Weyl Anomalies of Boundaries and Defects

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The Royal Society

Science & Technology Facilities Council

Conformal Anomalies: Theory and Applications
Université de Tours
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Credits

1309.4523 O'B. + Jensen
1509.02160 O'B. + Jensen
1812.00923 O'B. + Estes, Krym, Robinson, Rodgers
1812.08745 O'B. + Jensen, Robinson, Rodgers
2003.02857 O'B. + Chalabi, Robinson, Sisti
2111.14713 O'B. + Chalabi, Herzog, Robinson, Sisti

1205.1573 Nozaki, Takayanagi, Ugajin
1305.2334 Fursaev
1510.00021 Herzog, Huang, Jensen
1510.01427 Fursaev
1510.04566 Solodukhin
1511.06713 Bianchi, Meineri, Myers, Smolkin
1601.06418 Fursaev + Solodukhin
1601.02883 Billò, Gonçalves, Lauria, Meineri
1604.07571 Bethiere + Solodukhin
1702.00566 Astaneh + Solodukhin
1703.04186 Astaneh, Berthiere, Fursaev, Solodukhin
1707.06224 Herzog + Huang
1709.07431 Herzog, Huang, Jensen
1804.01974 Prochazka
1810.06995 Kobayashi, Nishioka, Sato, Watanabe
1812.08183 Casini, Landea, Torroba
1906.11281 Herzog + Shamir
1907.04952 Herzog + Shamir
1907.06193 Bianchi
1911.05082 Bianchi + Lemos
2010.04995 Herzog + Shrestha
2012.06574 Wang
2101.12648 Wang
2102.07661 Astaneh + Solodukhin

See also A. F. Astaneh's talk!

Outline:

- Review: Boundaries and Defects
- Review: Weyl Anomalies
- 1d Boundary or Defect
- 2d Boundary or Defect
- 3d Boundary or Defect
- 4d Boundary or Defect
- Summary and Outlook

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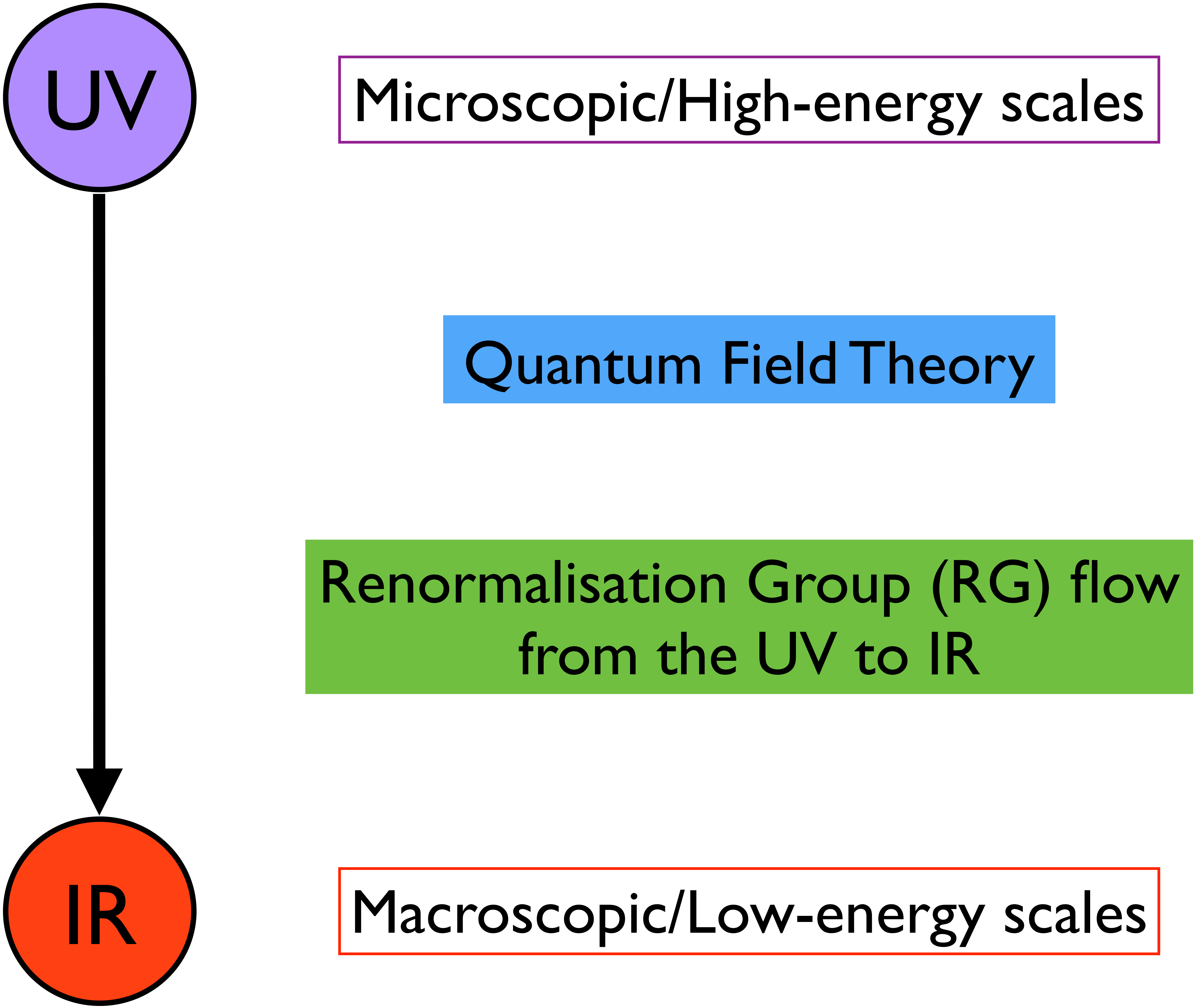
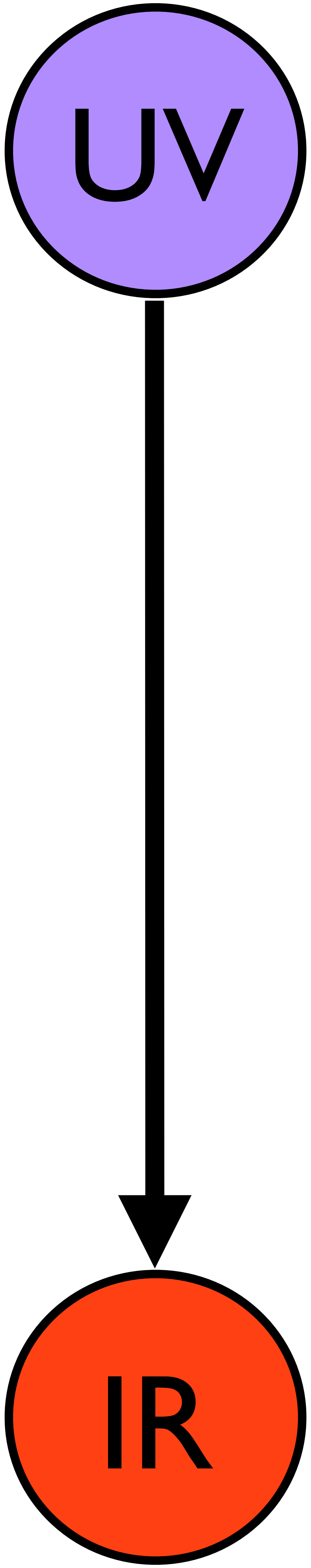

UV
Microscopic/High-energy scales
Quantum Field Theory
Renormalisation Group (RG) flow from the UV to IR
IR
Macroscopic/Low-energy scales



UV CFT

Conformal Field Theories (CFTs)

RG fixed points

IR CFT

Quantum Field Theory

Generating functional Z

Non-dynamical background metric $g_{\mu\nu}(x)$

Stress-Energy Tensor

$$T_{\mu\nu} \equiv \frac{2}{\sqrt{g}} \frac{\delta}{\delta g^{\mu\nu}} \ln Z[g_{\mu\nu}]$$

Quantum Field Theory

Generating functional Z

Non-dynamical background metric $g_{\mu\nu}(x)$

Stress-Energy Tensor

$$g_{\mu\nu} \to \delta_{\mu\nu}$$

Translational and Rotational Symmetry

$$\partial_\mu T_{\mu\nu} = 0$$

Conformal Field Theory

Generating functional Z

Non-dynamical background metric $g_{\mu\nu}(x)$

Conformal Transformation

Diffeomorphism

$$x^\mu \to x'^\mu(x)$$

such that

$$g_{\mu\nu}(x) \to e^{2\Omega(x)} g_{\mu\nu}(x)$$

Conformal Field Theory

$$g_{\mu\nu}(x) \to e^{2\Omega(x)} g_{\mu\nu}(x)$$

$$T_\mu{}^\mu = -\frac{1}{\sqrt{g}} \frac{\delta}{\delta\Omega} \ln Z[g_{\mu\nu}]$$

Conformal invariance

$$T_\mu{}^\mu = 0$$

CFT

flat space $\mathbb{R}^d$

Rotations $\qquad x^\mu \to \Lambda^\mu{}_\nu \, x^\nu$

Translations $\qquad x^\mu \to x^\mu + c^\mu$

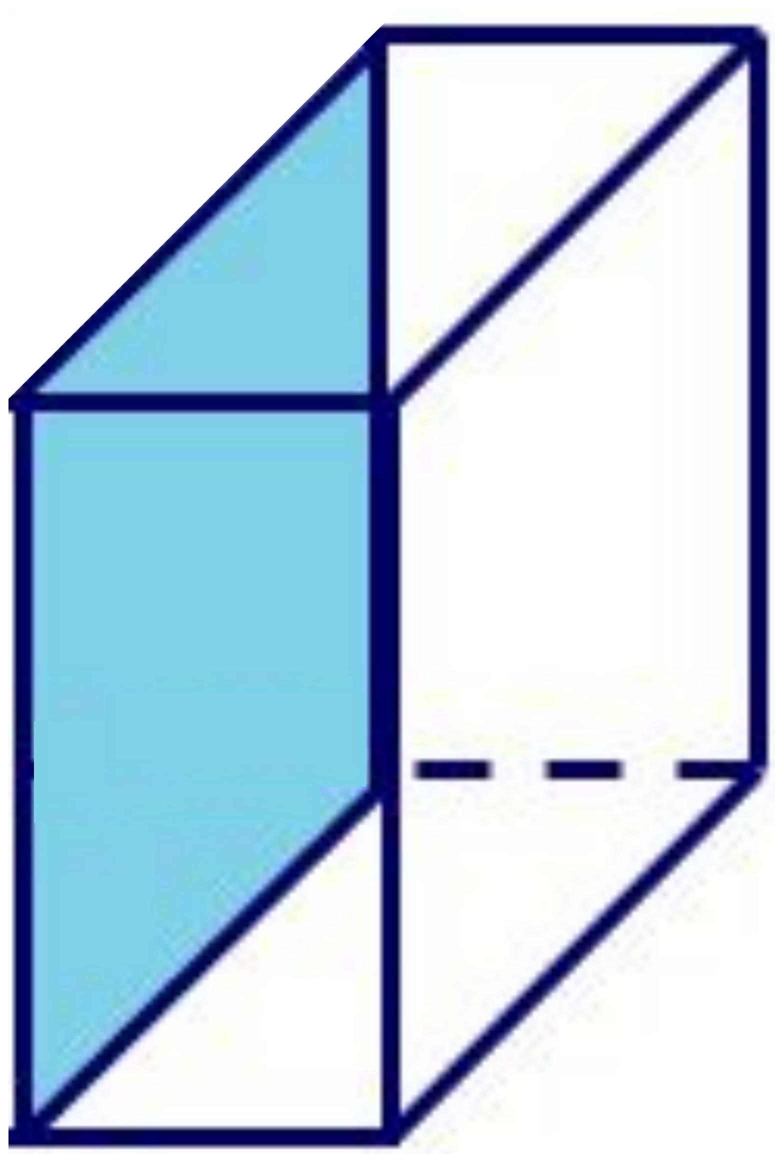
Dilatations $\qquad x^\mu \to \lambda \, x^\mu$

Special Conformal $\qquad x^\mu \to \dfrac{x^\mu + b^\mu x^2}{1 + 2x^\nu b_\nu + b^2 x^2}$

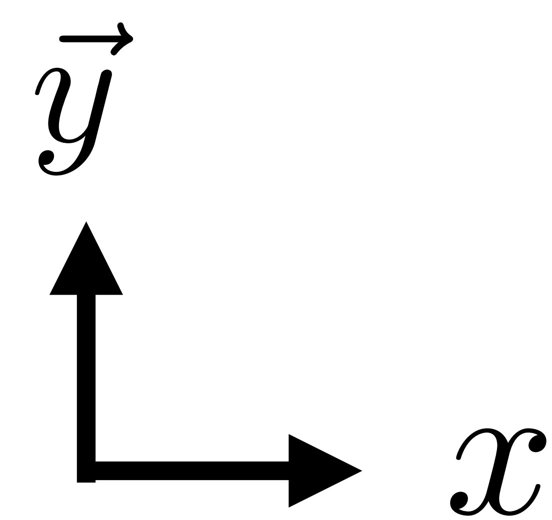
$$SO(d+1,1)$$

conformal field theory (CFT) with a boundary
+ conformally-invariant boundary conditions
possibly + massless degrees of freedom at the boundary

Boundary Conformal Field Theory (BCFT)



CFT dimension	d
Boundary dimension	$d-1$
Co-dimension	1



Examples

Experiments
graphene with a boundary

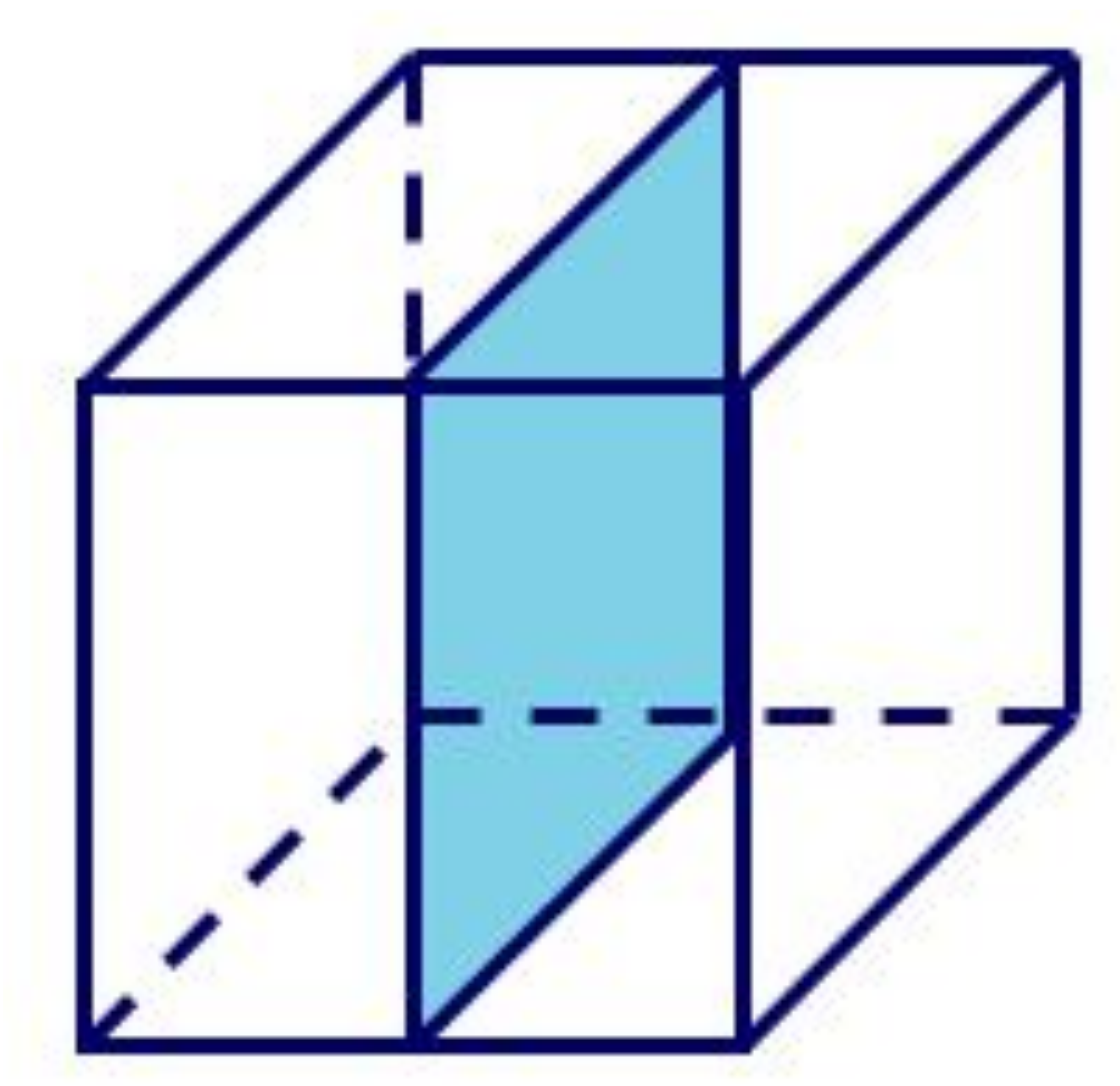
Quantum Field Theory
free fields, Ising model, Wilson-Fisher, etc.
with a boundary

String theory + M-theory
various branes ending on branes

Holography
many examples

CFT with a conformally-invariant defect:
conformally-invariant boundary conditions along a submanifold
and/or massless degrees of freedom supported along a submanifold

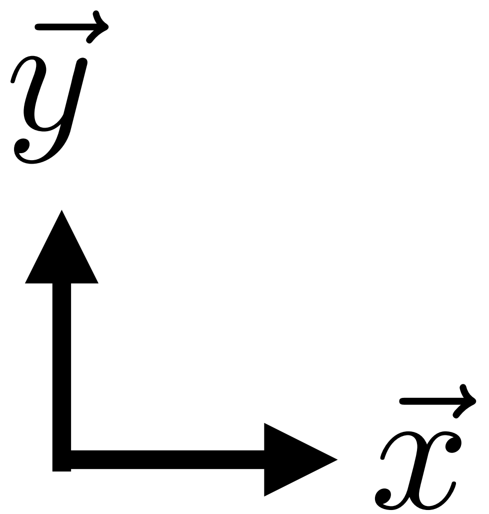
Defect Conformal Field Theory (DCFT)



CFT dimension d

Defect dimension p

Co-dimension $d - p$



Examples

Experiments
graphene with an impurity, line defect, etc.

Quantum Field Theory
Wilson lines, surface operators, etc.

String theory + M-theory
various branes intersecting branes

Holography
many examples

CFT

flat space $\mathbb{R}^d$

Rotations $\quad x^\mu \to \Lambda^\mu{}_\nu x^\nu$

Translations $\quad x^\mu \to x^\mu + c^\mu$

Dilatations $\quad x^\mu \to \lambda x^\mu$

Special Conformal $\quad x^\mu \to \dfrac{x^\mu + b^\mu x^2}{1 + 2x^\nu b_\nu + b^2 x^2}$

$$SO(d+1,1)$$

BCFT + DCFT

boundary or defect in flat space

Rotations $x^\mu \to \Lambda^\mu{}_\nu \, x^\nu$

Translations $x^\mu \to x^\mu + c^\mu$

Dilatations $x^\mu \to \lambda \, x^\mu$

Special Conformal $x^\mu \to \dfrac{x^\mu + b^\mu x^2}{1 + 2x^\nu b_\nu + b^2 x^2}$

$$SO(d+1,1)$$

Broken to subgroup
that preserves the boundary or defect

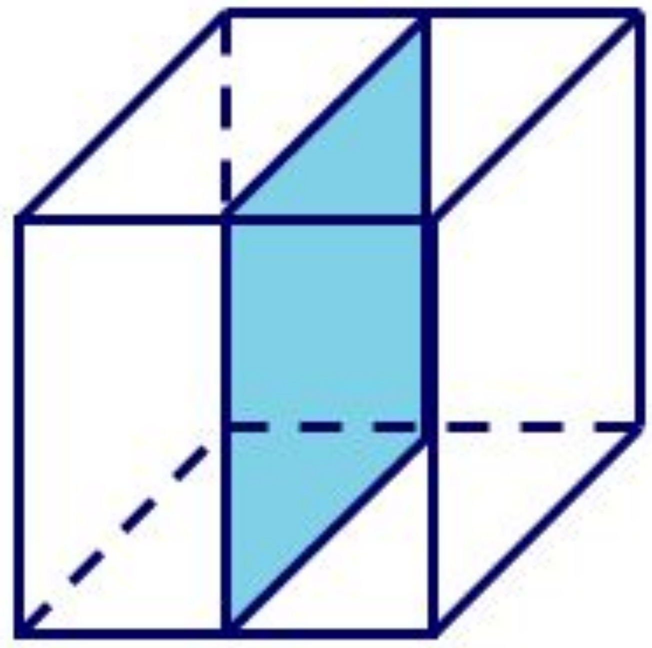
BCFT + DCFT

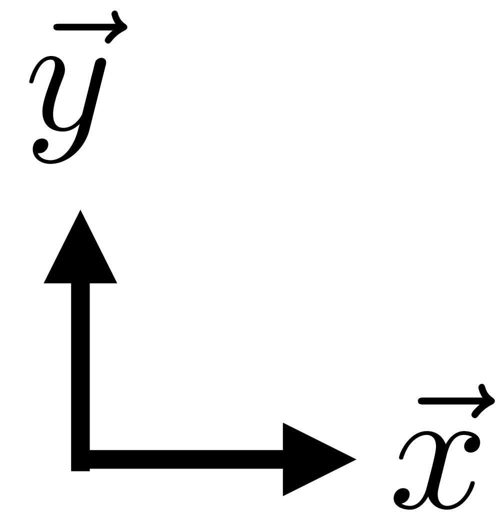
boundary or defect in flat space

Rotations

$$x^\mu \to \Lambda^\mu{}_\nu\, x^\nu$$

Broken to rotations in $\vec{y}$ + rotations in $\vec{x}$



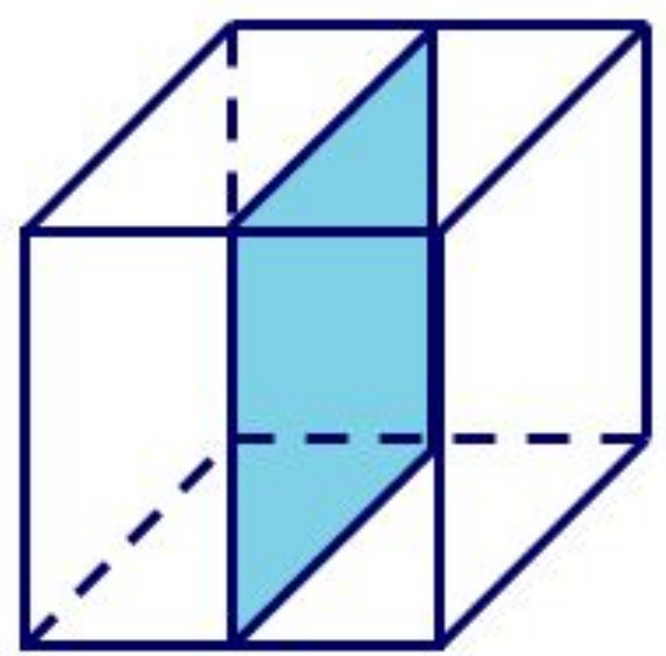


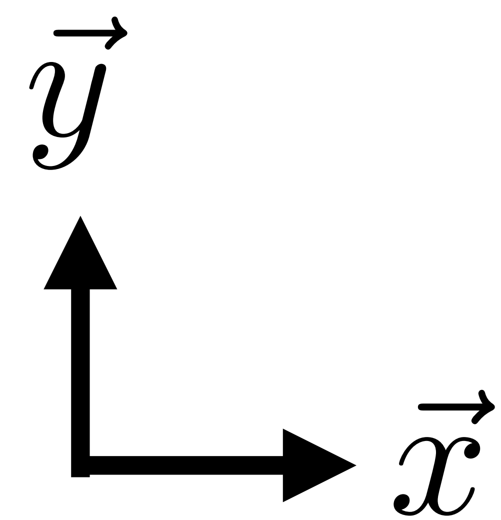
BCFT + DCFT

boundary or defect in flat space

Translations $x^\mu \to x^\mu + c^\mu$

Broken to translations along $\vec{y}$



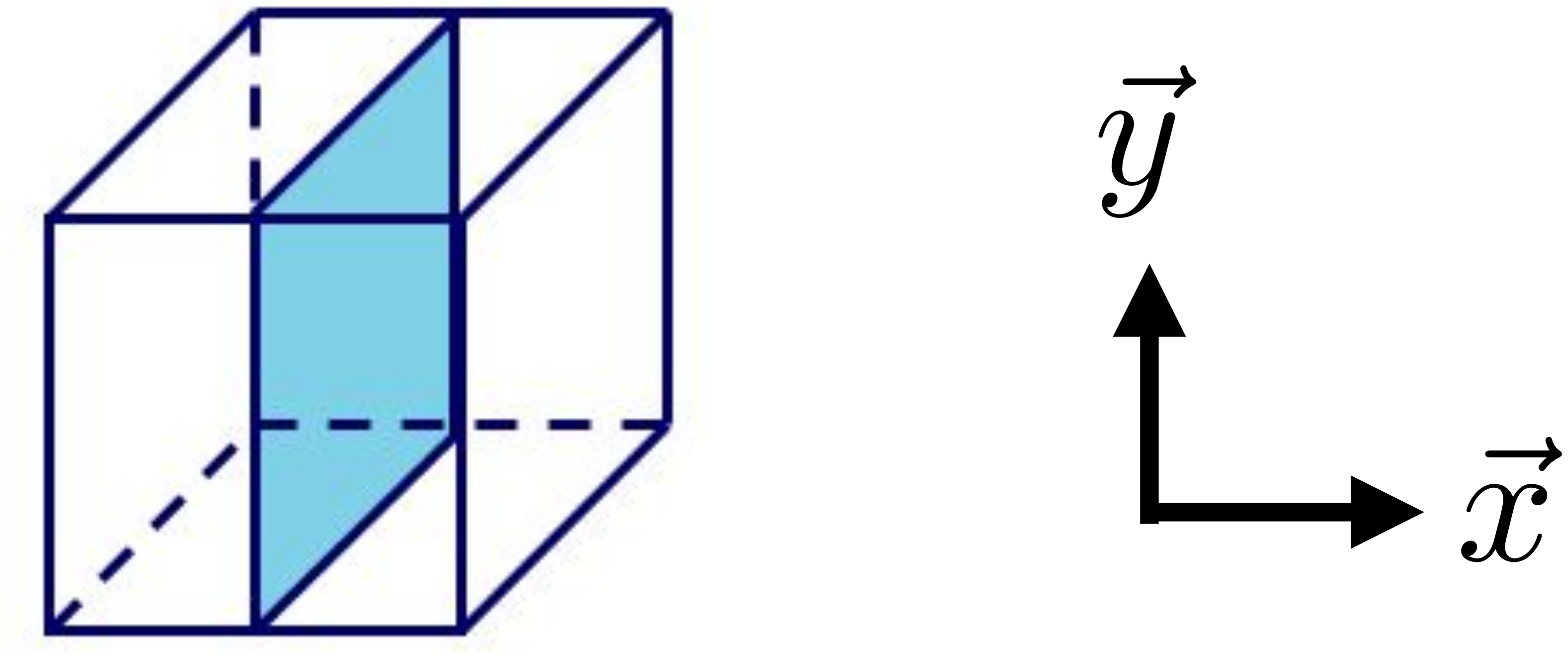


BCFT + DCFT

boundary or defect in flat space

Dilatations $x^\mu \to \lambda\, x^\mu$

Unbroken

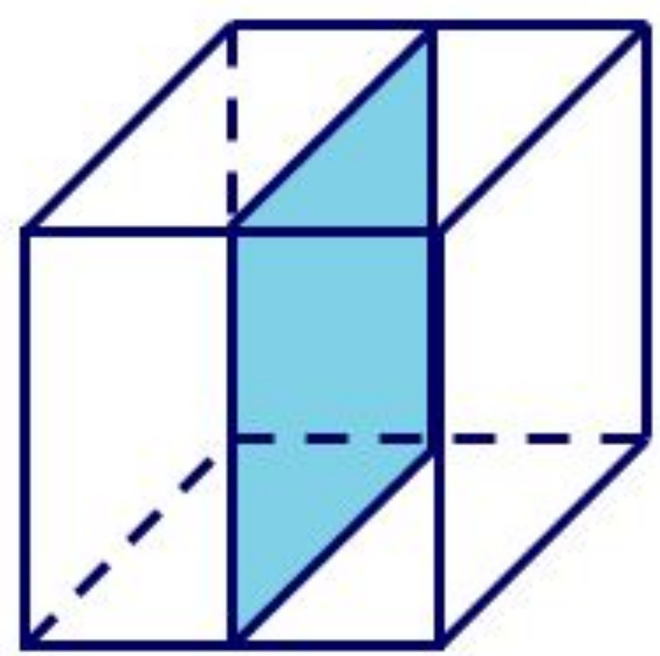


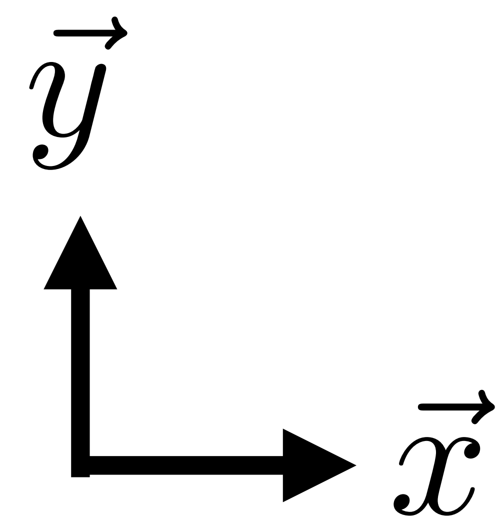
BCFT + DCFT

boundary or defect in flat space

Special Conformal $$x^\mu \to \frac{x^\mu + b^\mu x^2}{1 + 2x^\nu b_\nu + b^2 x^2}$$

Broken to $b^\perp = 0$





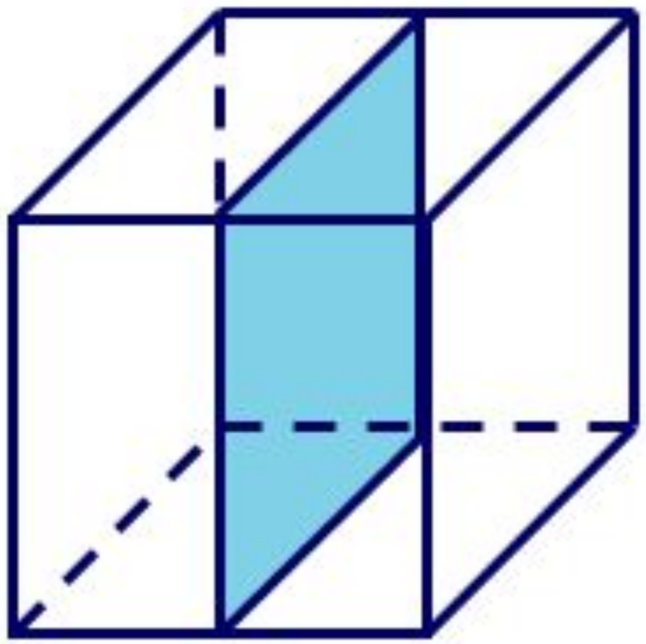
BCFT + DCFT

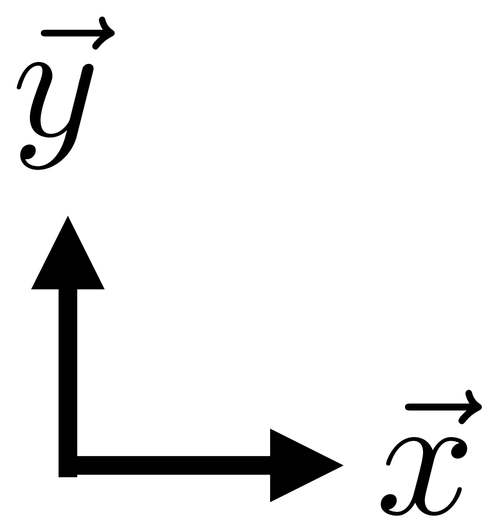
boundary or defect in flat space

$$SO(d+1,1) \to SO(p+1,1) \times SO(d-p)$$

conformal transformations preserving the boundary/defect

rotations around the defect





BCFT + DCFT

boundary or defect in flat space

x^μ with $\mu = 1, 2, 3, \ldots, d$

along the boundary/defect

y^a with $a = 1, 2, \ldots, p$

transverse to the boundary/defect

x^i with $i = p+1, p+2, \ldots, d$

BCFT + DCFT

boundary or defect in flat space

$$\partial_\mu T^{\mu i} = \delta^{d-p}(\vec{x}) D^i$$

Displacement operator D^i

$$T^{\mu\nu} = \left[T^{\mu\nu}\right]_{\text{bulk}} + \delta^{d-p}(\vec{x}) \left[T^{\mu\nu}\right]_p$$

Gaussian pillbox integration around boundary/defect

Boundary case:

$$D \propto \left[T^{\perp\perp}\right]^{\partial}_{\text{bulk}}$$

$$\partial_a \left[T^{ab}\right]_{p=d-1} \propto \left[T^{b\perp}\right]^{\partial}_{\text{bulk}}$$

BCFT + DCFT

boundary or defect in flat space

$$\partial_\mu T^{\mu i} = \delta^{d-p}(\vec{x}) D^i$$

Displacement operator D^i

$$T^{\mu\nu} = \left[T^{\mu\nu}\right]_{\text{bulk}} + \delta^{d-p}(\vec{x}) \left[T^{\mu\nu}\right]_p$$

Conformal invariance

$$T_\mu{}^\mu = 0 \quad \Rightarrow \quad \left[T_\mu{}^\mu\right]_{\text{bulk}} = 0 \qquad \left[T_\mu{}^\mu\right]_p = 0$$

BCFT + DCFT

boundary or defect in flat space

$$\partial_\mu T^{\mu i} = \delta^{d-p}(\vec{x}) D^i$$

Displacement operator D^i

$$T^{\mu\nu} = [T^{\mu\nu}]_{\text{bulk}} + \delta^{d-p}(\vec{x})\, [T^{\mu\nu}]_p$$

Conformal invariance

$$\langle D^i(y^a) D^j(0) \rangle \propto \frac{\delta^{ij}}{|y^a|^{2(p+1)}}$$

BCFT + DCFT

boundary or defect in flat space

Co-dimension $d-p>1$

$$SO(d+1,1) \to SO(p+1,1) \times SO(d-p)$$

symmetry allows

$$\langle [T^{ab}]_{\mathrm{bulk}} \rangle = -h(d-p-1)\frac{\delta^{ab}}{|x_\perp|^d} \qquad \langle [T^{ai}]_{\mathrm{bulk}} \rangle = 0$$

$$\langle [T^{ij}]_{\mathrm{bulk}} \rangle = h\,\frac{(p+1)\delta^{ij} - d\frac{x_\perp^i x_\perp^j}{|x_\perp|^2}}{|x_\perp|^d}$$

BCFT + DCFT

Average Null Energy Condition (ANEC)

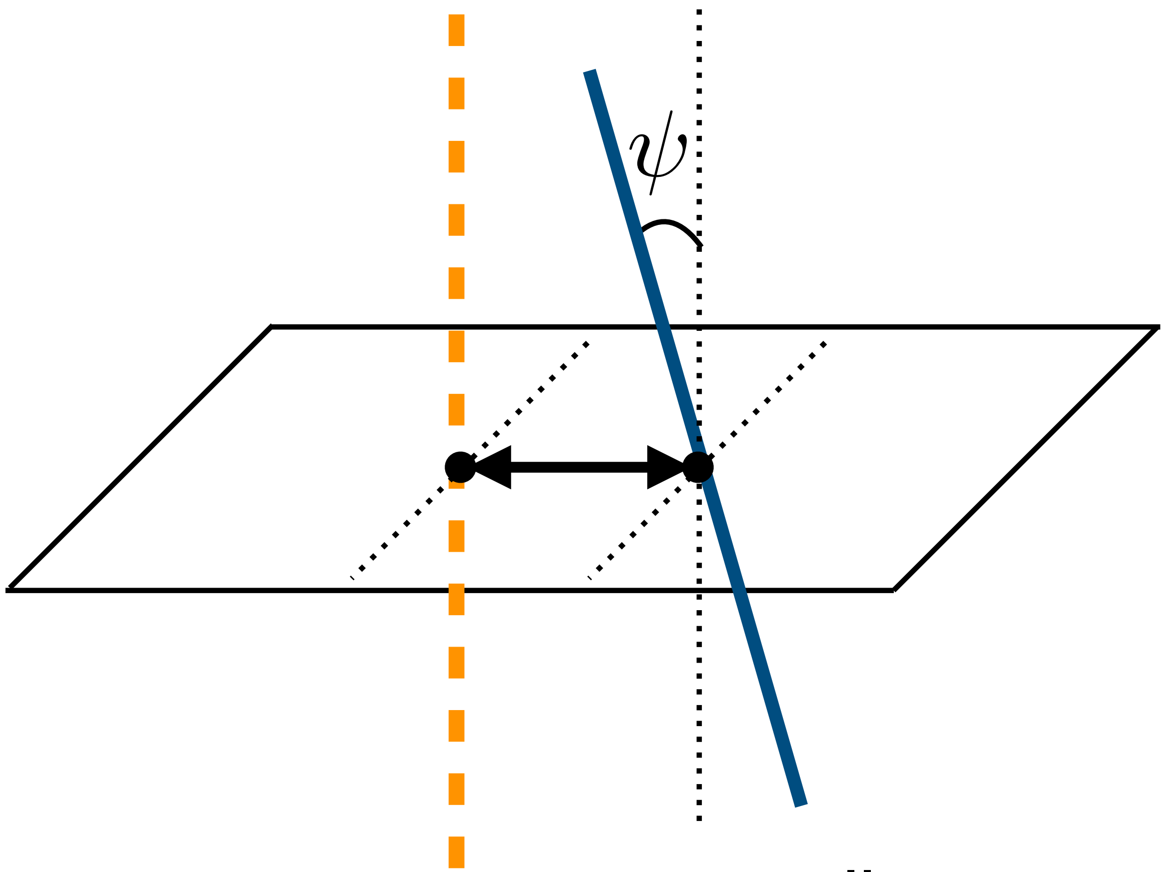
Jensen, O'Bannon, Robinson, Rodgers 1812.08745

Lorentzian BCFT/DCFT

along any null ray

$$\int_{-\infty}^{\infty} du \langle T_{uu} \rangle \geq 0$$

$$h \geq 0$$



CFT

Average Null Energy Condition (ANEC)

Faulkner, Leigh, Parrikar, Wang 1605.08072

Hartman, Kundu, Tajdini 1610.05308

Lorentzian CFT

excited state

along any null ray

$$\int_{-\infty}^{\infty} du \langle T_{uu} \rangle \geq 0$$

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Weyl Anomaly

Conformal Field Theory

$$g_{\mu\nu} = \delta_{\mu\nu}$$

Conformal invariance

$$T_{\mu}{}^{\mu} = 0$$

Weyl Anomaly

Conformal Field Theory

$$g_{\mu\nu} \neq \delta_{\mu\nu}$$

Quantum Effects
Break Conformal Invariance

$$T_\mu{}^\mu \neq 0$$

Weyl Anomaly

What is the general form of $T_\mu{}^\mu$?

Step #1

Write down all curvature invariants
built from $g_{\mu\nu}$
with the correct dimension

$d = 4$ CFT

$$T_\mu{}^\mu = c_1 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + c_2 R_{\mu\nu} R^{\mu\nu} + c_3 R^2 + c_4 \Box R$$

Weyl Anomaly

What is the general form of $T_\mu{}^\mu$?

Step #2

Wess-Zumino consistency

$$g_{\mu\nu} \to e^{2\Omega_1} e^{2\Omega_2} g_{\mu\nu} \quad = \quad g_{\mu\nu} \to e^{2\Omega_2} e^{2\Omega_1} g_{\mu\nu}$$

Fixes some coefficients

$$T_\mu{}^\mu = c_1 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + c_2 R_{\mu\nu} R^{\mu\nu} + c_3 R^2 + c_4 \Box R$$

Weyl Anomaly

What is the general form of $T_\mu{}^\mu$?

Step #3

Add local counterterms

Determine how they enter $T_\mu{}^\mu$

Fixes more coefficients

$$T_\mu{}^\mu = c_1 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + c_2 R_{\mu\nu} R^{\mu\nu} + c_3 R^2 + c_4 \Box R$$

0

Weyl Anomaly

Conformal Field Theory

What is the general form of $T_\mu{}^\mu$?

d odd $\qquad T_\mu{}^\mu = 0$

d even $\qquad T_\mu{}^\mu \neq 0$

Weyl Anomaly

$$d = 2 \qquad T_\mu{}^\mu = \frac{1}{6}\, c_{2d}\, E_2$$

$$d = 4 \qquad T_\mu{}^\mu = \frac{1}{16\pi^2}\left(-a_{4d}\, E_4 + c_{4d}\, W_{\mu\nu\rho\sigma} W^{\mu\nu\rho\sigma}\right)$$

$$d = 6 \qquad T_\mu{}^\mu = \frac{1}{(4\pi)^3}\left(a_{6d}\, E_6 + c_{6d}^{(1)}\, I_1 + c_{6d}^{(2)}\, I_2 + c_{6d}^{(3)}\, I_3\right)$$

Deser, Duff, Isham NPB 111 (1976) 45

Bonora, Pasti, Bregola CQG 3 (1986) 635

Duff hep-th/9308075

Bastianelli, Frolov, Tseytlin hep-th/0001041

Weyl Anomaly

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$$d = 6 \quad T_\mu{}^\mu = \frac{1}{(4\pi)^3}\left(a_{6d}\, E_6 + c^{(1)}_{6d}\, I_1 + c^{(2)}_{6d}\, I_2 + c^{(3)}_{6d}\, I_3\right)$$

Euler density

$$E_d = \frac{1}{2^{d/2}}\, \delta^{\mu_1\nu_1\ldots\mu_{d/2}\nu_{d/2}}_{\rho_1\sigma_1\ldots\rho_{d/2}\sigma_{d/2}}\, R^{\rho_1\sigma_1}{}_{\mu_1\nu_1}\, R^{\rho_2\sigma_2}{}_{\mu_2\nu_2} \ldots R^{\rho_{d/2}\sigma_{d/2}}{}_{\mu_{d/2}\nu_{d/2}}$$

Weyl Anomaly

$$d = 2 \quad T_\mu{}^\mu = \frac{1}{6}\, c_{2d}\, E_2$$

$$d = 4 \quad T_\mu{}^\mu = \frac{1}{16\pi^2}\,(-a_{4d}\, E_4 + c_{4d}\, W_{\mu\nu\rho\sigma} W^{\mu\nu\rho\sigma})$$

$$d = 6 \quad T_\mu{}^\mu = \frac{1}{(4\pi)^3}\left(a_{6d}\, E_6 + c_{6d}^{(1)}\, I_1 + c_{6d}^{(2)}\, I_2 + c_{6d}^{(3)}\, I_3\right)$$

Weyl tensor

$$W_{\mu\nu\rho\sigma}$$

Weyl Anomaly

$$d = 2 \quad T_\mu{}^\mu = \frac{1}{6}\, c_{2d}\, E_2$$

$$d = 4 \quad T_\mu{}^\mu = \frac{1}{16\pi^2}\left(-a_{4d}\, E_4 + c_{4d}\, W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma}\right)$$

$$d = 6 \quad T_\mu{}^\mu = \frac{1}{(4\pi)^3}\left(a_{6d}\, E_6 + c_{6d}^{(1)}\, I_1 + c_{6d}^{(2)}\, I_2 + c_{6d}^{(3)}\, I_3\right)$$

$$I_1 = W_{\mu\lambda\rho\nu}W^{\lambda\sigma\tau\rho}W_\sigma{}^{\mu\nu}{}_\tau$$

$$I_2 = W_{\mu\nu}{}^{\lambda\rho}W_{\lambda\rho}{}^{\sigma\tau}W_{\sigma\tau}{}^{\mu\nu}$$

$$I_3 = W_{\mu\nu\lambda\rho}\left(D^2\,\delta^\nu{}_\sigma - \frac{6}{5}R\,\delta^\nu{}_\sigma + 4R^\nu{}_\sigma\right)W^{\sigma\nu\lambda\rho}$$

Weyl Anomaly

$$d = 2 \qquad T_\mu{}^\mu = \frac{1}{6} c_{2d} E_2$$

$$d = 4 \qquad T_\mu{}^\mu = \frac{1}{16\pi^2} \left(-a_{4d} E_4 + c_{4d} W_{\mu\nu\rho\sigma} W^{\mu\nu\rho\sigma} \right)$$

$$d = 6 \qquad T_\mu{}^\mu = \frac{1}{(4\pi)^3} \left(a_{6d} E_6 + c_{6d}^{(1)} I_1 + c_{6d}^{(2)} I_2 + c_{6d}^{(3)} I_3 \right)$$

"Weyl anomaly coefficients"

"central charges"

Wess-Zumino consistency

$\int d^d x \sqrt{g}\, T_\mu{}^\mu$ is conformally invariant

$$d = 4 \quad T_\mu{}^\mu = \frac{1}{16\pi^2}\left(-a_{4d}\, E_4 + c_{4d}\, W_{\mu\nu\rho\sigma} W^{\mu\nu\rho\sigma}\right)$$

Type A

$$\sqrt{g}\, E$$

Changes by a total derivative

Type B

$$\sqrt{g}\, W^2$$

Invariant

Weyl Anomaly

$$d=2 \quad T_\mu{}^\mu = \frac{1}{6} c_{2d}\, E_2$$

$$d=4 \quad T_\mu{}^\mu = \frac{1}{16\pi^2}\left(-a_{4d}\, E_4 + c_{4d}\, W_{\mu\nu\rho\sigma} W^{\mu\nu\rho\sigma}\right)$$

$$d=6 \quad T_\mu{}^\mu = \frac{1}{(4\pi)^3}\left(a_{6d}\, E_6 + c_{6d}^{(1)}\, I_1 + c_{6d}^{(2)}\, I_2 + c_{6d}^{(3)}\, I_3\right)$$

Type A

Type B

Weyl Anomaly

$$d = 2 \qquad T_\mu{}^\mu = \frac{1}{6}\, c_{2d}\, E_2$$

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Type A

RG Flows

Monotonicity Theorems

Euclidean symmetry (Poincaré symmetry)
Reflection positivity (Unitarity)
Locality

Weyl Anomaly

$$d = 2 \qquad T_\mu{}^\mu = \frac{1}{6} c_{2d} E_2$$

$$d = 4 \qquad T_\mu{}^\mu = \frac{1}{16\pi^2} (-a_{4d} E_4 + c_{4d} W_{\mu\nu\rho\sigma} W^{\mu\nu\rho\sigma})$$

$$d = 6 \qquad T_\mu{}^\mu = \frac{1}{(4\pi)^3} \left(a_{6d} E_6 + c_{6d}^{(1)} I_1 + c_{6d}^{(2)} I_2 + c_{6d}^{(3)} I_3 \right)$$

Type A

c-theorem

$$c_{2d}^{\mathrm{UV}} \geq c_{2d}^{\mathrm{IR}}$$

A.B. Zamolodchikov
JETP Vol. 43 No. 12 p. 565, 1986

a-theorem

$$a_{4d}^{\mathrm{UV}} \geq a_{4d}^{\mathrm{IR}}$$

Cardy PLB 215 (1988) 749
Komargodski + Schwimmer JHEP 12 (2011) 099

Weyl Anomaly

$$d = 2 \qquad T_\mu{}^\mu = \frac{1}{6} c_{2d} E_2$$

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$$d = 6 \qquad T_\mu{}^\mu = \frac{1}{(4\pi)^3} \left(a_{6d} E_6 + c_{6d}^{(1)} I_1 + c_{6d}^{(2)} I_2 + c_{6d}^{(3)} I_3\right)$$

Type A

c-theorem

$$c_{2d}^{\mathrm{UV}} \geq c_{2d}^{\mathrm{IR}}$$

a-theorem

$$a_{4d}^{\mathrm{UV}} \geq a_{4d}^{\mathrm{IR}}$$

Counts the number of degrees of freedom

Weyl Anomaly

$$d=2 \qquad T_\mu{}^\mu = \frac{1}{6}\, c_{2d}\, E_2$$

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Entanglement Entropy (EE)

spherical region of radius ℓ

$$S_{\rm EE} = \#\frac{\rm Area}{\varepsilon^{d-2}} + \frac{\#}{\varepsilon^{d-4}} + \ldots + \#\, a\, \ln\left(\ell/\varepsilon\right) + \ldots$$

Weyl Anomaly

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$$d = 6 \qquad T_\mu{}^\mu = \frac{1}{(4\pi)^3}\left(a_{6d}\, E_6 + c_{6d}^{(1)}\, I_1 + c_{6d}^{(2)}\, I_2 + c_{6d}^{(3)}\, I_3\right)$$

fixes normalisation of

$$\langle T_{\mu\nu}(x) T_{\rho\sigma}(0) \rangle$$

reflection positivity (unitarity)

$$c_{2d} \geq 0$$

Weyl Anomaly

$$d = 2 \qquad T_\mu{}^\mu = \frac{1}{6}\, c_{2d}\, E_2$$

$$d = 4 \qquad T_\mu{}^\mu = \frac{1}{16\pi^2}\left(-a_{4d}\, E_4 + c_{4d}\, W_{\mu\nu\rho\sigma} W^{\mu\nu\rho\sigma}\right)$$

$$d = 6 \qquad T_\mu{}^\mu = \frac{1}{(4\pi)^3}\Big(a_{6d}\, E_6 + c_{6d}^{(1)}\, I_1 + c_{6d}^{(2)}\, I_2 + c_{6d}^{(3)}\, I_3\Big)$$

Type A

Wess-Zumino consistency

type A coefficients are independent of marginal couplings

Osborn NPB 363 (1991) 486

Question

How do boundaries and defects contribute to the Weyl anomaly?

$$T_\mu{}^\mu = \left[T_\mu{}^\mu\right]_{\rm bulk} + \delta^{d-p}(\vec{x}) \left[T_\mu{}^\mu\right]_p$$

What is the general form of $\left[T_\mu{}^\mu\right]_p$?

Geometry of Submanifolds

boundary/defect	σ^a
"target space"	x^μ
embedding	$x^\mu(\sigma^a)$
induced metric	$\hat{g}_{ab} = g_{\mu\nu} \frac{\partial x^\mu}{\partial \sigma^a} \frac{\partial x^\nu}{\partial \sigma^b}$
Intrinsic curvature	$\hat{R}_{abcd} \Rightarrow \hat{R}_{ab} \Rightarrow \hat{R}$
second fundamental form	$\mathrm{I\!I}^\mu_{ab}$
trace	$\mathrm{I\!I}^\mu \equiv \hat{g}^{ab} \mathrm{I\!I}^\mu_{ab}$
traceless	$\mathring{\mathrm{I\!I}}^\mu_{ab} \equiv \mathrm{I\!I}^\mu_{ab} - \hat{g}_{ab} \mathrm{I\!I}^\mu / p$

Question

How do boundaries and defects contribute to the Weyl anomaly?

$$T_\mu{}^\mu = \left[T_\mu{}^\mu\right]_{\rm bulk} + \delta^{d-p}(\vec{x})\left[T_\mu{}^\mu\right]_p$$

What is the general form of $\left[T_\mu{}^\mu\right]_p$?

Question

How do boundaries and defects contribute to the Weyl anomaly?

$$T_\mu{}^\mu = \left[T_\mu{}^\mu\right]_{\text{bulk}} + \delta^{d-p}(\vec{x}) \left[T_\mu{}^\mu\right]_p$$

What is the general form of $\left[T_\mu{}^\mu\right]_p$?

Depends on p and d

Question

How do boundaries and defects contribute to the Weyl anomaly?

Do we get boundary/defect central charges?

Do they obey monotonicity theorems?

Do they appear in entanglement entropy?

Do they appear in stress-energy correlators?

Do they obey bounds?

and so on…

Outline:

- Review: Boundaries and Defects
- Review: Weyl Anomalies
- 1d Boundary or Defect
- 2d Boundary or Defect
- 3d Boundary or Defect
- 4d Boundary or Defect
- Summary and Outlook

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1d Boundary or Defect

$$p = 1 \quad d \geq 2$$

What is the general form of $\left[T_\mu{}^\mu\right]_{p=1}$?

$$\left[T_\mu{}^\mu\right]_{p=1} = \begin{cases} 0 & d > 2 \\ -\frac{1}{12\pi}\, c_{2d}\, \mathbb{I}^\perp & d = 2 \end{cases}$$

Boundary contribution to Euler density E_2

No new central charge(s)

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2d Boundary or Defect

$$p = 2 \quad d \geq 3$$

What is the general form of $\left[T_\mu{}^\mu\right]_{p=2}$?

Berenstein, Corrado, Fischler, Maldacena hep-th/9809188

Graham + Witten hep-th/9901021

Henningson + Skenderis hep-th/9905163

Gustavsson hep-th/0310037, 0404150

Asnin 0801.1469

Schwimmer + Theisen 0802.1017

2d Boundary or Defect

$$\left[T_\mu{}^\mu\right]_{p=2} = \frac{b}{6}\hat{E}_2 + \frac{d_1}{24\pi}\mathring{\mathrm{II}}^\mu{}_{ab}\mathring{\mathrm{II}}_\mu{}^{ab} - \frac{d_2}{24\pi}W^{ab}{}_{ab}$$

boundary/defect central charges

$b \qquad d_1 \qquad d_2$

2d Boundary or Defect

$$\left[T_\mu{}^\mu\right]_{p=2} = \frac{b}{6}\hat{E}_2 + \frac{d_1}{24\pi}\mathring{\mathrm{I\!I}}^\mu{}_{ab}\mathring{\mathrm{I\!I}}_\mu{}^{ab} - \frac{d_2}{24\pi}W^{ab}{}_{ab}$$

Type A

$$\sqrt{\hat{g}}\,\hat{R}$$

Changes by a total derivative

Type B

$$\sqrt{\hat{g}}\,\mathring{\mathrm{I\!I}}^2 \qquad \sqrt{\hat{g}}\,W_{ab}{}^{ab}$$

Invariant

2d Boundary or Defect

$$[T_\mu{}^\mu]_{p=2} = \frac{b}{6}\hat{E}_2 + \frac{d_1}{24\pi}\mathring{\mathrm{I\!I}}^\mu{}_{ab}\mathring{\mathrm{I\!I}}_\mu{}^{ab} - \frac{d_2}{24\pi}W^{ab}{}_{ab}$$

b-theorem

Jensen and O'Bannon 1509.02160
Casini, Landea, Torroba 1812.08183

RG flow on the boundary/defect

$$b_{\rm UV} \geq b_{\rm IR}$$

Euclidean symmetry (Poincaré symmetry) along the boundary/defect
Reflection positivity (Unitarity)
Locality

Counts the number of degrees of freedom

2d Boundary or Defect

$$\left[T_{\mu}{}^{\mu}\right]_{p=2} = \frac{b}{6}\hat{E}_2 + \frac{d_1}{24\pi}\mathring{\mathbb{I}}^{\mu}{}_{ab}\mathring{\mathbb{I}}_{\mu}{}^{ab} - \frac{d_2}{24\pi}W^{ab}{}_{ab}$$

Wess-Zumino Consistency

b is independent of boundary/defect marginal couplings

in general, b can depend on *bulk* marginal couplings

Herzog and Shamir 1906.11281, 1907.04952
Bianchi 1907.06193

2d Boundary or Defect

$$\left[T_\mu{}^\mu\right]_{p=2} = \frac{b}{6}\hat{E}_2 + \frac{d_1}{24\pi}\mathring{\mathrm{II}}^\mu{}_{ab}\mathring{\mathrm{II}}_\mu{}^{ab} - \frac{d_2}{24\pi}W^{ab}{}_{ab}$$

Bianchi, Meineri, Myers, Smolkin 1511.06713
Herzog + Huang 1707.06224
Herzog, Huang, Jensen 1709.07431

Displacement operator D^i

$$\langle D^i(y^a)D^j(0)\rangle \propto \frac{d_1\,\delta^{ij}}{|y^a|^4}$$

Reflection positivity (unitarity)

$$d_1 \geq 0$$

2d Boundary or Defect

$$\left[T_\mu{}^\mu\right]_{p=2} = \frac{b}{6}\hat{E}_2 + \frac{d_1}{24\pi}\mathring{\mathrm{I\!I}}^\mu{}_{ab}\mathring{\mathrm{I\!I}}_\mu{}^{ab} - \frac{d_2}{24\pi}W^{ab}{}_{ab}$$

Weyl tensor

$$d = 3 \;\; \Rightarrow \;\; W_{\mu\nu\rho\sigma} \equiv 0$$

d_2 is only defined for $d > 3$

$\Rightarrow$ defect co-dimension > 1

2d Boundary or Defect

$$\left[T_\mu{}^\mu\right]_{p=2} = \frac{b}{6}\hat{E}_2 + \frac{d_1}{24\pi}\mathring{\mathrm{I\!I}}^\mu{}_{ab}\mathring{\mathrm{I\!I}}_\mu{}^{ab} - \frac{d_2}{24\pi}W^{ab}{}_{ab}$$

$$\langle [T^{ab}]_{\text{bulk}}\rangle = -h(d-p-1)\frac{\delta^{ab}}{|x_\perp|^d} \qquad \langle [T^{ai}]_{\text{bulk}}\rangle = 0$$

$$\langle [T^{ij}]_{\text{bulk}}\rangle = h\frac{(p+1)\delta^{ij} - d\frac{x_\perp^i x_\perp^j}{|x_\perp|^2}}{|x_\perp|^d}$$

$$h = \frac{d-3}{3(d-1)\mathrm{vol}(\mathbb{S}^d)}d_2$$

Bianchi, Meineri, Myers, Smolkin 1511.06713
Jensen, O'Bannon, Robinson, Rodgers 1812.08745

2d Boundary or Defect

$$\left[T_\mu{}^\mu\right]_{p=2} = \frac{b}{6}\hat{E}_2 + \frac{d_1}{24\pi}\mathring{\mathrm{I\!I}}^\mu{}_{ab}\mathring{\mathrm{I\!I}}_\mu{}^{ab} - \frac{d_2}{24\pi}W^{ab}{}_{ab}$$

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$$\langle [T^{ij}]_{\text{bulk}}\rangle = h\frac{(p+1)\delta^{ij} - d\frac{x_\perp^i x_\perp^j}{|x_\perp|^2}}{|x_\perp|^d}$$

Average Null Energy Condition (ANEC)

$$h \geq 0 \quad \Rightarrow \quad d_2 \geq 0$$

Jensen, O'Bannon, Robinson, Rodgers 1812.08745

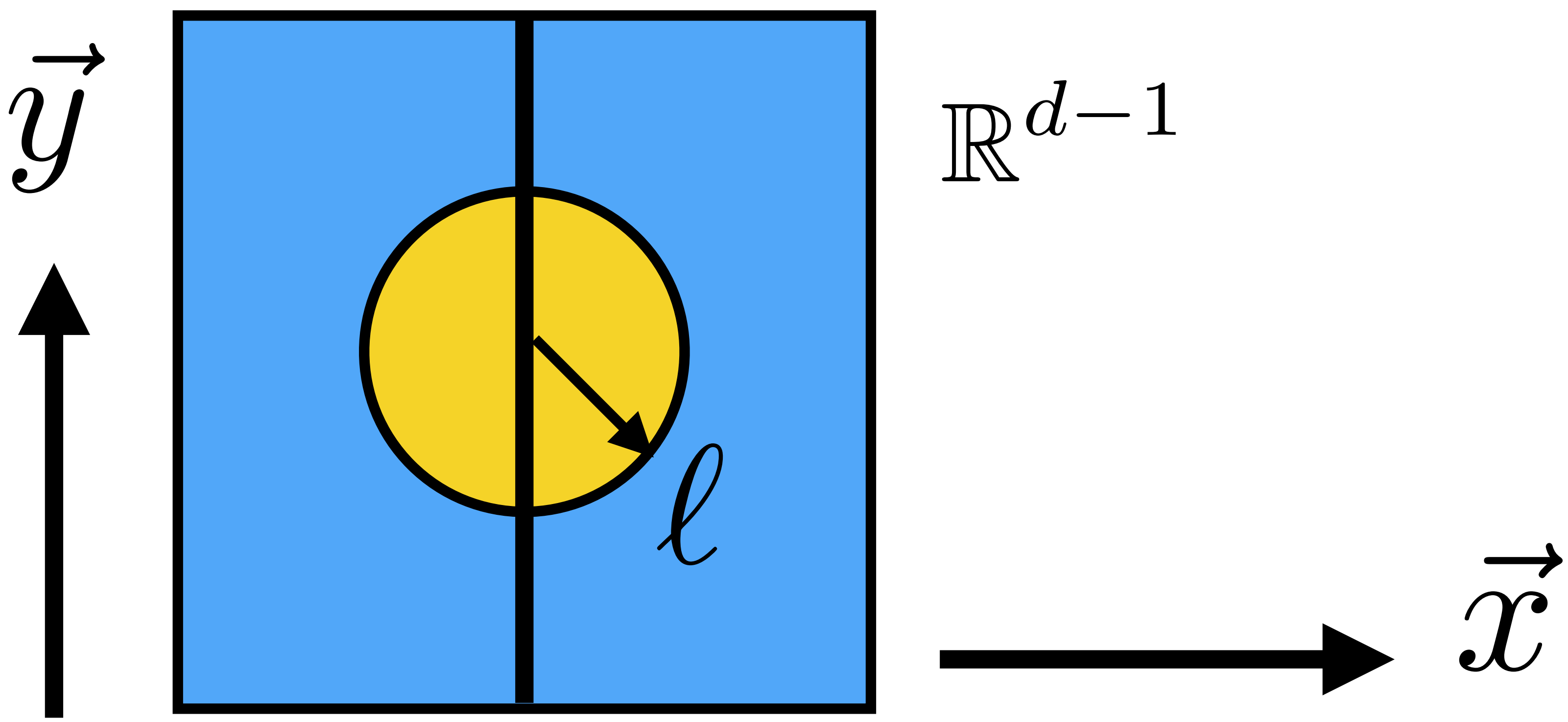
2d Boundary or Defect

$$\left[T_\mu{}^\mu\right]_{p=2} = \frac{b}{6}\hat{E}_2 + \frac{d_1}{24\pi}\mathring{\mathbb{I}}^\mu{}_{ab}\mathring{\mathbb{I}}_\mu{}^{ab} - \frac{d_2}{24\pi}W^{ab}{}_{ab}$$

Entanglement Entropy (EE)

spherical region of radius ℓ

centered on the boundary/defect



2d Boundary or Defect

$$\left[T_\mu{}^\mu\right]_{p=2} = \frac{b}{6}\hat{E}_2 + \frac{d_1}{24\pi}\mathring{\mathrm{II}}^\mu{}_{ab}\mathring{\mathrm{II}}_\mu{}^{ab} - \frac{d_2}{24\pi}W^{ab}{}_{ab}$$

Entanglement Entropy (EE)

$$S_{\mathrm{EE}} = S^d_{\mathrm{EE}} + S^{p=2}_{\mathrm{EE}}$$

Kobayashi, Nishioka, Sato, Watanabe 1810.06995
Jensen, O'Bannon, Robinson, Rodgers 1812.08745

$$S^{\mathrm{CFT}}_{\mathrm{EE}} = \#\frac{\mathrm{Area}}{\varepsilon^{d-2}} + \frac{\#}{\varepsilon^{d-4}} + \ldots + \#a\ln\left(\ell/\varepsilon\right) + \# + \ldots$$

$$S^{p=2}_{\mathrm{EE}} = \frac{1}{3}\left(b - \frac{d-3}{d-1}d_2\right)\ln\left(\ell/\varepsilon\right) + \# + \ldots$$

Examples

$d = 3$ BCFTs

free massless real scalar or Dirac fermion

Theory	BC	b	d_1	d_2
Scalar	Dirichlet	$-1/16$	$3/32$	N/A
Scalar	Neumann	$1/16$	$3/32$	N/A
Fermion	Mixed	0	$3/16$	N/A

Nozaki, Takayanagi, Ugajin 1205.1573

Jensen and O'Bannon 1509.02160

Fursaev and Solodukhin 1601.06418

Examples

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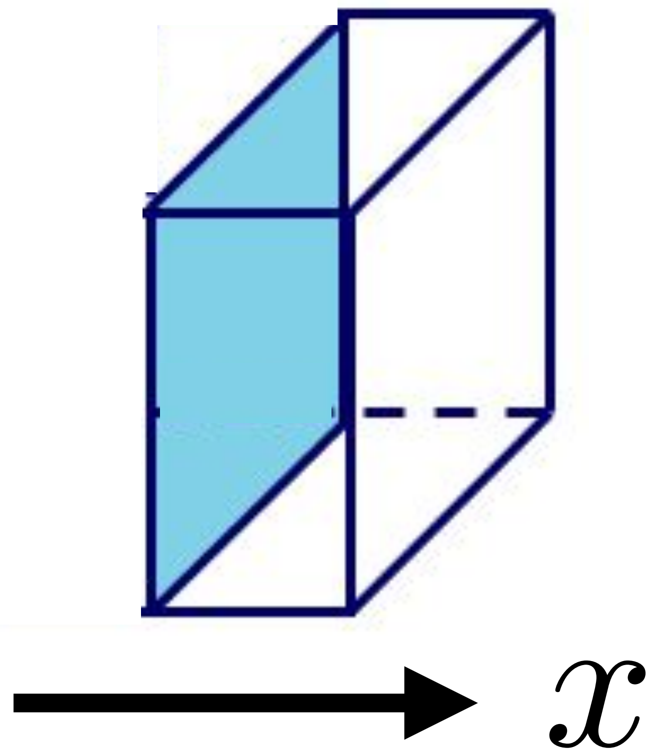

"mixed"

$$\Psi_\pm \equiv \frac{1}{2}\left(1 \pm \gamma^x\right)\Psi$$

Dirichlet for Ψ_+ or Ψ_-

$+$ Neumann for Ψ_- or Ψ_+

Examples

$d = 3$ BCFTs

free massless real scalar or Dirac fermion

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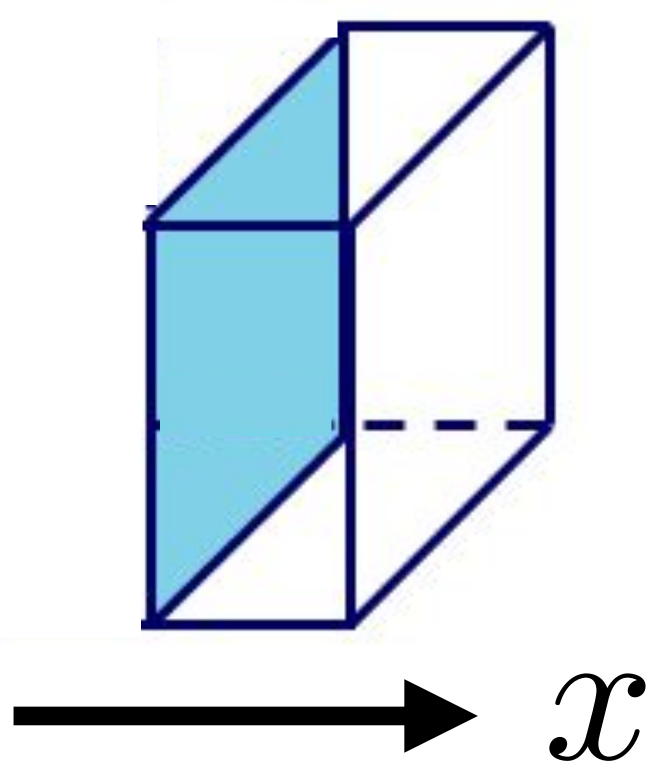

"mixed"

$$\Psi_\pm \equiv \frac{1}{2}\left(1 \pm \gamma^x\right)\Psi$$

Dirichlet for Ψ_+ or Ψ_-

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Examples

$d = 3$ BCFTs

free massless real scalar or Dirac fermion

Theory	BC	b	d_1	d_2
Scalar	Dirichlet	$-1/16$	$3/32$	N/A
Scalar	Neumann	$1/16$	$3/32$	N/A
Fermion	Mixed	0	$3/16$	N/A

"fermion = Scalar Dirichlet + Scalar Neumann"

$b = 0$ because:

b is independent of boundary marginal couplings

$m\bar{\Psi}\Psi\big|_\partial$ is marginal

Examples

$d = 3$ BCFTs

free massless real scalar or Dirac fermion

Theory	BC	b	d_1	d_2
Scalar	Dirichlet	$-1/16$	$3/32$	N/A
Scalar	Neumann	$1/16$	$3/32$	N/A
Fermion	Mixed	0	$3/16$	N/A

$b < 0$ in reflection-positive theory

Does b have a lower bound?

$d_1 \geq 0$

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3d Boundary or Defect

$$p = 3 \quad d \geq 4$$

What is the general form of $\left[T_\mu{}^\mu\right]_{p=3}$?

$$\left[T_\mu{}^\mu\right]_{p=3} \begin{cases} = 0 & d > 4 \\ \neq 0 & d = 4 \end{cases}$$

Herzog, Huang, Jensen 1510.00021, 1709.07431

3d Boundary

$$\left[T_\mu{}^\mu\right]_{p=3} = \frac{1}{16\pi^2}\left(a_{4d}\, E_4|_\partial - d_1\, \mathring{\mathrm{II}}^3 - d_2\, \mathring{\mathrm{II}}^{ab} W^c{}_{acb}\right)$$

Herzog, Huang, Jensen 1510.00021, 1709.07431

3d Boundary

$$\left[T_\mu{}^\mu\right]_{p=3} = \frac{1}{16\pi^2}\left(a_{4d}\, E_4\big|_\partial - d_1\, \mathring{\mathrm{I\!I}}^3 - d_2\, \mathring{\mathrm{I\!I}}^{ab} W^c{}_{acb}\right)$$

Herzog, Huang, Jensen 1510.00021, 1709.07431

Boundary contribution to Euler density E_4

No new central charge

3d Boundary

$$\left[T_\mu{}^\mu\right]_{p=3} = \frac{1}{16\pi^2}\left(a_{4d}\, E_4|_\partial - d_1\, \hat{K}^3 - d_2\, \hat{K}^{ab} W^c{}_{acb}\right)$$

Herzog, Huang, Jensen 1510.00021, 1709.07431

boundary central charges

$d_1 \qquad d_2$

Type B

3d Boundary

$$\left[T_\mu{}^\mu\right]_{p=3} = \frac{1}{16\pi^2}\left(a_{4d}\, E_4|_\partial - d_1\, \mathring{\mathrm{II}}^3 - d_2\, \mathring{\mathrm{II}}^{ab} W^c{}_{acb}\right)$$

Herzog, Huang, Jensen 1510.00021, 1709.07431

Displacement operator D

$$\langle D(y^a) D(0)\rangle \propto \frac{d_1}{|y^a|^8}$$

Reflection positivity (unitarity)

$$d_1 \geq 0$$

3d Boundary

$$[T_\mu{}^\mu]_{p=3} = \frac{1}{16\pi^2}\left(a_{4d}\, E_4|_\partial - d_1\, \mathring{\mathrm{II}}^3 - d_2\, \mathring{\mathrm{II}}^{ab} W^c{}_{acb}\right)$$

Herzog, Huang, Jensen 1510.00021, 1709.07431

Displacement operator D

$$\langle D(\mathbf{y}_1) D(\mathbf{y}_2) D(0)\rangle \propto \frac{d_2}{|\mathbf{y}_1|^4 |\mathbf{y}_2|^4 |\mathbf{y}_1 - \mathbf{y}_2|^4}$$

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4d Boundary or Defect

$$p = 4 \quad d \geq 5$$

What is the general form of $\left[T_\mu{}^\mu\right]_{p=4}$?

$$p = 4 \quad d = 5$$

Astaneh + Solodukhin 2102.07661

See A. F. Astaneh's talk!

$$p = 4 \quad d \geq 5$$

Chalabi, Herzog, O'Bannon, Robinson, Sisti 2111.14713

4d Boundary or Defect

$$\left[T_\mu{}^\mu\right]_{p=4} = \frac{1}{(4\pi)^2}\Big(- b\,\hat{E}_4 + d_1\mathcal{J}_1 + d_2\mathcal{J}_2 + d_3 W_{abcd}W^{abcd} + d_4(W_{ab}{}^{ab})^2$$
$$+ d_5 W_{aibj}W^{aibj} + d_6 W^b{}_{iab}W_c{}^{iac} + d_7 W_{ijkl}W^{ijkl} + d_8 W_{aijk}W^{aijk}$$
$$+ d_9 W_{abjk}W^{abjk} + d_{10} W_{iabc}W^{iabc} + d_{11} W^c{}_{acb}W_d{}^{adb} + d_{12} W^a{}_{iaj}W_b^{ibj}$$
$$+ d_{13} W_{ab}{}^{ab}\mathring{\mathrm{I\!I}}^i_{cd}\mathring{\mathrm{I\!I}}_i^{cd} + d_{14} W^a{}_{bij}\mathring{\mathrm{I\!I}}^i_{ac}\mathring{\mathrm{I\!I}}^{jbc} + d_{15} W^a{}_{ibj}\mathring{\mathrm{I\!I}}^i{}_{ac}\mathring{\mathrm{I\!I}}^{jbc}$$
$$+ d_{16} W^{abcd}\mathring{\mathrm{I\!I}}^i_{ac}\mathring{\mathrm{I\!I}}_{ibd} + d_{17} W_a{}^{bac}\mathring{\mathrm{I\!I}}^i_{bd}\mathring{\mathrm{I\!I}}_{ic}{}^d + d_{18} W^c{}_{icj}\mathring{\mathrm{I\!I}}^i_{ab}\mathring{\mathrm{I\!I}}^{jab}$$
$$+ d_{19}\mathrm{Tr}\,\mathring{\mathrm{I\!I}}^i\mathring{\mathrm{I\!I}}_i\mathring{\mathrm{I\!I}}^j\mathring{\mathrm{I\!I}}_j + d_{20}\mathrm{Tr}\,\mathring{\mathrm{I\!I}}^i\mathring{\mathrm{I\!I}}^j\mathring{\mathrm{I\!I}}_i\mathring{\mathrm{I\!I}}_j + d_{21}(\mathrm{Tr}\,\mathring{\mathrm{I\!I}}^i\mathring{\mathrm{I\!I}}_i)^2 + d_{22}(\mathrm{Tr}\,\mathring{\mathrm{I\!I}}^i\mathring{\mathrm{I\!I}}^j)(\mathrm{Tr}\,\mathring{\mathrm{I\!I}}_i\mathring{\mathrm{I\!I}}_j)\Big)$$

boundary/defect central charges

$$b \qquad d_1, d_2, \ldots, d_{22}$$

4d Boundary or Defect

$$\left[T_{\mu}{}^{\mu}\right]_{p=4} = \frac{1}{(4\pi)^2}\Big(- b\,\hat{E}_4 + d_1\mathcal{J}_1 + d_2\mathcal{J}_2 + d_3 W_{abcd}W^{abcd} + d_4 (W_{ab}{}^{ab})^2$$

W^2

$$+ d_5 W_{aibj}W^{aibj} + d_6 W^{b}{}_{iab}W_{c}{}^{iac} + d_7 W_{ijkl}W^{ijkl} + d_8 W_{aijk}W^{aijk}$$

$$+ d_9 W_{abjk}W^{abjk} + d_{10} W_{iabc}W^{iabc} + d_{11} W^{c}{}_{acb}W_{d}{}^{adb} + d_{12} W^{a}{}_{iaj}W_{b}{}^{ibj}$$

$W\mathrm{II}^2$

$$+ d_{13} W_{ab}{}^{ab}\mathring{\mathrm{II}}^{i}_{cd}\mathring{\mathrm{II}}_{i}^{cd} + d_{14} W^{a}{}_{bij}\mathring{\mathrm{II}}^{i}_{ac}\mathring{\mathrm{II}}^{jbc} + d_{15} W^{a}{}_{ibj}\mathring{\mathrm{II}}^{i}{}_{ac}\mathring{\mathrm{II}}^{jbc}$$

$$+ d_{16} W^{abcd}\mathring{\mathrm{II}}^{i}_{ac}\mathring{\mathrm{II}}_{ibd} + d_{17} W_{a}{}^{bac}\mathring{\mathrm{II}}^{i}_{bd}\mathring{\mathrm{II}}_{ic}{}^{d} + d_{18} W^{c}{}_{icj}\mathring{\mathrm{II}}^{i}_{ab}\mathring{\mathrm{II}}^{jab}$$

II^4

$$+ d_{19}\mathrm{Tr}\,\mathring{\mathrm{II}}^{i}\mathring{\mathrm{II}}_{i}\mathring{\mathrm{II}}^{j}\mathring{\mathrm{II}}_{j} + d_{20}\mathrm{Tr}\,\mathring{\mathrm{II}}^{i}\mathring{\mathrm{II}}^{j}\mathring{\mathrm{II}}_{i}\mathring{\mathrm{II}}_{j} + d_{21}(\mathrm{Tr}\,\mathring{\mathrm{II}}^{i}\mathring{\mathrm{II}}_{i})^2 + d_{22}(\mathrm{Tr}\,\mathring{\mathrm{II}}^{i}\mathring{\mathrm{II}}^{j})(\mathrm{Tr}\,\mathring{\mathrm{II}}_{i}\mathring{\mathrm{II}}_{j})$$

boundary/defect central charges

$$b \qquad d_1, d_2, \ldots, d_{22}$$

4d Boundary or Defect

$$\left[T_\mu{}^\mu\right]_{p=4} = \frac{1}{(4\pi)^2}\Big(- b\,\hat{E}_4 + d_1\mathcal{J}_1 + d_2\mathcal{J}_2 + d_3 W_{abcd}W^{abcd} + d_4 (W_{ab}{}^{ab})^2$$

W^2

$$+ d_5 W_{aibj}W^{aibj} + d_6 W^b{}_{iab}W_c{}^{iac} + d_7 W_{ijkl}W^{ijkl} + d_8 W_{aijk}W^{aijk}$$

$$+ d_9 W_{abjk}W^{abjk} + d_{10} W_{iabc}W^{iabc} + d_{11} W^c{}_{acb}W_d{}^{adb} + d_{12} W^a{}_{iaj}W_b^{ibj}$$

$W\mathrm{I\!I}^2$

$$+ d_{13} W_{ab}{}^{ab}\mathring{\mathrm{I\!I}}^i_{cd}\mathring{\mathrm{I\!I}}_i^{cd} + d_{14} W^a{}_{bij}\mathring{\mathrm{I\!I}}^i_{ac}\mathring{\mathrm{I\!I}}^{jbc} + d_{15} W^a{}_{ibj}\mathring{\mathrm{I\!I}}^i{}_{ac}\mathring{\mathrm{I\!I}}^{jbc}$$

$$+ d_{16} W^{abcd}\mathring{\mathrm{I\!I}}^i_{ac}\mathring{\mathrm{I\!I}}_{ibd} + d_{17} W_a{}^{bac}\mathring{\mathrm{I\!I}}^i_{bd}\mathring{\mathrm{I\!I}}_{ic}{}^d + d_{18} W^c{}_{icj}\mathring{\mathrm{I\!I}}^i_{ab}\mathring{\mathrm{I\!I}}^{jab}$$

$\mathrm{I\!I}^4$

$$+ d_{19}\mathrm{Tr}\,\mathring{\mathrm{I\!I}}^i\mathring{\mathrm{I\!I}}_i\mathring{\mathrm{I\!I}}^j\mathring{\mathrm{I\!I}}_j + d_{20}\mathrm{Tr}\,\mathring{\mathrm{I\!I}}^i\mathring{\mathrm{I\!I}}^j\mathring{\mathrm{I\!I}}_i\mathring{\mathrm{I\!I}}_j + d_{21}(\mathrm{Tr}\,\mathring{\mathrm{I\!I}}^i\mathring{\mathrm{I\!I}}_i)^2 + d_{22}(\mathrm{Tr}\,\mathring{\mathrm{I\!I}}^i\mathring{\mathrm{I\!I}}^j)(\mathrm{Tr}\,\mathring{\mathrm{I\!I}}_i\mathring{\mathrm{I\!I}}_j)\Big)$$

$$\begin{aligned}
\mathcal{J}_1 = {}& \frac{1}{d-1}R\mathring{\mathrm{I\!I}}^i_{ab}\mathring{\mathrm{I\!I}}_i^{ab} - \frac{1}{d-2}N^{\mu\nu}R_{\mu\nu}\mathring{\mathrm{I\!I}}^i_{ab}\mathring{\mathrm{I\!I}}_i^{ab} - \frac{2}{d-2}R^a{}_b\mathring{\mathrm{I\!I}}^i_{ac}\mathring{\mathrm{I\!I}}_i^{bc} - \frac{1}{2}W^c{}_{acb}\mathrm{I\!I}_i\mathring{\mathrm{I\!I}}^{iab} \\
& + \frac{4}{9}W^c{}_{ica}\overline{D}^b\mathring{\mathrm{I\!I}}^i_{ab} + \mathring{\mathrm{I\!I}}^{iab}D_iW^c{}_{acb} - \frac{1}{2}\mathrm{I\!I}^i\mathrm{Tr}\,\mathring{\mathrm{I\!I}}_i\mathring{\mathrm{I\!I}}^j\mathring{\mathrm{I\!I}}_j + \frac{1}{16}\mathrm{I\!I}^i\mathrm{I\!I}_i\mathrm{Tr}\,\mathring{\mathrm{I\!I}}^j\mathring{\mathrm{I\!I}}_j \\
& + \frac{2}{9}\overline{D}^b\mathring{\mathrm{I\!I}}^i_{ab}\overline{D}^c\mathring{\mathrm{I\!I}}_{ic}{}^a\,,
\end{aligned}$$

$$\begin{aligned}
\mathcal{J}_2 = {}& \frac{d-4}{d-2}W_{ab}{}^{ab}N^{\mu\nu}R_{\mu\nu} - \frac{d-4}{d-1}RW_{ab}{}^{ab} + \frac{4(d-5)}{3(d-2)}R_{ab}W_c{}^{acb} \\
& - \frac{5(d-4)}{48}W_{ab}{}^{ab}\mathrm{I\!I}^i\mathrm{I\!I}_i + \frac{2(d-5)}{3}W^c{}_{ica}\overline{D}^b\mathring{\mathrm{I\!I}}^i_{ab} + \frac{4(d+1)}{9}\mathring{\mathrm{I\!I}}^{iab}D_iW^c{}_{acb} \\
& - \frac{1}{3}W_{ic}{}^{ac}\overline{D}_a\mathrm{I\!I}^i - \frac{2(d-5)}{3}\mathrm{I\!I}^i\mathrm{Tr}\,\mathring{\mathrm{I\!I}}_i\mathring{\mathrm{I\!I}}^j\mathring{\mathrm{I\!I}}_j + \frac{(d-10)}{12}\mathrm{I\!I}^iD_iW_{ab}{}^{ab} + \frac{1}{3}D^iD_iW_{ab}{}^{ab}
\end{aligned}$$

4d Boundary or Defect

$$
\begin{aligned}
\left[T_\mu{}^\mu\right]_{p=4} = \frac{1}{(4\pi)^2}\Big(& - b\,\hat{E}_4 + d_1\mathcal{J}_1 + d_2\mathcal{J}_2 + d_3 W_{abcd}W^{abcd} + d_4 (W_{ab}{}^{ab})^2 \\
& + d_5 W_{aibj}W^{aibj} + d_6 W^b{}_{iab}W_c{}^{iac} + d_7 W_{ijkl}W^{ijkl} + d_8 W_{aijk}W^{aijk} \\
& + d_9 W_{abjk}W^{abjk} + d_{10} W_{iabc}W^{iabc} + d_{11} W^c{}_{acb}W_d{}^{adb} + d_{12} W^a{}_{iaj}W_b^{ibj} \\
& + d_{13} W_{ab}{}^{ab}\mathring{\Pi}^i_{cd}\mathring{\Pi}_i^{cd} + d_{14} W^a{}_{bij}\mathring{\Pi}^i_{ac}\mathring{\Pi}^{jbc} + d_{15} W^a{}_{ibj}\mathring{\Pi}^i{}_{ac}\mathring{\Pi}^{jbc} \\
& + d_{16} W^{abcd}\mathring{\Pi}^i_{ac}\mathring{\Pi}_{ibd} + d_{17} W_a{}^{bac}\mathring{\Pi}^i_{bd}\mathring{\Pi}_{ic}{}^d + d_{18} W^c{}_{icj}\mathring{\Pi}^i_{ab}\mathring{\Pi}^{jab} \\
& + d_{19}\mathrm{Tr}\ \mathring{\Pi}^i\mathring{\Pi}_i\mathring{\Pi}^j\mathring{\Pi}_j + d_{20}\mathrm{Tr}\ \mathring{\Pi}^i\mathring{\Pi}^j\mathring{\Pi}_i\mathring{\Pi}_j + d_{21}(\mathrm{Tr}\ \mathring{\Pi}^i\mathring{\Pi}_i)^2 + d_{22}(\mathrm{Tr}\ \mathring{\Pi}^i\mathring{\Pi}^j)(\mathrm{Tr}\ \mathring{\Pi}_i\mathring{\Pi}_j)
\end{aligned}
$$

boundary/defect central charges

$$b \qquad d_1, d_2, \ldots, d_{22}$$

Type A

Type B

4d Boundary or Defect

$$
\begin{aligned}
\left[T_\mu{}^\mu\right]_{p=4} = \frac{1}{(4\pi)^2}\Big(&-b\,\hat{E}_4 + d_1\mathcal{J}_1 + d_2\mathcal{J}_2 + d_3 W_{abcd}W^{abcd} + d_4 (W_{ab}{}^{ab})^2 \\
&+ d_5 W_{aibj}W^{aibj} + d_6 W^b{}_{iab}W_c{}^{iac} + d_7 W_{ijkl}W^{ijkl} + d_8 W_{aijk}W^{aijk} \\
&+ d_9 W_{abjk}W^{abjk} + d_{10} W_{iabc}W^{iabc} + d_{11} W^c{}_{acb}W_d{}^{adb} + d_{12} W^a{}_{iaj}W_b^{ibj} \\
&+ d_{13} W_{ab}{}^{ab}\mathring{\mathrm{II}}^i_{cd}\mathring{\mathrm{II}}^{cd}_i + d_{14} W^a{}_{bij}\mathring{\mathrm{II}}^i_{ac}\mathring{\mathrm{II}}^{jbc} + d_{15} W^a{}_{ibj}\mathring{\mathrm{II}}^i{}_{ac}\mathring{\mathrm{II}}^{jbc} \\
&+ d_{16} W^{abcd}\mathring{\mathrm{II}}^i_{ac}\mathring{\mathrm{II}}_{ibd} + d_{17} W_a{}^{bac}\mathring{\mathrm{II}}^i_{bd}\mathring{\mathrm{II}}_{ic}{}^d + d_{18} W^c{}_{icj}\mathring{\mathrm{II}}^i_{ab}\mathring{\mathrm{II}}^{jab} \\
&+ d_{19}\mathrm{Tr}\,\mathring{\mathrm{II}}^i\mathring{\mathrm{II}}_i\mathring{\mathrm{II}}^j\mathring{\mathrm{II}}_j + d_{20}\mathrm{Tr}\,\mathring{\mathrm{II}}^i\mathring{\mathrm{II}}^j\mathring{\mathrm{II}}_i\mathring{\mathrm{II}}_j + d_{21}(\mathrm{Tr}\,\mathring{\mathrm{II}}^i\mathring{\mathrm{II}}_i)^2 + d_{22}(\mathrm{Tr}\,\mathring{\mathrm{II}}^i\mathring{\mathrm{II}}^j)(\mathrm{Tr}\,\mathring{\mathrm{II}}_i\mathring{\mathrm{II}}_j)
\end{aligned}
$$

b-theorem

Wang 2101.12648

RG flow on the boundary/defect

$$b_{\mathrm{UV}} \geq b_{\mathrm{IR}}$$

Counts the number of degrees of freedom

4d Boundary or Defect

$$\left[T_\mu{}^\mu\right]_{p=4} = \frac{1}{(4\pi)^2}\Big(- b\,\hat{E}_4 + d_1\mathcal{J}_1 + d_2\mathcal{J}_2 + d_3 W_{abcd}W^{abcd} + d_4 (W_{ab}{}^{ab})^2$$
$$+ d_5 W_{aibj}W^{aibj} + d_6 W^b{}_{iab}W_c{}^{iac} + d_7 W_{ijkl}W^{ijkl} + d_8 W_{aijk}W^{aijk}$$
$$+ d_9 W_{abjk}W^{abjk} + d_{10} W_{iabc}W^{iabc} + d_{11} W^c{}_{acb}W_d{}^{adb} + d_{12} W^a{}_{iaj}W_b^{ibj}$$
$$+ d_{13} W_{ab}{}^{ab}\mathring{\Pi}^i_{cd}\mathring{\Pi}^{cd}_i + d_{14} W^a{}_{bij}\mathring{\Pi}^i_{ac}\mathring{\Pi}^{jbc} + d_{15} W^a{}_{ibj}\mathring{\Pi}^i{}_{ac}\mathring{\Pi}^{jbc}$$
$$+ d_{16} W^{abcd}\mathring{\Pi}^i_{ac}\mathring{\Pi}_{ibd} + d_{17} W_a{}^{bac}\mathring{\Pi}^i_{bd}\mathring{\Pi}_{ic}{}^d + d_{18} W^c{}_{icj}\mathring{\Pi}^i_{ab}\mathring{\Pi}^{jab}$$
$$+ d_{19}\mathrm{Tr}\,\mathring{\Pi}^i\mathring{\Pi}_i\mathring{\Pi}^j\mathring{\Pi}_j + d_{20}\mathrm{Tr}\,\mathring{\Pi}^i\mathring{\Pi}^j\mathring{\Pi}_i\mathring{\Pi}_j + d_{21}(\mathrm{Tr}\,\mathring{\Pi}^i\mathring{\Pi}_i)^2 + d_{22}(\mathrm{Tr}\,\mathring{\Pi}^i\mathring{\Pi}^j)(\mathrm{Tr}\,\mathring{\Pi}_i\mathring{\Pi}_j)$$

Wess-Zumino Consistency

b is independent of boundary/defect marginal couplings

in general, b can depend on *bulk* marginal couplings

4d Boundary or Defect

$$\left[T_\mu{}^\mu\right]_{p=4} = \frac{1}{(4\pi)^2}\Big(- b\,\hat{E}_4 + d_1\mathcal{J}_1 + d_2\mathcal{J}_2 + d_3 W_{abcd}W^{abcd} + d_4 (W_{ab}{}^{ab})^2$$
$$+ d_5 W_{aibj}W^{aibj} + d_6 W^b{}_{iab}W_c{}^{iac} + d_7 W_{ijkl}W^{ijkl} + d_8 W_{aijk}W^{aijk}$$
$$+ d_9 W_{abjk}W^{abjk} + d_{10} W_{iabc}W^{iabc} + d_{11} W^c{}_{acb}W_d{}^{adb} + d_{12} W^a{}_{iaj}W_b^{ibj}$$
$$+ d_{13} W_{ab}{}^{ab}\mathring{\Pi}^i_{cd}\mathring{\Pi}_i^{cd} + d_{14} W^a{}_{bij}\mathring{\Pi}^i_{ac}\mathring{\Pi}^{jbc} + d_{15} W^a{}_{ibj}\mathring{\Pi}^i{}_{ac}\mathring{\Pi}^{jbc}$$
$$+ d_{16} W^{abcd}\mathring{\Pi}^i_{ac}\mathring{\Pi}_{ibd} + d_{17} W_a{}^{bac}\mathring{\Pi}^i_{bd}\mathring{\Pi}_{ic}{}^d + d_{18} W^c{}_{icj}\mathring{\Pi}^i_{ab}\mathring{\Pi}^{jab}$$
$$+ d_{19}\mathrm{Tr}\,\mathring{\Pi}^i\mathring{\Pi}_i\mathring{\Pi}^j\mathring{\Pi}_j + d_{20}\mathrm{Tr}\,\mathring{\Pi}^i\mathring{\Pi}^j\mathring{\Pi}_i\mathring{\Pi}_j + d_{21}(\mathrm{Tr}\,\mathring{\Pi}^i\mathring{\Pi}_i)^2 + d_{22}(\mathrm{Tr}\,\mathring{\Pi}^i\mathring{\Pi}^j)(\mathrm{Tr}\,\mathring{\Pi}_i\mathring{\Pi}_j)$$

Displacement operator D^i

$$\langle D^i(y^a)D^j(0)\rangle \propto \frac{d_1\,\delta^{ij}}{|y^a|^{10}}$$

Reflection positivity (unitarity)

$$d_1 \geq 0$$

4d Boundary or Defect

$$\left[T_\mu{}^\mu\right]_{p=4} = \frac{1}{(4\pi)^2}\Big(- b\,\hat{E}_4 + d_1\mathcal{J}_1 + d_2\mathcal{J}_2 + d_3 W_{abcd}W^{abcd} + d_4 (W_{ab}{}^{ab})^2$$
$$+ d_5 W_{aibj}W^{aibj} + d_6 W^b{}_{iab}W_c{}^{iac} + d_7 W_{ijkl}W^{ijkl} + d_8 W_{aijk}W^{aijk}$$
$$+ d_9 W_{abjk}W^{abjk} + d_{10} W_{iabc}W^{iabc} + d_{11} W^c{}_{acb}W_d{}^{adb} + d_{12} W^a{}_{iaj}W_b^{ibj}$$
$$+ d_{13} W_{ab}{}^{ab}\mathring{\Pi}^i_{cd}\mathring{\Pi}_i^{cd} + d_{14} W^a{}_{bij}\mathring{\Pi}^i_{ac}\mathring{\Pi}^{jbc} + d_{15} W^a{}_{ibj}\mathring{\Pi}^i{}_{ac}\mathring{\Pi}^{jbc}$$
$$+ d_{16} W^{abcd}\mathring{\Pi}^i_{ac}\mathring{\Pi}_{ibd} + d_{17} W_a{}^{bac}\mathring{\Pi}^i_{bd}\mathring{\Pi}_{ic}{}^d + d_{18} W^c{}_{icj}\mathring{\Pi}^i_{ab}\mathring{\Pi}^{jab}$$
$$+ d_{19}\mathrm{Tr}\,\mathring{\Pi}^i\mathring{\Pi}_i\mathring{\Pi}^j\mathring{\Pi}_j + d_{20}\mathrm{Tr}\,\mathring{\Pi}^i\mathring{\Pi}^j\mathring{\Pi}_i\mathring{\Pi}_j + d_{21}(\mathrm{Tr}\,\mathring{\Pi}^i\mathring{\Pi}_i)^2 + d_{22}(\mathrm{Tr}\,\mathring{\Pi}^i\mathring{\Pi}^j)(\mathrm{Tr}\,\mathring{\Pi}_i\mathring{\Pi}_j)$$

$$\langle [T^{ab}]_{\text{bulk}}\rangle = -h(d-p-1)\frac{\delta^{ab}}{|x_\perp|^d} \qquad \langle [T^{ai}]_{\text{bulk}}\rangle = 0$$

$$\langle [T^{ij}]_{\text{bulk}}\rangle = h\frac{(p+1)\delta^{ij} - d\frac{x_\perp^i x_\perp^j}{|x_\perp|^2}}{|x_\perp|^d}$$

4d Boundary or Defect

$$\left[T_\mu{}^\mu\right]_{p=4} = \frac{1}{(4\pi)^2}\Big(- b\,\hat{E}_4 + d_1\mathcal{J}_1 + d_2\mathcal{J}_2 + d_3 W_{abcd}W^{abcd} + d_4 (W_{ab}{}^{ab})^2$$
$$+ d_5 W_{aibj}W^{aibj} + d_6 W^b{}_{iab}W_c{}^{iac} + d_7 W_{ijkl}W^{ijkl} + d_8 W_{aijk}W^{aijk}$$
$$+ d_9 W_{abjk}W^{abjk} + d_{10} W_{iabc}W^{iabc} + d_{11} W^c{}_{acb}W_d{}^{adb} + d_{12} W^a{}_{iaj}W_b^{ibj}$$
$$+ d_{13} W_{ab}{}^{ab}\mathring{\text{II}}^i_{cd}\mathring{\text{II}}_i^{cd} + d_{14} W^a{}_{bij}\mathring{\text{II}}^i_{ac}\mathring{\text{II}}^{jbc} + d_{15} W^a{}_{ibj}\mathring{\text{II}}^i{}_{ac}\mathring{\text{II}}^{jbc}$$
$$+ d_{16} W^{abcd}\mathring{\text{II}}^i_{ac}\mathring{\text{II}}_{ibd} + d_{17} W_a{}^{bac}\mathring{\text{II}}^i_{bd}\mathring{\text{II}}_{ic}{}^d + d_{18} W^c{}_{icj}\mathring{\text{II}}^i_{ab}\mathring{\text{II}}^{jab}$$
$$+ d_{19}\text{Tr}\,\mathring{\text{II}}^i\mathring{\text{II}}_i\mathring{\text{II}}^j\mathring{\text{II}}_j + d_{20}\text{Tr}\,\mathring{\text{II}}^i\mathring{\text{II}}^j\mathring{\text{II}}_i\mathring{\text{II}}_j + d_{21}(\text{Tr}\,\mathring{\text{II}}^i\mathring{\text{II}}_i)^2 + d_{22}(\text{Tr}\,\mathring{\text{II}}^i\mathring{\text{II}}^j)(\text{Tr}\,\mathring{\text{II}}_i\mathring{\text{II}}_j)$$

$$h = \frac{\Gamma\left(\frac{d-p}{2}+1\right)}{\pi^{\frac{d-p}{2}+2}\,(d-p+3)}\,d_2$$

Average Null Energy Condition (ANEC)

$$h \geq 0 \quad \Rightarrow \quad d_2 \geq 0$$

4d Boundary or Defect

$$\left[T_\mu{}^\mu\right]_{p=4} = \frac{1}{(4\pi)^2}\Big(-b\,\hat{E}_4 + d_1\mathcal{J}_1 + d_2\mathcal{J}_2 + d_3 W_{abcd}W^{abcd} + d_4(W_{ab}{}^{ab})^2$$
$$+ d_5 W_{aibj}W^{aibj} + d_6 W^b{}_{iab}W_c{}^{iac} + d_7 W_{ijkl}W^{ijkl} + d_8 W_{aijk}W^{aijk}$$
$$+ d_9 W_{abjk}W^{abjk} + d_{10}W_{iabc}W^{iabc} + d_{11}W^c{}_{acb}W_d{}^{adb} + d_{12}W^a{}_{iaj}W_b^{ibj}$$
$$+ d_{13}W_{ab}{}^{ab}\mathring{\text{II}}^i_{cd}\mathring{\text{II}}^{cd}_i + d_{14}W^a{}_{bij}\mathring{\text{II}}^i_{ac}\mathring{\text{II}}^{jbc} + d_{15}W^a{}_{ibj}\mathring{\text{II}}^i{}_{ac}\mathring{\text{II}}^{jbc}$$
$$+ d_{16}W^{abcd}\mathring{\text{II}}^i_{ac}\mathring{\text{II}}_{ibd} + d_{17}W_a{}^{bac}\mathring{\text{II}}^i_{bd}\mathring{\text{II}}_{ic}{}^d + d_{18}W^c{}_{icj}\mathring{\text{II}}^i_{ab}\mathring{\text{II}}^{jab}$$
$$+ d_{19}\text{Tr}\,\mathring{\text{II}}^i\mathring{\text{II}}_i\mathring{\text{II}}^j\mathring{\text{II}}_j + d_{20}\text{Tr}\,\mathring{\text{II}}^i\mathring{\text{II}}^j\mathring{\text{II}}_i\mathring{\text{II}}_j + d_{21}(\text{Tr}\,\mathring{\text{II}}^i\mathring{\text{II}}_i)^2 + d_{22}(\text{Tr}\,\mathring{\text{II}}^i\mathring{\text{II}}^j)(\text{Tr}\,\mathring{\text{II}}_i\mathring{\text{II}}_j)$$

Entanglement Entropy (EE)

spherical region of radius ℓ

centered on the boundary/defect

4d Boundary or Defect

$$\left[T_\mu{}^\mu\right]_{p=4} = \frac{1}{(4\pi)^2}\Big(-b\,\hat{E}_4 + d_1\mathcal{J}_1 + d_2\mathcal{J}_2 + d_3 W_{abcd}W^{abcd} + d_4 (W_{ab}{}^{ab})^2$$
$$+ d_5 W_{aibj}W^{aibj} + d_6 W^b{}_{iab}W_c{}^{iac} + d_7 W_{ijkl}W^{ijkl} + d_8 W_{aijk}W^{aijk}$$
$$+ d_9 W_{abjk}W^{abjk} + d_{10} W_{iabc}W^{iabc} + d_{11} W^c{}_{acb}W_d{}^{adb} + d_{12} W^a{}_{iaj}W_b^{ibj}$$
$$+ d_{13} W_{ab}{}^{ab}\mathring{\text{II}}^i_{cd}\mathring{\text{II}}_i^{cd} + d_{14} W^a{}_{bij}\mathring{\text{II}}^i_{ac}\mathring{\text{II}}^{jbc} + d_{15} W^a{}_{ibj}\mathring{\text{II}}^i{}_{ac}\mathring{\text{II}}^{jbc}$$
$$+ d_{16} W^{abcd}\mathring{\text{II}}^i_{ac}\mathring{\text{II}}_{ibd} + d_{17} W_a{}^{bac}\mathring{\text{II}}^i_{bd}\mathring{\text{II}}_{ic}{}^d + d_{18} W^c{}_{icj}\mathring{\text{II}}^i_{ab}\mathring{\text{II}}^{jab}$$
$$+ d_{19}\,\text{Tr}\,\mathring{\text{II}}^i\mathring{\text{II}}_i\mathring{\text{II}}^j\mathring{\text{II}}_j + d_{20}\,\text{Tr}\,\mathring{\text{II}}^i\mathring{\text{II}}^j\mathring{\text{II}}_i\mathring{\text{II}}_j + d_{21}(\text{Tr}\,\mathring{\text{II}}^i\mathring{\text{II}}_i)^2 + d_{22}(\text{Tr}\,\mathring{\text{II}}^i\mathring{\text{II}}^j)(\text{Tr}\,\mathring{\text{II}}_i\mathring{\text{II}}_j)\Big)$$

Entanglement Entropy (EE)

$$S_{\text{EE}} = S^d_{\text{EE}} + S^{p=4}_{\text{EE}}$$

$$S^{p=4}_{\text{EE}} = \ldots + \left(4\,b - \frac{(d-5)(d-4)}{d-1}\,d_2\right)\ln(\ell/\varepsilon) + \ldots$$

Outline:

- Review: Boundaries and Defects
- Review: Weyl Anomalies
- 1d Boundary or Defect
- 2d Boundary or Defect
- 3d Boundary or Defect
- 4d Boundary or Defect
- Summary and Outlook

Outline:

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Question

How do boundaries and defects contribute to the Weyl anomaly?

Do we get boundary/defect central charges?

Do they obey monotonicity theorems?

Do they appear in entanglement entropy?

Do they appear in stress-energy correlators?

Do they obey bounds?

and so on…

Outlook

c-theorems? bounds? constraints?

Stress-energy tensor correlators?

thermal entropy?

Examples?

Applications?

Thank You.