IHES : Le Monde Quantique
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THEORY OF DECOHERENCE IN THE ALGEBRAIC FRAMEWORK

≥ 1993 Wrocław R. Olkiewicz P. Ługiewicz
Bielefeld M. Hellmich Ph. B.

- Introducing decoherence
- Unified algebraic framework
- Definition of decoherence for finite and infinite systems
- Models and scenarios of decoherence
- Conclusions

"Unified"

$\mathcal{M}$ = (v. Neumann) Algebra of operators

$\mathcal{Z}(\mathcal{M}) = \mathcal{M} \cap \mathcal{M}'$ Center of $\mathcal{M}$

$\mathcal{M}' = \{A \in \mathcal{M} \mid [A, \mathcal{M}] = 0\}$

Factor : $\mathcal{Z}(\mathcal{M})$ trivial $\Leftrightarrow$ $\mathcal{Z}(\mathcal{M}) = \{\mathbb{C}\,\mathbb{1}\}$

genuine Quantum!

C - system

- $\mathcal{M}$ commutative $\Leftrightarrow$ Classical probability $(\Omega, \mathcal{B}, \mathbb{P})$

Q - system $\Leftrightarrow$ $\mathcal{M}$ non commutative
$\Leftrightarrow$ Q probability $(\mathcal{M}, \varphi)$

S - finite $\mathcal{M}$ factor of type I $\sim \mathcal{L}(\mathcal{H})$
S - infinite $\mathcal{M}$ factor of type II or III

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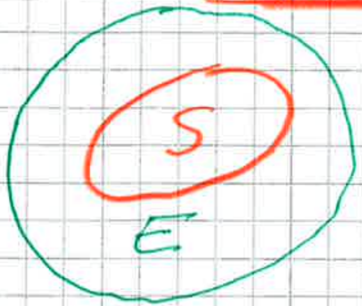
Introducing Decoherence



"good" experience:
Maximum information on S with minimum of environment noise

Idealization: closed (isolated) systems

Q-World: Decoherence emphasizes the necessity of considering S as "OPEN"

- Unavoidable interaction of virtually all S with E ⇒ Entanglement
- Unitary global dynamics of S×E ⇒ irreversibly increasing integlement
- Phase relations are preserved globally but are no longer locally accessible
  ⇒ apparent violation of the Q-superposition principle
  ⇒ Entanglement fundamentally alters what we may observe at the level of S
- E as defining the observable properties of S

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Interaction $S \times E$

1) Dissipation (Energy exchange $S \leftrightarrow E$)
2) Fluctuations (acting like "noises")
3) Decoherence (purely Quantum)

Some aspects of decoherence

1) Destruction of macroscopic interferences
2) Selection of privileged states : Pointer states ~ " events "
3) Pointer states evolve classically
4) Coupling $S \times E$ defines the observable properties of S
5) Q-coherence is delocalized in the non observable state of $S \times E$

Questions about Decoherence

1) Observing experimentally decoherence
2) Finding a reliable theory
3) Evaluating order of magnitudes : time scales, ...
4) Making precise the conditions in which

$$\rho_{ij}(t) \xrightarrow[t \to \infty]{} 0$$

5) Why the importance of interactions $S \times E$ had been overlooked for so long?

"historical accident" ? (Joos)

- Physics is about idealized closed systems
  ~ deeply incrusted idea
- In C world states are local ⇒ Justification of the closed system idealization
  E perturbs S but not alters the "nature" of S
- In QT still a "local theory" but states generated by local interactions generate non local Q-correlations changing what we may observe at the level of S
- Standard QM formalism applied to the closed system $S \times E$ ⇒ Decoherence
  ⇒ Decoherence must be taken into account

Very brief "historical" surway

- 1935 : Schrodinger's Cat "Gedanken" cat experiment
  Cat ⇔ Poison ⇔ Unstable atom via the hammer
  Observer only required to "open" the "closed system
- H. D Zeh (1970)
- K. Hepp (1972)
- W. Zurek (1981)
- E. Joos H. Zeh (1985)
- Zurek's "Physics Today" article (1991)
- ENS experiment (1996)
- R. Omnés (1997) ... and many others ...

Books

- B. Deplantier J. M. Raymond V. Rivasseau
  Editors
  Quantum Decoherence
  Poincaré Seminar (2005) PMP 48 Birkhäuser
  (2007)
- M. Schlosshauer
  Decoherence and the Q-to-C Transition
  Springer (2008)
- R. Omnès
  Understanding QM
  Princeton University Press

Physics = Real part (Experiments)
∪ Imaginary part (Theory)

Theory = Physical phenomena ∪ Formalism
∪ Interpretation

Inventory of a theory

$$(\mathcal{M}, \Sigma, \mathcal{P}_{\mathbb{R}})$$

↑ Observables ↑ States

$$\mathcal{P}_{\mathbb{R}} = \{ \mathbb{P} : \mathcal{M} \times \Sigma \to \text{probability measure on } \mathbb{R} \}$$

- Additional structure
  - Σ is convex

$$\varphi_1, \varphi_2 \in \Sigma \Rightarrow \lambda_1 \varphi_1 + \lambda_2 \varphi_2 \in \Sigma$$
$$\lambda_i \geq 0 \qquad \lambda_1 + \lambda_2 = 1$$

  - $\mathcal{M}$ is an algebra + Topology

Formal structure of standard QM

mathematically precise rendering of the formalism of Born, Heisenberg, Jordan, Dirac .. codified by v. Neumann

Questions to be addressed

- Meaning independ of "observers" ?
- "Intrinsic randomness" ?
- Understanding probabilistic phenomena in QM?
- Information loss

2) An Unified Algebraic Framework

valid just as well for C-Physics and Q-Physics

SQM incredibly succesfull :

" no experimental fact, not a single one, that contradicts a quantum theoretical prediction "

Englert (2013)

⇒ Justification FAPP (for all practical purpose) of the Q-Rules starting from v. Neumann's epistemic principle

⇕

only one kind of physical laws given by the Q-principles

Algebraic framework :

Heisenberg, Jordan, v. Neumann, Haag, Kastler, Araki .....

Physical system $(\mathcal{A}, \varphi, \alpha_t)$

state: φ; observable: $\mathcal{A}$; automorphism (dynamics): α_t

States $\varphi : \mathcal{A} \to \mathbb{C}$

φ linear, normalized $\varphi(\mathbb{1}) = 1$,

positive $\varphi(A^*A) \geq 0$

$\Rightarrow \varphi$ is continuous

Connections to experiments

Experiments more and more sophisticated

History, Non-demolition experiments, ...

$A = A^*$ $\varphi(A) \in \mathbb{R}$ To be computed

$\mathrm{Prob}\,[A, \varphi \mid \varphi(A) \in I]$ $\mathrm{Prob}\,[\mathrm{History}] = \mathrm{Prob}\,[A(t_1) \cdots A(t_n)]$

$I \subset \mathbb{R}$ $t_1 < t_2 < \cdots < t_n$

- C^* – algebra $\mathcal{A}$
  - Complex Banach Algebra such that

$$\| A^* A \| = \| A \|^2 \quad \text{Gelfand condition}$$

$\mathcal{A}$ commutative $\Rightarrow$ $\mathcal{A} = C(K)$ K compact

$$\| f \| = \sup_x | f(x) |$$

$\mathcal{A}$ non-commutative $\mathcal{A} = \mathcal{B}(\mathcal{H})$

φ state on $\mathcal{A}$

$$\langle \, , \, \rangle_\varphi : \langle A, B \rangle_\varphi = : \varphi(A^* B) \to \mathcal{H}_\varphi$$

GNS: Representation $\Pi_\varphi(\mathcal{A})$ in $\mathcal{B}(\mathcal{H}_\varphi)$

- Von Neumann algebra $\mathcal{M}$

$\mathcal{M} \subset \mathcal{B}(\mathcal{H})$ such that $\mathcal{M} = \mathcal{M}''$

$$\mathcal{M}' = \{ A \in \mathcal{M} \mid [A, B] = 0 \ \forall B \in \mathcal{M} \}$$

$\Leftrightarrow$ $\mathcal{M}$ is weakly closed "physical topology"!

Algebraic Probability Theory

$$(\mathcal{M}, \varphi)$$

$\uparrow$

- von Neumann algebra
- φ faithful normal state $\approx$ non-commutative measure

$$\Leftrightarrow \begin{cases} \varphi(\sum P_i) = \sum \varphi(P_i) & P_i \perp P_j \ \forall i \neq j \\ \varphi(A) = \mathrm{tr}(\rho A) \end{cases}$$

- $\mathcal{M}$ commutative $\Leftrightarrow$ $\mathcal{M} = L^\infty(\Omega)$

Kolmogorov probability theory (1932)

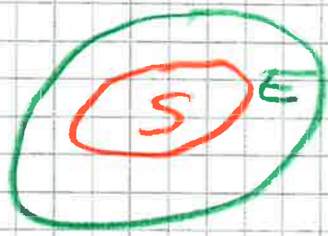
$\mathcal{M}$ non-commutative "Quantum Probability theory"

The two logics of science (Maxwell)!

3) Decoherence

Emergent phenomenon :

- multiscale
- interaction $S \times E$
- time



$\uparrow$ Environment

- The total system $S \vee E$ is a closed quantum system in Hilbert Space $\mathcal{H}_{S+E} = \mathcal{H}_S \otimes \mathcal{H}_E$
- Observables $\Rightarrow$ v. Neumann algebra $\mathcal{N}$

$$\mathcal{N} = \mathcal{M} \otimes \mathcal{M}_E$$

- Dynamics

$$\alpha_t : \mathcal{N} \to \mathcal{N}$$

*automorphism $\Updownarrow$ α_t unitary

- Reference state of E
- Embedding $i : \mathcal{M} \to \mathcal{N}$

$$i(A) = A \otimes \mathbb{1}_E$$

- Conditional expectation $E_{\omega(E)}$

$E_{\omega_E}(A \otimes B) = A \otimes E_{\omega_E}(B)$

$$E_{\omega_E} : \mathcal{N} \to \mathcal{M}$$

- Reduced dynamics $T_t : \mathcal{M} \to \mathcal{M}$

$$\boxed{T_t = E_{\omega_E} \otimes \alpha_t \otimes i}$$

$\varrho \otimes \omega_E$ state on $\mathcal{N}$

$$T_t(\varrho) = (E_{\omega_E} \circ \alpha_t \circ i)(\varrho \otimes \omega_E)$$

$$\begin{tikzcd}
\mathcal{N} \arrow[r, "\alpha_t \circ i"] & \mathcal{N} \arrow[d, "E_{\omega_E}"] \\
\mathcal{M} \arrow[u, "i"] \arrow[r, "T_t"] & \mathcal{M}
\end{tikzcd}$$

Theorem

T_t is completely positive as combination of three CP-maps.

$$T_t(A)\varrho = \mathrm{tr}_{\mathcal{H}_E}\left[e^{itH}(A \otimes 1\!\!1_E)e^{-itH}(\varrho \otimes \omega_E)\right]$$

Partial trace ~ classical "integrating out" Marginals

$T : \mathcal{M} \to \mathcal{M}$ CP map

if

$$1\!\!1_n \otimes \mathcal{M} \to M_n(\mathbb{C}) \otimes \mathcal{M}$$

for $n = 2, 3, 4, \dots$

is positive.

Mathematical definition of Decoherence

$\mathcal{M}$ observables of S

- B.O Reviews in Math. Phys. 15 (2003) 217–243
- Mario Hellmich thesis

$\exists$ splitting $\mathcal{M} = \mathcal{M}_1 \oplus \mathcal{M}_2$

$\mathcal{M}_1$ v. Neumann subalgebra of $\mathcal{M}$

$A = A_1 \oplus A_2$

$\mathcal{M}_1 \approx$ effective observables, decoherence-free part

$\beta_t = \alpha_t \restriction_{\mathcal{M}_1}$ is an automorphism

$\mathcal{M}_2 \Leftrightarrow \lim_{t\to\infty} \varphi[\alpha_t(A_2)] = 0$

not detectable by measurements for $t \to \infty$

$\lim_{t\to\infty} \varphi[\alpha_t(A_2)] = 0$ Borel principle

$\mathcal{M}_2 \equiv 0$ FAPP

- Automorphism $\alpha_t : \mathcal{M} \otimes \mathcal{M}_E$

$\downarrow$ E_{ω_E} : conditional expectation averaging over $\mathcal{M}_E$

- CP-map $T_t : \mathcal{M} \longrightarrow \mathcal{M}$

$\downarrow$ $\lim t \to \infty$ asymptotic decoherence (Hepp...)

- Automorphism FAPP $\beta_t = T_t \restriction_{\mathcal{M}_1}$

DECOHERENCE IS NOTHING ELSE AS THE STRICT APPLICATION OF V. NEUMANN RULES

Remark In realistic model T_t not Markov

Try to perform a "Markovian limit" $\Rightarrow$ a kind of Markov process

Physically E has no memory

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System - environment models

$$\mathcal{N} = \mathcal{M} \otimes \mathcal{M}_E \quad \text{acting on}$$

$$\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_E$$

α_t total time evolution hamiltonian

$$\alpha_t = e^{itH} A e^{-itH}$$

$$H = H_1 \otimes \mathbb{1}_E + \mathbb{1}_S \otimes H_2 + H_I$$

coupling $S \times E$

E_{ω_E} Conditional expectation w.r.t. reference state ω_E of E

Scenarios of decoherence

$\lim_{t\to\infty} \mathcal{M}_1 \sim$ attractors of the dynamics

$$\mathcal{Z}(\mathcal{M}) = \mathcal{M} \cap \mathcal{M}' = \mathbb{C}\mathbb{1}$$

$\Leftrightarrow$ The system S is a genuine (purely) Q-System

- **Pointer states**

$\mathcal{M}_1$ commutative $\simeq L^\infty(\mathbb{R}), \ell^\infty$

$\{\beta_t\}_{t\in\mathbb{R}}$ trivial $\beta_t = 1$

Emergence of a pointer basis of the system selected by the stability criterion

Physical quantities most easily "read off"

$p_n = p_n^* = p_n^2$

$\mathcal{M}_1 = \bigoplus p_n \mathcal{M}$

$$\left.\begin{array}{l} L^\infty \simeq \text{continuous} \\ \ell^\infty \simeq \text{discrete} \end{array}\right\} \text{Pointer states}$$

- **SSR SuperSelection Rules**

$\mathcal{M}_1$ non commutative

$\mathcal{Z}(\mathcal{M}_1) \neq \mathbb{C}\mathbb{1}$ Superselection observables

$$\left.\begin{array}{l} \mathcal{M}_1 = \bigoplus_i \mathcal{B}_i \\ \mathcal{H} = \bigoplus_i \mathcal{H}_i \end{array}\right\} \text{acting on}$$

$\psi_i \in \mathcal{H}_i \qquad \psi_j \in \mathcal{H}_j$

$$\langle \psi_i, x\, \psi_j \rangle = 0 \quad \forall x$$

- **New genuine Quantum system**

$$\left.\begin{array}{l} \mathcal{M}_1 \text{ non commutative} \\ \mathcal{Z}(\mathcal{M}_1) = \mathbb{C}\mathbb{1} \end{array}\right\} \text{Q-computing!}$$

- **Classical system**

  $\mathcal{M}_1$ commutative $\sim$ L^∞, l^∞

Emergence via decoherence of the classical world

Q and C conceptions of the world being almost opposite in every respect

BUT C (our daily experience)

projected shadow in a Q-universe

$\sim$ Plato's cave

Q-dyn. system $\underset{\text{Decoherence}}{\Longrightarrow}$ FAPP C-dyn. system

- **Ergodicity**

  $\mathcal{M}_1 = \mathbb{C}\,\mathbb{1}$ trivial

- **Lüders - v. Neumann Reduction Formula**

  $A = A^* \in \mathcal{M}$ $\quad \sigma(A) = \{a_1, \dots a_n\}$

  Spectral projector $\{q_i\}_{1 \le i \le n}$

  State ρ

  Apparatus + Decoherence $\Rightarrow$ SSR

  Pointer Observable $X \Rightarrow \sigma(X) = \{x_1 \dots x_n\}$ of apparatus $\quad x_i \Leftrightarrow a_i$

  Measurement

  1) Interaction S x Apparatus

  $\rho = \sum \lambda_i q_i \quad \lambda_i \geq 0 \quad \sum \lambda_i = 1$

  $\rho \to T(\rho)$ CP map

  2) Reading by the observer $\quad \boxed{\dfrac{\sum_i q_i \rho q_i}{tr(\sum q_i \rho q_i)}}$

MEASUREMENTS

1) System Algebra $\mathcal{M}_S$

Observable $A = A^* \in \mathcal{M}_S$

$\sigma(A) = \{a_1, a_2, \dots a_n\}$

$A\psi_i = a_i \psi_i$ $\quad \psi_i$ non degenerate

State $\rho = \rho^* \geq 0$ $\quad \operatorname{tr} \rho = 1$

2) Measuring apparatus Algebra $\mathcal{M}_A$

$\Downarrow$

macroscopic, open system subject to decoherence

Pointer $\Rightarrow$ $X = X^* \in \mathcal{M}_A$

$\sigma(X) = \{x_1, x_2, \dots x_n\}$

$x_i \Leftrightarrow a_i$

Decoherence $\Rightarrow$ SSR between pointer positions

$$\mathcal{M}_{S,1} = \bigoplus_{i=1}^{n} p_i \mathcal{M}_S$$

$\{\alpha_t\}_{t \in \mathbb{R}}$ trivial on $\mathcal{M}_{S,1}$

Measurement proceeds in two steps

a) Interaction S – A

$$\rho = \sum_i \lambda_i P_{\psi_i} \qquad \lambda_i \geq 0 \quad \sum \lambda_i = 1$$

Coupling $\Leftrightarrow$ CCP map T on

$$\mathcal{M} = \mathcal{M}_S \otimes \mathcal{M}_A$$

$$\rho \to T_*(\rho)$$

If $\quad T = \sum_{i=1}^{n} P_{\psi_i} X P_{\psi_i}$

T ideal measurement (quantum non demolition measurement)

b) Reading of the dial

- Gain of information : x_i (corresponding to a_i)

  $\Rightarrow$ Condition probability distribution $\{\lambda_j\}_{1 \leq j \leq n}$ according to the observation

  $\Rightarrow$ distribution δ_{x_i}

  $\Rightarrow$ System in state P_{ψ_i}

- Only partial information

  Event $E \subsetneq \{x_1, \dots x_n\}$

  State given by

$$\frac{\displaystyle\sum_{i,\, x_i \in E} P_{\psi_i} \rho P_{\psi_i}}{\mathrm{tr} \displaystyle\sum_{i,\, x_i \in E} P_{\psi_i} \rho P_{\psi_i}}$$

$\Rightarrow$ Lüders – von Neumann reduction formula

Sufficient Conditions for Decoherence

Concentrate on the Markovian case

$\Longleftrightarrow$ T_t reduced dynamics on $\mathcal{M}$

$$T_t = e^{tZ}$$

Lindblad + Kraus

$$ZA = i[H, A] - \frac{1}{2}\{\Phi(\mathbb{1}), A\} + \Phi(A)$$

$$\Phi(A) = \sum_j \tilde{A}_j^* A \tilde{A}_j$$

$$\tilde{A}_j \in \mathcal{B}(\mathcal{H})$$

$\{T_t\}_{t\geq 0}$ is uniformly continuous

+ Tomita-Takesaki modular group

$$\{\sigma_t^\omega\}_{t\in\mathbb{R}}$$

Theorem (M. Hellmich)

$\{T_t\}_t$ MSG on $\mathcal{M}$ such that

1) $\omega \circ T_t \leq \omega \quad \forall t \geq 0$ $\quad \omega$ normal faithful state on $\mathcal{M}$

2) $[\sigma_s^\omega, T_t] = 0 \quad \forall s \in \mathbb{R} \quad \forall t \geq 0$

Then $\{T_t\}$ displays decoherence $\mathcal{M} = \mathcal{M}_1 \oplus \mathcal{M}_2$

$\exists$ conditional expectation $Q : \mathcal{M} \to \mathcal{M}_1$

$$\omega \circ Q = \omega$$

$$\mathcal{M}_1 = \{A \in \mathcal{M} : T_t(AB) = T_t(A)\,T_t(B) \ \forall B \in \mathcal{M}\ t\geq 0\}$$
$$\cap \{A \in \mathcal{M} : T_t(BA) = T_t(B)\,T_t(A) \ \forall B \in \mathcal{M}\ t \geq 0\}$$

The Araki–Zurek model

$$H = H_S \otimes \mathbb{1}_E + \mathbb{1}_S \otimes H_E + A \otimes B \quad \text{on } \mathcal{H}_S \otimes \mathcal{H}_E$$

$$\mathcal{H}_E = L^2(\mathbb{R}, da)$$

- $[H_E, B] = 0$
- $B = p$
- $A = \sum \lambda_n P_n \quad \lambda_n \in \mathbb{R} \quad \lambda_n \neq \lambda_m \quad P_n$ one dimensional projections mutually orthogonal
- $[H_S, A] = 0$ i.e $H_S = \sum \varepsilon_n P_n \quad \varepsilon_n \in \mathbb{R}$
- $\omega_E = \psi_E(a) = \frac{1}{\sqrt{2\pi}} \int \frac{e^{iap}}{\sqrt{\pi(1+p^2)}} dp$

$X \in \mathcal{B}(\mathcal{H}_S)$

$$T_t(X) = \sum_{n,m=1}^{\infty} \chi_{n,m}(t)\, e^{it(\varepsilon_n - \varepsilon_m)} \cdot P_n X P_m$$

$$\chi_{n,m}(t) = \frac{1}{\pi} \int e^{it(\lambda_n - \lambda_m)p} \frac{dp}{1+p^2} = e^{-|\lambda_n - \lambda_m| t}$$

$$\chi_{n,m}(t)\, \chi_{n,m}(s) = \chi_{n,m}(t+s)$$

$\Rightarrow T_t$ is a Q-Markov Semigroup

$\{b_n\}_{n \in \mathbb{N}} \quad b_n > 0 \quad \sum_n \lambda_b = 1$

$\rho_0 = \sum b_n P_n \quad T_t$ invariant density matrix

$B \in \mathcal{M}_2 \quad \mathbb{E}(T_t B) \xrightarrow[t \to \infty]{} 0$

$$\boxed{\mathcal{M}_1} = \sum_{n=1}^{\infty} P_n \mathcal{B}(\mathcal{H}_S) P_n = \boxed{\ell^\infty(\mathbb{N})} \qquad T_t\big|_{\mathcal{M}_1} = \mathbb{1}$$

Discrete Pointer states

Infinite Systems of spin - $\frac{1}{2}$

$\mathcal{A}_n = M_{2^n \times 2^n}(\mathbb{C}) \iff n$ - spin $\frac{1}{2}$ observables

$\mathcal{A}_n \subset \mathcal{A}_{n+1}$

$$\mathcal{A} = \overline{\bigcup_{n=0}^{\infty} \mathcal{A}_n}^{\|\ \|} \qquad C^*\text{- Algebra}$$

$\|\ \|\restriction_{\mathcal{A}_n} = \|\ \|_n =$ bigest eigenvalue of $x^* x$, $x \in \mathcal{A}_n$

$\mathcal{M} = \overline{\pi_{\omega_0}(\mathcal{A})}''$ is a factor of type II_1

Different models are possible

- $\mathcal{M}$ coupled to a particle

  $\mathcal{M}_1 = L^\infty$

  Uniform Newtonian motion on $\to$ the Circle S_1 / on $\mathbb{R}^3$

- $\mathcal{M}$ coupled to a Phonon Bath

  – $\mathcal{M}_1 = \{ \mathbb{C}\mathbb{1} \}$ Ergodic system

  – Non trivial effective algebra $\mathcal{M}_1$ with non trivial effective dynamics

$\Rightarrow$ From Q — to q

<u>$\mathcal{M}$ and $\mathcal{M}_1$ are both factors</u>

$\Updownarrow$

Genuine Quantum Systems

CONCLUSIONS

Quantum theory makes manifest aspects

Decoherence Scenarios
- Superselection rules
- Discrete and continuous pointer states
- Ergodic properties
- Reduction of degrees of freedom

- Q-probility theory $(\mathcal{M}, \varphi)$ cannot be imbedded in classical probability theory as soon as $\dim \mathcal{H} \geq 3$
- " intrinsich fundamental randomness " resulting from coupling $S \times E$

  CP-maps for states $\Rightarrow$ Stochastic dynamics for individual paths

  ~ Probabitiscic phenomena in Q-World

  q-jumps , particles tracks ...
- The classical notions making QT meaningful – " events ", " histories " ... – arise by decoherence and are " idealizations " applying FAPP

C. STANLEY OGILVY

For All Practical Purposes

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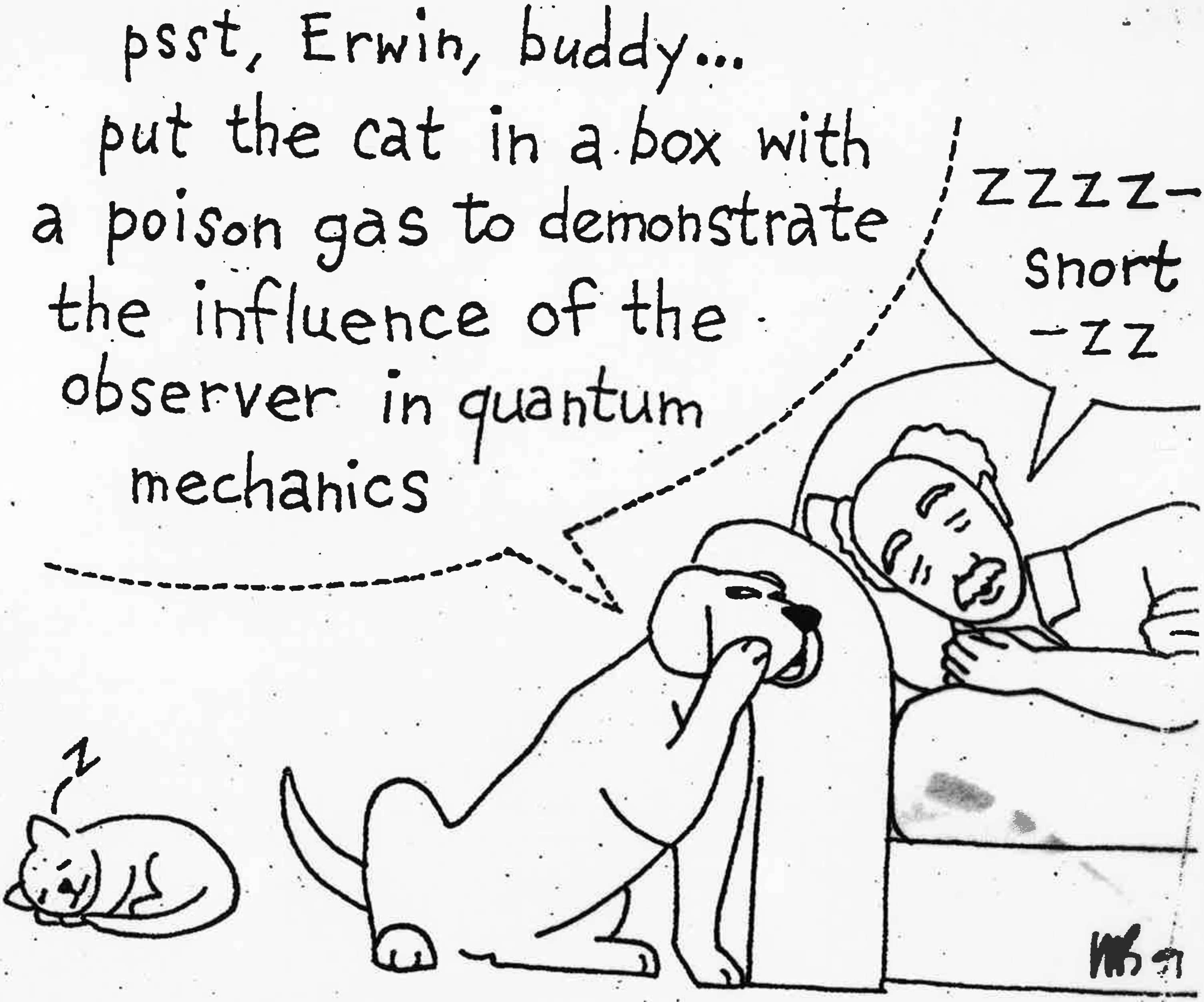
+

A PROFESSOR, asked what he meant by the phrase ["for all practical purposes"] explained:

"Suppose all the young men in this class were to line up on one side of the room, and all the young ladies on the other. At a given signal, the two lines move toward each other, halving the distance between them. At a second signal, they move forward again, halving the remaining distance; and so on at each succeeding signal. Theoretically, the boys would never reach the girls; but actually, after a relatively small number of moves, they would be close enough for all practical purposes."

[FROM *Through the Mathescope*, © 1956, C. STANLEY OGILVY, OXFORD UNIVERSITY PRESS, LONDON]



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