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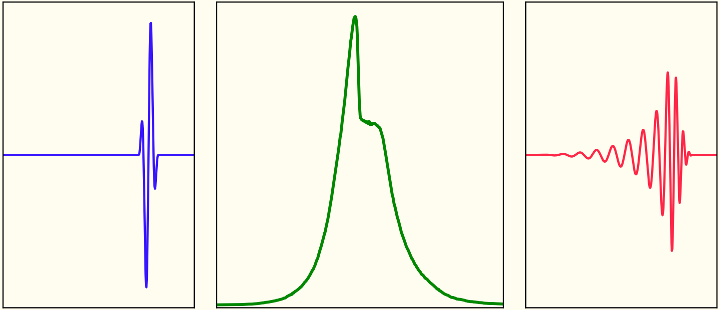

Looking into solid-density plasmas using attosecond dispersion

Andréas Sundström*

In collaboration with Lund Laser Centre

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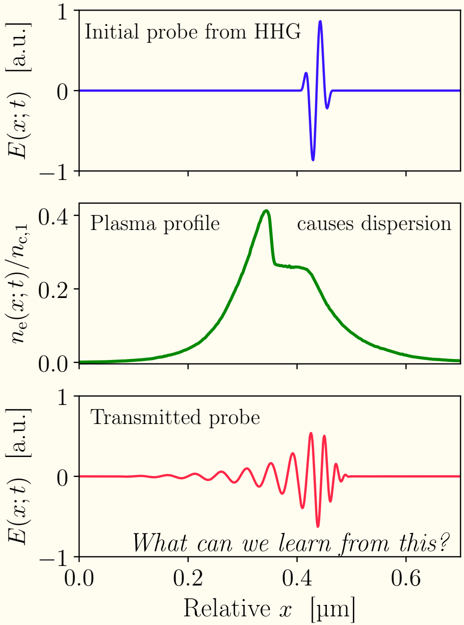
*andsunds@chalmers.se

Knut och Alice Wallenberg Foundation

- High-harmonic generation (HHG) can generate ~300 attosecond, few-cycle XUV pulses.
  - XUV frequencies allow us to probe solid-density plasmas $\lesssim 10^{24}$ cm^{-3}.
- Each frequency component is delayed differently when it passes though the plasmas $\implies$ **dispersed pulse**

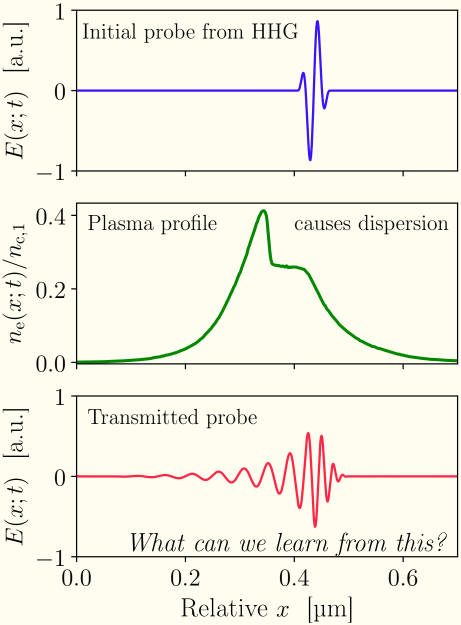
We shall see what information can be extracted from this dispersion!



- Dispersion occurs due to differences in **group velocity** $v_{gr}(\omega) = c\sqrt{1 - \omega_p^2/\omega^2}$
  - $\omega_p^2 = \omega_p^2(x;t) \sim n_e(x;t)$

But how do we measure the disperesed pulse?

- RABBIT[1] or streak-camera[2] methods can measure the delays of each freq.[3].
  - Combine the XUV pulse with an IR field
  - The XUV photons ionize atoms in a gas
  - The phase of the IR field is a timestamp for the ionization-generated electrons



[1]Paul *et al.* Science (2001)
[2]Itatani *et al.* PRL (2002)
[3]López-Martens *et al.* PRL (2005)

In this talk we will see[4]:

- how to generate **synthetic dispersion diagnostics** in solid-density plasmas.
  - **Combining PIC and pseudo-spectral** simulations
- what information can be deduced from the XUV dispersion:
  - **Peak plasma density / shock waves**
  - **Expansion dynamics**
  - **Line-integrated density**

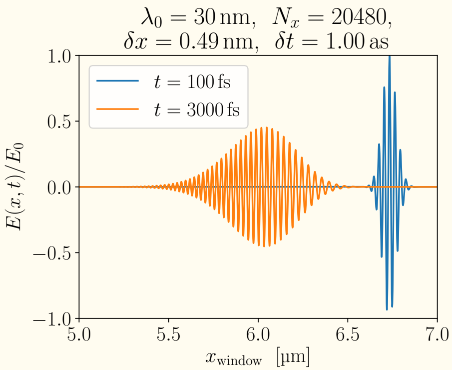
[4] Sundström *et al.* Submitted to JPP (2022)

Numerics

- **Finite difference time-domain** (FDTD) methods suffer from numerical dispersion.
  - PIC codes normally use FDTD solvers, e.g. the Yee mesh.
- The time resolution required for XUV pulses would become **prohibitively expensive** in a full PIC simulation.
- Need alternative free of numerical dispersion.
  - **Spectral solvers!**

We can use a **two-step process**:

1. Simulate the plasma dynamics with PIC
2. Propagate XUV **externally** in **spectral solver**.



Numerical dispersion of a pulse in vacuum after 3000 fs (∼1 mm) propagation, simulated with Smilei.

Algorithm for solving the wave equation for each frequency component.

The full spectral wave equation reads

$$\frac{\mathrm{d}^2 \hat{A}_k}{\mathrm{d}t^2} + c^2 k^2 \hat{A}_k = -\underbrace{\sum_{k'} \hat{\omega}^2_{\mathrm{p},k'} \hat{A}_{(k-k')}}_{\equiv \hat{S}_k}$$

- Wave equation for vector potential $\hat{A}_k$ (Coulomb guage).
- We can evolve $\hat{A}_k$ in time using common methods (e.g. RK45)
- But we get a nasty source term $\hat{S}_k$ that is very expensive

Source term most efficiently evaluated using FFT!

$$\hat{S}_k(t) = \mathrm{FFT}\left[\tilde{\omega}^2_{\mathrm{p}}(x,t) \times \mathrm{FFT}^{-1}[\hat{A}_{k'}(t)]\right]$$

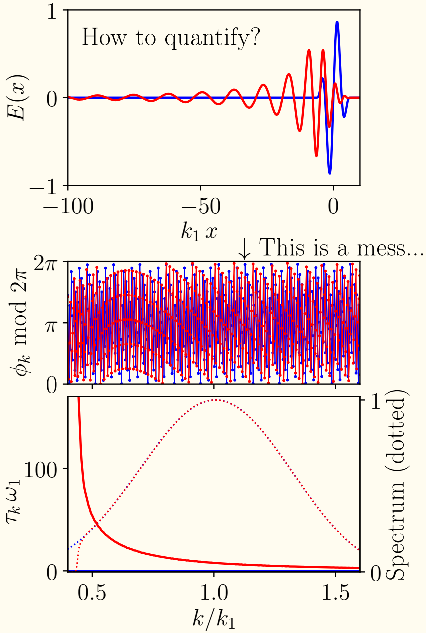
- It's from $\tilde{\omega}^2_{\mathrm{p}}(x,t)$, that the plasma affects the wave.
- This is the **pseudo-spectral** method.

- The phase information is encoded by the complex phase of the spectrum $P_k = \hat{E}_k / |\hat{E}_k| = \exp(\mathrm{i}\phi_k)$.

- But, we only have information about ϕ_k mod 2π.
  - This makes it hard to directly extract usable information about ϕ_k.
  - We need a way to accurately **unwrap** the pahse.

- We can study $-\mathrm{i}\,\Delta P_k / P_k$, from which we get $\Delta\phi_k$.

- **Group delay:** $\tau_k = \dfrac{\partial \phi}{\partial \omega} \approx \dfrac{\Delta \phi_k}{c \Delta k}$.
  - This is also what is measured experimentally.



Simulation results

2D PIC simulations of thin-foil targets

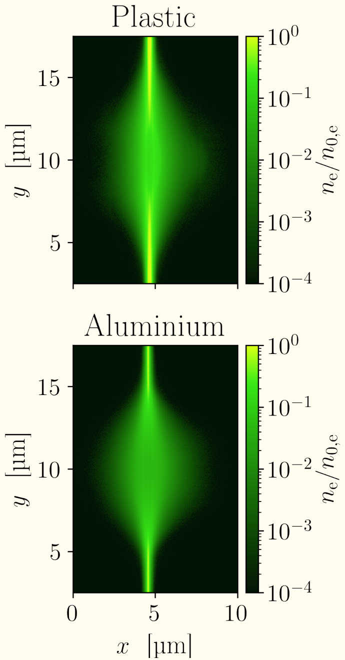
Pump pulse: $\lambda_0 = 800$ nm, $a_0 = 3.0$,
30 fs duration, 6 µm waist Gaussian pulse

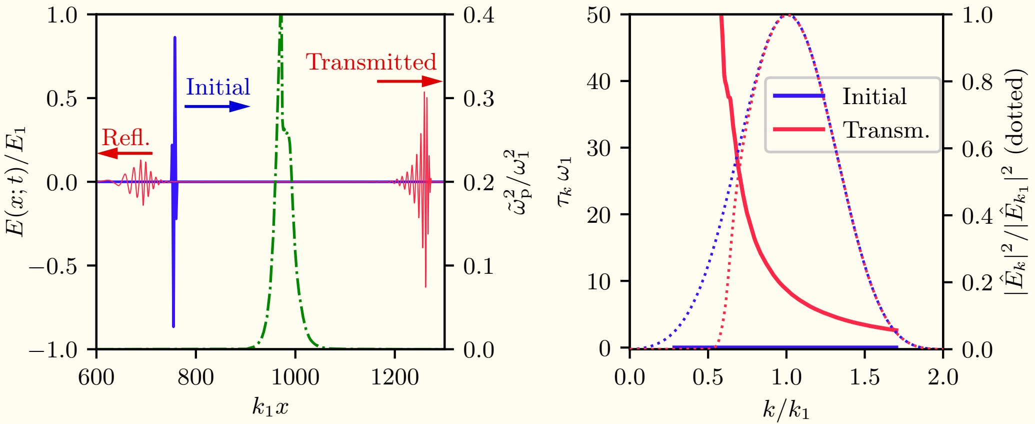
Plastic: 0.25 µm CH_2 foil at solid-density
$n_{e,0} \approx 3 \times 10^{23}$ cm^{-3} (fully ionized)

Aluminium: 0.1 µm Al foil at solid-density
$n_{e,0} \approx 8 \times 10^{23}$ cm^{-3} (fully ionized)

Same initial line-integrated densities.

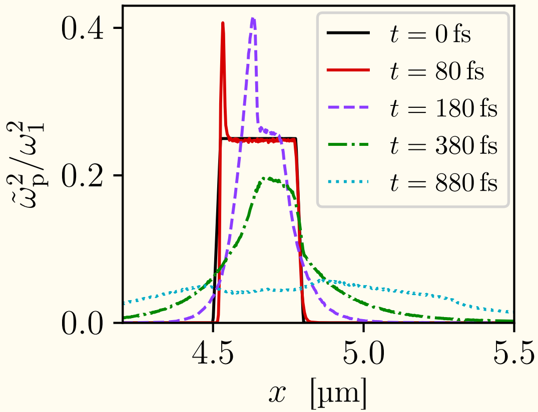
Next, we extract the on-axis plasma profile and calculate the probe-pulse dispersion

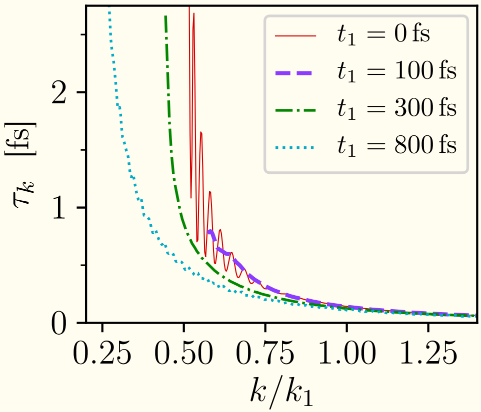




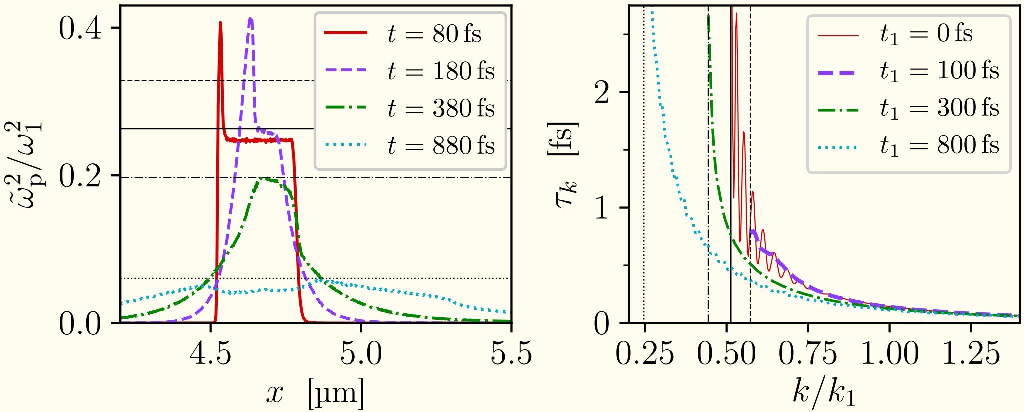
- $\lambda_1 = 30\,\mathrm{nm}$ ($\sim$40 eV), 4 cycle $\cos^2$ pulse, $\sim 10^{13}\,\mathrm{W\,cm^{-2}}$
- On-axis plasma profile (from PIC) is interpolated in t and x.
- We solve the propagation of the XUV pulse using the pseudo-spectral solver*.
- The group delay of the *transmitted* pulse is computed.

*https://github.com/andsunds/PseudoSpectral





- We can redo the procedure for different pump–probe delays t_1.
- Low-frequency cut-off $\Longrightarrow$ "peak density".



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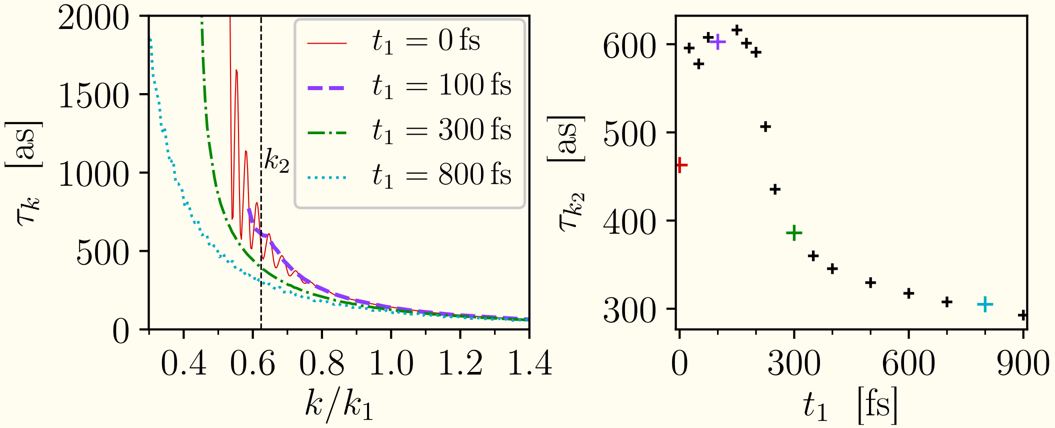
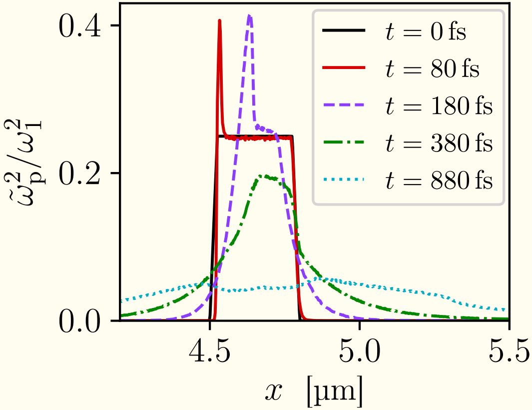
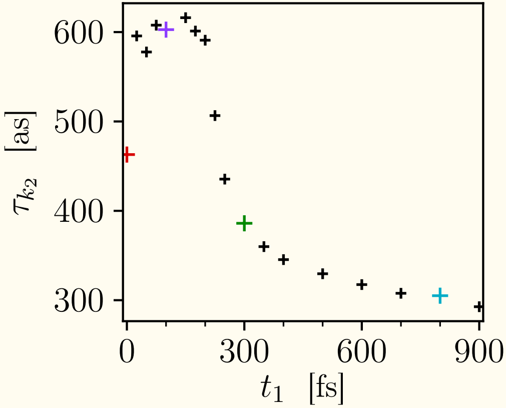


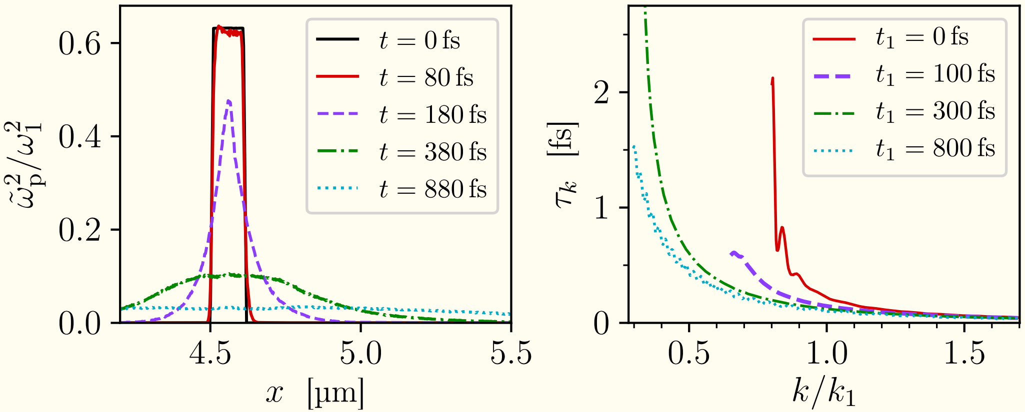
- We can also analyze the group delay τ_{k_2} for different pump–probe delays t_1 at a single frequency $k_2 = 0.625\,k_1$.
- By following τ_{k_2}, we can indirectly follow the evolution of the plasma profile.



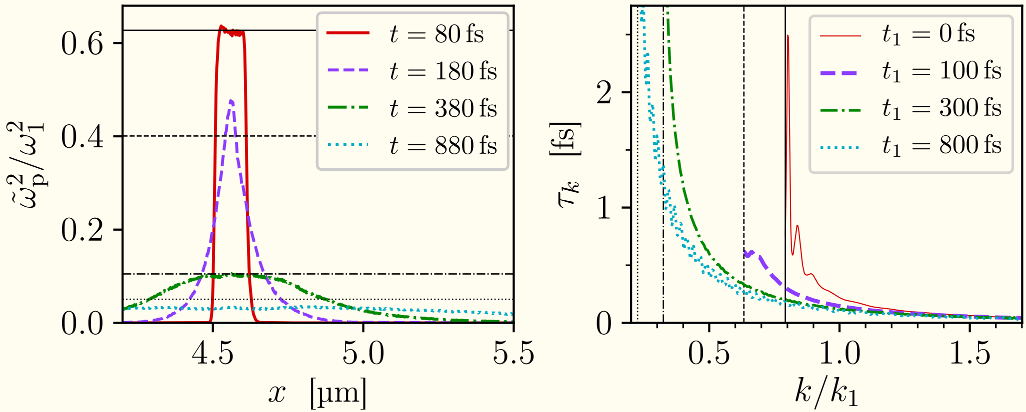


- Initial rise in τ_{k_2} due to laser compression.
  - Broad peak in τ_{k_2} at $t_1 = 50–200$ fs indicate the life-span of the shock/blast wave.
- Following decay of τ_{k_2} is a result of plasma expansion.

Compare aluminium target



- Redo the analysis for **aluminium** target
- Peak density deduced from low-frequency spectral cut-off.
- Study single-frequency group delay τ_{k_3}, at $k_3 = 0.9\,k_1$



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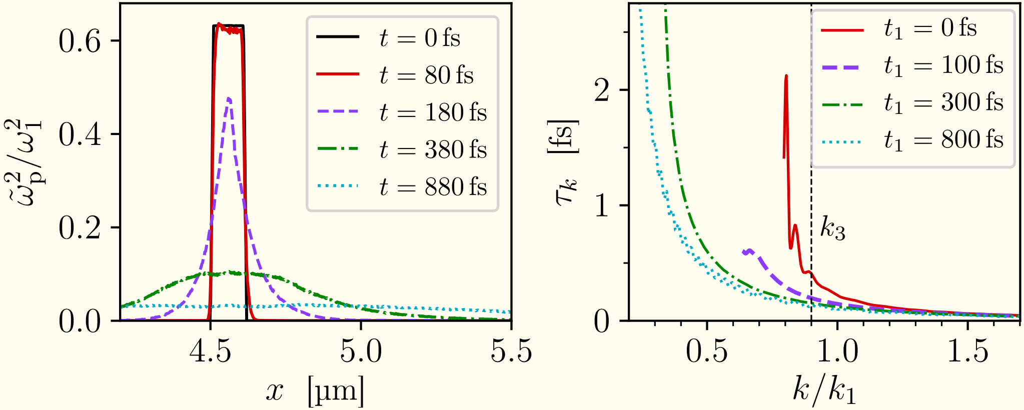

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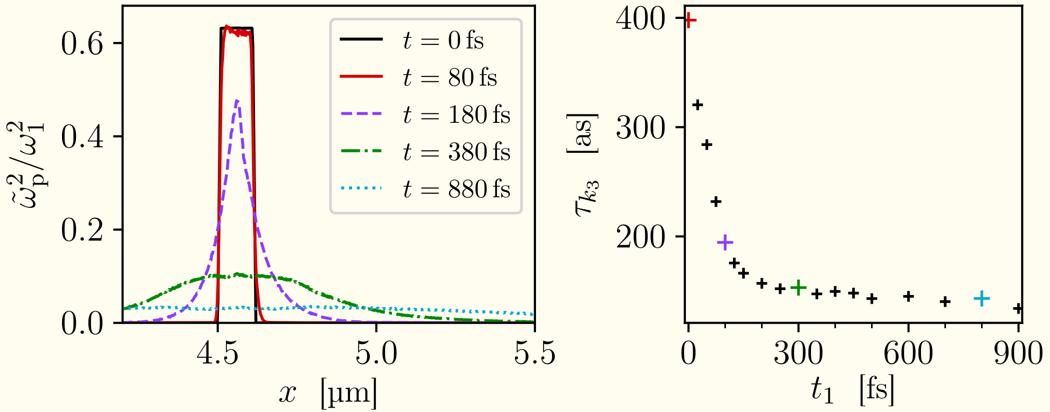
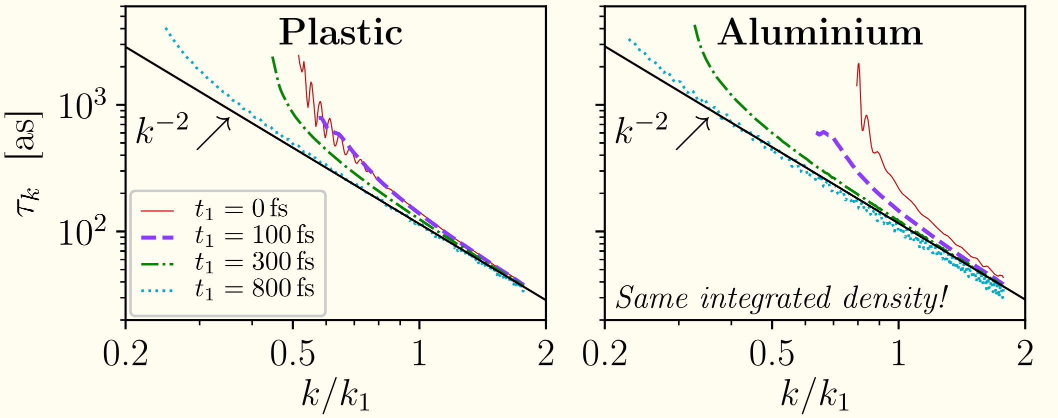


- No initial rise in τ_{k_3} $\Longrightarrow$ Lack of laser compression
- The sharp fall in τ_{k_3} is broken in a knee at $t_1 \sim 150\,\mathrm{fs}$
  - Meeting of expansion fronts, and flattening of plasma profile.

Compare plastic target



- High-frequency asymptote: $\tau_k \propto N_{\text{int}}/k^2$, where N_{int} is line-integrated density
- The two target thicknesses were chosen to give the same N_{int}.
- There is little plasma loss transversally (on-axis) – in this 2D PIC simulation.

Conclusions and outlook

- We have illustrated the possibility to use the **dispersion of attosecond XUV** pulses to **probe solid-density plasmas**.
  - Sundström *et al.* Submitted to JPP (2022) – `arxiv.org/abs/2202.00406`
- We use the **group delay** as our measure of dispersion
  - It can also measured directly in experiments using RABBIT or streak-camera.
- By studying the group delays for different pump–probe delays, we can **infer information** about
  - **Plasma compression** / **peak-density**
  - **Line-integrated density**

Get in touch! `andsunds@chalmers.se`

These are our first steps toward XUV plasma diagnostics. We are also looking into:

- **Spectral interference effects** – multiple pulses
- Studying **rapid ionization** on sub-femtosecond time scales.
- **Polarization spectroscopy** – plasma birefringence, Faraday rotation

Bonus slides

ITATANI, J. *et al.* 2002 "Attosecond streak camera". *Phys. Rev. Lett.* **88**, 173 903, DOI: `10.1103/PhysRevLett.88.173903`

LÓPEZ-MARTENS, R. *et al.* 2005 "Amplitude and phase control of attosecond light pulses". *Phys. Rev. Lett.* **94**, 033 001, DOI: `10.1103/PhysRevLett.94.033001`

MANSCHWETUS, B. *et al.* 2016 "Two-photon double ionization of neon using an intense attosecond pulse train". *Phys. Rev. A* **93**, 061 402, DOI: `10.1103/PhysRevA.93.061402`

PAUL, P. M. *et al.* 2001 "Observation of a train of attosecond pulses from high harmonic generation". *Science* **292** (5522), 1689–1692, DOI: `10.1126/science.1059413`

SUNDSTRÖM, A., PUSZTAI, I., ENG-JOHNSSON, P. & FÜLÖP, T. 2022 "Attosecond dispersion as a diagnostics tool for solid-density laser-generated plasmas". *Submitted to JPP* `https://arxiv.org/abs/2202.00406`

- Plasma dispersion is normally derived for a cold-fluid plasma.
- How do we modify it for a kinetic, relativistic plasma?

Conservation of transverse canonical momentum $\delta p = -e\delta A$.

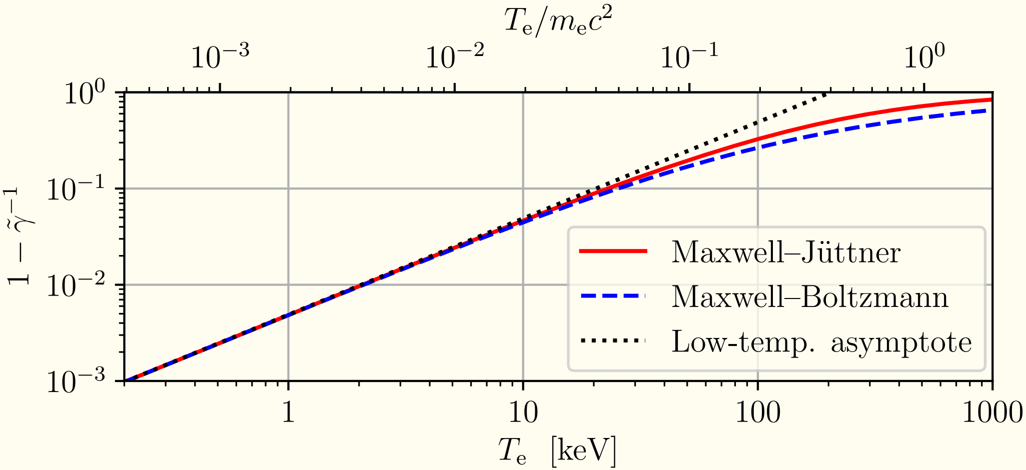
Small-amplitude momentum perturbations:

$$\delta v_y \approx \delta p_y \frac{\mathrm{d}v_y}{\mathrm{d}p_y} = \frac{\delta p_y}{\gamma}\left(1 - \frac{p_y^2}{\gamma^2}\right)$$

$$\tilde{\omega}_\mathrm{p}^2 \equiv \frac{e^2 n_\mathrm{e}}{\epsilon_0 \tilde{\gamma} m_\mathrm{e}}$$

$$\tilde{\gamma}^{-1} = \int \mathrm{d}^3 p \, \frac{\tilde{f}(\boldsymbol{p})}{\gamma}\left(1 - \frac{p_y^2}{\gamma^2}\right).$$

- Same momentum perturbation, but different velocity (current) response.
- Results in a modified (effective) relativistic plasma frequency,
- with an effective gamma factor.



- For the cases considered, $T_e \lesssim 10\,\text{keV}$ means that the effect of the relativistic dispersion is $\lesssim 5\,\%$.
- We therefore mainly **probe the plasma density** with the method presented.

- In the high-wavenumber limit, the plasma variaitons are slow compared to the wavelength.
  - We can therefore use the WKB approximation!

- The group delay can be expressed by integrating the group velocity:

$$\tau(\omega') + \frac{L}{c} \sim \int_0^L \frac{\mathrm{d}x}{v_{\mathrm{gr}}(x;\omega')} = \int_0^L \frac{\mathrm{d}x}{c\sqrt{1-\tilde{\omega}_{\mathrm{p}}^2(x)/\omega'^2}}.$$

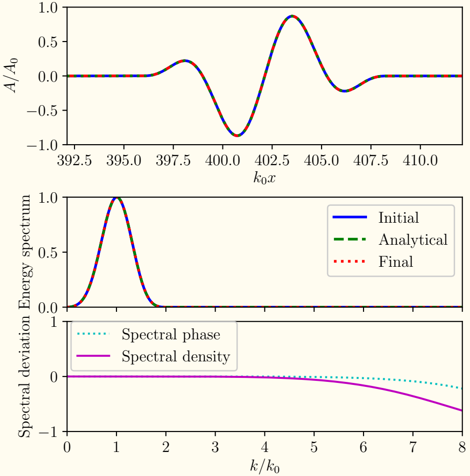
- For ${\omega'}^2 \gg \tilde{\omega}_{\mathrm{p}}^2$, we can approximate

$$\tau(\omega' \gg \tilde{\omega}_{\mathrm{p}}^2) \approx \frac{1}{2c}\int_0^L \frac{\tilde{\omega}_{\mathrm{p}}^2(x)}{\omega'^2}\,\mathrm{d}x \simeq \frac{1}{2c}\left(\frac{\omega_1}{\omega'}\right)^2 \int_0^L \frac{n_{\mathrm{e}}(x)}{n_{\mathrm{c},1}}\,\mathrm{d}x\,.$$

- The solid density at keV temperature will radiate **bremsstrahlung** (BS) in the same spectral band as the probe pulse.
- The probe pulse is **highly directional**: $\lesssim 1$ mrad divergence
- With similar directional discimination of BS, we estimate the detected BS energy to be $\sim$1 nJ, while on-target XUV energy can be as high as 40 nJ[5].
- While challenging, we belive that **the probe signal can be separated from the BS noise.**

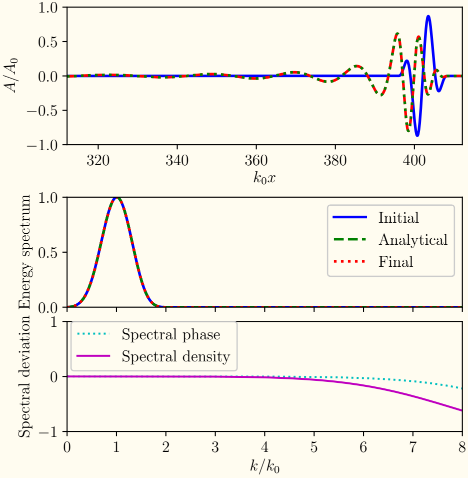
[5]Manschwetus *et al.* PRA (2016)

Benchmark in vacuum



In vacuum we don't expect any dispersion.

- We propagate the pulse for 300 box lengths $= 38000\lambda_0 = 1.1$ mm.
  - $\lambda_0 = 30$ nm
  - $L_{box} = 128\lambda_0 = 3.84\,\mu$m
  - $N = 2048$
  - $\delta x \approx 1.9$ nm
- The propagated pulse is not visibly changed (good!).
- The energy spectrum stays the same.
- Looking at deviations in the spectrum, we see some deviation starting at $k \simeq 4k_0$
  - Well outside pulse bandwidth.



In a homogeneous plasma we can calculate the dispersion analytically.

- We propagate the pulse for 300 box lengths $= 38000\lambda_0 = 1.1$ mm.
  - $\lambda_0 = 30$ nm
  - $L_{\text{box}} = 128\lambda_0 = 3.84\,\mu$m
  - $N = 2048$
  - $\delta x \approx 1.9$ nm
- The propagated pulse is clearly changed, but fits with the analytical form.
- The energy spectrum stays the same (as it should!).
- Looking at deviations in the spectrum, we see some deviation starting at $k \simeq 4k_0$.
  - Same as in vacuum, this is mostly a resolution issue.