SFQEDtoolkit: a high-performance library for the accurate modelling of strong-field QED effects in PIC and Monte Carlo codes

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Presented by Samuele Montefiori

Smilei Workshop

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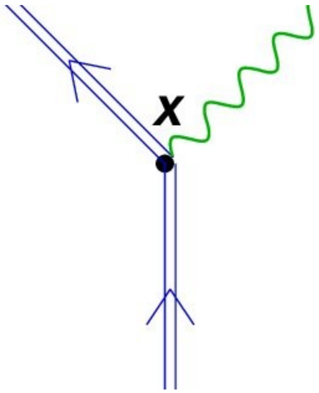
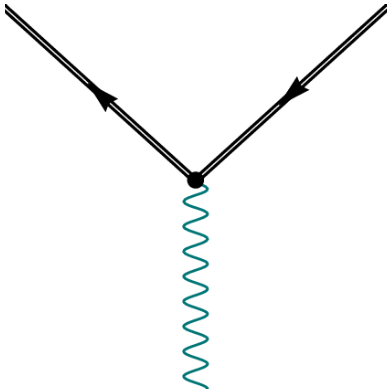
Max-Planck-Institut für Kernphysik

Introduction



Non-linear
Compton scattering



Breit-Wheeler pair production

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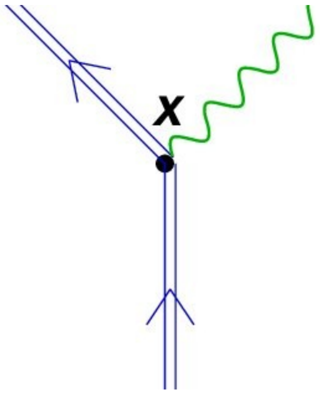

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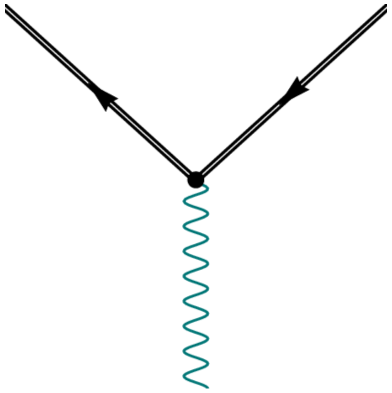

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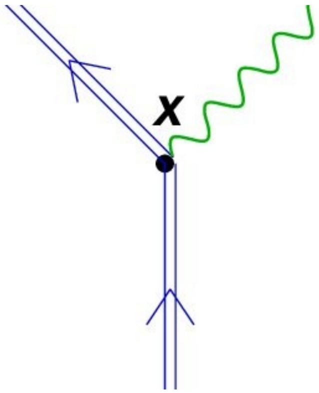

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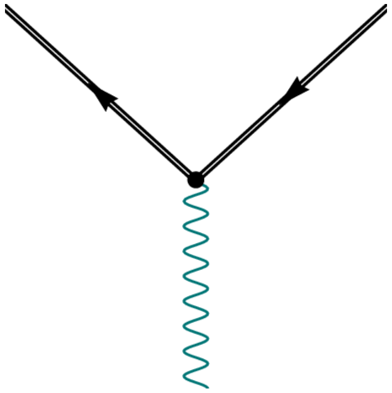

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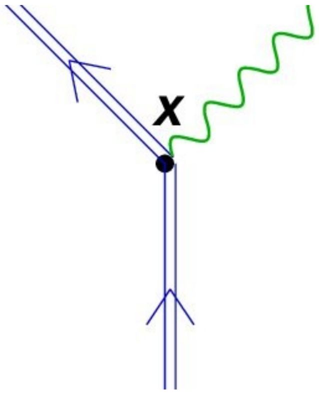

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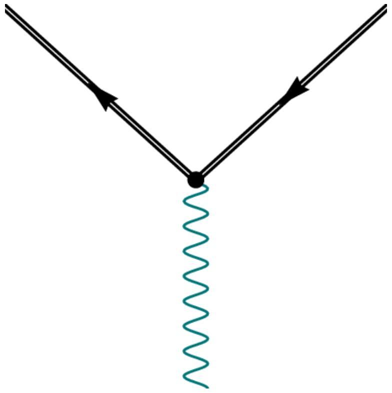

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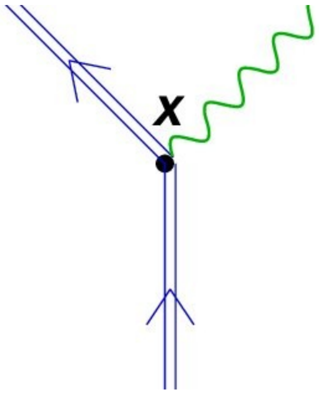

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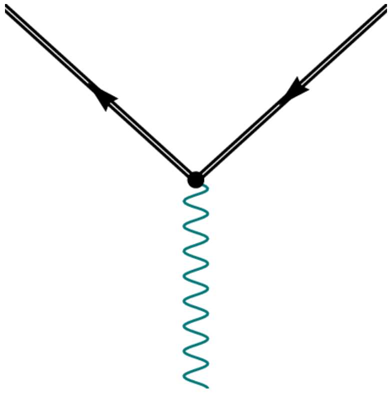

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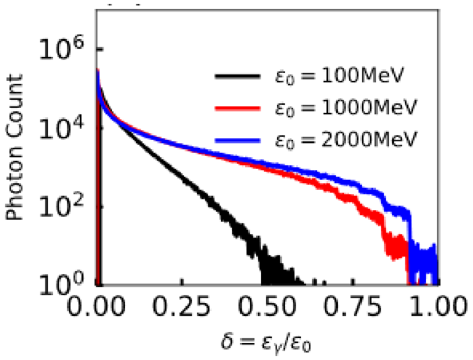

Yinlong Guo et al.
Phys. Rev. E 105, 025309

Introduction

Smilei Radiation reaction models

Regime	χ value	Description	Models
Classical radiation emission	$\chi \sim 10^{-3}$	$\gamma_\gamma \ll \gamma$, radiated energy overestimated for $\chi > 10^{-2}$	Landau-Lifshitz
Semi-classical radiation emission	$\chi \sim 10^{-2}$	$\gamma_\gamma \ll \gamma$, no stochastic effects	Corrected Landau-Lifshitz
Weak quantum regime	$\chi \sim 10^{-1}$	$\gamma_\gamma < \gamma$, $\gamma_\gamma \gg mc^2$	Stochastic model of Niel *et al* / Monte-Carlo
Quantum regime	$\chi \sim 1$	$\gamma_\gamma \gtrsim \gamma$	Monte-Carlo

Com ... duction

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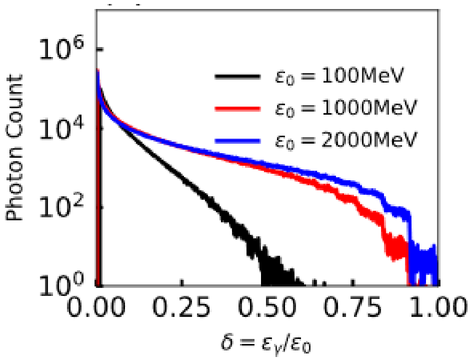

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addressed by **SFQEDtoolkit**

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$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x) \qquad T_0(x) = 1, \;\; T_1(x) = x,$$

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Chebyshev transform → $Ch[f(x)]$

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$$\tilde{W}_{rad}(\chi_e)$$

Emission rate (What tells us if the event occurs)

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SFQEDtoolkit is loaded with the coefficients of

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over the range

$$0 \leq \chi_e \leq 2000$$

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Exponentially fast convergence for C^∞ functions

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Emitted photon energy

Starting from:

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Less coefficients for

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Analytically inverted into: $\bar{w} = r\tilde{W}_{rad}(\chi_e) \left[\frac{9}{2^{1/3}} \Gamma\left(\frac{2}{3}\right)\right]^{-1}$

Emitted photon energy (final part)

$$\bar{w} = \texttt{inverse}\left[\tilde{I}_{pe}(\bar{w}, \chi_e) - r\tilde{W}_{rad}(\chi_e) = 0\right]$$

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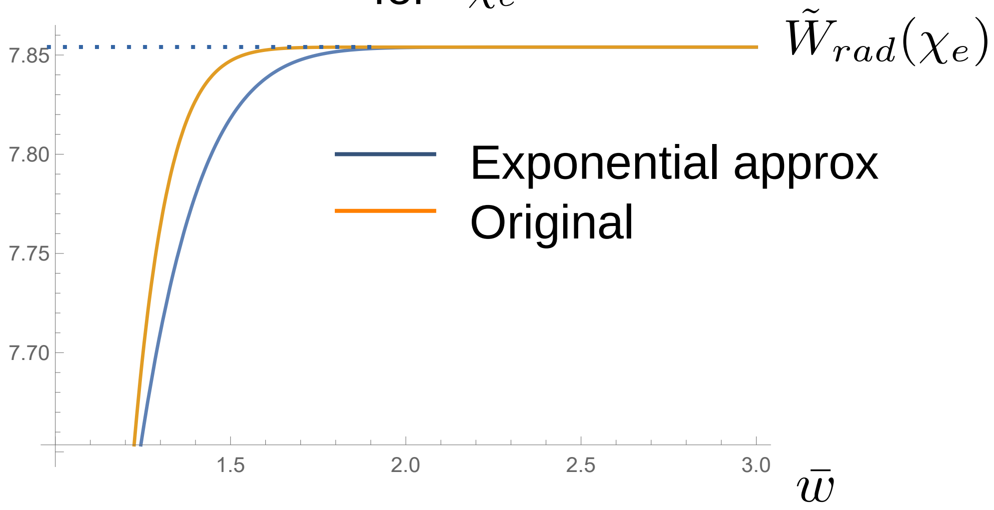

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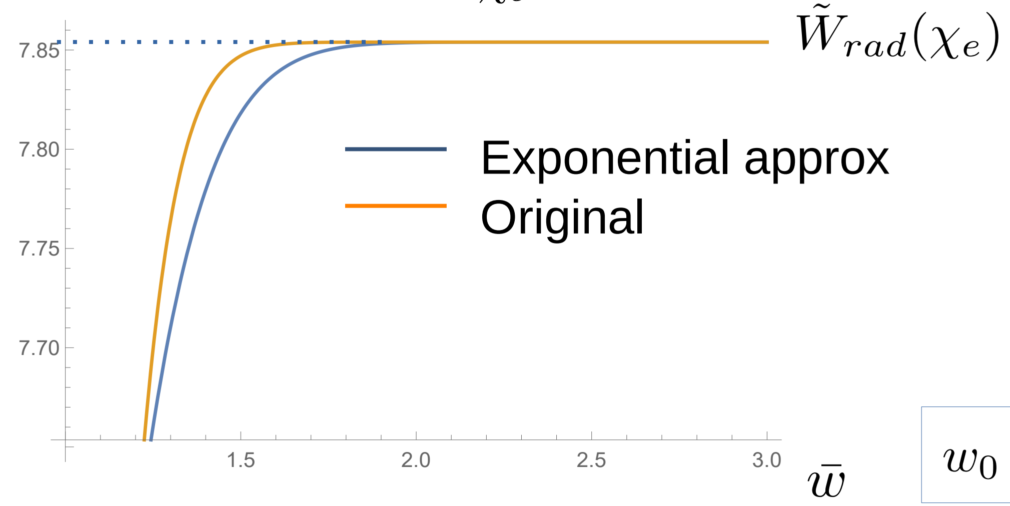

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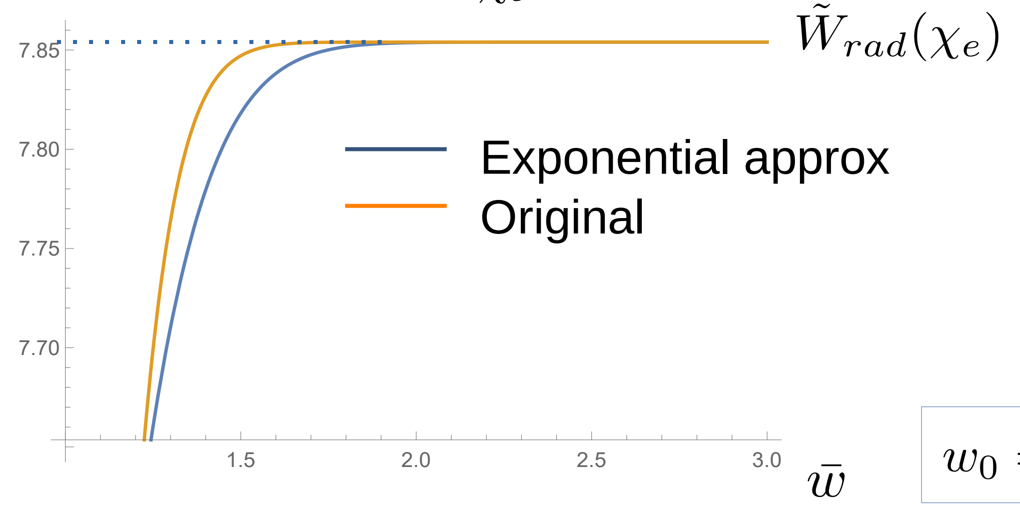

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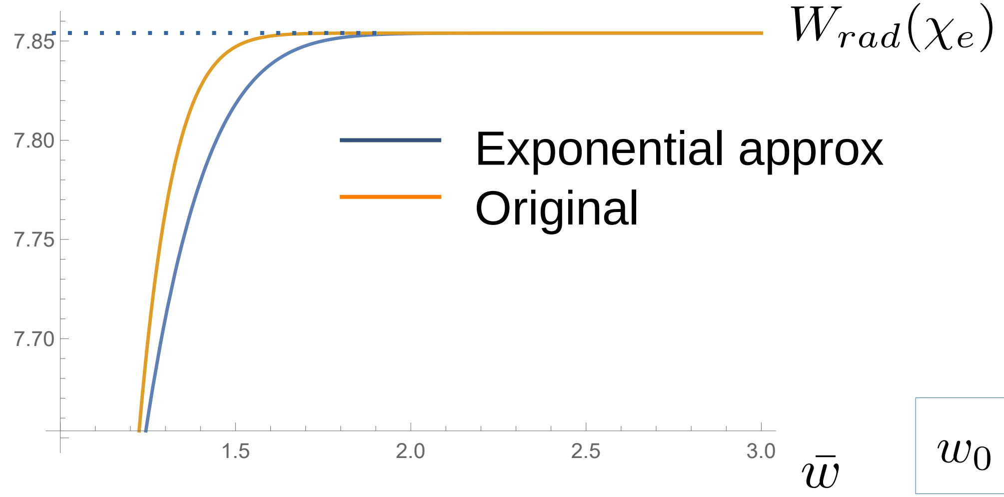

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<u>Analytically inverted</u> into:

$$\bar{w} = \sqrt[3]{w_0^3 - \log\left[\frac{\tilde{W}_{rad}(\chi_e)(1-r)}{\tilde{W}_{rad}(\chi_e) - \tilde{I}_{pe}(w_0,\chi_e)}\right]}$$

Accuracy and Results

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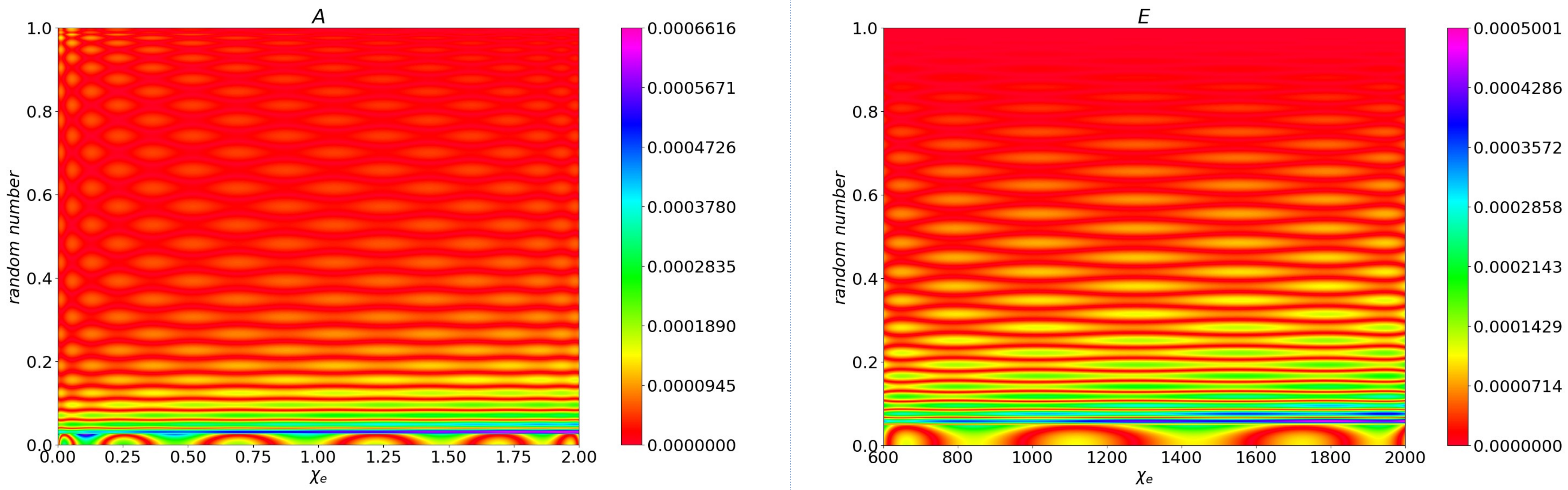

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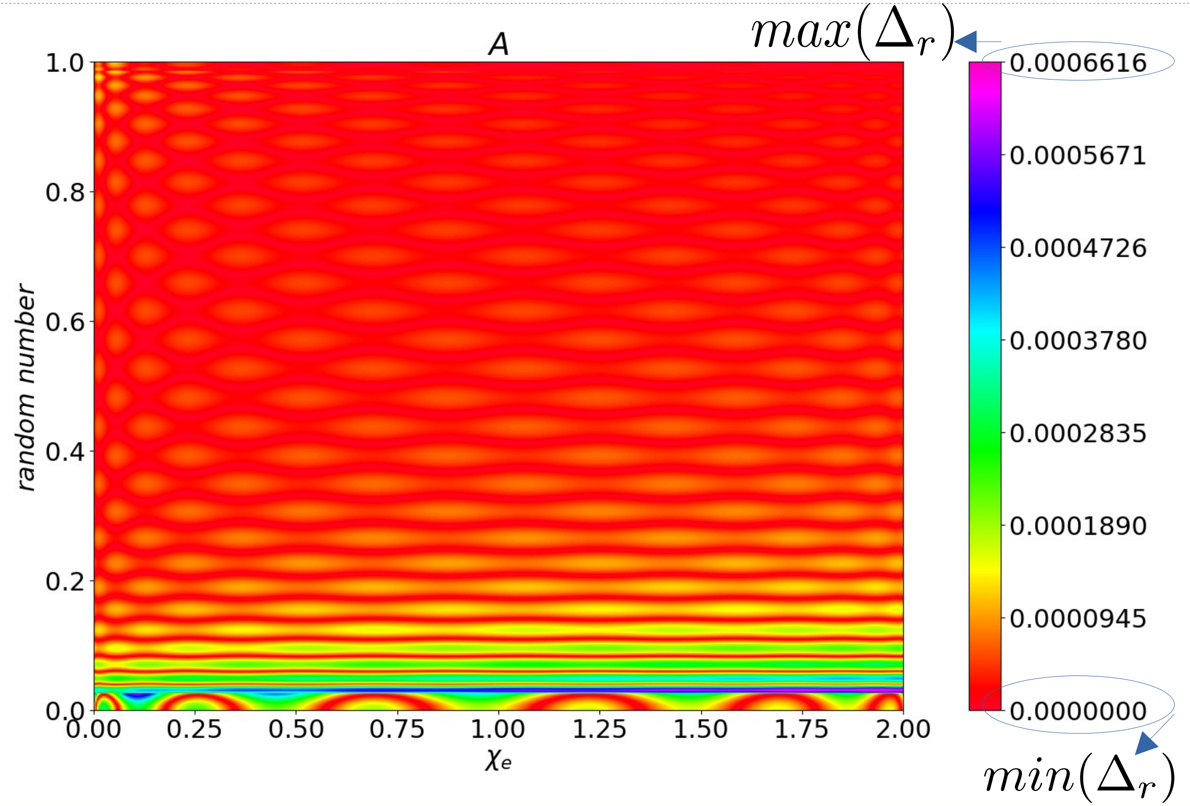

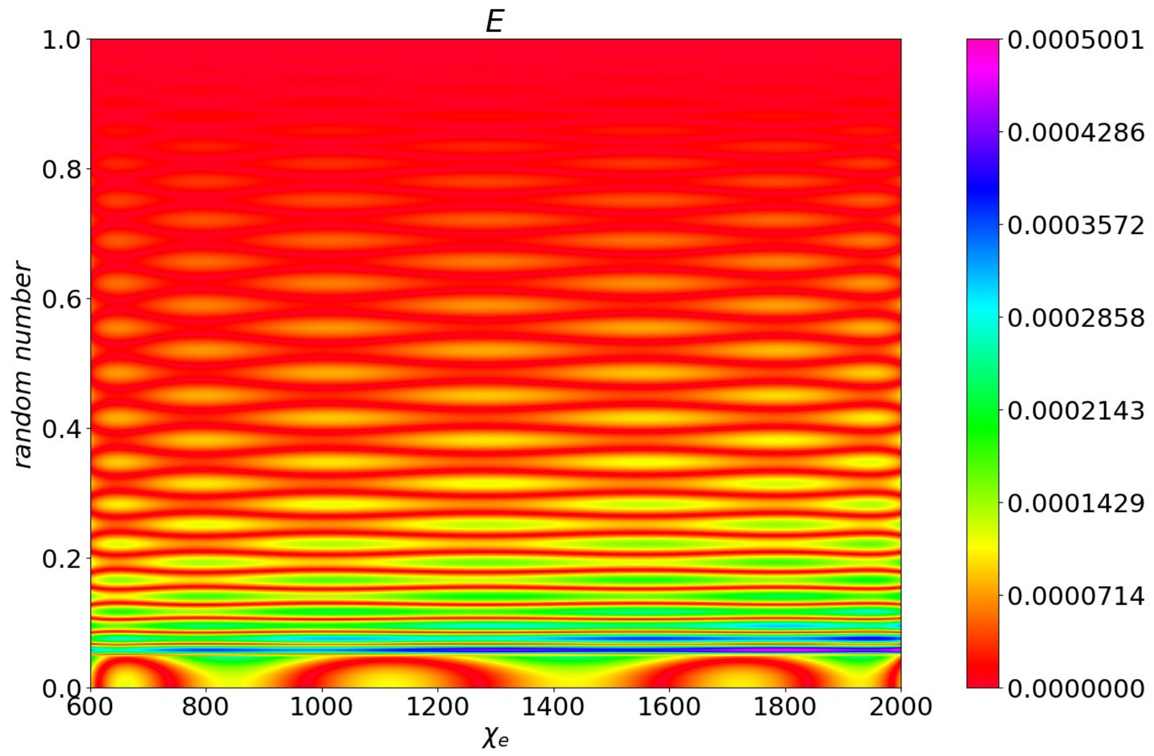

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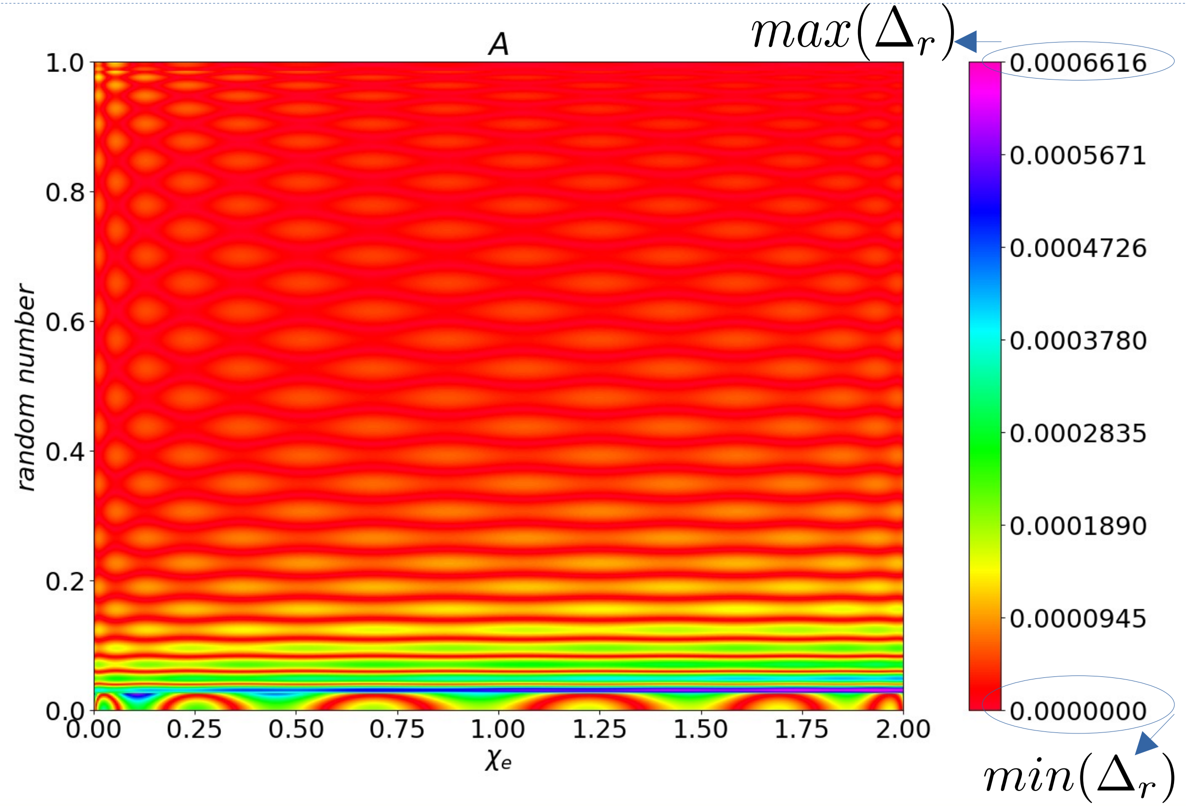
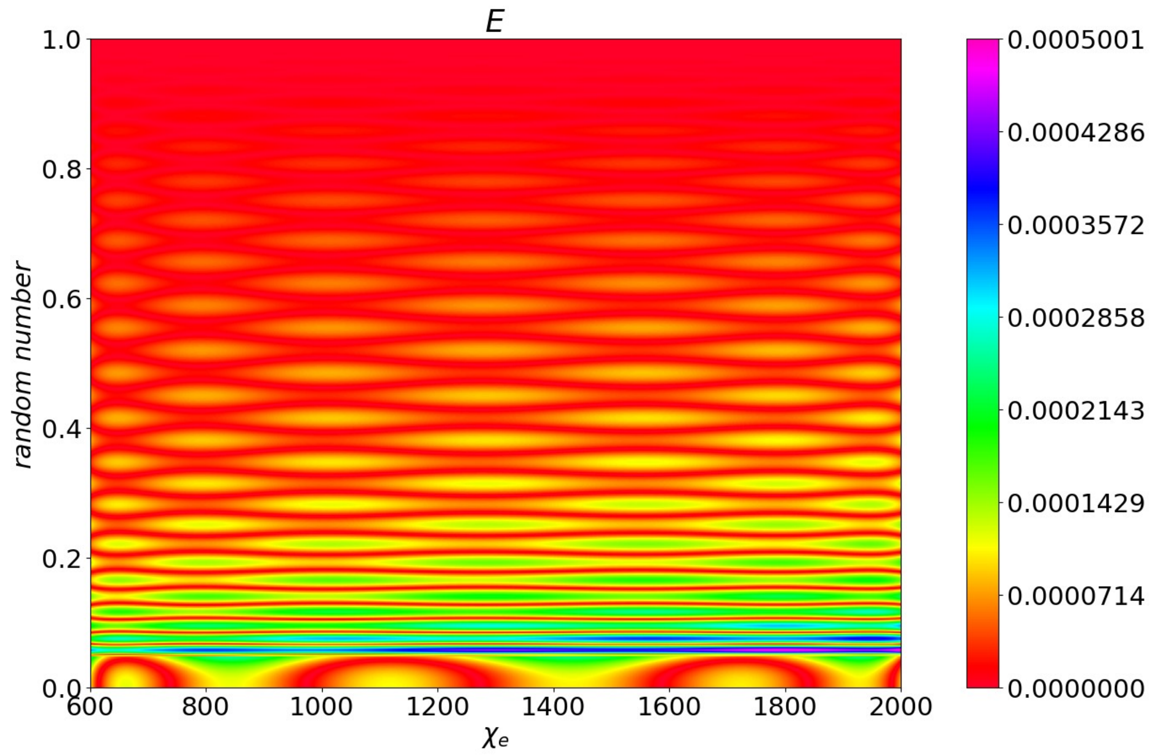
The percentage error $\Delta_r \cdot 100$ is well below 0.1%



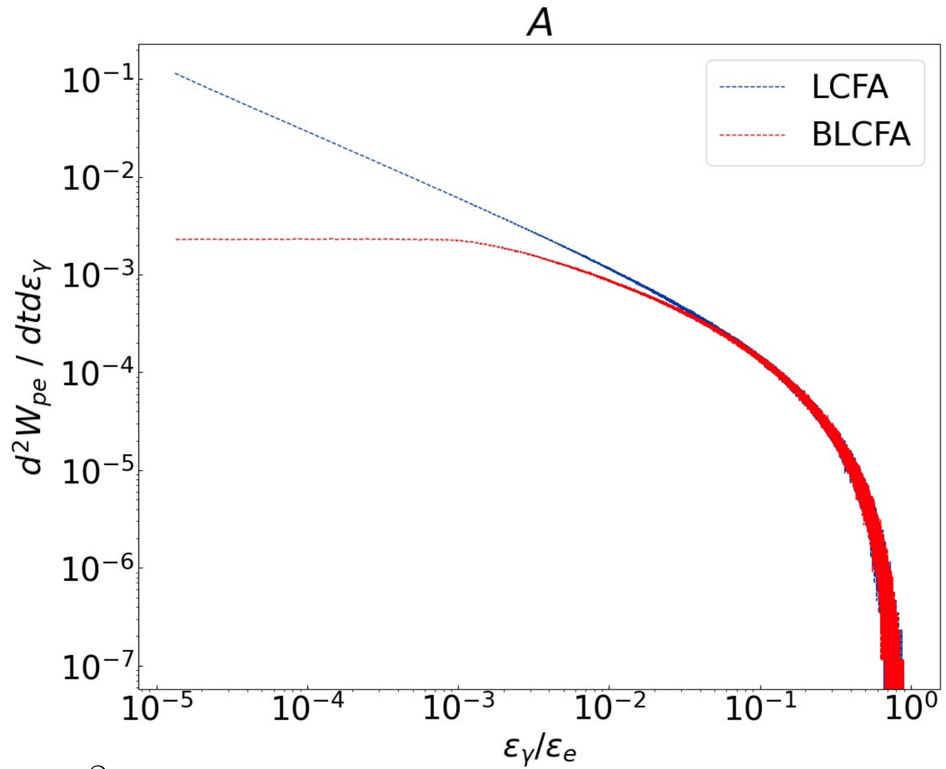


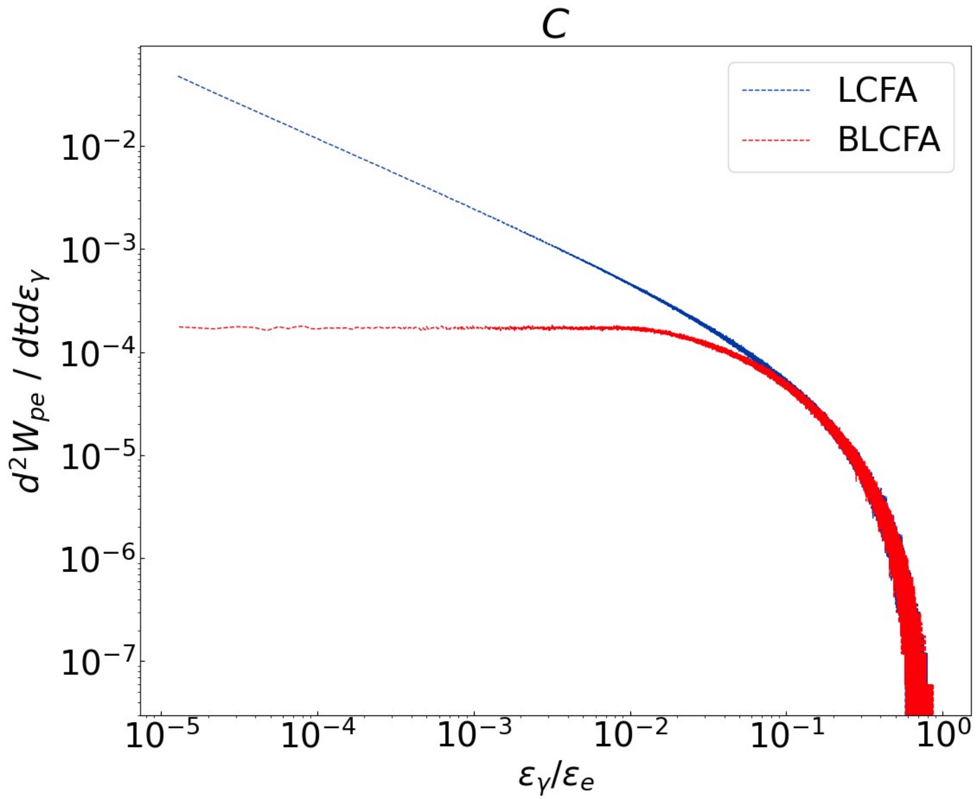
from

Beyond the LCFA

Improved local-constant-field approximation for strong-field QED codes, Di Piazza et al.

Fails at low energies!





10^8 electrons

$E_e = 5GeV$

Plane wave

$a_0 = 8$

$FWHM = 5 \cdot 10^{-15} s$

10^8 electrons

$E_e = 10GeV$

Plane wave

$a_0 = 3$

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Pair production precision

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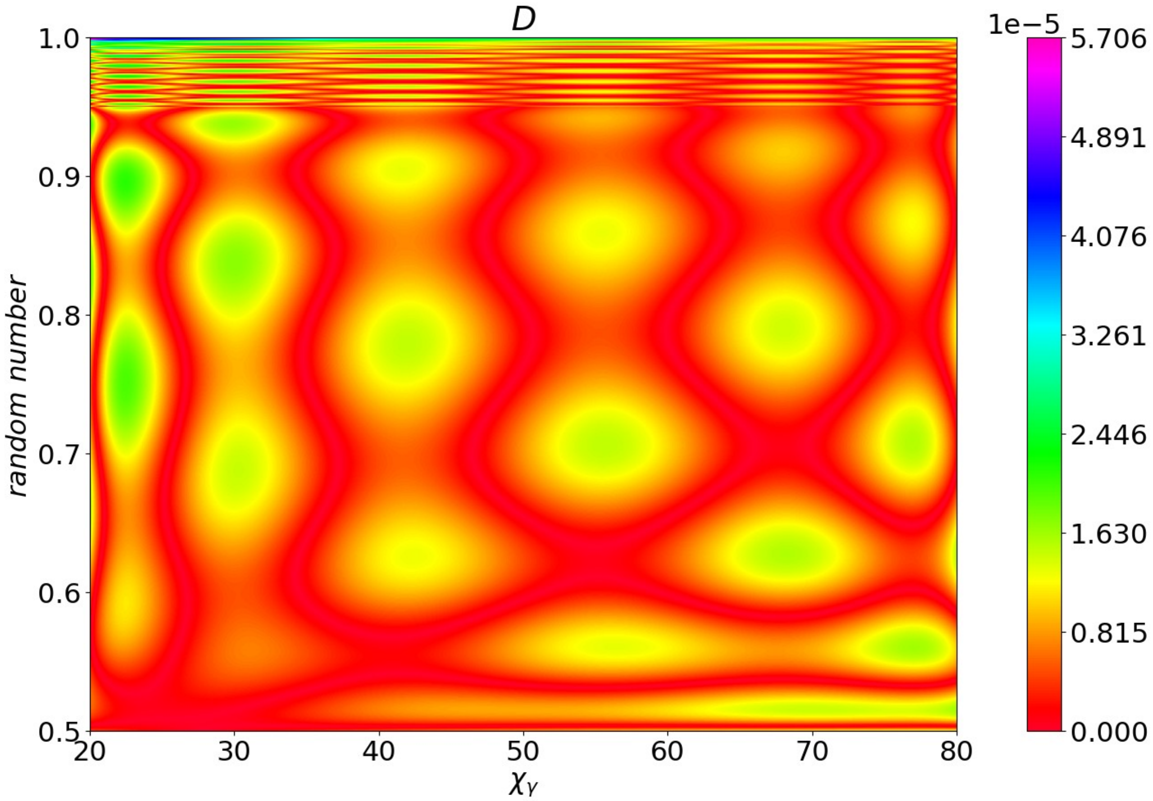

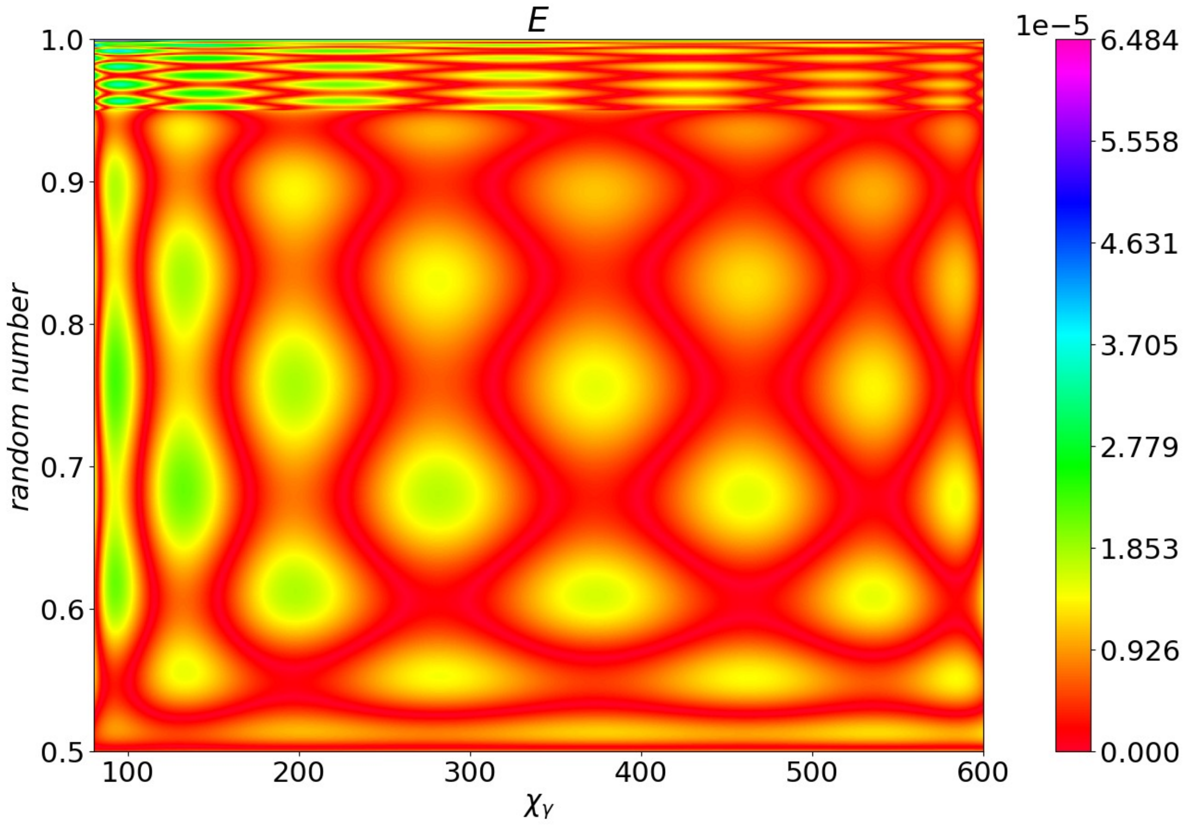

Conclusions

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Outlook

- Will account for the full angular distribution
- Will include spin and polarization effects