Holographic 4-pt functions in momentum space:

renormalisation, anomalies and a reduction scheme

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Introduction

- Conformal invariance imposes strong constraints on correlation functions.
- It determines two- and three-point functions of scalars, conserved vectors and the stress-energy tensor [Polyakov (1970)] ... [Osborn, Petkou (1993)]. For example,

$$\langle \mathcal{O}_1(\boldsymbol{x}_1)\mathcal{O}_2(\boldsymbol{x}_2)\mathcal{O}_3(\boldsymbol{x}_3)\rangle = \frac{c_{123}}{|\boldsymbol{x}_1-\boldsymbol{x}_2|^{\Delta_1+\Delta_2-\Delta_3}|\boldsymbol{x}_2-\boldsymbol{x}_3|^{\Delta_2+\Delta_3-\Delta_1}|\boldsymbol{x}_3-\boldsymbol{x}_1|^{\Delta_3+\Delta_1-\Delta_2}}.$$

- It determines the form of higher point functions up to functions of cross-ratios.

Introduction

- These results (and many others) were obtained in position space.
- This is in stark contrast with general QFT were Feymnan diagrams are typically computed in momentum space.
- While position space methods are powerful, typically they
  - provide results that hold only at separated points ("bare" correlators).
  - are hard to extend beyond CFTs
- Momentum space results were needed in several recent applications:
  - Holographic cosmology [McFadden, KS](2010)(2011) [Bzowski, McFadden, KS (2011)(2012)] [Pimentel, Maldacena (2011)][Mata, Raju,Trivedi (2012)] [Kundu, Shukla,Trivedi (2014)][Arkani-Hamed, Maldacena (2015)] .... [Arkani-Hamed, Baumann, Lee, Pimentel Cosmological Bootstrap (2018)] [Baumann et al (2019) (2020)(2021) ....
  - Studies of 3d critical phenomena [Sachdev et al (2012)(2013)][Myers et al (2016)] [Lucas et al (2017)]....

References

This talk will be based on work with Adam Bzowski, Paul McFadden:

- Implications of conformal invariance in momentum space 1304.7760
- Scalar 3-point functions in CFT: renormalisation, beta functions and anomalies , 1510.08442
- Evaluation of conformal integrals, 1511.02357
- Renormalised 3-point functions of stress tensors and conserved currents in CFT, 1711.09105
- Renormalised CFT 3-point functions of scalars, currents and stress tensors, 1805.12100.
- Conformal n-point functions in momentum space, 1910.10162
- Conformal correlators as simplex integrals in momentum space, 2008.07543.
- A handbook of holographic 4-point functions, *to appear*

- Related work: [Coriano et al] [Maldacena-Pimentel] . . . [Gillioz] [Bautista, Godazgar] . . .

Outline

Outline

Position space

- In position space the solution of the conformal Ward identities is well known:

$$\langle O_1(\boldsymbol{x}_1) \dots O_n(\boldsymbol{x}_n) \rangle = \prod_{1 \le i < j \le n} x_{ij}^{2\alpha_{ij}} f(\boldsymbol{u}),$$

where $n \geq 1$, $x_{ij}^2 = (\boldsymbol{x}_i - \boldsymbol{x}_j)^2$ and α_{ij} are related to the scaling dimensions by

$$\Delta_m = -\sum_{j=1}^{n} \alpha_{mj}, \qquad m = 1, 2, \dots, n.$$

with $\alpha_{ji} = \alpha_{ij}$ and $\alpha_{ii} = 0$.

- $\boldsymbol{u}$ are the $n(n-3)/2$ cross-ratios,

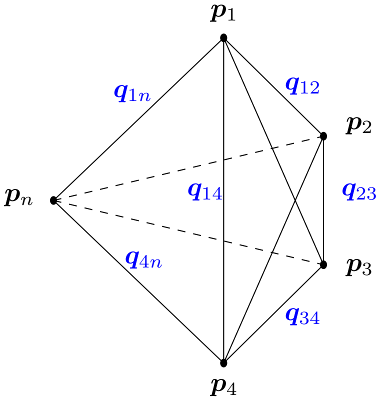
$$u_{[pqrs]} = \frac{x_{pr}^2 x_{qs}^2}{x_{pq}^2 x_{rs}^2},$$

where $p, q, r, s = 1, 2, \dots, n$.

- Conformal invariance does not restrict $f(\boldsymbol{u})$.

Momentum space [Bzowski, McFadden, KS (2019)]

$$\langle O_1(\boldsymbol{p}_1)\ldots O_n(\boldsymbol{p}_n)\rangle = \prod_{1\le i<j\le n}\int \frac{d^d\boldsymbol{q}_{ij}}{(2\pi)^d}\frac{\hat{f}(\hat{\boldsymbol{u}})}{q_{ij}^{2\alpha_{ij}+d}}\prod_{k=1}^{n}(2\pi)^d\delta\Big(\boldsymbol{p}_k+\sum_{l=1}^{n}\boldsymbol{q}_{lk}\Big),$$



- Momentum conservation at each vertex.
- Momentum space cross-ratios

$$\hat{u}_{[pqrs]} = \frac{q_{pq}^2 q_{rs}^2}{q_{pr}^2 q_{qs}^2}$$

- Conformal invariance does not restrict $\hat{f}(\hat{\boldsymbol{u}})$.

Outline

Outline

2-point function

Specialising the general solution

$$\langle O_1(\boldsymbol{p}_1)\dots O_n(\boldsymbol{p}_n)\rangle = \prod_{1\le i<j\le n}\int \frac{d^d\boldsymbol{q}_{ij}}{(2\pi)^d}\frac{\hat{f}(\hat{\boldsymbol{u}})}{q_{ij}^{2\alpha_{ij}+d}}\prod_{k=1}^{n}(2\pi)^d\delta\Big(\boldsymbol{p}_k+\sum_{l=1}^{n}\boldsymbol{q}_{lk}\Big),$$

to the 2-point function we get:

$$\Delta_m = -\sum_{j=1}^{2}\alpha_{mj},\quad m=1,2 \quad\Rightarrow\quad \Delta_1=\Delta_2=-\alpha_{12}$$

$$\begin{aligned}\langle O_1(\boldsymbol{p}_1)O_2(\boldsymbol{p}_2)\rangle &= c_{12}\int\frac{d^d\boldsymbol{q}_{12}}{(2\pi)^d}\frac{1}{q_{12}^{2\alpha_{12}+d}}\delta\Big(\boldsymbol{p}_1+\boldsymbol{q}_{12}\Big)\delta\Big(\boldsymbol{p}_2-\boldsymbol{q}_{12}\Big)\\ &= c_{12}p_1^{2\Delta_1-d}\delta(\boldsymbol{p}_1+\boldsymbol{p}_2)\end{aligned}$$

Scalar 2-point function

The general solution of the conformal Ward identities is:

$$\langle O_\Delta(\boldsymbol{p}) O_\Delta(-\boldsymbol{p})\rangle = c_{12} p^{2\Delta-d}.$$

- This solution is trivial when

$$\Delta = \frac{d}{2} + k, \qquad k = 0, 1, 2, \ldots$$

  because then correlator is local,

$$\langle O(\boldsymbol{p}) O(-\boldsymbol{p})\rangle = c p^{2k} \to \langle O(\boldsymbol{x}_1) O(\boldsymbol{x}_2)\rangle \sim \Box^k \delta(x_1 - x_2)$$

- Let ϕ_0 be the source of O. It has dimension $d{-}\Delta{=}d/2{-}k$. The term

$$\phi_0 \Box^k \phi_0$$

  has dimension d and can act as a local counterterm.

Position space

- In position space, at first it appears that nothing special happens for any Δ:

$$\langle \mathcal{O}(\boldsymbol{x})\mathcal{O}(0)\rangle = \frac{C_{\mathcal{O}}}{x^{2\Delta}}$$

- This expression however is valid only at separated points, $x^2 \neq 0$.
- Correlation functions should be well-defined as distributions and they should have well-defined Fourier transform.
- Fourier transforming we find:

$$\int d^d\boldsymbol{x}\, e^{-i\boldsymbol{p}\cdot\boldsymbol{x}} \frac{1}{x^{2\Delta}} = \frac{\pi^{d/2}2^{d-2\Delta}\Gamma\left(\frac{d-2\Delta}{2}\right)}{\Gamma(\Delta)} p^{2\Delta-d},$$

- This is well-behaved, except when $\Delta = d/2 + k$, where k is a positive integer.

Regularization

- We use dimensional regularisation to regulate the theory

$$d \mapsto d + 2u\epsilon, \qquad \Delta_j \mapsto \Delta_j + (u + v_j)\epsilon$$

u, v_j are constants that characterize the scheme.

- In the regulated theory, the solution of the Ward identities is the same as before but the integration constant may depend on the regulator,

$$\langle O(\boldsymbol{p}) O(-\boldsymbol{p}) \rangle_{\text{reg}} = c(\epsilon, u, v) p^{2\Delta - d + 2v\epsilon}.$$

- In local CFTs:

$$c(\epsilon, u, v) = \frac{c^{(-1)}(u, v)}{\epsilon} + c^{(0)}(u, v) + O(\epsilon)$$

- This leads to

$$\langle O(\boldsymbol{p}) O(-\boldsymbol{p}) \rangle_{\text{reg}} = p^{2k} \left[\frac{c^{(-1)}}{\epsilon} + c^{(-1)} v \log p^2 + c^{(0)} + O(\epsilon) \right].$$

Renormalization

- Let ϕ_0 the source that couples to O,

$$S[\phi_0] = S_0 + \int d^{d+2u\epsilon}\boldsymbol{x}\, \phi_0 O.$$

- The divergence in the 2-point function can be removed by the addition of the counterterm action

$$S_{\text{ct}} = a_{\text{ct}}(\epsilon, u, v) \int d^{d+2u\epsilon}\boldsymbol{x}\, \phi_0 \Box^k \phi_0 \mu^{2v\epsilon},$$

- Removing the cut-off we obtain the renormalised correlator:

$$\langle O(\boldsymbol{p}) O(-\boldsymbol{p})\rangle_{ren} = p^{2k} \left[C \log \frac{p^2}{\mu^2} + C_1 \right]$$

Anomalies

- The counterterm breaks scale invariance and as result the theory has a conformal anomaly.
- The 2-point function depends on a scale

$$\mathcal{A}_2 = \mu \frac{\partial}{\partial \mu} \langle O(\boldsymbol{p}) O(-\boldsymbol{p}) \rangle = c p^{2\Delta - d},$$

- The integrated anomaly is Weyl invariant

$$A = \int d^d \boldsymbol{x} \, \phi_0 \Box^k \phi_0$$

On a curved background, $\Box^k$ is replaced by the "k-th power of the conformal Laplacian", P^k.

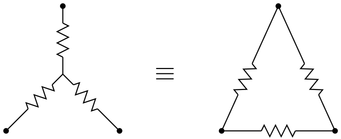
General solution for 3-pt functions

- The general solution reduces to

$$\langle O_1(\boldsymbol{p}_1)O_2(\boldsymbol{p}_2)O_3(\boldsymbol{p}_3)\rangle = c_3 \int \frac{d^d\boldsymbol{q}}{(2\pi)^d} \frac{1}{q^{2\alpha_{12}+d}|\boldsymbol{q}-\boldsymbol{p}_1|^{2\alpha_{13}+d}|\boldsymbol{q}+\boldsymbol{p}_2|^{2\alpha_{23}+d}}$$

  with $2\alpha_{12} = 2\Delta_3 - \Delta_t = -\Delta_1 - \Delta_2 + \Delta_3$, etc.

- One can use a star-triangle relation:



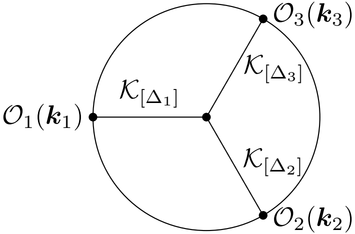
  to express the answer as *triple-K integral*:

$$\langle \mathcal{O}_1(\boldsymbol{p}_1)\mathcal{O}_2(\boldsymbol{p}_2)\mathcal{O}_3(\boldsymbol{p}_3)\rangle = C_{123} p_1^{\Delta_1-\frac{d}{2}} p_2^{\Delta_2-\frac{d}{2}} p_3^{\Delta_3-\frac{d}{2}} \int_0^\infty dx\, x^{\frac{d}{2}-1} K_{\Delta_1-\frac{d}{2}}(p_1 x) K_{\Delta_2-\frac{d}{2}}(p_2 x) K_{\Delta_3-\frac{d}{2}}(p_3 x),$$

  where $K_\nu(p)$ is a Bessel function and C_{123} is a constant.

Holographic 3-point functions

- The same expression can be obtained by computing the standard Witten diagram:



where

$$\mathcal{K}_{[\Delta]} = \frac{k^{\Delta-\frac{d}{2}} z^{\frac{d}{2}} K_{\Delta-\frac{d}{2}}(kz)}{2^{\Delta-\frac{d}{2}-1}\Gamma\left(\Delta-\frac{d}{2}\right)}$$

is the scalar bulk-to-boundary propagator.

Triple K-integrals

- Triple-K integrals,

$$I_{\alpha\{\beta_1\beta_2\beta_3\}}(p_1, p_2, p_3) = \int_0^\infty dx\, x^\alpha \prod_{j=1}^{3} p_j^{\beta_j} K_{\beta_j}(p_j x),$$

  are the building blocks of all 3-point functions.
- The integral converges provided

$$\alpha > \sum_{j=1}^{3} |\beta_j| - 1$$

- The integral can be defined by analytic continuation when

$$\alpha + 1 \pm \beta_1 \pm \beta_2 \pm \beta_3 \neq -2k,$$

  where k is any non-negative integer.

Renormalization and anomalies

- If the equality holds,

$$\alpha + 1 \pm \beta_1 \pm \beta_2 \pm \beta_3 = -2k,$$

  the integral cannot be defined by analytic continuation.
- Non-trivial subtractions and renormalization may be required and this may result in conformal anomalies and beta functions.
- Physically when this equality holds, there are new terms of dimension d that one can add to the action (counterterms) and/or new terms that can appear in T^{μ}_{μ} (conformal anomalies).

Anomalies and beta functions

- $(- - -)$ case: $\Delta_1 + \Delta_2 + \Delta_3 = 2d + 2k$. This the analogue of the $\Delta = d/2 + k$ case in 2-point functions.
- There are possible counterterms: $\int d^d\boldsymbol{x}\, \Box^{k_1}\phi_1\Box^{k_2}\phi_2\Box^{k_3}\phi_3$.
- ➠ This leads to new conformal anomalies

- $(- - +)$ case: $\Delta_1 + \Delta_2 - \Delta_3 = d + 2k$
- There are possible counterterms: $\int d^d x \Box^{k_1}\phi_1\Box^{k_2}\phi_2 O_3$
- ➠ This leads to beta functions.

- $(+ + +)$ and $(- + +)$ cases. In these cases it is the representation of the correlator in terms of the triple-K integral that is singular, not the correlator itself.

Callan-Symanzik equation

- The quantum effective action $\mathcal{W}$ (the generating functional of renormalised connected correlators) obeys the equation

$$\mu \frac{d}{d\mu}\mathcal{W} = \Big(\mu \frac{\partial}{\partial\mu} + \sum_i \int d^d\vec{x}\, \beta_i \frac{\delta}{\delta\phi_i(\vec{x})}\Big)\mathcal{W} = \int d^d\vec{x}\, \mathcal{A},$$

- This implies that for 3-point functions we have

$$\mu \frac{\partial}{\partial\mu}\langle O_i(p_1)O_j(p_2)O_j(p_3)\rangle = \beta_{j,ji}\big(\langle O_j(p_2)O_j(-p_2)\rangle + \langle O_j(p_3)O_j(-p_3)\rangle\big) + \mathcal{A}^{(3)}_{ijj},$$

$$\beta_{i,jk} = \frac{\delta^2\beta_i}{\delta\phi_j\delta\phi_k}\Big|_{\{\phi_l\}=0}, \quad \mathcal{A}^{(3)}_{ijk}(\vec{x}_1,\vec{x}_2,\vec{x}_3) = -\frac{\delta^3}{\delta\phi_i(\vec{x}_1)\delta\phi_j(\vec{x}_2)\delta\phi_k(\vec{x}_3)}\int d^d\vec{x}\,\mathcal{A}(\{\phi_l(\vec{x})\})$$

Example: $\Delta_1 = 4, \Delta_2 = \Delta_3 = 3$ in $d = 4$

- $\Delta_1 + \Delta_2 + \Delta_3 = 10 = 2d + 2k$, which satisfies the $(---)$-condition with $k = 1$.
- ➠ There is an anomaly

$$\int d^4 x \phi_0 \phi_1 \Box \phi_1$$

- $\Delta_1 + \Delta_2 - \Delta_3 = 4 = d + 2k$, which satisfies the $(--+)$ condition with $k = 0$. The following counterterm is needed,

$$\int d^4 x \phi_0 \phi_1 O_3$$

- ➠ There is a beta function for ϕ_1.

$\langle O_4 O_3 O_3 \rangle$

$$\langle O_4(\boldsymbol{p}_1) O_3(\boldsymbol{p}_2) O_3(\boldsymbol{p}_3) \rangle = \alpha \left(2 - p_1 \frac{\partial}{\partial p_1} \right) I^{\text{(non-local)}}$$
$$+ \frac{\alpha}{8} \left[(p_2^2 - p_3^2) \log \frac{p_1^2}{\mu^2} \left(\log \frac{p_3^2}{\mu^2} - \log \frac{p_2^2}{\mu^2} \right) - (p_2^2 + p_3^2) \log \frac{p_2^2}{\mu^2} \log \frac{p_3^2}{\mu^2} \right.$$
$$\left. (p_1^2 - p_2^2) \log \frac{p_3^2}{\mu^2} + (p_1^2 - p_3^2) \log \frac{p_2^2}{\mu^2} + p_1^2 \right]$$

It satisfies

$$\mu \frac{\partial}{\partial \mu} \langle O_4(\boldsymbol{p}_1) O_3(\boldsymbol{p}_2) O_3(\boldsymbol{p}_3) \rangle = \frac{\alpha}{2} \left(p_2^2 \log \frac{p_2^2}{\mu^2} + p_3^2 \log \frac{p_3^2}{\mu^2} - p_1^2 + \frac{1}{2}(p_2^2 + p_3^2) \right).$$

4-point functions

- Counterterms exist when

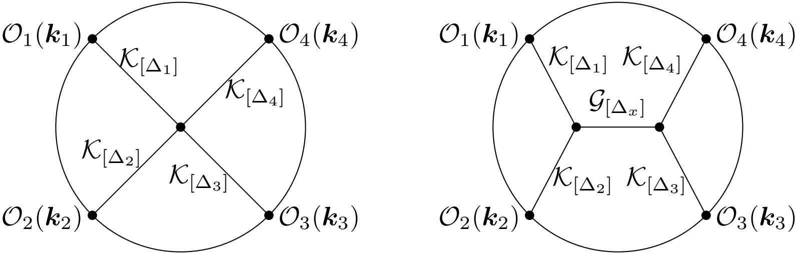
$$d + \sum_{j=1}^{4} \sigma_j (\Delta_j - d/2) = -2n$$

  for some n non-negative integer, with signs σ_j whose values are either all minus, or else three minus and one plus.
  - $(- - - -)$: anomalies
  - $(- - - +)$: beta functions
- The general solution exhibits a UV singularity when the condition holds.

Outline

1. Solution of conformal Ward identities
2. Renormalisation, beta functions and anomalies
3. Holographic 4-point functions

Tree-level holographic 4-point functions



where

$$\mathcal{G}_{[\Delta]} = \begin{cases} (z\zeta)^{\frac{d}{2}} I_{\Delta-\frac{d}{2}}(kz) K_{\Delta-\frac{d}{2}}(k\zeta) & \text{for } z < \zeta, \\ (z\zeta)^{\frac{d}{2}} K_{\Delta-\frac{d}{2}}(kz) I_{\Delta-\frac{d}{2}}(k\zeta) & \text{for } z > \zeta. \end{cases}$$

is the scalar bulk-to-bulk propagator and I_β and K_β are the modified Bessel functions.

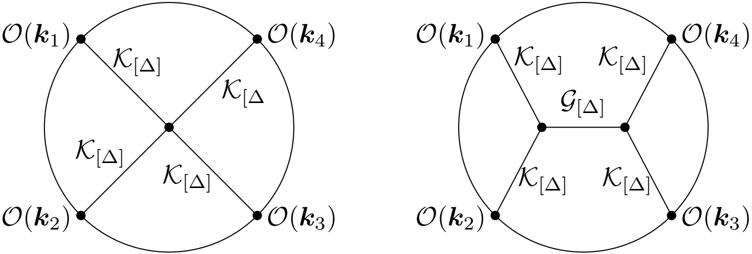
Each tree-level Witten diagram is a CFT correlator

- Usually in QFT one needs to add of all Feynman diagrams contributing to given correlator to obtain an expression that satisfies Ward identities.
- It turns out that each tree-level Witten diagram on it own satisfies the conformal Ward identities.
- This is the case because one can show that there is always a bulk action in AdS where the given diagram is the only diagram contributing to a given correlator.
- Invariance under AdS isometries then implies that the diagram satisfies the conformal Ward identities on its own.

- This implies that the IR divergences of each diagram separately should have the right analytic structure to correspond to CFT UV divergences.

Example: all diagrams contributing

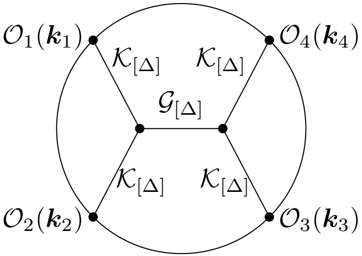
$$S = \frac{1}{2} \int d^d x \sqrt{g} \left[\partial_\mu \Phi \partial^\mu \Phi + m_\Delta^2 \Phi^2 + \lambda_{123} \Phi^3 - \lambda_{1234} \Phi^4 \right],$$



+ t and u channels

Example: only one of the diagrams contributes

$$S^{\text{asym}} = \frac{1}{2} \int d^d x \sqrt{g} \sum_{j=1,2,3,4,x} \left[\partial_\mu \Phi_j \partial^\mu \Phi_j + m_\Delta^2 \Phi_j^2 \right] + \int d^d x \sqrt{g} \left[\lambda_{12x} \Phi_1 \Phi_2 \Phi_x + \lambda_{34x} \Phi_x \Phi_3 \Phi_4 \right],$$



Evaluation

- The evaluation of the diagrams is straightforward when $\beta_i = \Delta_i - d/2$ are half-integral, because the Bessel functions reduce to elementary functions.
- There is preferred regularisation scheme, the 'half-integer' scheme,

$$d \longmapsto \hat{d} = d + 2\epsilon, \qquad \Delta_j \longmapsto \hat{\Delta}_j = \Delta_j + \epsilon,$$

  which preserves the indices on all Bessel functions.
- In the paper we compute the correlators in the 'half-integral' scheme and show to find the answer in any other scheme.

Results

- We present results for all tree-level correlators with external dimensions 2 or 3 and exchange scalar of dimension 2 or 3 in $d = 3$.
- These cases are particularly relevant in the context of holographic cosmology/cosmological bootstrap: $\Delta = 2$ is dual to a bulk conformal scalar and $\Delta = 3$ is dual to a massless bulk scalar.
- Correlators exhibit infinities that require renormalisation.

Divergences

Degree of divergence n for Witten diagrams dimensionally regulated.

External operator	Contact	s-exchange $\Delta_x = 2$	s-exchange $\Delta_x = 3$
$[22, 22]$	0	0	0
$[32, 22]$	1	2	1
$[33, 22]$	1	1	2
$[32, 32]$	1	2	1
$[32, 33]$	1	2	2
$[33, 33]$	1	1	2

Each diagram diverges as ϵ^{-n} for $\epsilon \to 0$, and is labelled by the dimensions $[\Delta_1\Delta_2, \Delta_3\Delta_4]$ of the external operators and that of the exchanged operator Δ_x where present.

Anomalies and beta function

$$d + \sum_{j=1}^{4} \sigma_j (\Delta_j - d/2) = -2n$$

- $[33, 33]$: beta function

$$(- - - +): \qquad 3 - (3 - 3/2) - (3 - 3/2) - (3 - 3/2) + (3 - 3/2) = 0$$

- $[33, 22]$: beta function

$$(- - - +): \qquad 3 - (3 - 3/2) - (3 - 3/2) - (2 - 3/2) + (2 - 3/2) = 0$$

- $[32, 22]$: anomaly

$$(- - - -): \qquad 3 - (3 - 3/2) - (2 - 3/2) - (2 - 3/2) - (2 - 3/2) = 0$$

- $[33, 32]$: anomaly

$$(- - - -): \qquad 3 - (3 - 3/2) - (3 - 3/2) - (3 - 3/2) - (2 - 3/2) = -2$$

Anomalies and beta functions

- Beta functions

$$\beta_{\phi_{[0]}} = \frac{1}{6}\lambda_{[333]}\phi_{[0]}^2 + \left[-\frac{1}{18}\lambda_{[3333]} + \frac{1}{12}\lambda_{[332]}^2 - \frac{1}{54}\lambda_{[333]}^2\right]\phi_{[0]}^3 + O(\phi_{[0]}^4),$$
$$\beta_{\phi_{[1]}} = \lambda_{[322]}\phi_{[0]}\phi_{[1]} + [\ldots]\,\phi_{[0]}^2\phi_{[1]} + O(\phi_{[0]}^3).$$

- Anomalies

$$\mathcal{A}^{(3)} = -\frac{1}{6}\lambda_{[222]}\phi_{[1]}^3 + \frac{1}{2}\lambda_{[332]}\phi_{[1]}\partial_\mu\phi_{[0]}\partial^\mu\phi_{[0]},$$
$$\mathcal{A}^{(4)} = \phi_{[0]}\phi_{[1]}^3\left[\frac{1}{2}\lambda_{[322]}\lambda_{[222]}(1 + a_{[222]}^{(1)} - a_{[322]}^{(1)}) + \frac{1}{4}\lambda_{[332]}\lambda_{[322]} + \frac{1}{6}\lambda_{[3222]}\right]$$
$$+ \left(\frac{1}{6}\phi_{[0]}^3\partial^2\phi_{[1]} - \frac{1}{2}\phi_{[0]}^2\phi_{[1]}\partial^2\phi_{[0]}\right)[\ldots]$$
$$+ \phi_{[0]}^3\partial^2\phi_{[1]}\left[\frac{1}{3}\lambda_{[332]}\lambda_{[322]} + \frac{1}{9}\lambda_{[333]}\lambda_{[332]}\right].$$

Reduction scheme

- Weight-shifting operators act on a solution of conformal Ward identities and produce another solution where the dimension of two operators has been shifted by ± 1. [Katareev et cl (2017)].
- One may use weight-shifting operators as a method to obtain the correlators discussed here and this has been pursued in [Baumann et al (2020)(2021)].
- There are however subtleties:
  - application is subtle when renormalisation is needed
  - weight-shifting operator yield both contact and exchange diagrams when act on exchange diagrams
- Taking into account the subtleties, all exchange diagrams can be obtained from a single exchange diagram, $i[33, 33x3]$ and all contact diagrams.

Conclusions/Outlook

- We discussed the general solution of conformal Ward identities in momentum space.
- We discussed renormalization, beta function and anomalies.
- We presented the computation of a class of tree-level holographic 4-point function.
- Apply the results to early universe cosmology.
- The last missing piece for CFT in momentum space is the general solution for tensorial n-point functions.
- It would interesting to setup the bootstrap program in momentum space.
- $\ldots$