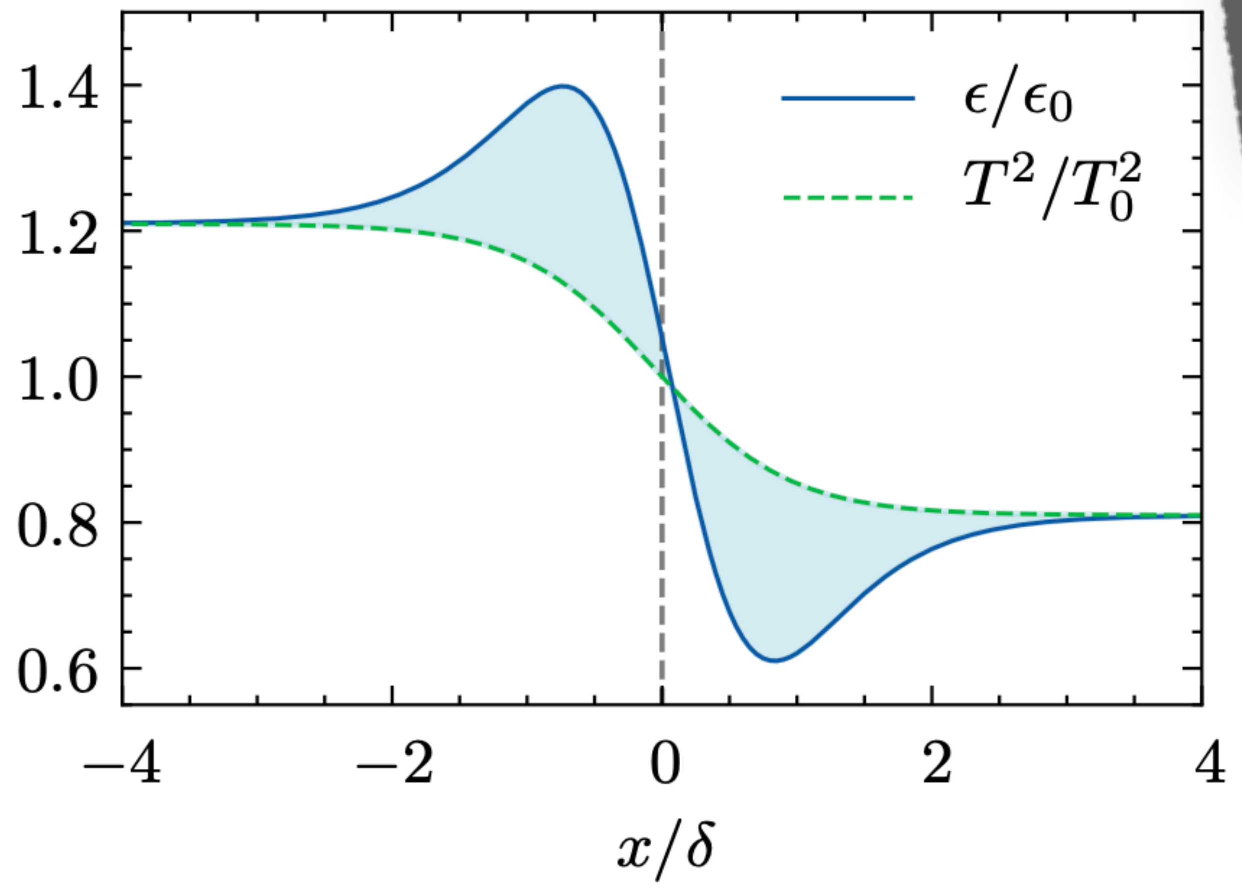




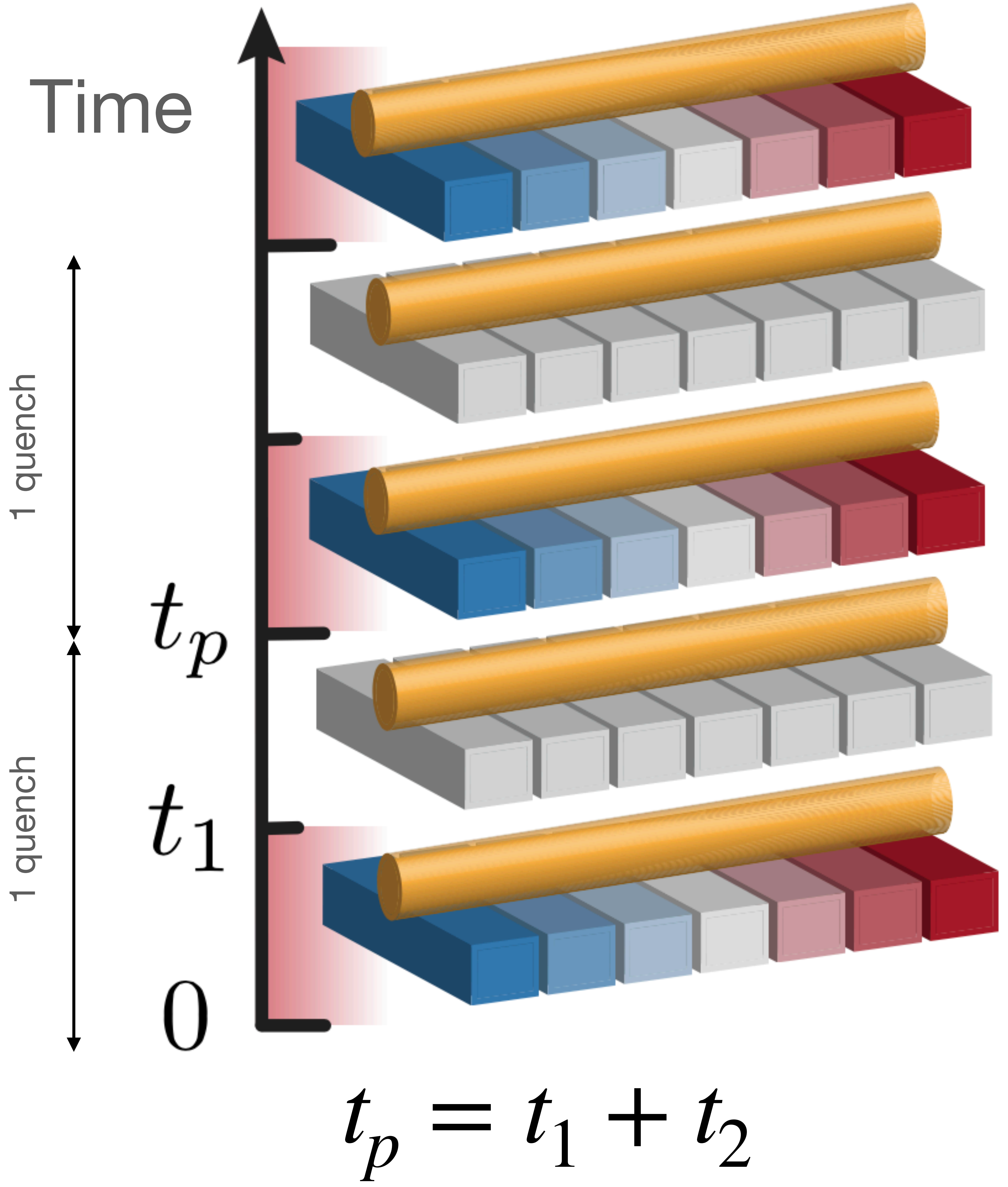




Consistent with numerical simulation (XXZ chain), perturbative analysis and CFT computations

4. From one to periodic quenches: Floquet heating states

Wen et al. (2018)
Lapierre et al. (2020)
Fan et al. (2020)



Protocol

- **Periodically** heat the system with $T(x)$ during time t_1 (curved spacetime)
- Let it evolve freely during time t_2 (flat spacetime)

Result

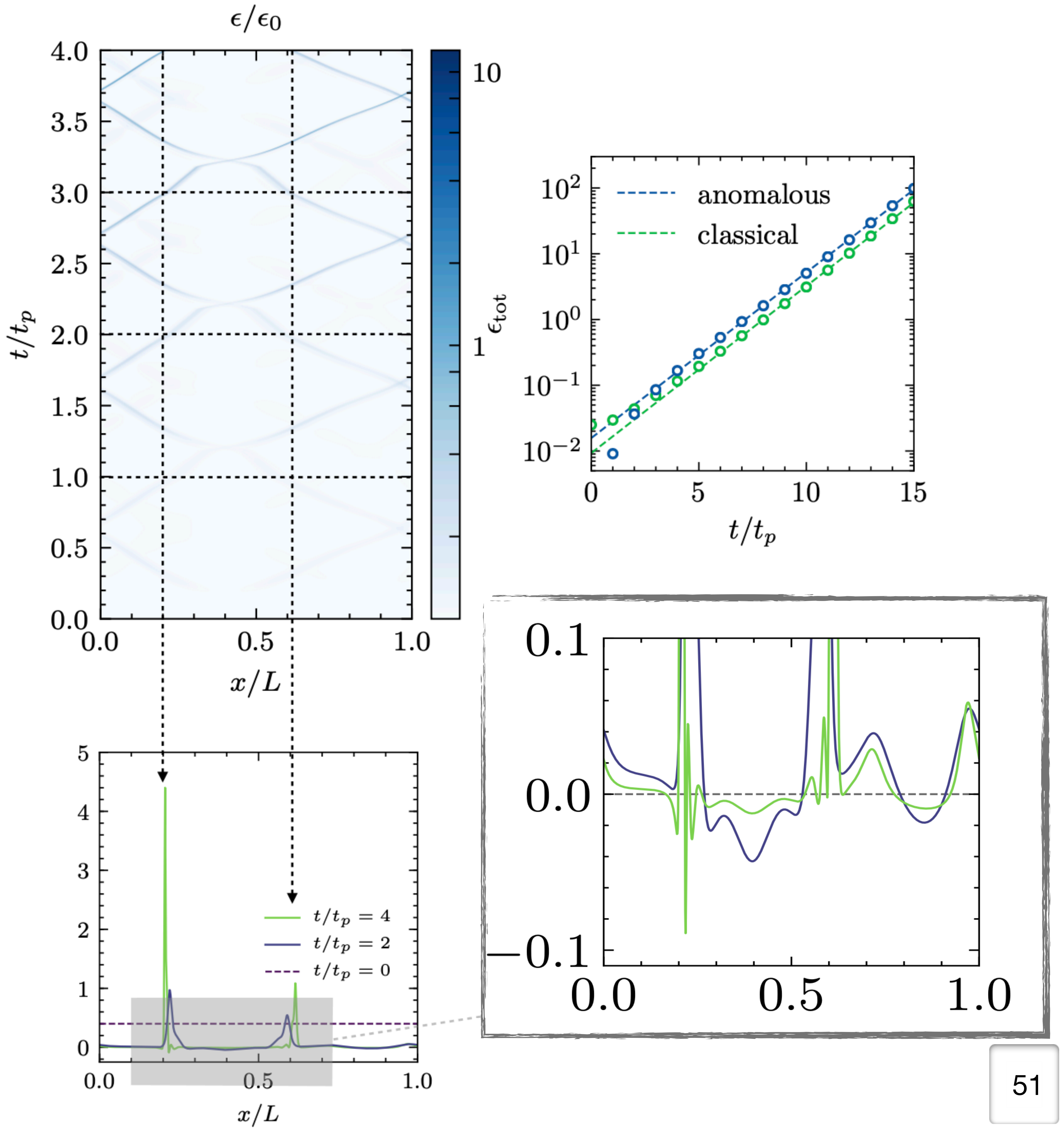
- For $t_p \approx L/v_F$: **Heating phase**

$E_{tot}(t) = \int dx\ \varepsilon(x,t)$ grows exp. with time

4. From one to periodic quenches: Floquet heating states

Lapierre et al. (2020)

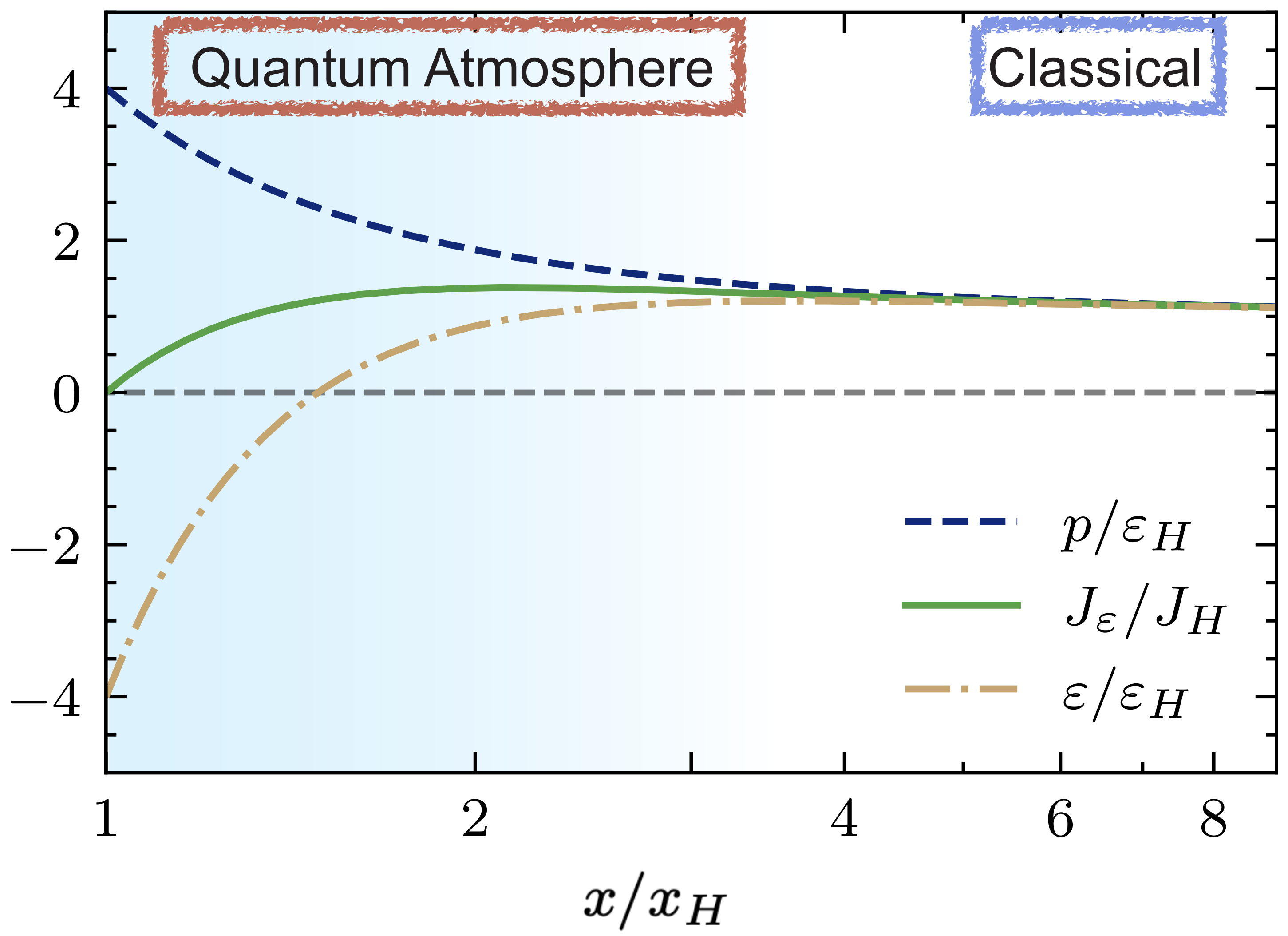
- Energy increases (**heating**) and **localizes**
- Amplitude of growth and energy profile affected by gravitational anomalies
    - Focusing points ≈ 2 **black holes**
    - Energy density negative close to their vicinity
    - Similar to **black-holes atmosphere**



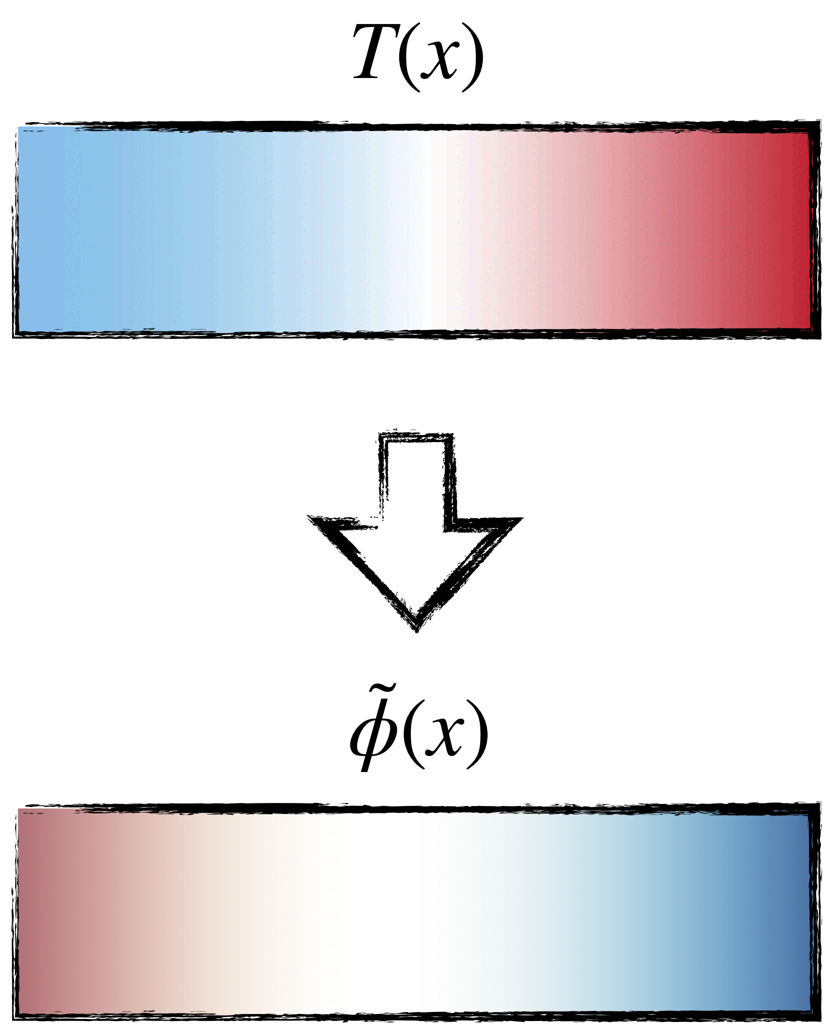
Conclusion

- Spacetime curvature lead to anomalous quantum fluctuations (Gravitational anomalies)
- Sizable consequences on heat transport in:

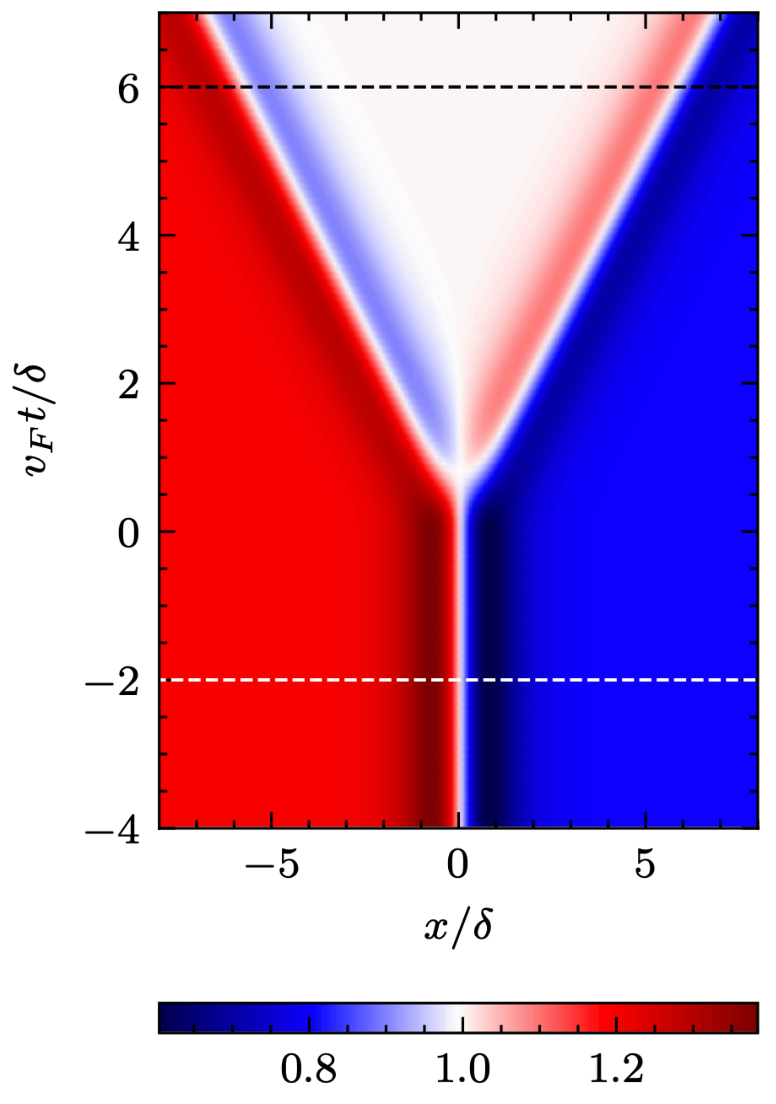
General relativity:
Black holes atmosphere



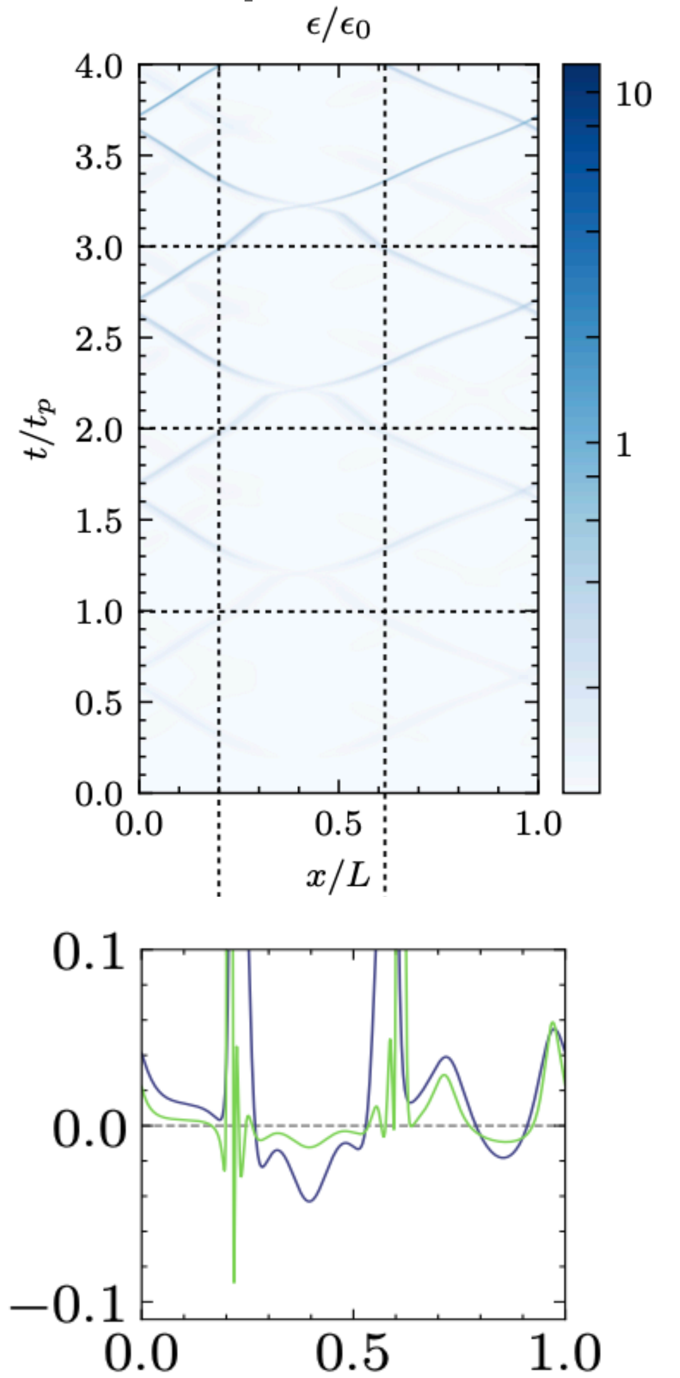
Condensed matter
Response theory



Out of equilibrium physics



Floquet heating phase



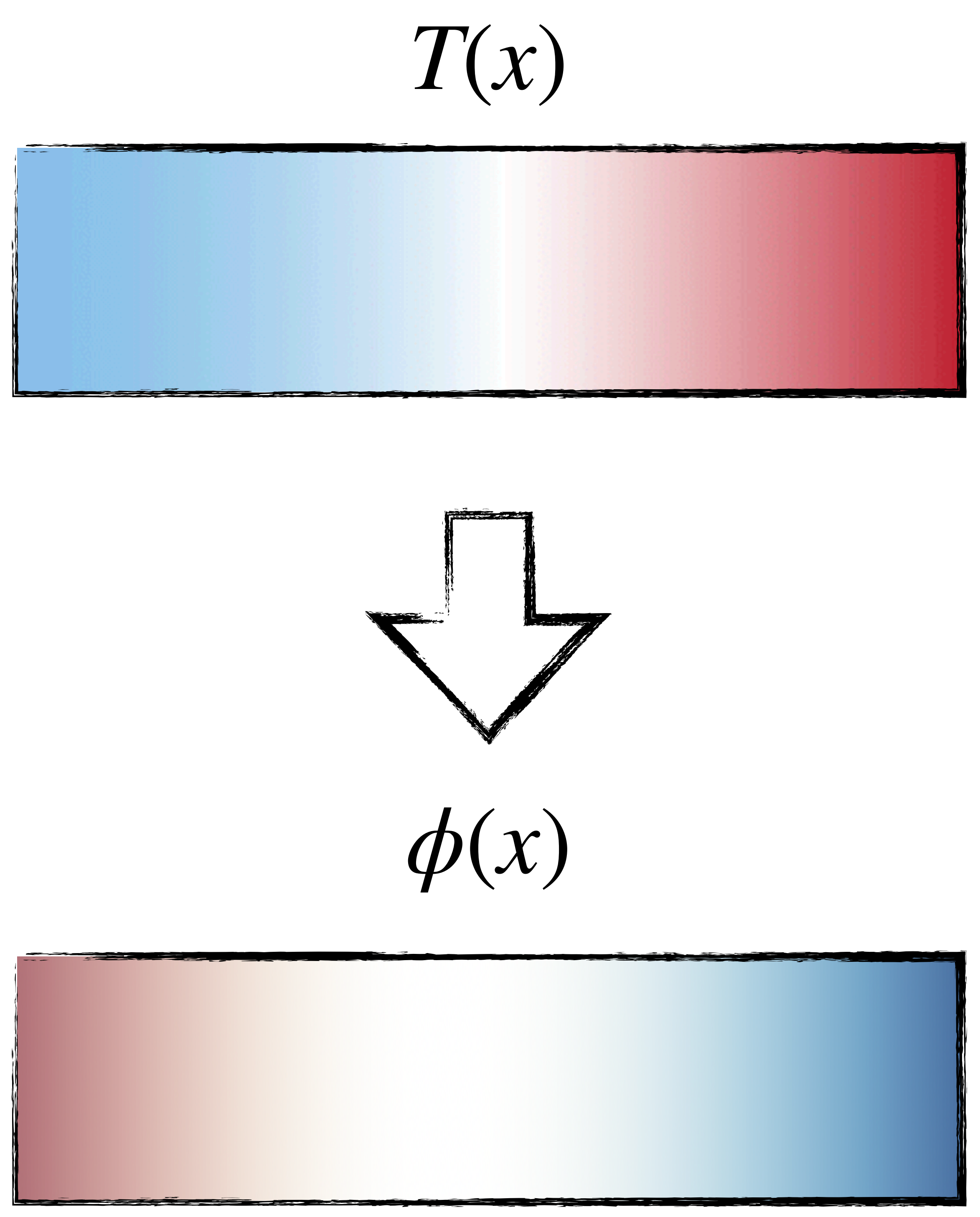
Anomalous Luttinger relation for energy transport: From black hole's atmosphere to thermal quenches
Bermond, Chernodub, Grushin, Carpentier (To be published)

2. Generalized luttinger trick and response theory

Static metric $g_{\mu\nu} = \begin{pmatrix} e^{2\phi(x)} & 0 \\ 0 & -1 \end{pmatrix}$

Inhomogeneous temperature

Equivalent gravitational potential



In **classical** situation

Tolman-Ehrenfest Temperature

$$\gamma T^2(x) = \gamma T_0^2 e^{-2\phi(x)}$$

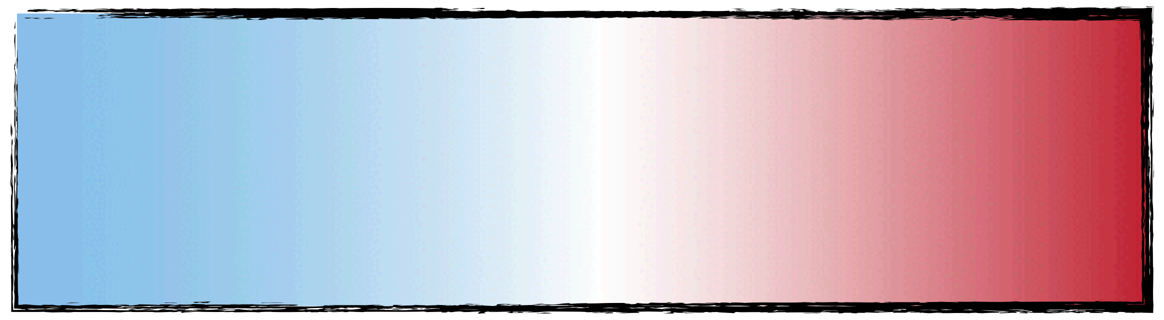
$$\Rightarrow \quad \frac{\partial_x T(x)}{T(x)} = -\partial_x \phi(x)$$

2. Generalized luttinger trick and response theory

Static metric $g_{\mu\nu} = \begin{pmatrix} e^{2\phi(x)} & 0 \\ 0 & -1 \end{pmatrix}$

Inhomogeneous temperature

$T(x)$



$\tilde{\phi}(x)$

Modified equivalent gravitational potential

In **presence** of gravitational anomalies

Corrected Tolman-Ehrenfest Temperature

$$\gamma T^2(x) = \gamma T_0^2 e^{-2\phi(x)} + \frac{\hbar v_F}{24\pi}\partial_x^2\phi(x)$$

$\varepsilon_q^{(2)}$

$$\Rightarrow \frac{\partial_x T(x)}{T(x)} = -\partial_x\phi(x) + \lambda_T^2\left[\partial_x\phi(x)\partial_x^2\phi(x) + \frac{1}{2}\partial_x^3\phi(x)\right]$$

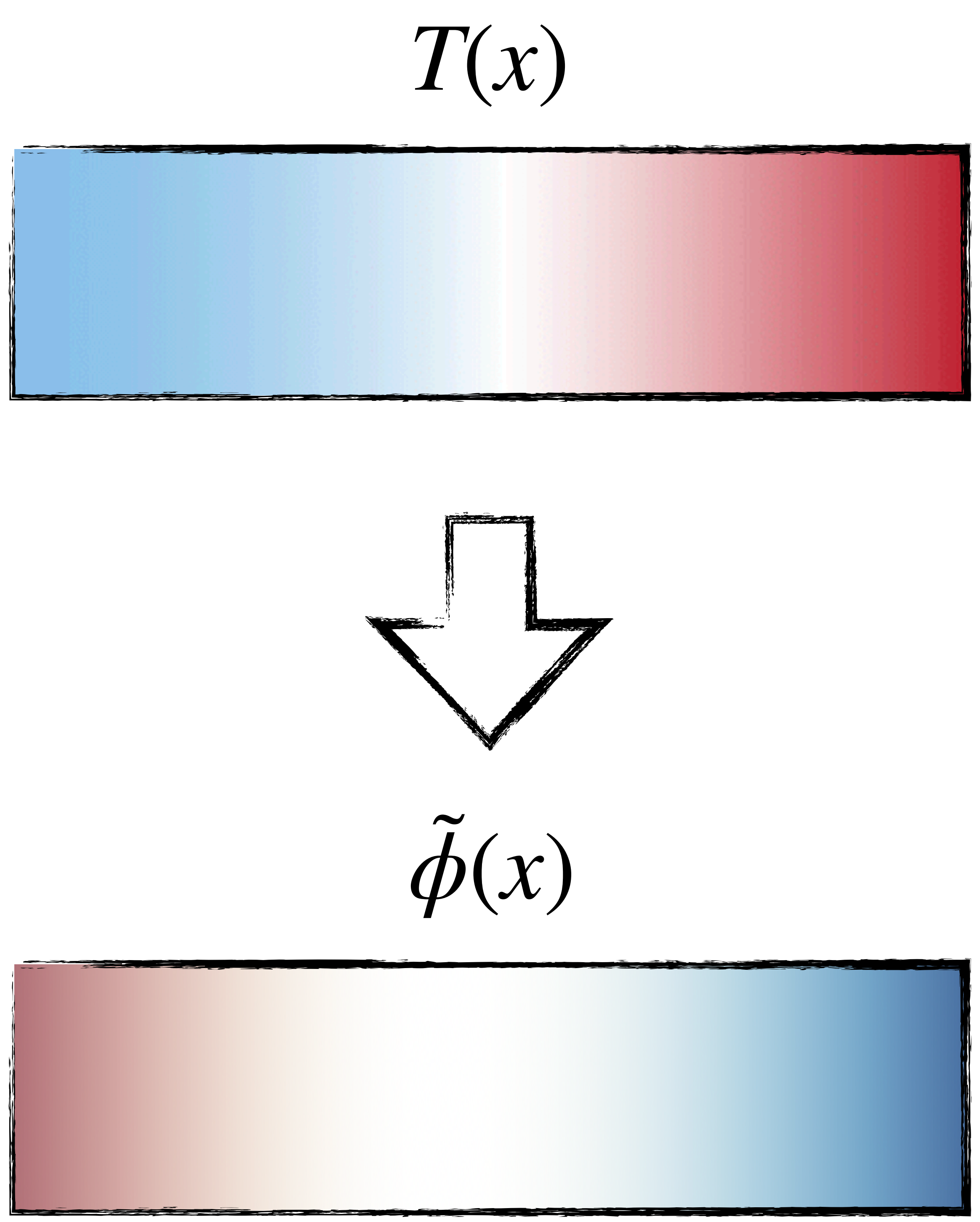
with $\lambda_T = \dfrac{\hbar v_F}{2\pi k_B T(x)}$

Corrected Luttinger relation

2. Generalized luttinger trick and response theory

Static metric $g_{\mu\nu} = \begin{pmatrix} e^{2\phi(x)} & 0 \\ 0 & -1 \end{pmatrix}$

Inhomogeneous temperature



In **presence** of gravitational anomalies

Corrected Tolman-Ehren…

$$\gamma T^2(x) = \gamma T_0^2 e^{-2\phi(x)} + \dots$$

Correction to linear response theory ?

Modified equivalent gravitational potential

$$\Rightarrow \quad \frac{\partial_x T(x)}{T(x)} = -\partial_x \phi(x) + \lambda_T^2 \left[\partial_x \phi(x) \partial_x^2 \phi(x) + \frac{1}{2} \partial_x^3 \phi(x) \right] \quad \text{with } \lambda_T = \frac{\hbar v_F}{2\pi k_B T(x)}$$

Corrected Luttinger relation

2. Generalized luttinger trick and response theory

Linear temperature profile

$$T(x) = T_0 + x\Delta T, \qquad x \in \left[-L/2, L/2\right]$$

Solve for ϕ perturbatively in $\alpha = \dfrac{\Delta T}{T}$

$$\phi(x) = \phi_0(x) + \delta\phi(x)$$

Classical gravitational potential

$$\phi_0(x) = \ln\left(T_0/T(x)\right)$$

Quantum corrections

$$\delta\phi(x) = \alpha^2 \frac{\lambda_{T_0}^2}{2L^2} - 2\alpha^3 \frac{\lambda_{T_0}^3}{L^3} x$$

$$\lambda_{T_0} = \frac{\hbar v_F}{2\pi k_B T_0}$$

2. Generalized luttinger trick and response theory

Linear temperature profile $T(x) = T_0 + x\Delta T, \qquad x \in \left[-L/2, L/2\right]$

Energy currents $J_\varepsilon(x) = J^0_\varepsilon(x) + \delta J_\varepsilon(x)$

Classical ballistic energy current

$$J^0_\varepsilon(x) = \frac{\pi}{6} k_B^2 T^2(x)$$

Quantum corrections

$$\delta J_\varepsilon(x) \propto \lambda_{T_0}^2 \left(\frac{\Delta T}{L\ T_0}\right)^2, \quad \lambda_{T_0} = \frac{\hbar v_F}{2\pi k_B T_0}$$

⇨ Correction of the **non-linear** response