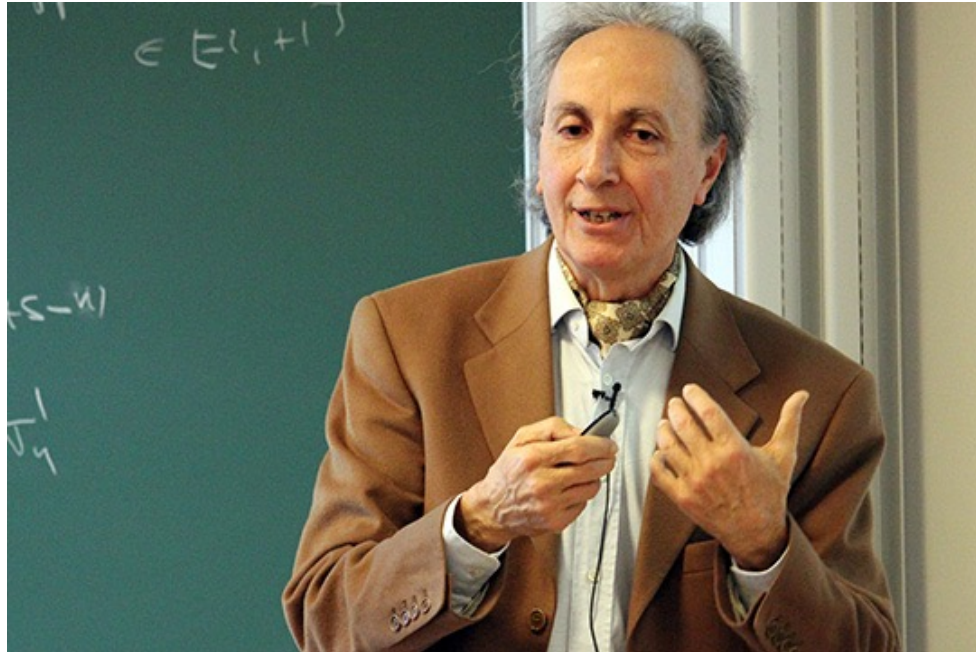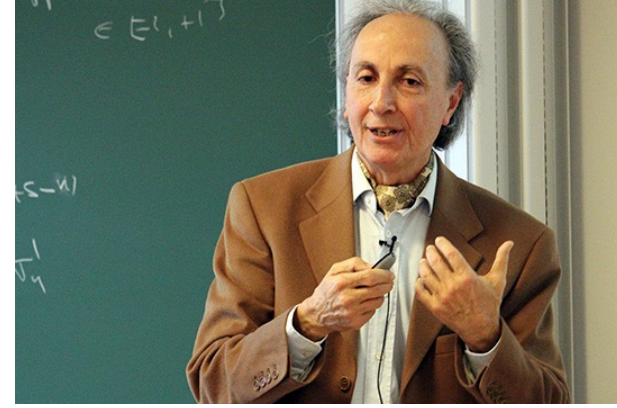# String scale black holes

Juan Maldacena

Damour Fest

October 2021

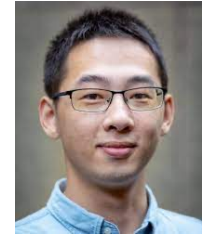# Self-gravitating fundamental strings and black holes

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Outline

Yiming Chen, JM, Edward Witten

- Consider black holes in weakly coupled string theory.

- When the black hole size is close to the string scale, they are supposed to turn into highly excited strings.

- We will discuss some aspects of this transition.

- Surprisingly, the situation is different in the heterotic and type II string cases.

- We will be considering a weakly coupled string theory, $g \ll 1$.
- Set ℓ_s =1= $\sqrt{\alpha'}$ and $G_N = l_p^2 \sim g^2$
- We can consider the type II or heterotic superstring.
- We work in D=4. Four non-compact dimensions, and six compact dimensions.

Highly excited string

$$S = \beta_H M, \qquad \beta_H = 2\pi\sqrt{2}l_s$$

β_H = Hagedorn (inverse) temperature.

$\sim \sqrt{M}$
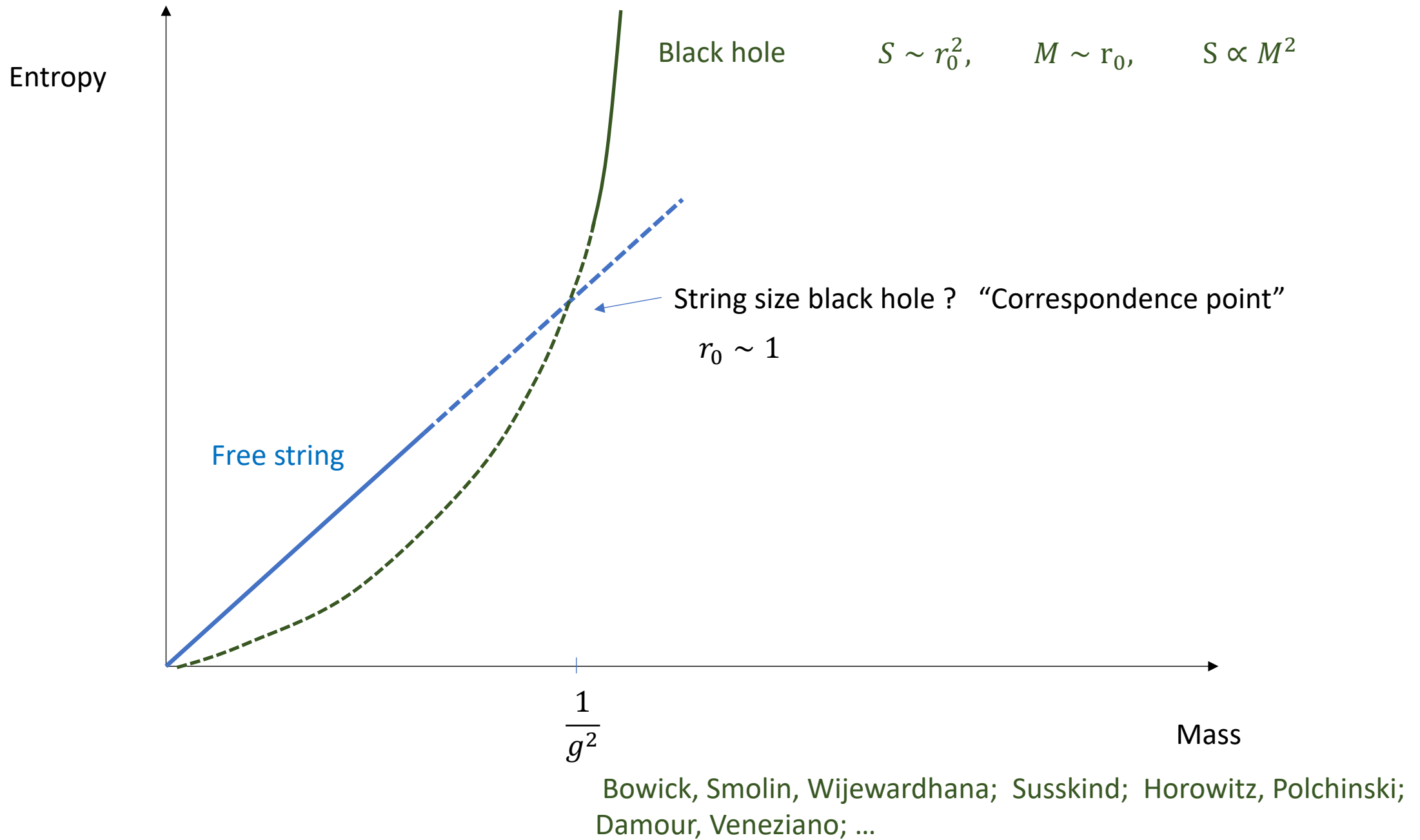
Size $\sim \sqrt{M}$
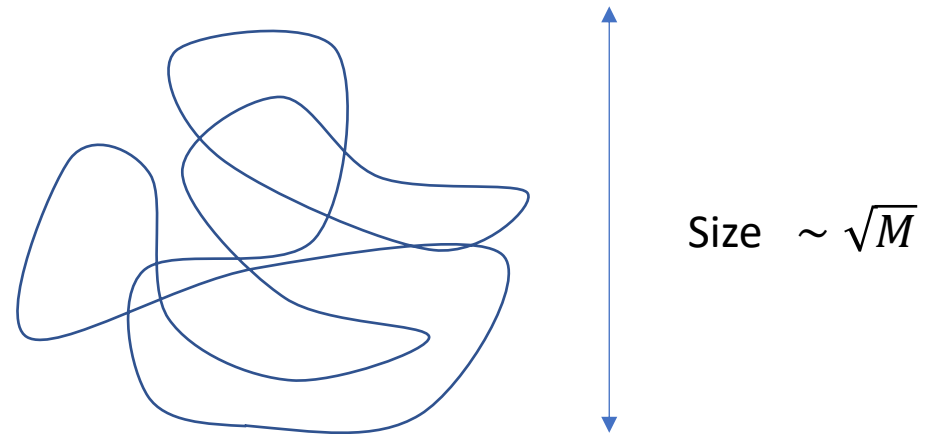
Thermal ensemble is well defined only for $\beta > \beta_H$

$$Z \sim \int dM \, M^p e^{(\beta_H - \beta)M}$$

Black holes

- Well defined if $r_s \gg 1$

- Leading order α' corrections were computed.

Callan, Myers, Perry; Myers, Chen (type II).

- What happens as they approach the string size?.

Entropy

Black hole $S \sim r_0^2$, $M \sim r_0$, $S \propto M^2$

String size black hole ?   "Correspondence point"

$r_0 \sim 1$

Free string

$\dfrac{1}{g^2}$

Mass

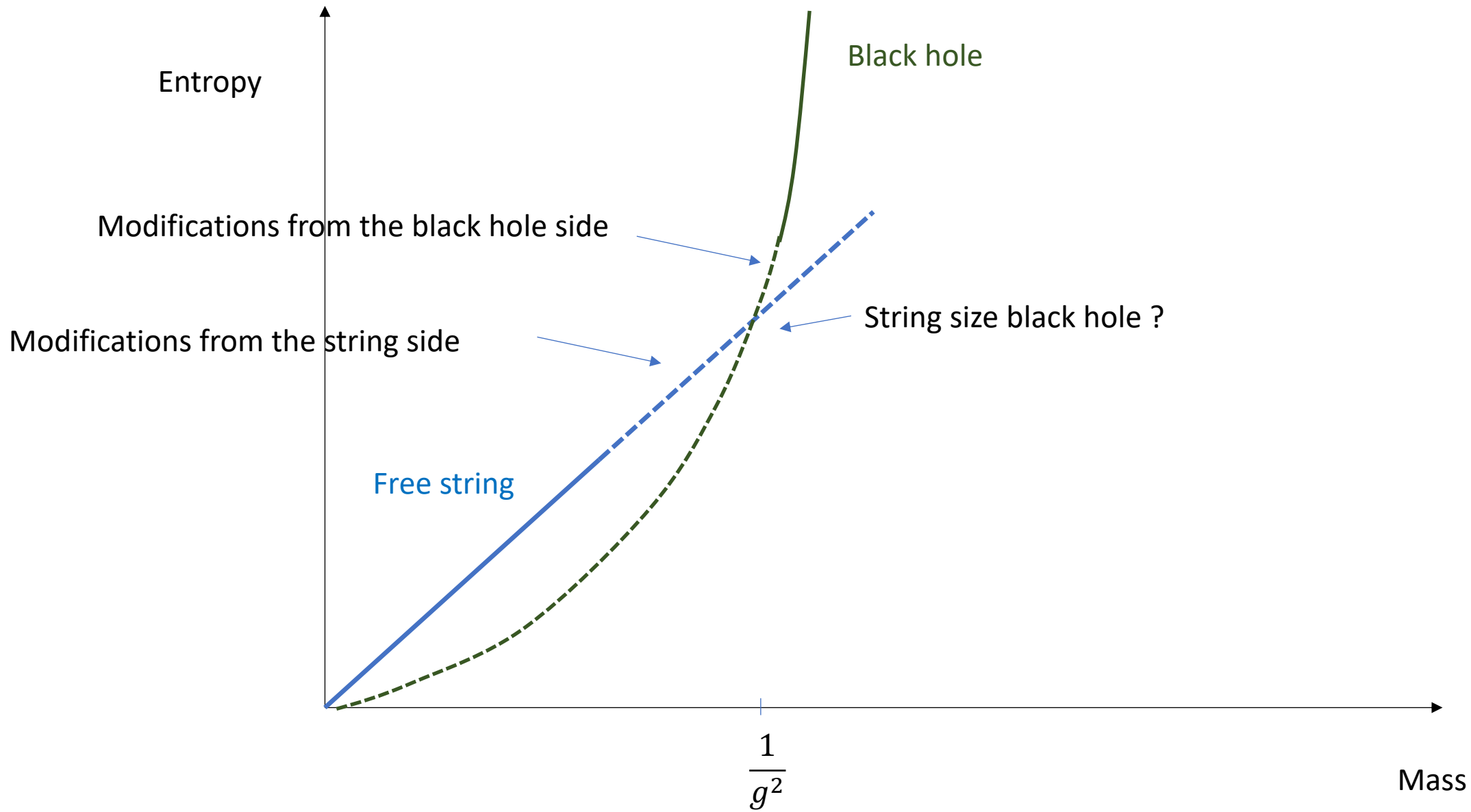Bowick, Smolin, Wijewardhana; Susskind; Horowitz, Polchinski;
Damour, Veneziano; …

Corrections are important before we can reach the correspondence point.

Is there a smooth transition between black holes and highly excited strings ?

Motivation:

- String picture: Microstates are explicit, but no interior.

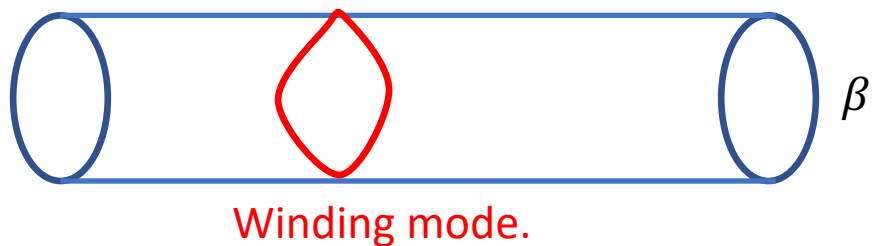- Black holes: there is an interior, but not obvious microstates.

Some comments on strings at finite temperature

Winding mode formalism

- Finite temperature → compactify the Euclidean time direction

Winding mode.

$$S = \frac{1}{g^2} \int |\nabla \chi|^2 + m^2(\beta)|\chi|^2, \qquad \mathrm{m}^2(\beta) \propto (\beta^2 - \beta_H^2) \qquad \text{When } \beta < \beta_H \text{ winding mode is tachyonic} \rightarrow \text{instability}$$

Self interactions → most important one is gravity.   It is attractive → somewhat like a negative χ^4 term.

$$S = \frac{1}{g^2} \int \sqrt{g}R + \frac{1}{g^2} \int |\nabla \chi|^2 + m^2(\beta)|\chi|^2,$$

For $\beta \sim \beta_H$ → winding mode is light and the field theory approximation is good.

This leads to an interesting solution

Self gravitating string

- Localized solution in 3 spatial dimensions. (D=4).

- Localized profile for the winding mode.

- Describes a self gravitating string in thermodynamic equilibrium.

- Size decreases as mass increases. Size $\sim \frac{1}{g^2 M} \propto \frac{1}{\sqrt{\beta - \beta_H}}$. (This size should be larger than 1 to trust the gravity approximation.) Breaks down before the correspondence point.
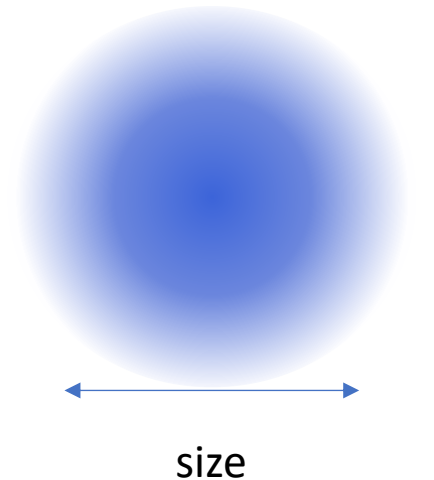
size
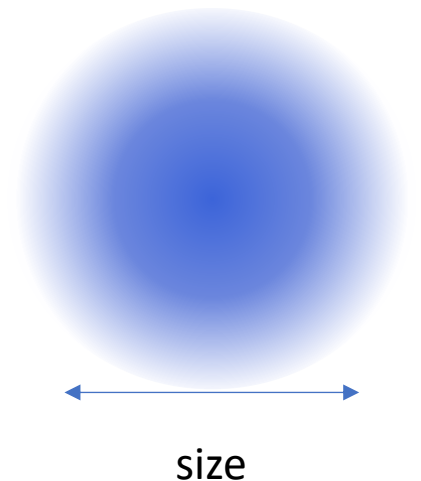
Entropy of the self gravitating string

- We can compute the entropy from the classical action.

- Entropy of order $\frac{1}{g^2}$.

$$S = (1 - \beta \partial_\beta)(-I) = \frac{\beta}{g^2} \int d^{D-1}x \, (\beta \, \partial_\beta m^2(\beta)) \, |\chi|^2 = 2\frac{\beta}{g^2} \int d^{D-1}x \, \left(\frac{\beta^2}{4\pi^2}\right) |\chi|^2$$
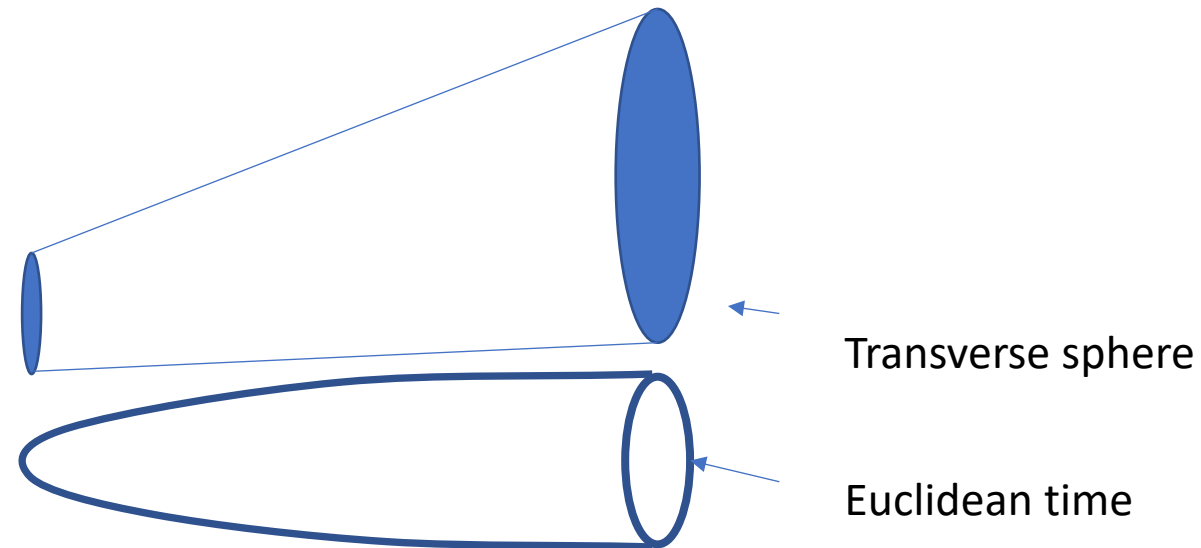
Only a contribution from the explicit dependence on β.
Non local term in D dimensions.

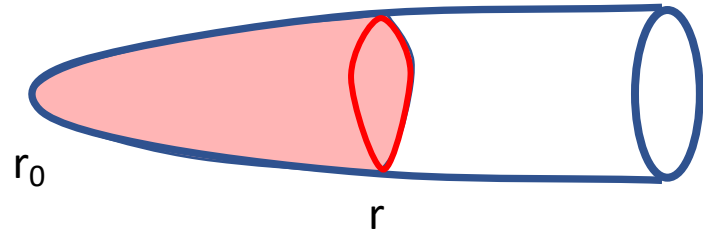To leading order this gives $S = \beta M + \cdots$

size

Euclidean black holes

- Geometry of the Euclidean black hole.

Transverse sphere

Euclidean time

Black holes and winding condensates

r_0

r

Winding one function → computed by a worldsheet wrapping the cigar.

$$\langle \chi(r) \rangle \propto e^{-T\,A} \sim e^{-\beta(r-r_0)}, \qquad \beta \gg \beta_H$$

Breaking of the winding symmetry.   Phase of χ → integral of $B_{\mu\nu}$ field over the cigar.
Integrating over it → restores symmetry. But $|\chi|^2 \neq 0$.
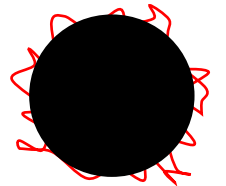
We can view it as a thermal atmosphere of strings.
Strings coming out of the horizon and back in.

It is a classical contribution to the entropy, formally of order $\frac{1}{g^2}$, but not calculable (to my knowledge),

since it is concentrated near the horizon.

Susskind, Uglum ; Dabholkar

(Some authors have conjectured, or claimed to prove, it is all the entropy. No convincing argument.)

Comment on the broken symmetries.

- Notice that the winding symmetry is spontaneously broken both on the black hole phase and the highly excited string phase.

- In both cases there is a winding condensate.

- This is consistent with the suggestion that they could be continuously connected.

- The black hole solution is good for $\beta \gg l_s$

- The HP solution is good for $(\beta - \beta_H) \ll l_s$

- Both have a non-zero classical entropy

- Is there an interpolating solution ?
  - Classical solution of string theory
  - Described by a 2d CFT (The worldsheet CFT_4). CFT_4 x CFT_6
  - With one parameter: β

β_H

HP   Interpolating solution?   Black hole

No smooth interpolation type II

- Consider an invariant of the worldsheet CFT. Invariant under deformations.

- Candidate: Witten index of the (1,1) supersymmetric worldsheet CFT.

- $I_W = Tr\left[(-1)^F e^{-\widetilde{\beta}H} \right]$

- On the HP side we find $I_W = 0$. We can consider a flat space target space, $S^1 \times R^3$.

- On the Black hole side we find $I_W = 2$. Index for a sigma model is equal to the Euler characteristic of the target space.

- Since they are different, there cannot be a continuous connection.

β_H

X

HP   Black hole

Heterotic theory

- Now we have a (1,0) CFT.

- The index can also be computed and it is equal to zero on both sides.

- Other known invariants are also the same.

- There is a linear sigma model analysis which suggests that they continuously connected.

Heterotic linear sigma model

- Construct the CFT as the IR limit of a simpler problem involving free fields + a potential.

$$S \sim S_{free} + \int d^2x\, d\theta_+ \Lambda_- W(X, Y) = S_{free} + \int d^2x\, W^2$$

$$W = (\vec{X}^2 - b)(\vec{Y}^2 + a) + c \qquad\qquad \vec{X} = (X_1, X_2)\,, \qquad \vec{Y} = (Y_1, Y_2, Y_3)$$

$$W = 0 \quad \longrightarrow \quad \vec{X}^2 = b - \frac{c}{\vec{Y}^2 + a}$$

$b - \dfrac{c}{a} > 0$ → HP topology

$b - \dfrac{c}{a} < 0$ → Black hole topology

As we vary the parameters the classical vacuum manifold similar to the HP or the black hole topology.

UV   Free fields + potential

Classical vacuum manifold given by W =0

Flow to a CFT , metric is adjusted. (we did not analyze this in detail)

IR   CFT

3 parameters in the UV theory: a,b,c.

In IR theory we expect 1 marginal parameter, β, and one relevant parameter that should be fine tuned.
This is because both the black hole and the HP solutions have one negative mode.
We expect that the third parameter is irrelevant, but we did not check it explicitly.

As we vary the parameters nothing special happens when b-c/a=0. There is no new branch appearing there. So we expect
that the flow to an IR CFT does not show any surprise.
In the intermediate stage the two theories are continuously connected, after we flow all the way to the IR we expect that to be the case too, but this is not a proof. We view it simply as an indication for a continuous connection.

Type II linear sigma model

- Construct the CFT as the IR limit of a simpler problem involving free fields + a potential.

$$S \sim S_{free} + \int d^2x d^2\theta \mathcal{W} = S_{free} + \int d^2x \sum_i (\partial_i \mathcal{W})^2$$

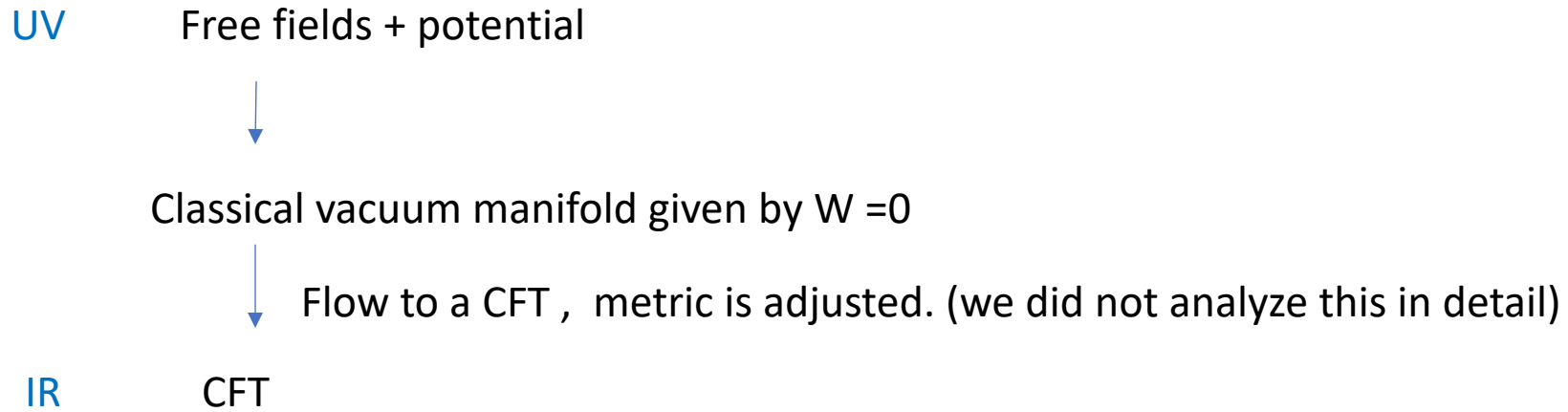$$\mathcal{W} = PW \qquad \text{P is a new superfield}$$

$$W = (\vec{X}^2 - b)(\vec{Y}^2 + a) + c \qquad \vec{X} = (X_1, X_2), \qquad \vec{Y} = (Y_1, Y_2, Y_3)$$

$$W = 0 \quad \longrightarrow \quad \vec{X}^2 = b - \frac{c}{\vec{Y}^2 + a}$$

$$P = 0$$

$$b - \frac{c}{a} > 0 \ \rightarrow \text{HP topology}$$

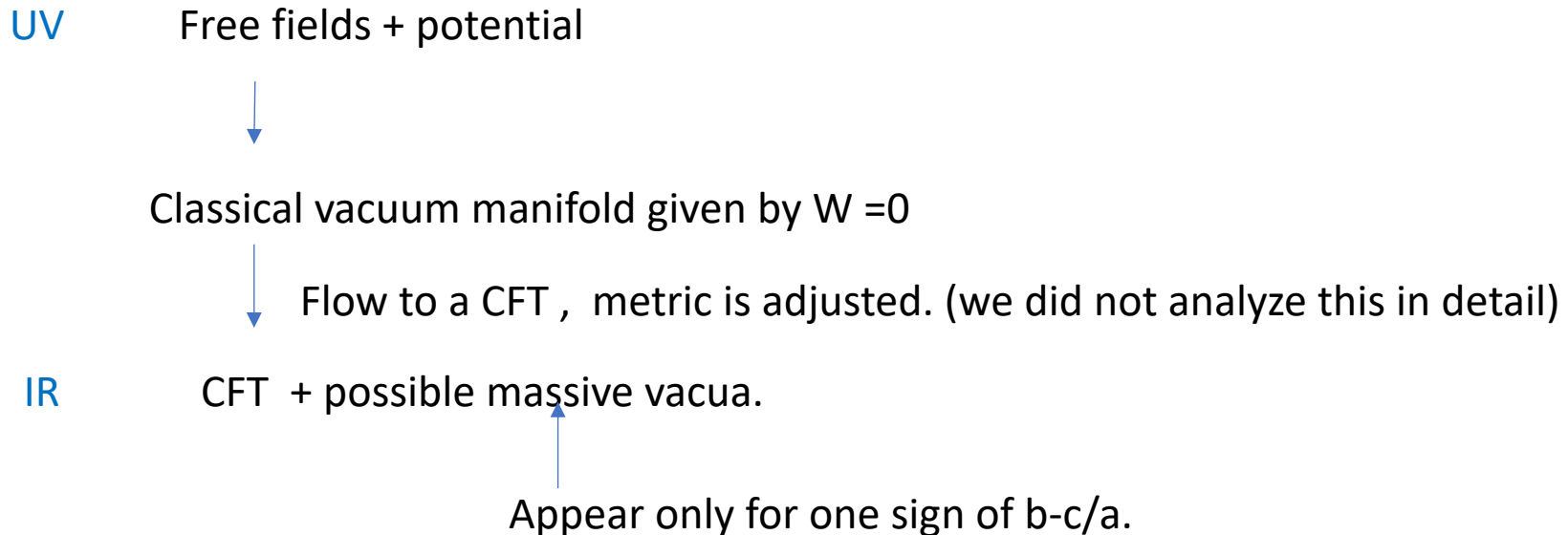$$b - \frac{c}{a} < 0 \ \rightarrow \text{Black hole topology}$$

When b-c/a =0 , a new classical branch appears with $P \neq 0, \vec{X} = \vec{Y} = 0$

On this P branch a superpotential is generated. Depending on the sign of b-c/a we do or do not have isolated vacua
 on this branch.

These isolated vacua are massive.

Now the index has the same value on both sides, in the full UV theory.

What happens is that the UV theory flows in the IR to a CFT + isolated massive vacua

UV   Free fields + potential

Classical vacuum manifold given by W =0

Flow to a CFT , metric is adjusted. (we did not analyze this in detail)

IR   CFT + possible massive vacua.

Appear only for one sign of b-c/a.

Index of the full theory is the same for both signs of b-c/a. But the index of the CFT part in the IR is different due to
the extra massive vacua.

Index and D-branes

There are some D-branes that exist on the HP side but not the BH side and vice versa.

Eg. A D0 brane localized at the origin and wrapping the time circle exists on the HP side.

There is no such brane on the black hole side (the Euclidean time circle is contractible)

These branes generate some fluxes at infinity. On the black hole side the flux is carried as an electrically charged black hole, without an explicit brane source.

β_H

X

HP

Black hole

Charged black holes

- If the internal theory has a circle → we can have black hole with momentum and fundamental string winding charges on this extra circle.

- These solutions are obtained from a a solution generating transformation that is a symmetry of the worldsheet CFT.

- Given the uncharged seed solution → obtain the charged solution.

- Thermodynamics of uncharged solution → thermodynamics of charged solution.

Charged black holes

- The transitions happen as we approach extremality.

- Go over to a free string phase as we approach extremality

- The HP or free string phases look a bit like a fuzzball (a configuration whose microstates are very clear)

Damour Veneziano
Mathur
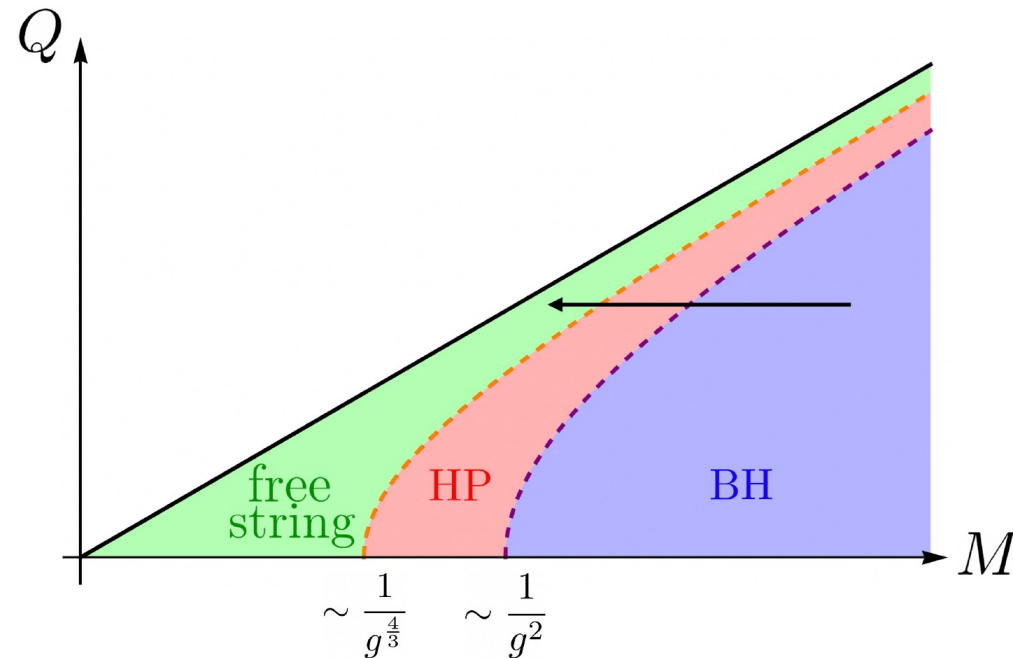
Conclusions

- We discussed the possible connection between the black hole and the self gravitating string solution of Horowitz and Polchinski (and further studied by Damour and Veneziano).

- For the type II case, we showed that the two could not be continuously connected as classical solutions.

- For the heterotic case, it seems that they could be continuously connected.

Questions

- Can we say more about the CFT at intermediate values of β?
- Can we track the picture of the microstates through this intermediate region.
- Why is the heterotic and type II picture different ?

Happy Birthday Thibault