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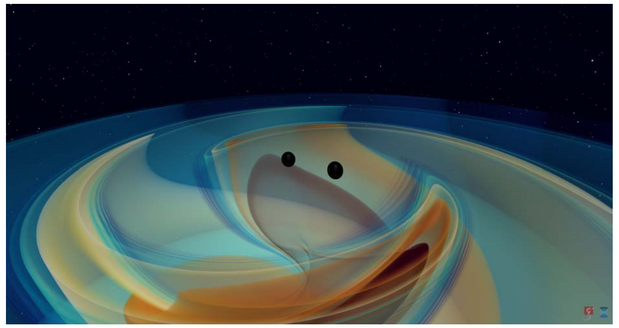

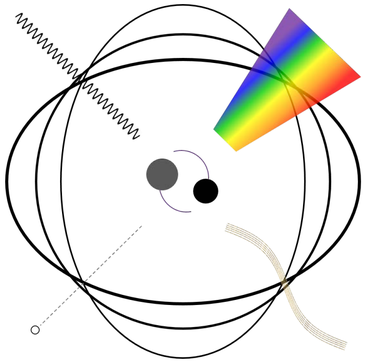

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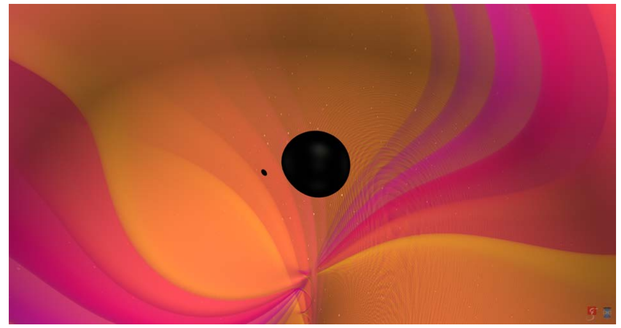


The Effective-One-Body Approach to the Relativistic Two-Body Problem: from Birth to Adulthood

Alessandra Buonanno

Max Planck Institute for Gravitational Physics

(Albert Einstein Institute)

Department of Physics, University of Maryland

"Damour Fest: Adventures in Gravitation at IHES"

October 12, 2021

Outline

- **My adventures in gravitation at IHES** in 1997-1999.

- GW research in late 90s and **original motivations to develop analytical** (approximate) **inspiral-merger-ringdown waveforms** from binary systems: the **effective-one-body** formalism.

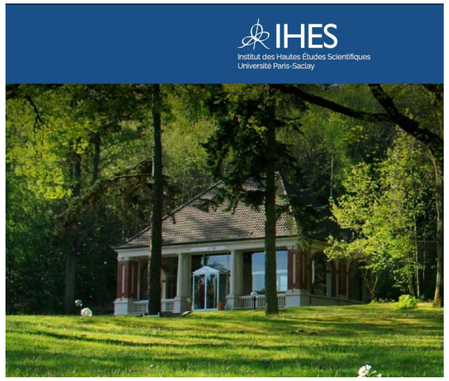
- The path to highly accurate **EOB waveforms for LIGO & Virgo searches/analyses:** synergy with **numerical relativity**.

- **Highlights on science from the latest observing run** of LIGO & Virgo.

- **Bright future** of gravitational-wave astronomy.

The first nine months at IHES …

I arrived at IHES from CERN in Sep 1997



PHYSICAL REVIEW D, VOLUME 60, 023517

On the gravitational, dilatonic, and axionic radiative damping of cosmic strings

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(Received 29 January 1998; published 25 June 1999)

We study the radiation reaction on cosmic strings due to the emission of dilatonic, gravitational and axionic waves. After verifying the (on average) conservative nature of the time-symmetric self-interactions, we concentrate on the finite radiation damping force associated with the half-retarded minus half-advanced ''reactive'' fields. We reexamine a recent proposal of using a ''local back reaction approximation'' for the reactive fields. Using *dimensional continuation* as a convenient technical tool, we find, contrary to previous claims, that this proposal leads to *antidamping* in the case of the axionic field, and to *zero* (integrated) *damping* in the case of the gravitational field. One gets normal *positive damping* only in the case of the dilatonic field. We propose to use a suitably modified version of the local dilatonic radiation reaction as a substitute for the exact (nonlocal) gravitational radiation reaction. The incorporation of such a local approximation to gravitational radiation reaction should allow one to complete, in a computationally nonintensive way, string network simulations and to give better estimates of the amount and spectrum of gravitational radiation emitted by a cosmologically evolving network of massive strings. [S0556-2821(99)07112-X]

ELSEVIER

23 July 1998

PHYSICS LETTERS B

Physics Letters B 432 (1998) 51–57

Effective action and tension renormalization for cosmic and fundamental strings

Alessandra Buonanno [a], Thibault Damour [a,b]

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Received 5 March 1998; revised 1 May 1998
Editor: P.V. Landshoff

Abstract

We derive the effective action for classical strings coupled to dilatonic, gravitational, and axionic fields. We show how to use this effective action for: (i) renormalizing the string tension, (ii) linking ultraviolet divergences to the infrared (long-range) interaction between strings, (iii) bringing additional light on the special cancellations that occur for fundamental strings, and (iv) pointing out the limitations of Dirac's celebrated field-energy approach to renormalization. © 1998 Elsevier Science B.V. All rights reserved.

JHEP HYPER VERSION

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Quantum inhomogeneities in string cosmology

Alessandra Buonanno[*], **Krzysztof A. Meissner**[†], **Carlo Ungarelli**[‡] **and Gabriele Veneziano**
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ABSTRACT: Within two specific string cosmology scenarios — differing in the way the pre- and post-big bang phases are joined — we compute the size and spectral slope of various types of cosmologically amplified quantum fluctuations that arise in generic compactifications of heterotic string theory. By further imposing that these perturbations become the dominant source of energy at the onset of the radiation era, we obtain physical bounds on the background's moduli, and discuss the conditions under which both a (quasi-) scale-invariant spectrum of axionic perturbations and sufficiently large seeds for the galactic magnetic fields are generated. We also point out a potential problem with achieving the exit to the radiation era when the string coupling is near its present value.

KEYWORDS: Physics of the Early Universe, Superstrings and Heterotic Strings, Models of Quantum Gravity.

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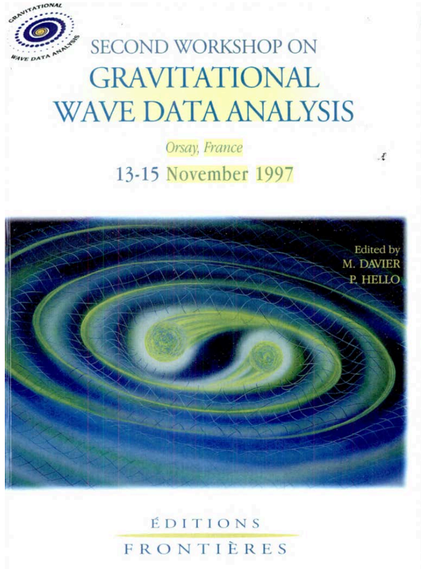

ELSEVIER

Nuclear Physics B 543 (1999) 275–320

NUCLEAR PHYSICS B

Pre-Big-Bang bubbles from the gravitational instability of generic string vacua

A. Buonanno [a], T. Damour [a,b], G. Veneziano [c,a]
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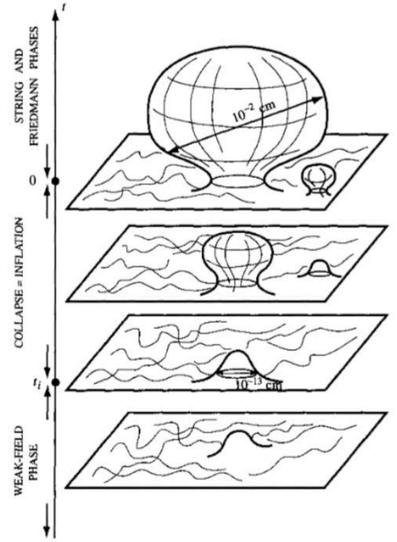
Abstract

We formulate the basic postulate of pre-Big-Bang cosmology as one of "asymptotic past triviality", by which we mean that the initial state is a generic perturbative solution of the tree-level low-energy effective action. Such a past-trivial "string vacuum" is made of an arbitrary ensemble of incoming gravitational and dilatonic waves, and is generically prone to gravitational instability, leading to the possible formation of many black holes hiding singular space-like hypersurfaces. Each such singular space-like hypersurface of gravitational collapse becomes, in the string-frame metric, the usual Big-Bang $t = 0$ hypersurface, i.e. the place of birth of a baby Friedmann universe after a period of dilaton-driven inflation. Specializing to the spherically symmetric case, we review and reinterpret previous work on the subject, and propose a simple, scale-invariant criterion for collapse/inflation in terms of asymptotic data at past null infinity. Those data should determine whether, when, and where collapse/inflation occurs, and, when it does, fix its characteristics, including anisotropies on the Big-Bang hypersurface whose imprint could have survived till now. Using Bayesian probability concepts, we finally attempt to answer some fine-tuning objections recently moved to the pre-Big-Bang scenario. © 1999 Elsevier Science B.V.



Summer of 1998 at IHES …

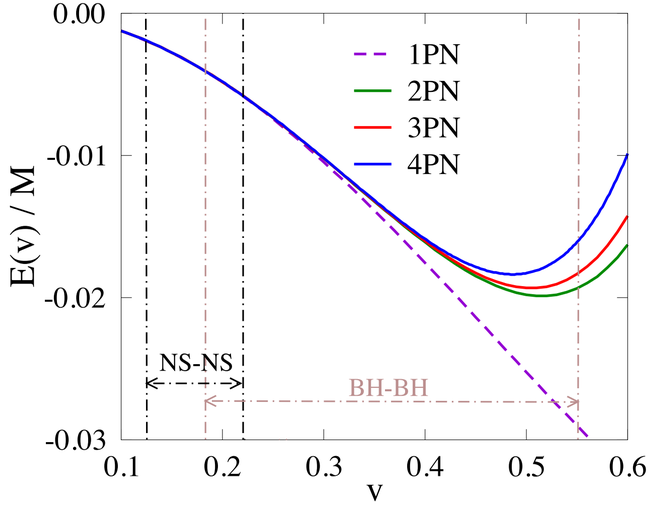
- Construction of **LIGO facilities was almost completed**. Virgo construction was underway.

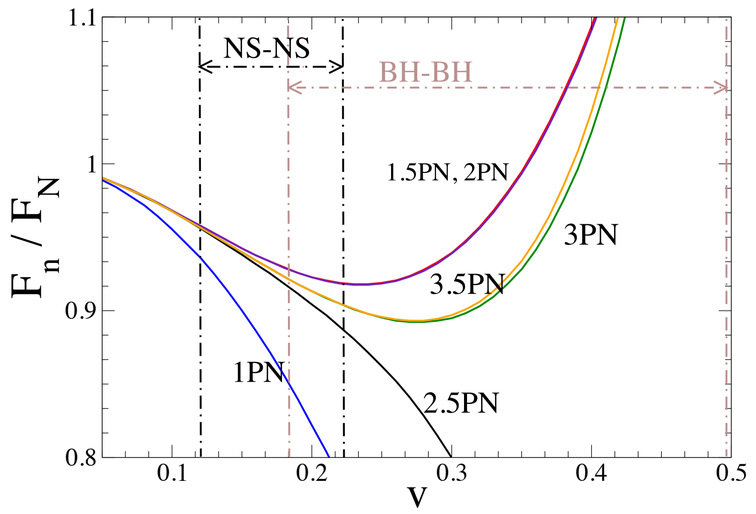
- **If BHs of** $30 - 40 M_\odot$ **existed** in binary systems, they **would be the strongest GW sources**, because would coalesce at $\sim$ few hundreds Hz (detector's sweet spot).

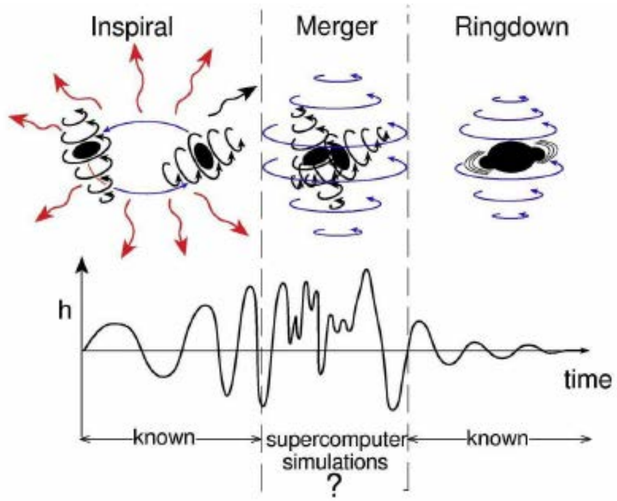
- To build **faithful templates for searches**, the **knowledge of merger signal was crucial**. Despite big efforts, the numerical-relativity community had not produced results, yet.

- **PN results** for binding energy and GW flux were **available only through 2PN and 2.5PN order**, respectively.

- Balance equation: $\dfrac{dE(\omega)}{dt} = -F(\omega) \longrightarrow \dot{\omega}(t) = -\dfrac{F(\omega)}{dE(\omega)/d\omega}$







(credit: Thorne)

The Birth of the Effective-One-Body Approach

PHYSICAL REVIEW D, VOLUME 59, 084006

Effective one-body approach to general relativistic two-body dynamics

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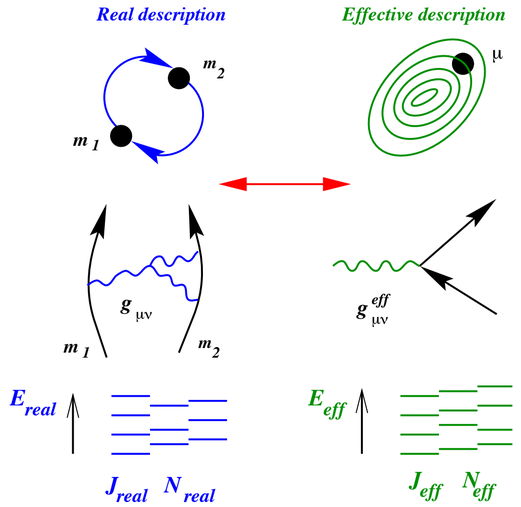
(Received 30 November 1998; published 8 March 1999)

We map the general relativistic two-body problem onto that of a test particle moving in an effective external metric. This effective-one-body approach defines, in a non-perturbative manner, the late dynamical evolution of a coalescing binary system of compact objects. The transition from the adiabatic inspiral, driven by gravitational radiation damping, to an unstable plunge, induced by strong spacetime curvature, is predicted to occur for orbits more tightly bound than the innermost stable circular orbit in a Schwarzschild metric of mass $M = m_1 + m_2$. The binding energy, angular momentum and orbital frequency of the innermost stable circular orbit for the time-symmetric two-body problem are determined as a function of the mass ratio.



$$E^2_{\rm real} = m_1^2 + m_2^2 + 2m_1 m_2 \left(\frac{E_{\rm eff}}{\mu} \right)$$

PHYSICAL REVIEW D, VOLUME 62, 064015

Transition from inspiral to plunge in binary black hole coalescences

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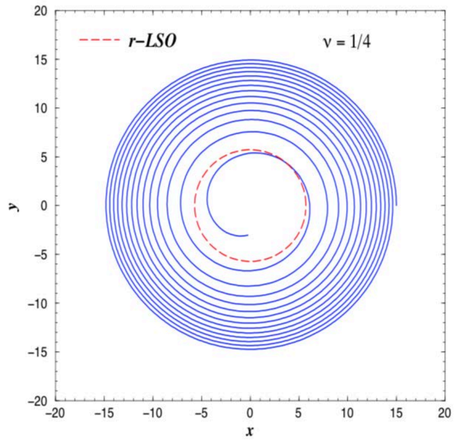
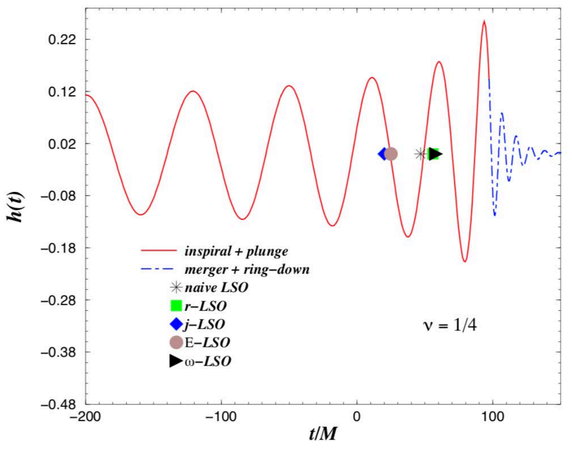
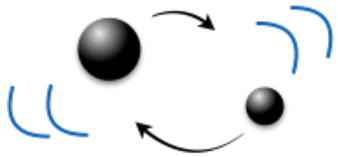
(Received 7 January 2000; published 10 August 2000)

Combining recent techniques giving nonperturbative resummed estimates of the damping and conservative parts of the two-body dynamics, we describe the transition between the adiabatic phase and the plunge, in coalescing binary black holes with comparable masses moving on quasicircular orbits. We give initial dynamical data for numerical relativity investigations, with a fraction of an orbit left, and provide, for data analysis purposes, an estimate of the gravitational waveform emitted throughout the inspiral, plunge and coalescence phases.





The Effective-One-Body Approach in a Nutshell

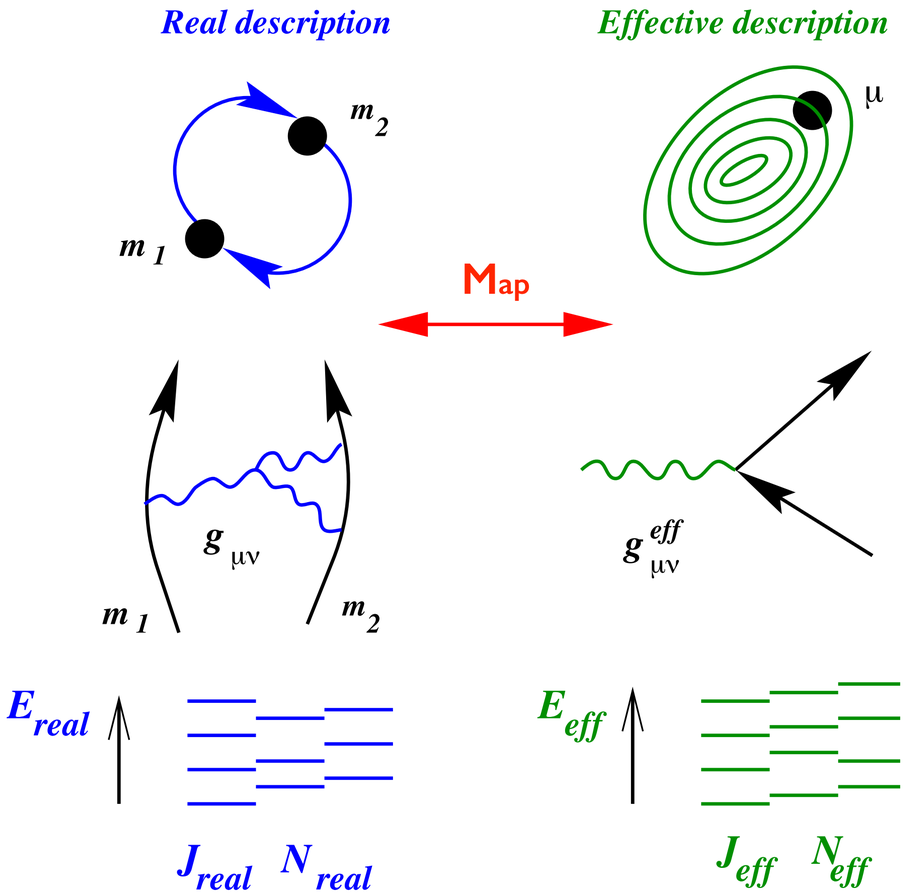


$$\nu = \frac{\mu}{M} \qquad 0 \leq \nu \leq 1/4$$

$$\mu = \frac{m_1 m_2}{M} \qquad M = m_1 + m_2$$

- **Two-body dynamics is mapped** into dynamics of **one-effective body** moving **in deformed black-hole spacetime**, deformation being the mass ratio.

- Some key **ideas** of EOB theory were **inspired by quantum** field **theory** when describing **energy** of **comparable-mass charged bodies**.



(AB & Damour 1998)

Energy for Comparable-Mass Black Holes

- Classical gravity *(AB & Damour 1998):*

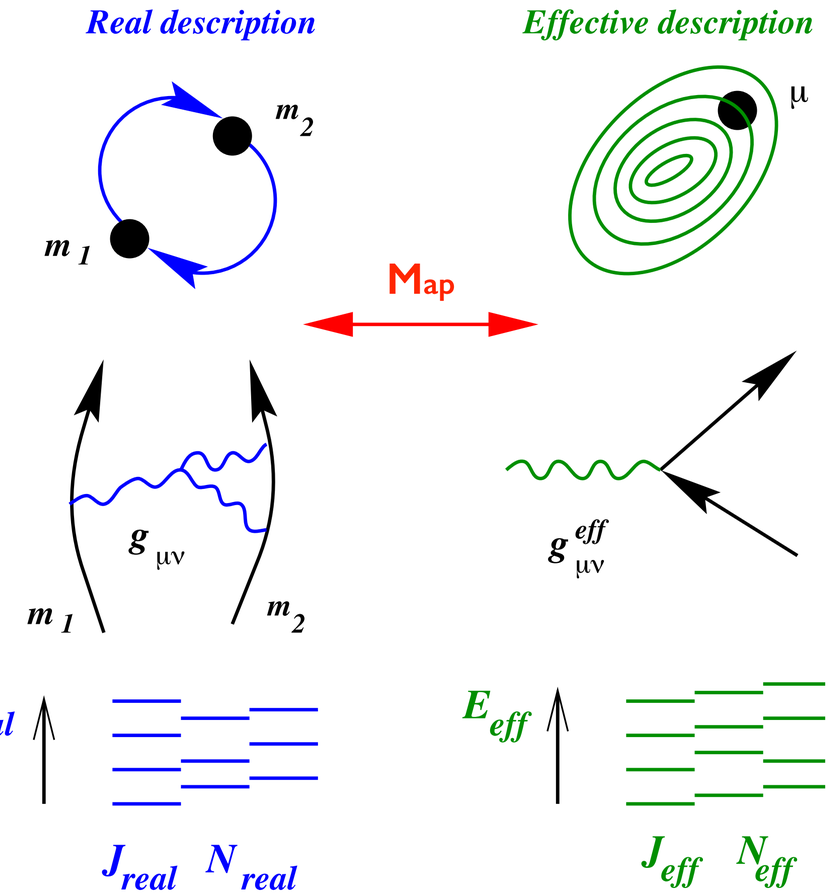
$$E^2_{\rm real} = m_1^2 + m_2^2 + 2m_1 m_2 \left(\frac{E_{\rm eff}}{\mu} \right)$$

- Quantum electrodynamics *(Brezin, Itzykson & Zinn-Justin 1970):*

$$E^2_{\rm real} = m_1^2 + m_2^2 + 2m_1 m_2 \frac{1}{\sqrt{1 + Z^2\,\alpha^2/(n-\epsilon_j)^2}}$$

- Considering scattering states:

$$\varphi(s) \equiv \frac{s - m_1^2 - m_2^2}{2m_1 m_2} = \frac{-(p_1+p_2)^2 - m_1^2 - m_2^2}{2m_1 m_2} = -\frac{p_1 \cdot p_2}{m_1 m_2}$$



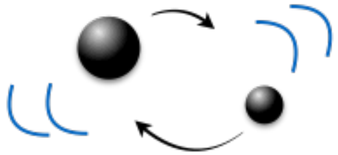
Most natural **symmetric function** of asymptotic **momenta** of two-particle system which **reduces in test-mass limit $m_2 \ll m_1$ to energy of m_2 in rest-frame of m_1.**

(AB & Damour 1998)

EOB Hamiltonian: Resummed Conservative Dynamics

- **Real Hamiltonian**

$$H^{\rm PN}_{\rm real} = H_{\rm Newt} + H_{\rm 1PN} + H_{\rm 2PN} + \cdots$$

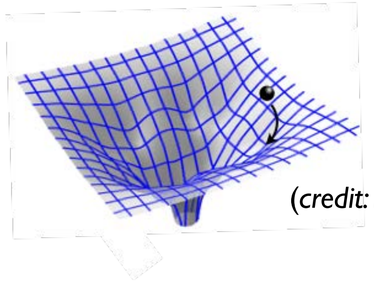


- **EOB Hamiltonian**

$$H^{\rm EOB}_{\rm real} = M\sqrt{1 + 2\nu\left(\frac{H_{\rm eff}}{\mu}^{\nu} - 1\right)}$$

- **Effective Hamiltonian**

$$H^{\nu}_{\rm eff} = \mu\sqrt{A_\nu(r)\left[1 + \frac{\mathbf{p}^2}{\mu^2} + \left(\frac{1}{B_\nu(r)} - 1\right)\frac{p_r^2}{\mu^2}\right]}$$



(credit: Hinderer)

$$ds^2_{\rm eff} = -A_\nu(r)dt^2 + B_\nu(r)dr^2 + r^2 d\Omega^2$$

- Dynamics condensed $A_\nu(r)$ and $B_\nu(r)$
- A_ν encodes the energetics of circular orbits. Through 4PN:

$$A_\nu(r) = 1 - \frac{2M}{r} + \frac{2M^3\nu}{r^3} + \left(\frac{94}{3} - \frac{41}{32}\pi^2\right)\frac{M^4\nu}{r^4} + \frac{a_5(\nu) + a_5^{\log}(\nu)\log(r)}{r^5} + \frac{a_6(\nu)}{r^6} + \cdots$$

5PN unknown as today

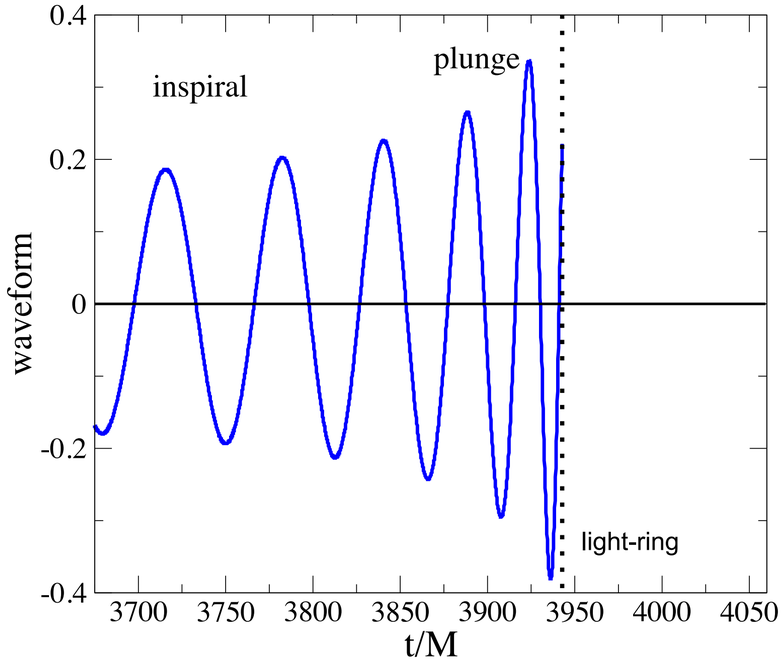
EOB Inspiral-Plunge Waveform & Frequency

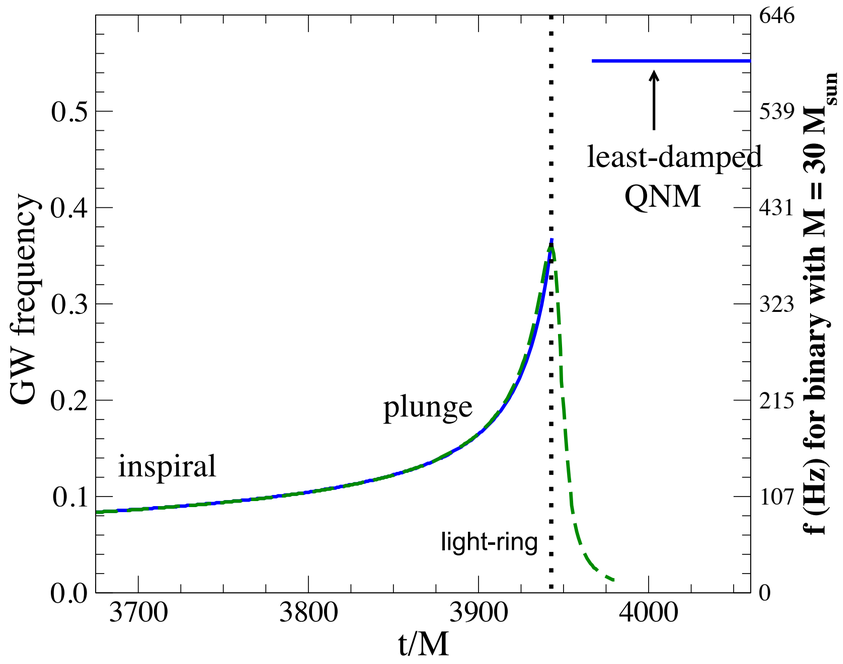
- **EOB equations of motion**

$$\dot{\mathbf{r}} = \frac{\partial H^{\rm EOB}_{\rm real}}{\partial \mathbf{p}} \qquad \dot{\mathbf{p}} = -\frac{\partial H^{\rm EOB}_{\rm real}}{\partial \mathbf{r}} + \mathbf{F} \qquad F \propto \frac{dE}{dt}$$

GW energy flux is also resummed

- Evolve **two-body dynamics up to light ring** (or photon orbit) and then …





- **Quasi-normal modes** excited at **light-ring crossing.** *(Goebel 1972, Davis, Ruffini & Tiomno 1972, Ferrari et al. 1984, Price and Pullin 1994)*

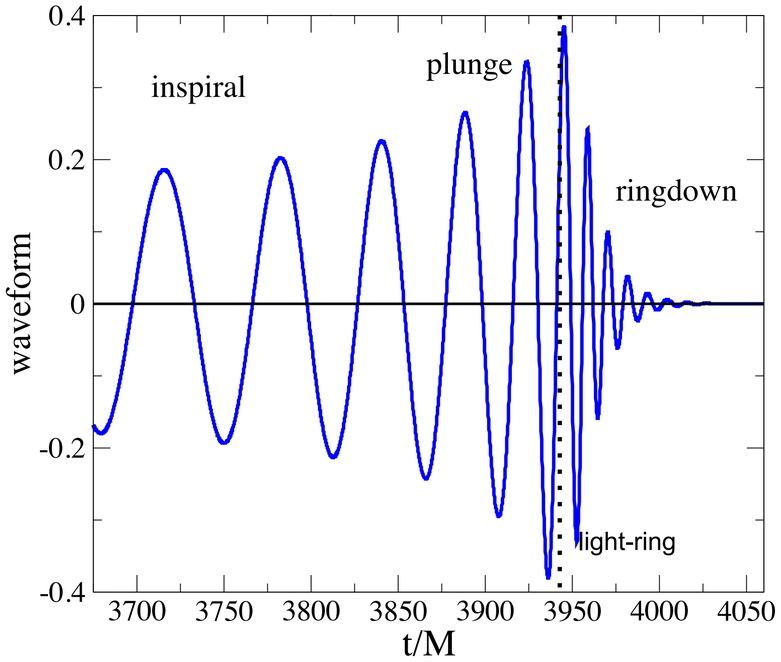
EOB Inspiral-Merger-Ringdown Waveform & Frequency

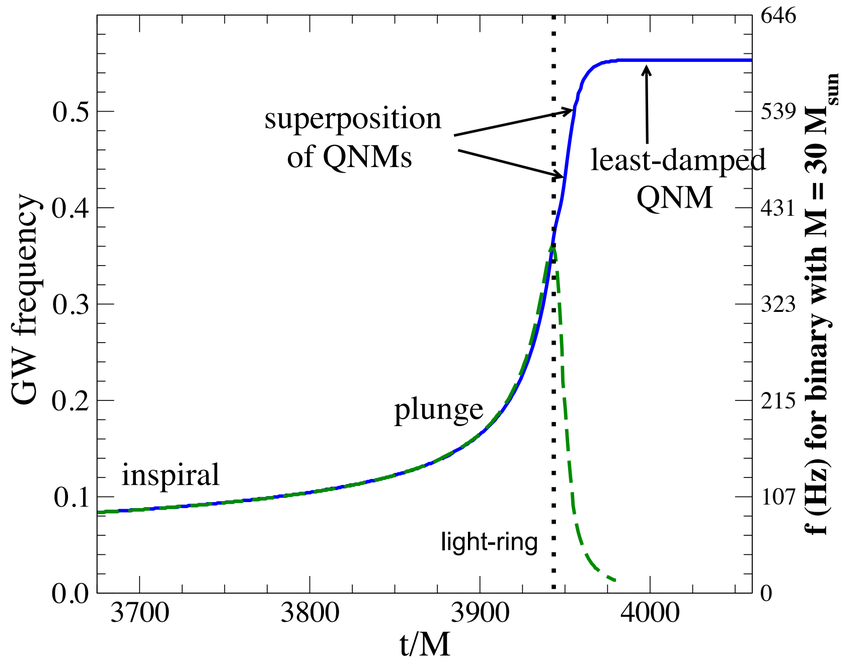
- **EOB equations of motion**

$$\dot{\mathbf{r}} = \frac{\partial H^{\rm EOB}_{\rm real}}{\partial \mathbf{p}} \qquad \dot{\mathbf{p}} = -\frac{\partial H^{\rm EOB}_{\rm real}}{\partial \mathbf{r}} + \mathbf{F} \qquad F \propto \frac{dE}{dt}$$

GW energy flux is also resummed

- … attach a **superposition of quasi-normal modes** of remnant BH.





- Very **short/simple transition** plunge-merger-ringdown

- **One single QNM** is matched with $M_{\rm BH} = E_{\rm LR} = 0.976M$, $a_{\rm BH} = J_{\rm LR}/M^2_{\rm BH} = 0.77$

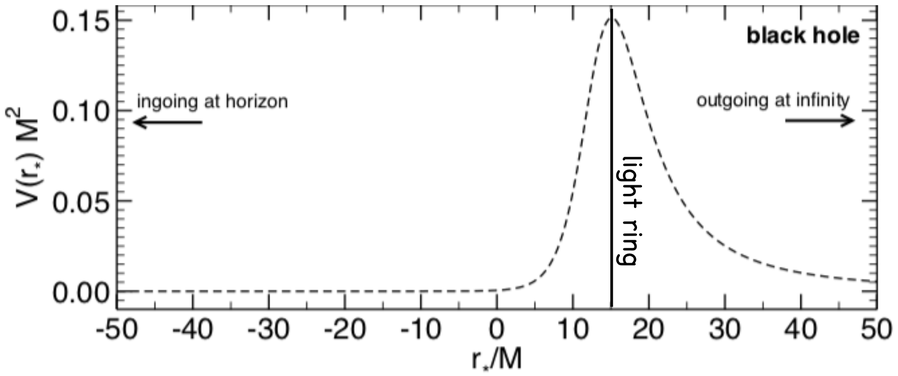
(AB & Damour 00)

On the Simplicity of the Merger Waveform: Test-Mass Limit

- Equation of **gravitational perturbations** in BH spacetime

(Regge & Wheeler 56, Zerilli 70, Teukolsky 72)

$$\frac{\partial^2\Psi}{\partial t^2} - \frac{\partial^2\Psi}{\partial r_\star^2} + V_{\ell m}\Psi = \mathcal{S}_{\ell m}$$



- **Peak** of BH potential **close to "light ring".**

- Once particle is inside potential, **direct gravitational radiation** from its motion is **strongly filtered** by potential barrier (**high-pass filter**).

- Only **BH spacetime vibrations** (quasi-normal modes) **leaks out** BH potential.

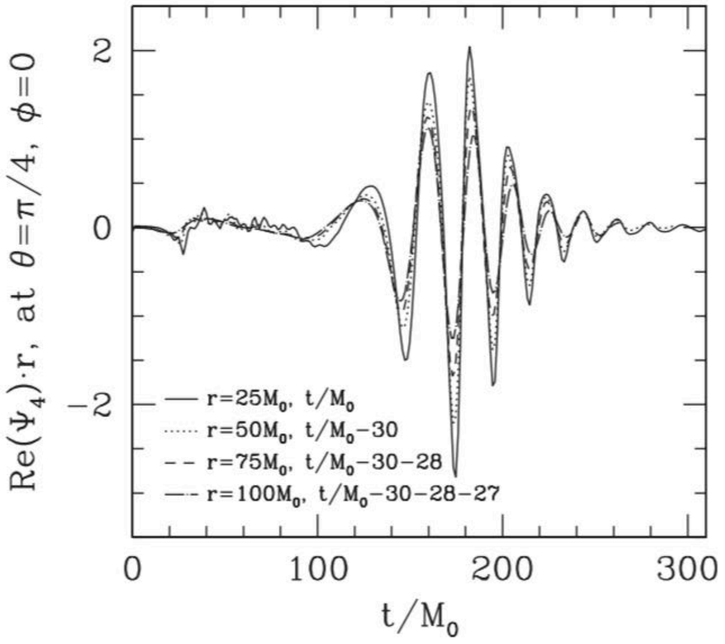
(Goebel 1972, Davis, Ruffini & Tiomno 1972, Ferrari & Mashhoon 1984)

Numerical Relativity: First Binary Black-Hole Simulations

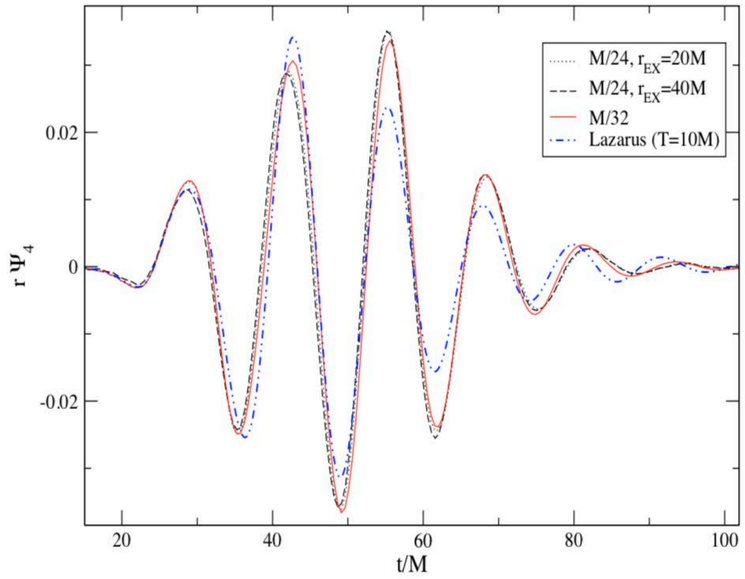
- **Breakthrough** in 2005 *(Pretorius 05, Campanelli et al. 06, Baker et al. 06)*

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

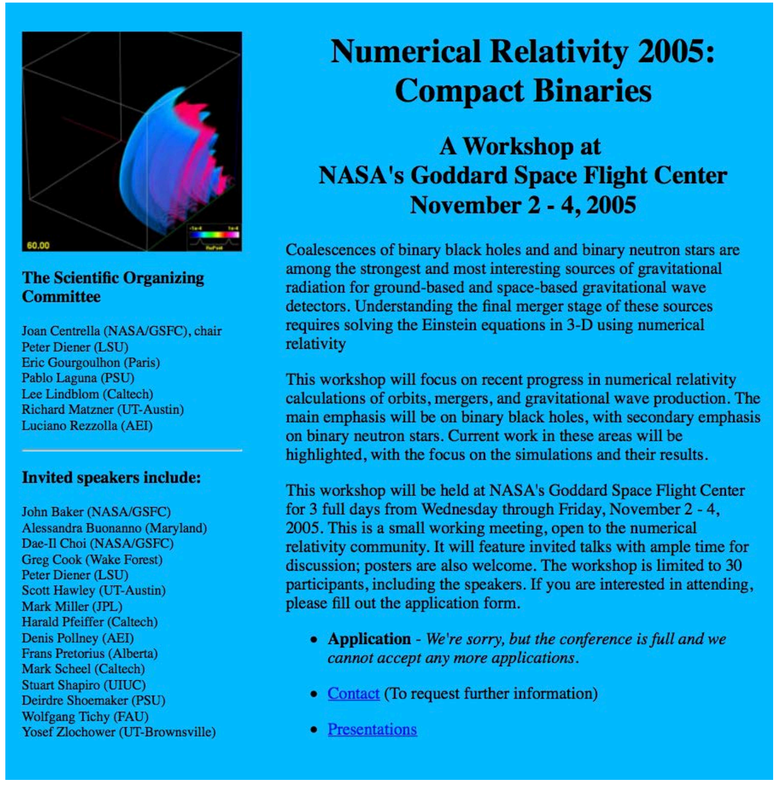
- After more than **40 years** of **steady work** *(Choquet-Bruhat, York, Smarr, Price, Pullin, Brügmann, Gundlach, Alcubierre, Brandt, Seidel, Shibata, Nakamura, Baumgarte, Shapiro, ….)*



(Pretorius 05)



(Baker et al. 06, see also Campanelli et al. 06)



Numerical Relativity 2005: Compact Binaries

A Workshop at NASA's Goddard Space Flight Center November 2 - 4, 2005

The Scientific Organizing Committee

Joan Centrella (NASA/GSFC), chair
Peter Diener (LSU)
Eric Gourgoulhon (Paris)
Pablo Laguna (PSU)
Lee Lindblom (Caltech)
Richard Matzner (UT-Austin)
Luciano Rezzolla (AEI)

Invited speakers include:

John Baker (NASA/GSFC)
Alessandra Buonanno (Maryland)
Dae-Il Choi (NASA/GSFC)
Greg Cook (Wake Forest)
Peter Diener (LSU)
Scott Hawley (UT-Austin)
Mark Miller (JPL)
Harald Pfeiffer (Caltech)
Denis Pollney (AEI)
Frans Pretorius (Alberta)
Mark Scheel (Caltech)
Stuart Shapiro (UIUC)
Deirdre Shoemaker (PSU)
Wolfgang Tichy (FAU)
Yosef Zlochower (UT-Brownsville)

Coalescences of binary black holes and and binary neutron stars are among the strongest and most interesting sources of gravitational radiation for ground-based and space-based gravitational wave detectors. Understanding the final merger stage of these sources requires solving the Einstein equations in 3-D using numerical relativity

This workshop will focus on recent progress in numerical relativity calculations of orbits, mergers, and gravitational wave production. The main emphasis will be on binary black holes, with secondary emphasis on binary neutron stars. Current work in these areas will be highlighted, with the focus on the simulations and their results.

This workshop will be held at NASA's Goddard Space Flight Center for 3 full days from Wednesday through Friday, November 2 - 4, 2005. This is a small working meeting, open to the numerical relativity community. It will feature invited talks with ample time for discussion; posters are also welcome. The workshop is limited to 30 participants, including the speakers. If you are interested in attending, please fill out the application form.

- **Application** - *We're sorry, but the conference is full and we cannot accept any more applications.*
- Contact (To request further information)
- Presentations

Effective-One-Body Approach: First Spinning-Precessing IMR Waveforms

PHYSICAL REVIEW D **74,** 104005 (2006)

Transition from inspiral to plunge in precessing binaries of spinning black holes

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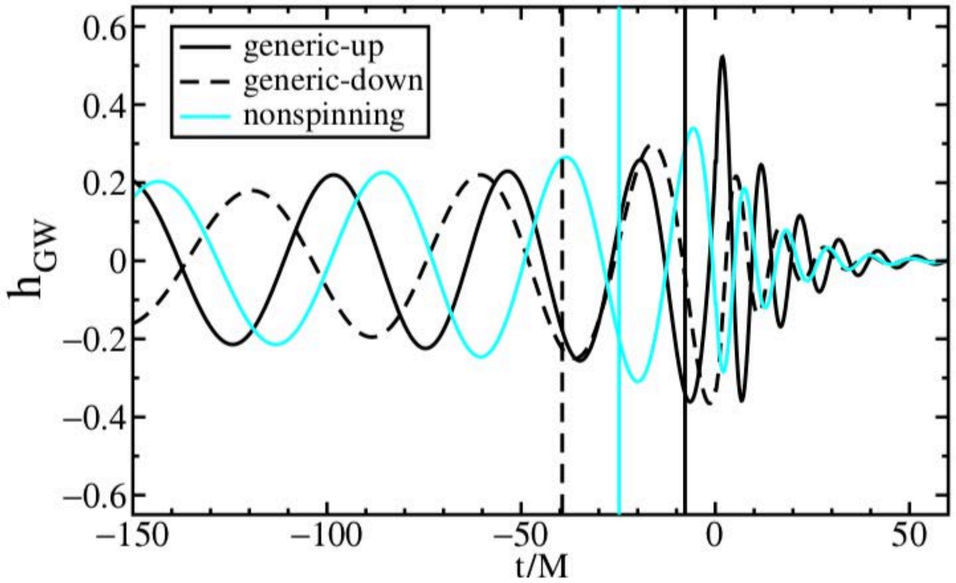
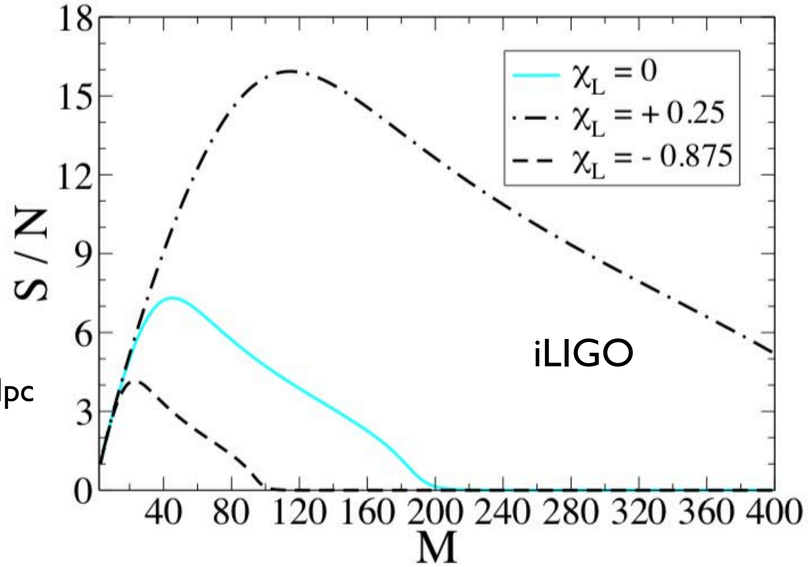
(Received 8 November 2005; published 2 November 2006)

We investigate the nonadiabatic dynamics of spinning black-hole binaries by using an analytical Hamiltonian completed with a radiation-reaction force, containing spin couplings, which matches the known rates of energy and angular-momentum losses on quasicircular orbits. We consider both a straightforward post-Newtonian-expanded Hamiltonian (including spin-dependent terms), and a version of the resummed post-Newtonian Hamiltonian defined by the effective one-body approach. We focus on the influence spin terms have on the dynamics and waveforms. We evaluate the energy and angular momentum released during the final stage of inspiral and plunge. For an equal-mass binary the energy released between 40 Hz and the frequency beyond which our analytical treatment becomes unreliable is found to be, when using the effective one-body dynamics, 0.6%M for antialigned maximally spinning black holes, 5%M for aligned maximally spinning black holes, and 1.8%M for nonspinning configurations. In confirmation of previous results, we find that, for all binaries considered, the dimensionless rotation parameter J/E^2 is always smaller than unity at the end of the inspiral, so that a Kerr black hole can form right after the inspiral phase. By matching a quasinormal mode ring down to the last reliable stages of the plunge, we construct complete waveforms approximately describing the gravitational-wave signal emitted by the entire process of coalescence of precessing binaries of spinning black holes.

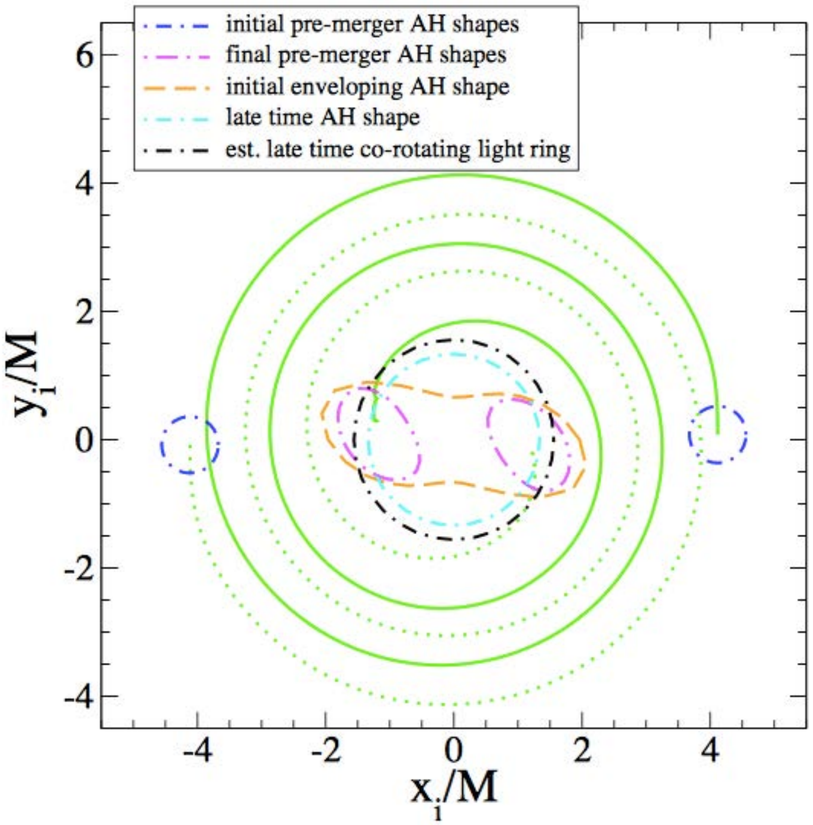


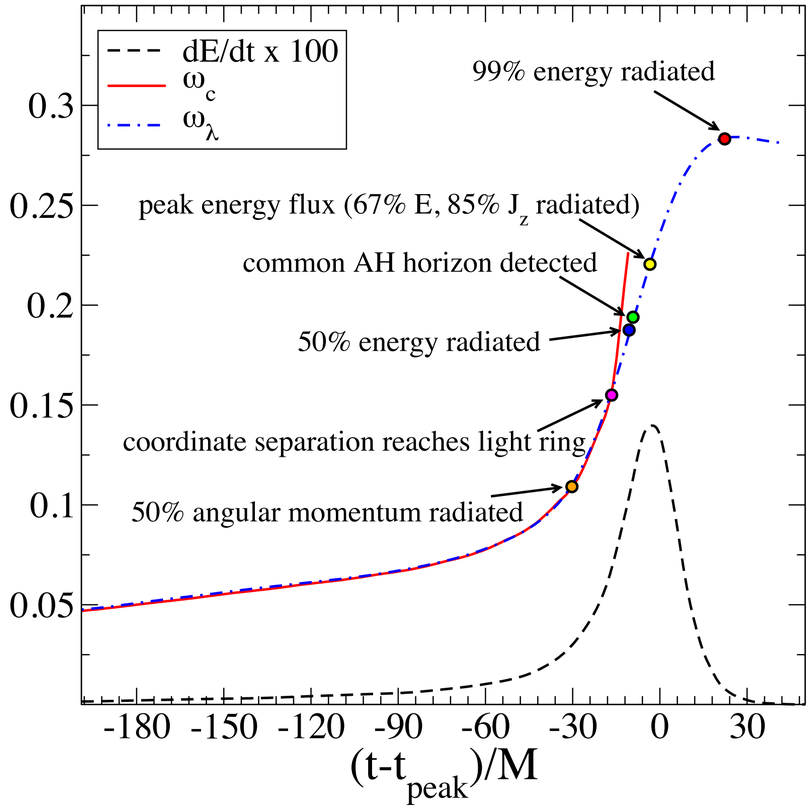


signal-to-noise at 100 Mpc

Plunge and Merger in First NR Simulations

(AB, Cook & Pretorius 07)

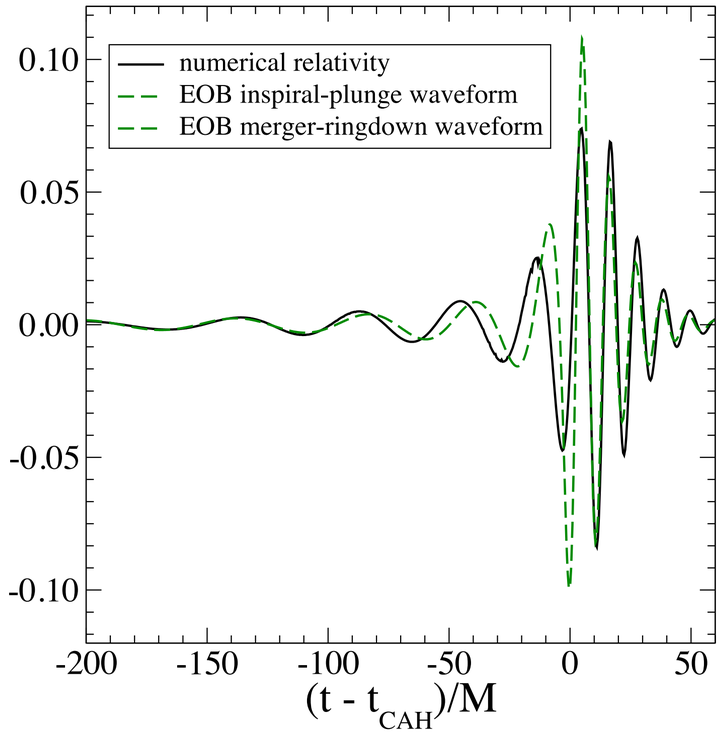




- Very **short/simple transition** plunge-merger-ringdown
- Energy **quickly released** during merger: $2\% - 12\,\% M$

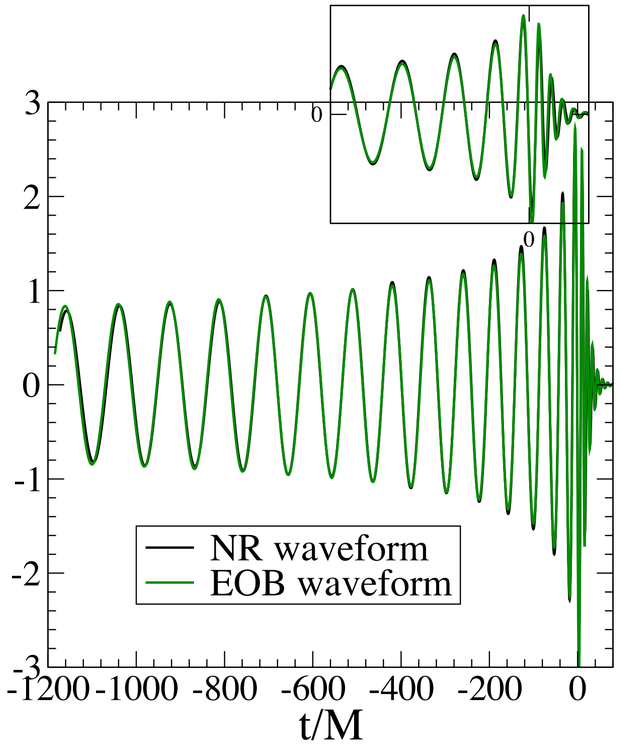
First Comparisons with NR — First EOBNR Waveforms for LIGO search

- **Uncalibrated** EOB waveform at 3.5PN order



(AB, Cook & Pretorius 07)

- **Calibrated** EOBNR waveform



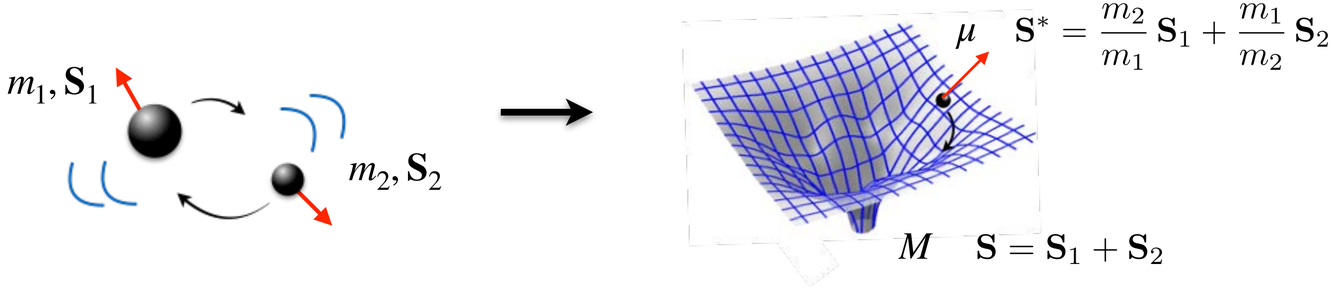
(AB, Pan, Baker, Centrella, Kelly et al. 08)

crucial collaboration with Sathyaprakash's group at Cardiff University

- EOBNR waveforms used in **iLIGO/Virgo BBH searches. Data taken from 2005 to 2007, and from 2009-2011.**

(Abadie et al. PRD 83 (2011) 122005, Aasi et al. PRD 87 (2013) 2, 022002)

EOB Formalism for Spinning Binaries: Conservative Dynamics



$$H^{\rm EOB}_{\rm real} = M\sqrt{1+2\nu\left(\frac{H^\nu_{\rm eff}}{\mu}-1\right)}$$

- What is $H^\nu_{\rm eff}$ when compact objects carry spin?

$$g^{\mu\nu}_{\rm Kerr}\,p_\mu\,p_\nu = -\mu^2 \qquad H^{\rm Kerr}_0 \equiv -p_0$$

(for simplicity we restrict to equatorial orbits)

- **Test spin** in Kerr spacetime:

$$H^{\rm Kerr} = \mu\sqrt{A^{\rm Kerr}\left(1+\frac{p_\varphi^2}{\mu^2 (r_c^{\rm Kerr})^2}+\frac{p_r^2}{\mu^2 B^{\rm Kerr}}\right)} + \left[G_{\rm S}^{\rm Kerr}(\mathbf{r})\,a + G_{\rm S_*}^{\rm Kerr}(\mathbf{r},\mathbf{p})\,a_*\right]p_\varphi$$

gyro-gravitomagnetic functions

$$a^* = \frac{S_*}{\mu M}$$

(Barausse, Racine & AB 10, Barausse & AB 11)

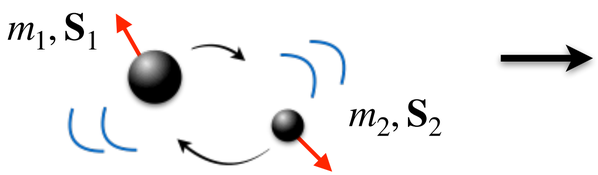
- **Test mass** in Kerr spacetime:

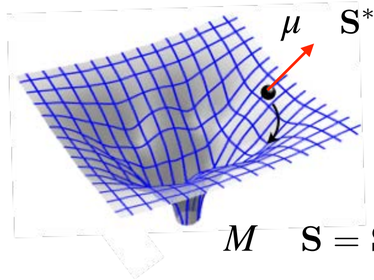
$$H^{\rm Kerr}_0 = \mu\sqrt{A^{\rm Kerr}\left(1+\frac{p_\varphi^2}{\mu^2 (r_c^{\rm Kerr})^2}+\frac{p_r^2}{\mu^2 B^{\rm Kerr}}\right)} + G_{\rm S}^{\rm Kerr}(\mathbf{r})\,a\,p_\varphi$$

$$a = \frac{S^{\rm Kerr}}{M^2}$$

(Damour 01, Rettegno et al. 20)

EOB Formalism for Spinning Binaries: Conservative Dynamics & Dissipation





$$\mathbf{S}^* = \frac{m_2}{m_1}\,\mathbf{S}_1 + \frac{m_1}{m_2}\,\mathbf{S}_2 \qquad \mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$$

$$H^{\rm EOB}_{\rm real} = M\sqrt{1 + 2\nu\left(\frac{H^{\nu}_{\rm eff}}{\mu} - 1\right)}$$

- What is $H^{\nu}_{\rm eff}$ when compact objects carry spin? Mapping is not unique, variants of $H^{\nu}_{\rm eff}$ exist.

- **Test spin**: $H^{\nu}_{\rm eff} = \mathbb{H}^{\text{Kerr-orb},\nu}_{\rm eff} + \left[g^{\nu}_{\rm S}(\mathbf{r},\mathbf{p})\,S + g^{\nu}_{\rm S^*}(\mathbf{r},\mathbf{p})\,S^*\right]p_{\varphi} + H^{\text{SS},\nu}_{\rm eff}$

(SEOBNR)

(Barausse, Racine & AB 10, Barausse & AB 11, 12, Vines et al. 16, Khalil et al. 20)

- **Test mass**: $H^{\nu}_{\rm eff} = H^{\text{Kerr-orb},\nu}_{\rm eff} + \left[G_{\rm S}(\mathbf{r},\mathbf{p})\,S + G_{\rm S^*}(\mathbf{r},\mathbf{p}), S^*\right]p_{\varphi}$

(TEOBResumS)

(Damour 01, Damour, Jaranowski & Schäfer 08; Damour & Nagar 14; Rettegno et al. 20)

- **Improved radiation-reaction force** and waveform modes:

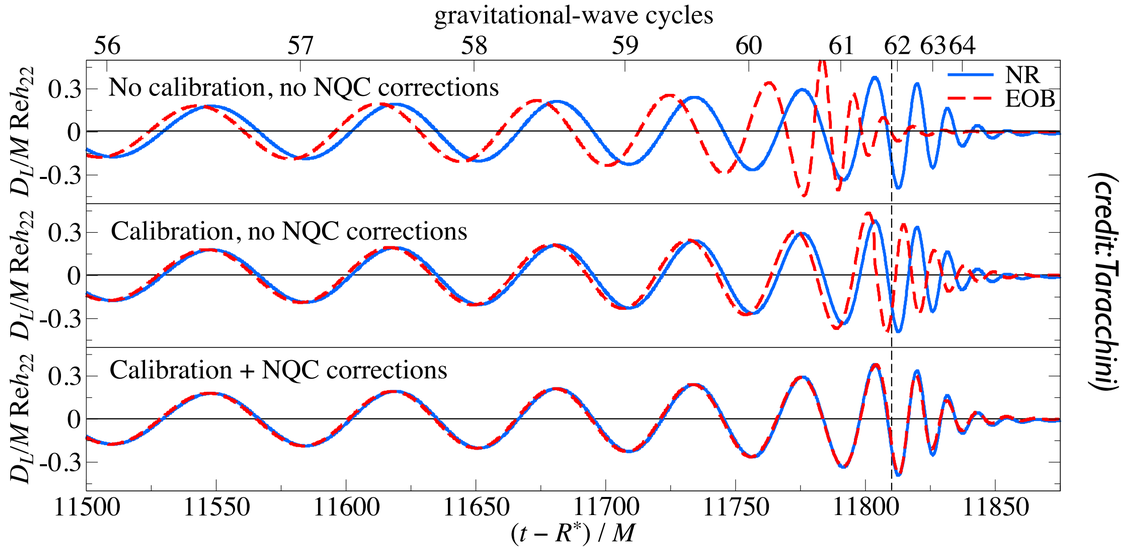
$$F \propto \frac{dE}{dt} \propto \sum_{\ell m} |\dot{h}_{\ell m}|^2$$

$$h^{\rm insp-plunge}_{\ell m} = h^{\rm Newt}_{\ell m}\, e^{-im\Phi}\, S_{\rm eff}\, T_{\ell m}\, e^{i\delta_{\ell m}}\, (\rho_{\ell m})^{\ell}\, h^{\rm NQC}_{\ell m}$$

(Damour, Iyer & Nagar et al. 09, 11; Pan, AB et al. 11)

Completing EOB Waveforms with NR Information & Template Bank

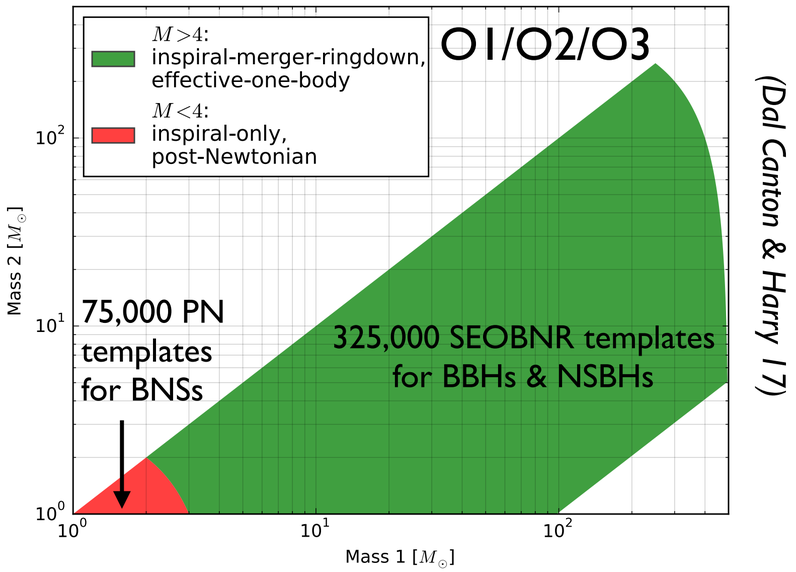
- EOB calibration to **inspiral-merger-ringdown NR** waveforms.



(credit: Taracchini)

(NQC: non-quasi-circular corrections)

- **Matched filtering** employed in LIGO/Virgo searches.



(Dal Canton & Harry 17)

Calibration of SEOBNR for O1-O2-O3 searches and inference studies

crucial collaboration with Simulating eXtreme Spacetimes Collaboration

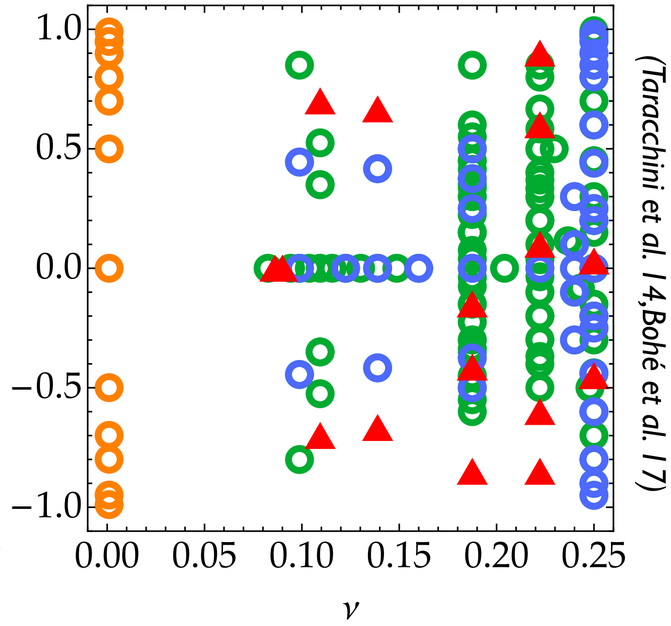
141 SXS simulations

SEOBNRv4
SEOBNRv2
Teukolsky
validation

$\chi_1 = S_1/m_1^2$

$\chi_2 = S_2/m_2^2$

$\chi_{\rm eff} = \frac{m_1}{M}\chi_1 + \frac{m_2}{M}\chi_2$



(Taracchini et al. 14, Bohé et al. 17)

(Pan, AB et al. 13, Taracchini, AB et al. 14, Pürrer 15, Bohé, Shao, Taracchini, AB & SXS 17, Babak et al. 16; Cotesta et al. 18, 20, Ossokine et al. 20; SEOBNR waveforms*)*

(see also Damour & Nagar 14, Nagar et al. 18, Nagar, Messina et al. 19, Nagar, Pratten et al. 20, Nagar, Riemenschneider et al. 20, Riemenschneider et al. 21; TEOBResumS waveforms)

Phenomenological & NR-Surrogate Waveforms

- **Fast, frequency-domain** waveform model hybridizing EOB & NR waveforms, and then fitting.

(Schmidt et al.12; Hannam et al. 13; Khan et al. 15; Husa et al. 15; Khan et al. 18-19; García-Quíros et al. 20, Pratten et al. 20)

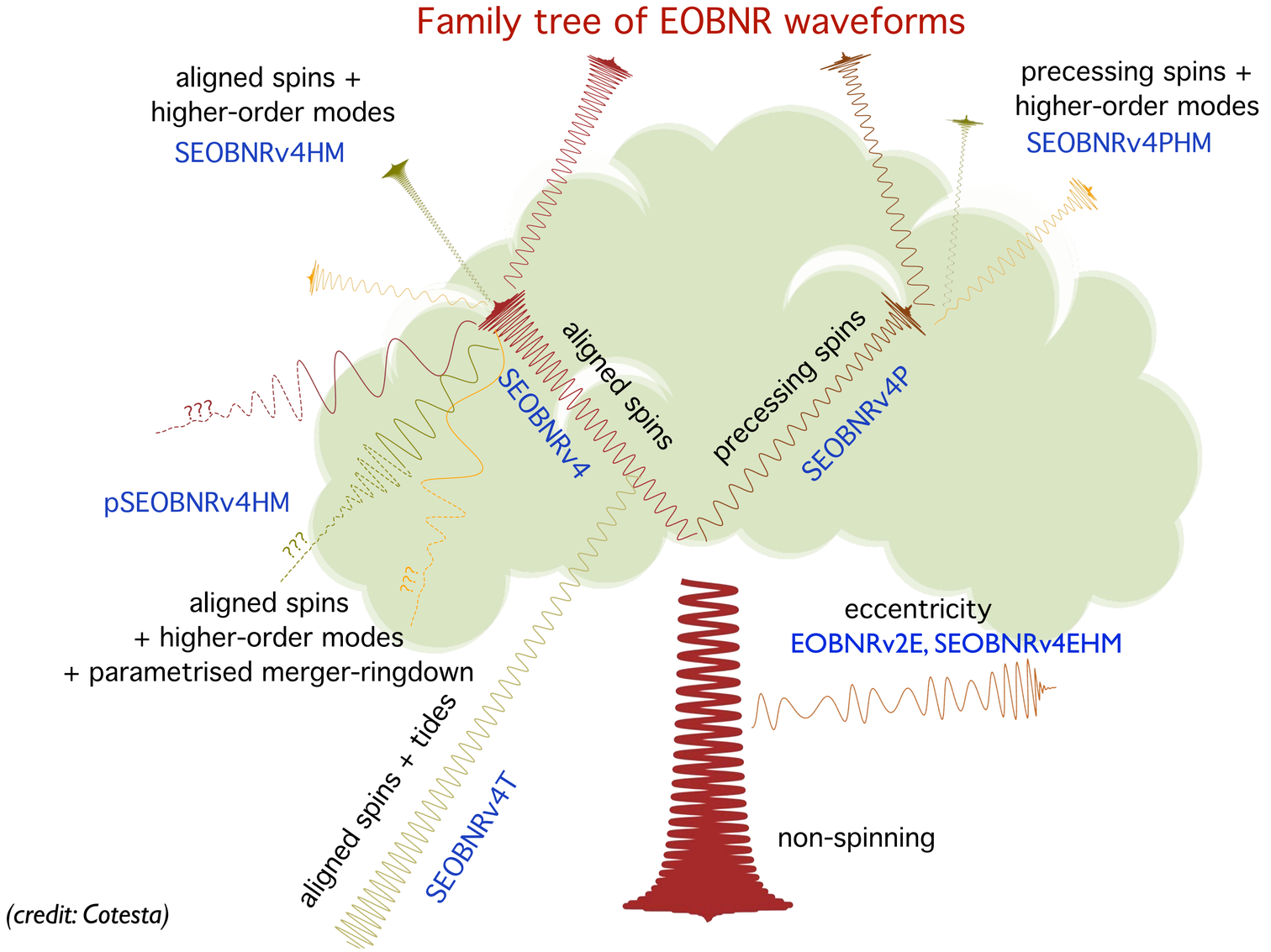
IMRPhenom $\tilde{h}(f;\lambda_i) = \mathcal{A}(f;\lambda_i)\, e^{i\phi(f;\lambda_i)}$

- **NR surrogate models** are built directly interpolating **NR simulations,** which are **selected in parameter space** using analytical waveform models.

- They are **highly accurate**, but **limited in parameter space** and **length** (~20 orbits), unless hybridized with EOBNR waveforms.

(Field et al. 11,14; Blackman et al. 15; Canizares et al. 15, Varma et al 19)

Ever More Physics in Waveform Models

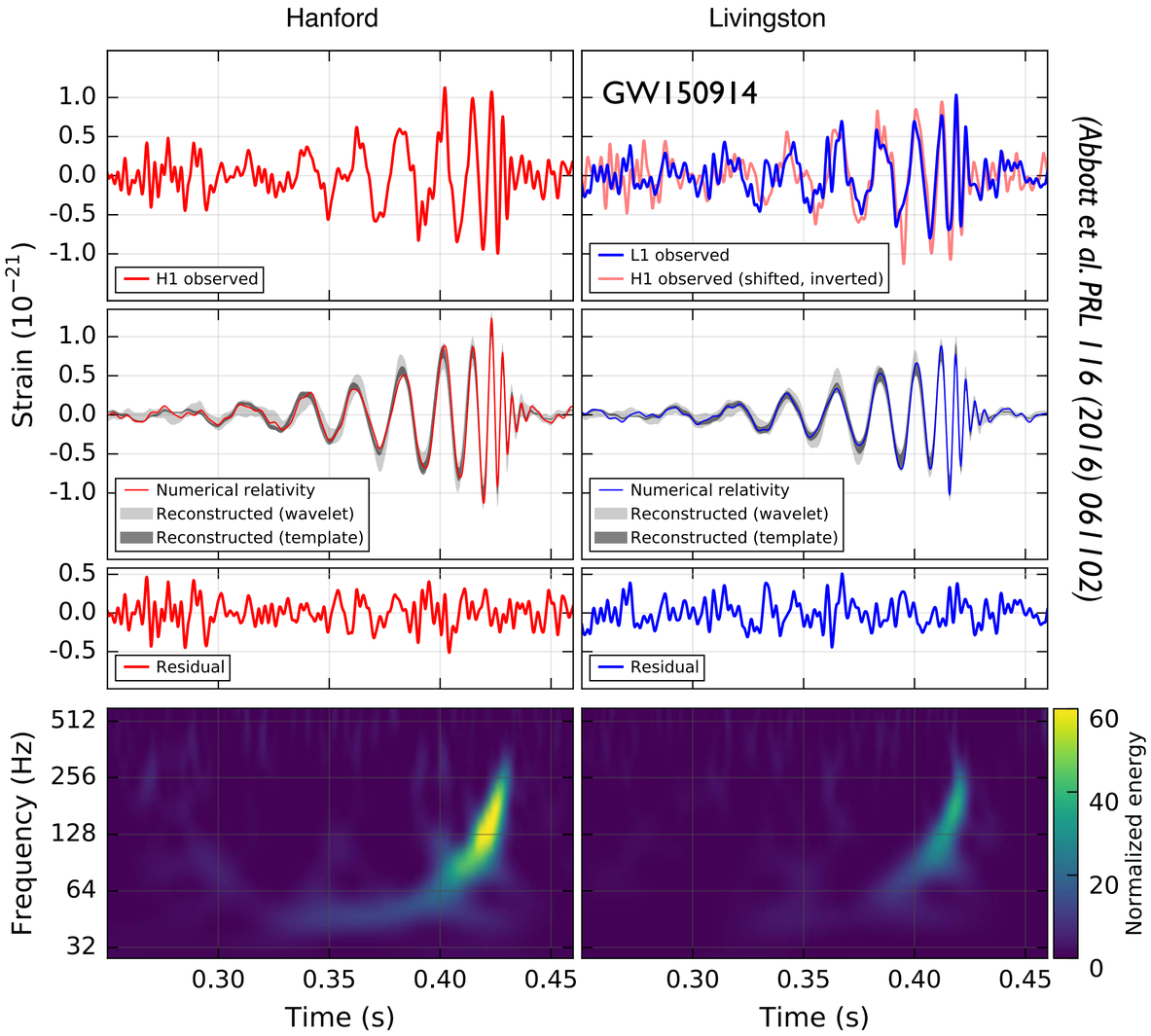


(credit: Cotesta)

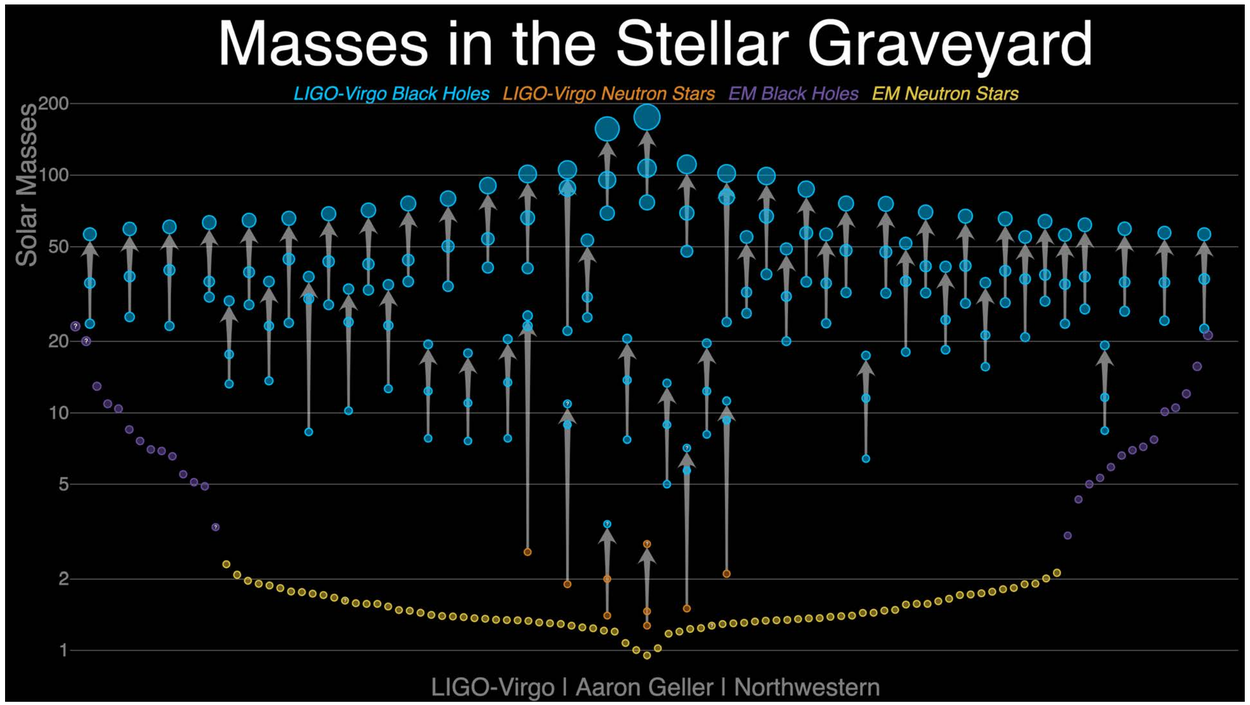
Other family trees for IMRPhenom, NRSur & TEOBResumS

Gravitational Waves Ushered in New Era of Astrophysics

- **Discovery** of **GW** from **a binary black-hole merger** by LIGO

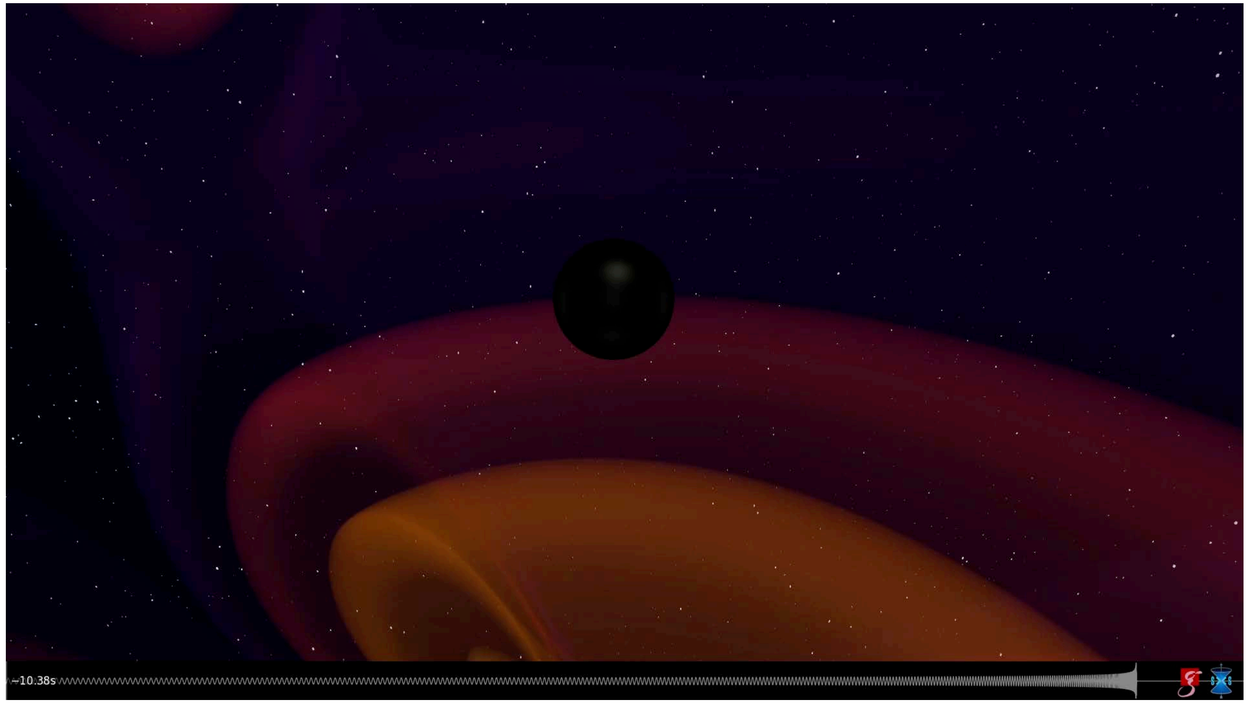


- **Since GW150914** was observed, **55 more binary black holes (BBH), 2 binary neutron stars (BNS) and 2 NSBH discovered** by LIGO/Virgo.



GW190814: a Binary with a Puzzling Companion

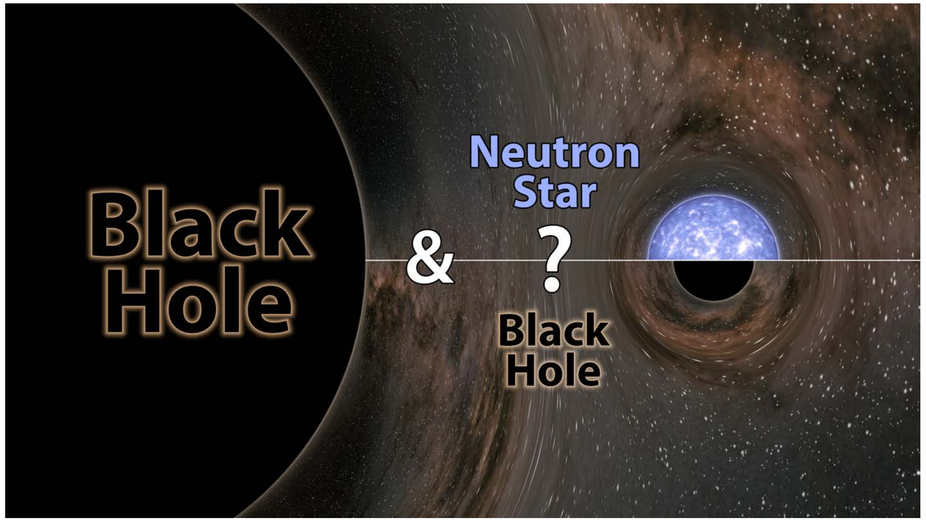
- The **more substructure and complexity** the binary has (e.g., masses or spins of BHs are different) **the richer is the spectrum of radiation** emitted.



(credit: Fischer, Pfeiffer, Ossokine & AB; SXS Collaboration)

- Either the **largest neutron star** or the **smallest black hole.**

$$m_1 = 23.2^{+1.1}_{-1.0}\,M_\odot \qquad m_2 = 2.59^{+0.08}_{-0.09}\,M_\odot$$



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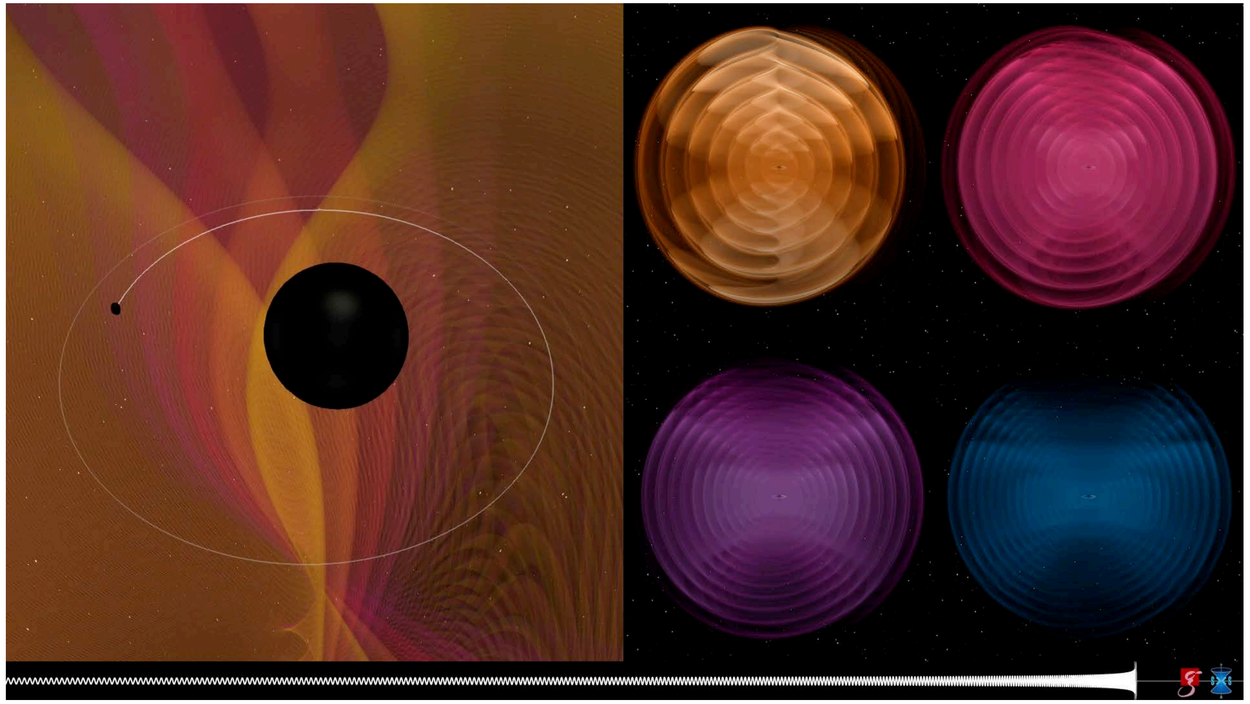

(credit: Fischer, Pfeiffer, Ossokine & AB; SXS Collaboration)

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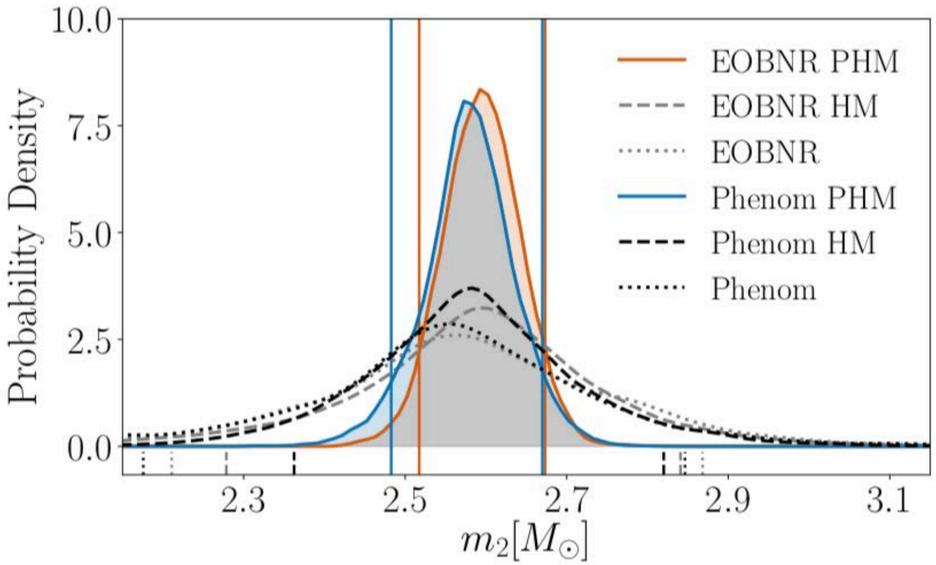


- Using waveform models with **higher-modes and spin-precession constrains more tightly** the **secondary mass.**

GW190521: a Signal Produced by the Largest BHs so far

(Abbott et al. PRL 125 (2020) 10, ApJ Lett 900 (2020) L13)

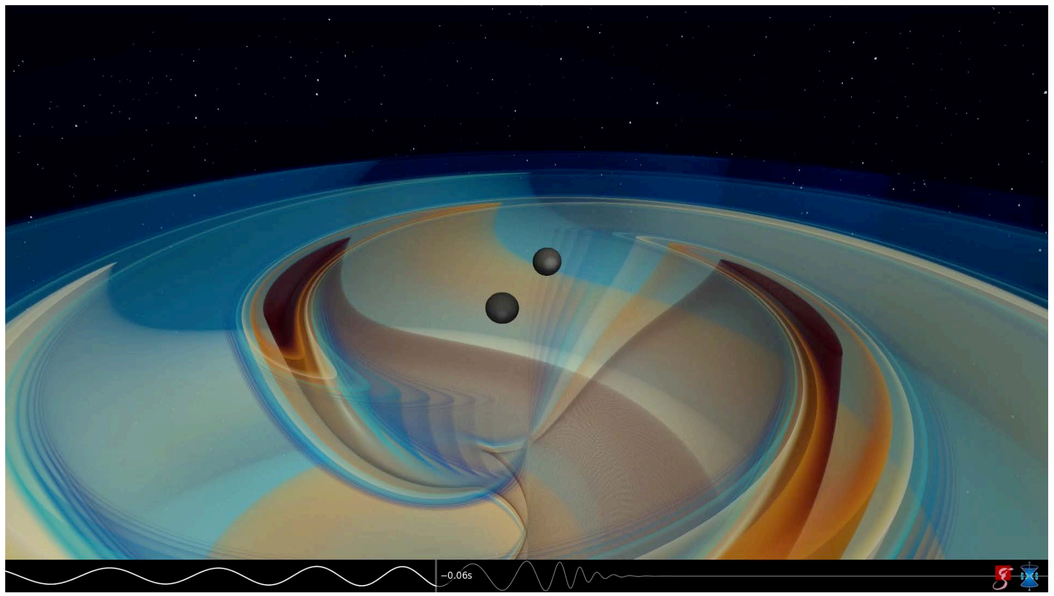
- Likely, BHs **too massive** to have been formed **from a collapsed star, because of Pair-Instability SN.**

$$m_1 = 91.4^{+29.3}_{-17.5}\,M_\odot \qquad m_2 = 66.8^{+20.7}_{-20.7}\,M_\odot$$

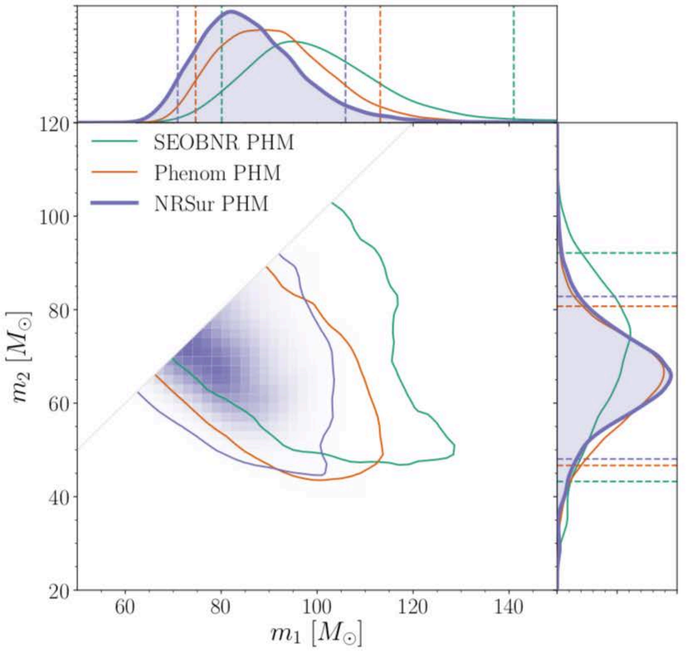
$$q = m_1/m_2$$
$$\chi_1 = S_1/m_1^2$$
$$\chi_2 = S_2/m_2^2$$

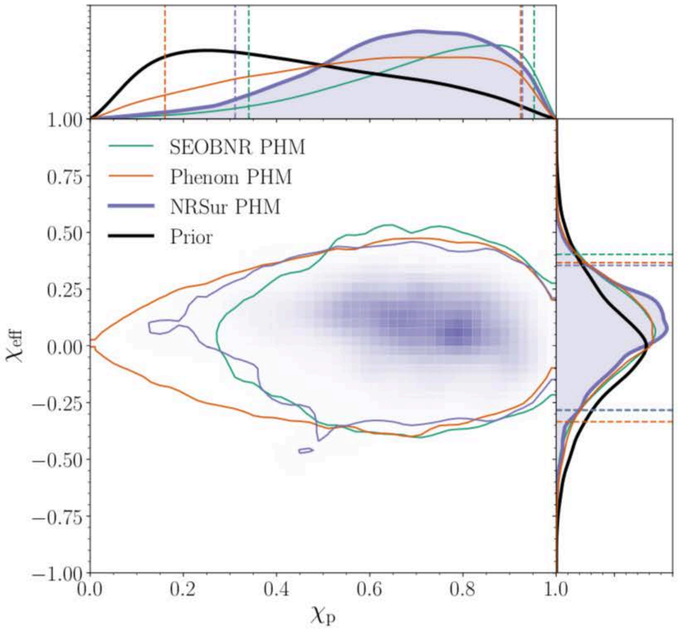
$$\chi_{\rm eff} = \left(\frac{m_1}{M}\chi_1 + \frac{m_2}{M}\chi_2\right)\cdot\hat{\mathbf{L}}$$

χ_p measures the spin components on the orbital plane



(credit: Fischer, Pfeiffer & AB; SXS Collaboration)





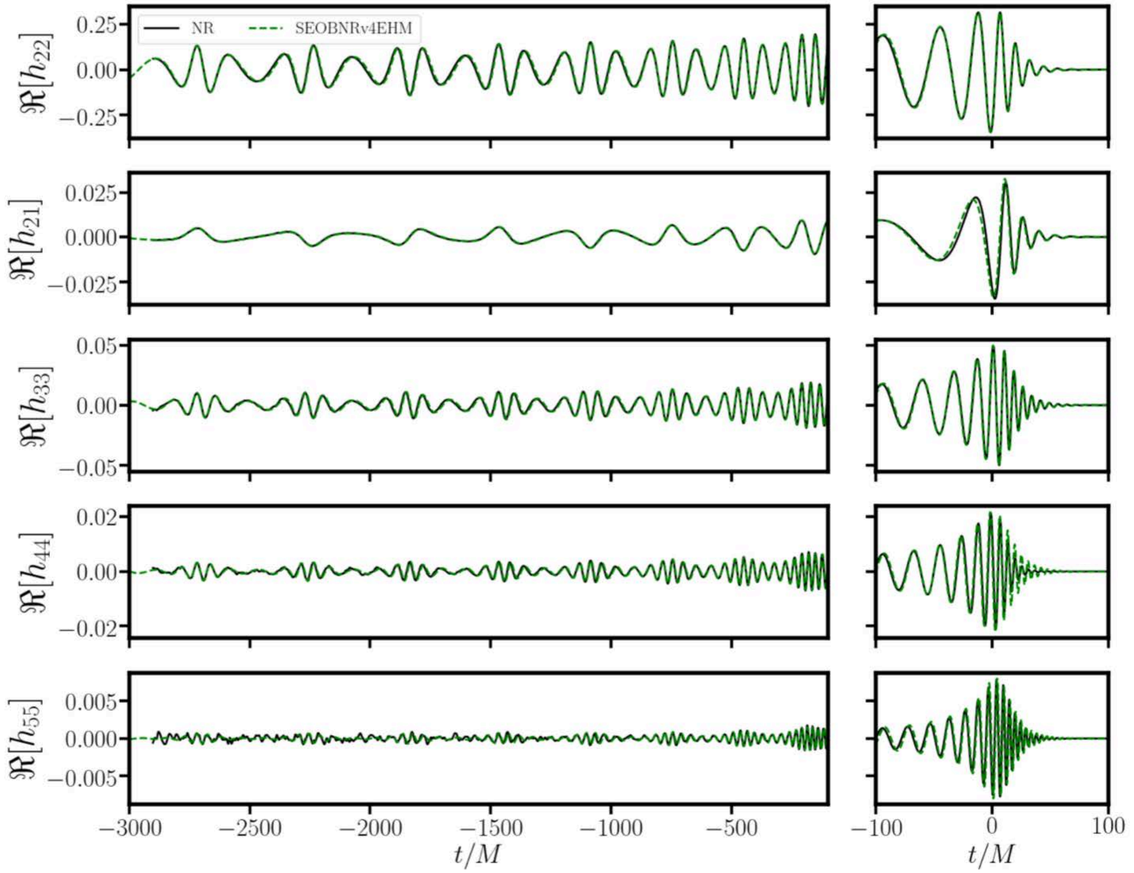
- **Systematics** due to waveform modeling **are not negligible when spin precession** and **higher modes are relevant,** but they are still **subdominant with respect to statistical uncertainty.**

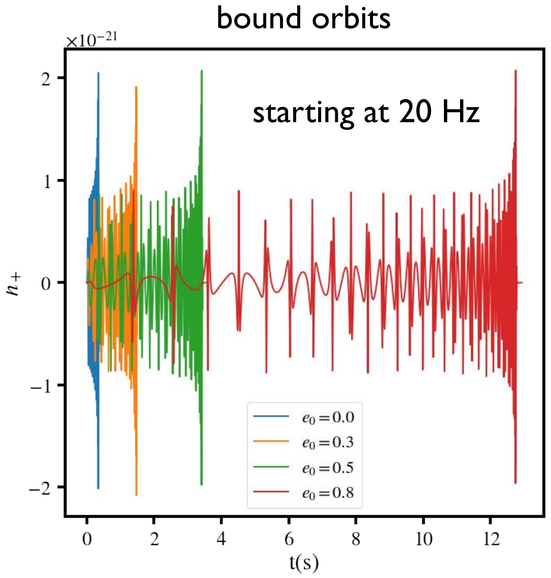
EOBNR Waveforms for Eccentric Non-Precessing Spinning BBHs

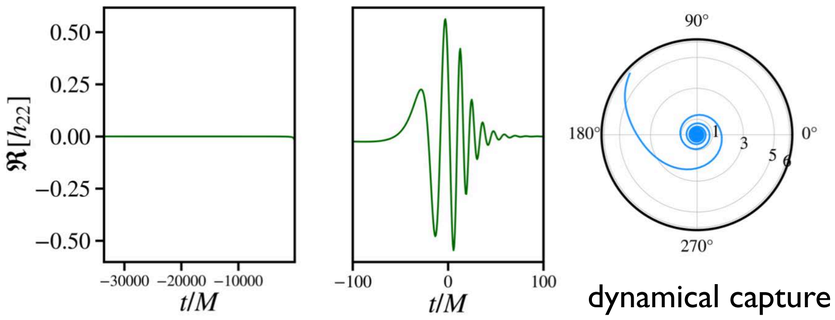
- **To avoid misinterpreting GW signals** and discriminate between different binary's formation scenarios, it is crucial to **extend waveform models to eccentric dynamics.**

(Khalil, AB, Steinhoff & Vines 21, Ramos-Buades, AB et al. in prep 21)

mass ratio = 2, non-spinning, e = 0.06







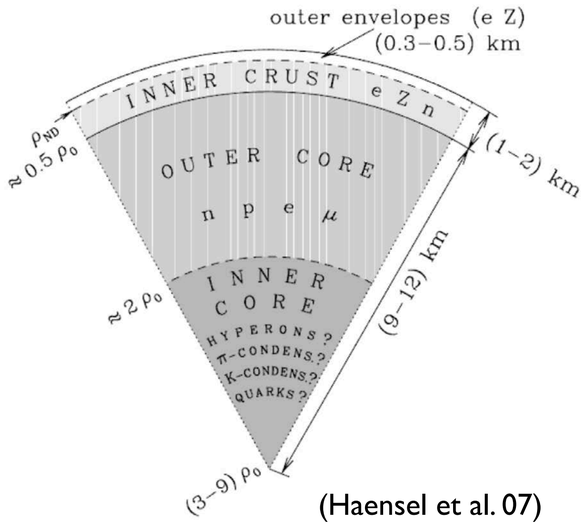
(see also Huerta et al. 14-19, Hinder et al. 17, Cao & Han 17; Loutrel & Yunes 16, 17, Ireland et al. 19, Moore & Yunes 19, Tiwari et al. 19, Chiaramello & Nagar 20, Ramos-Buades et al. 20, Liu et al. 21, Nagar et al. 20, 21, Islam et al. 21, Nagar & Rettegno 21)

Probing Extreme-Matter with Gravitational Waves

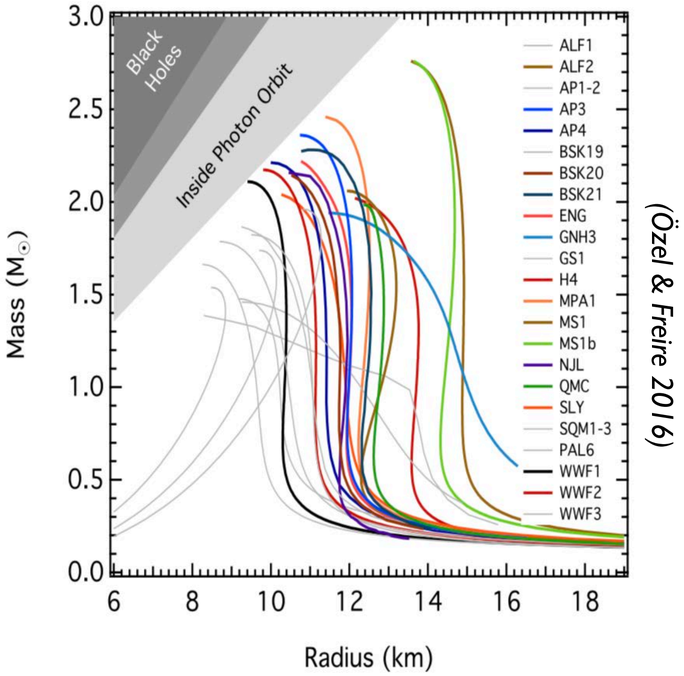
- **Neutron-star (NS)** properties:

- mass: $1 - 3\,M_{\rm Sun}$
- radius: $9 - 15\,\mathrm{km}$
- inner core density > $2 \times (2.8 \times 10^{14})\,\mathrm{g/cm^3}$
- magnetic field: ~$10^{15}\times$ @Earth
- surface temperature: ~$10^{3}\times$ @Earth
- pressure: ~$10^{27}\times$ @Earth

nuclear density



(Haensel et al. 07)



(Özel & Freire 2016)

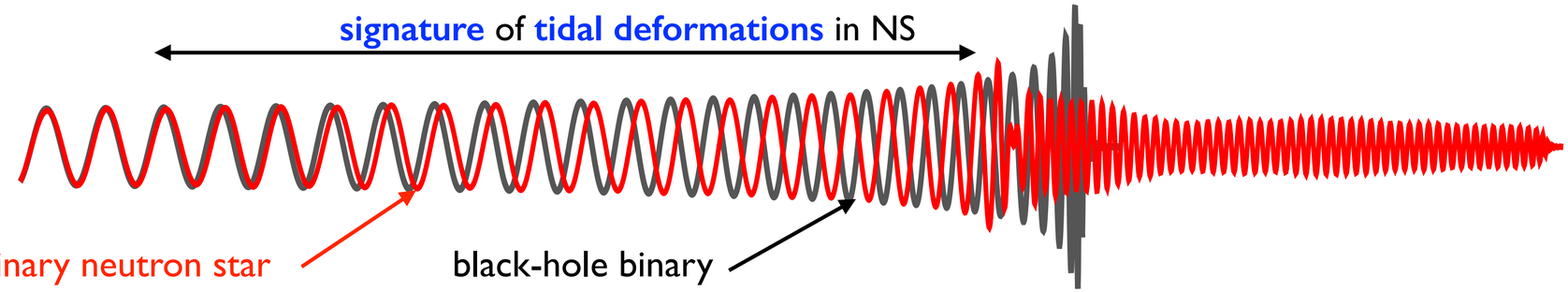
- What is the **internal structure** and **composition** of **neutron stars**?

- New parameter in BNS: **tidal deformability** λ.

$$Q_{ij} = -\lambda\,\mathscr{E}_{ij}$$

quadrupole moment external tidal field

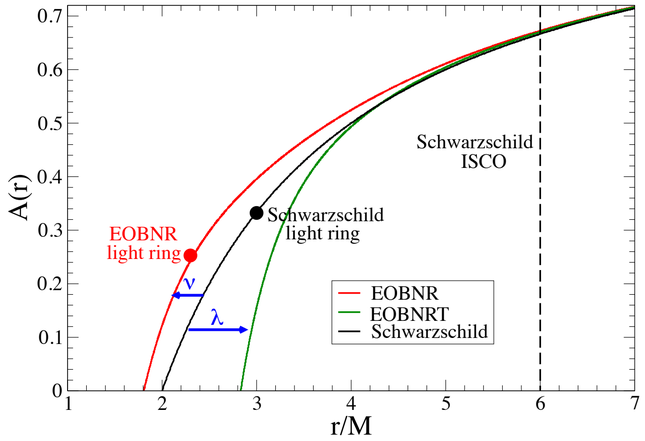
- **NS equation of state** (EOS) affects gravitational **waveform** during **late inspiral, merger** and **post-merger**.



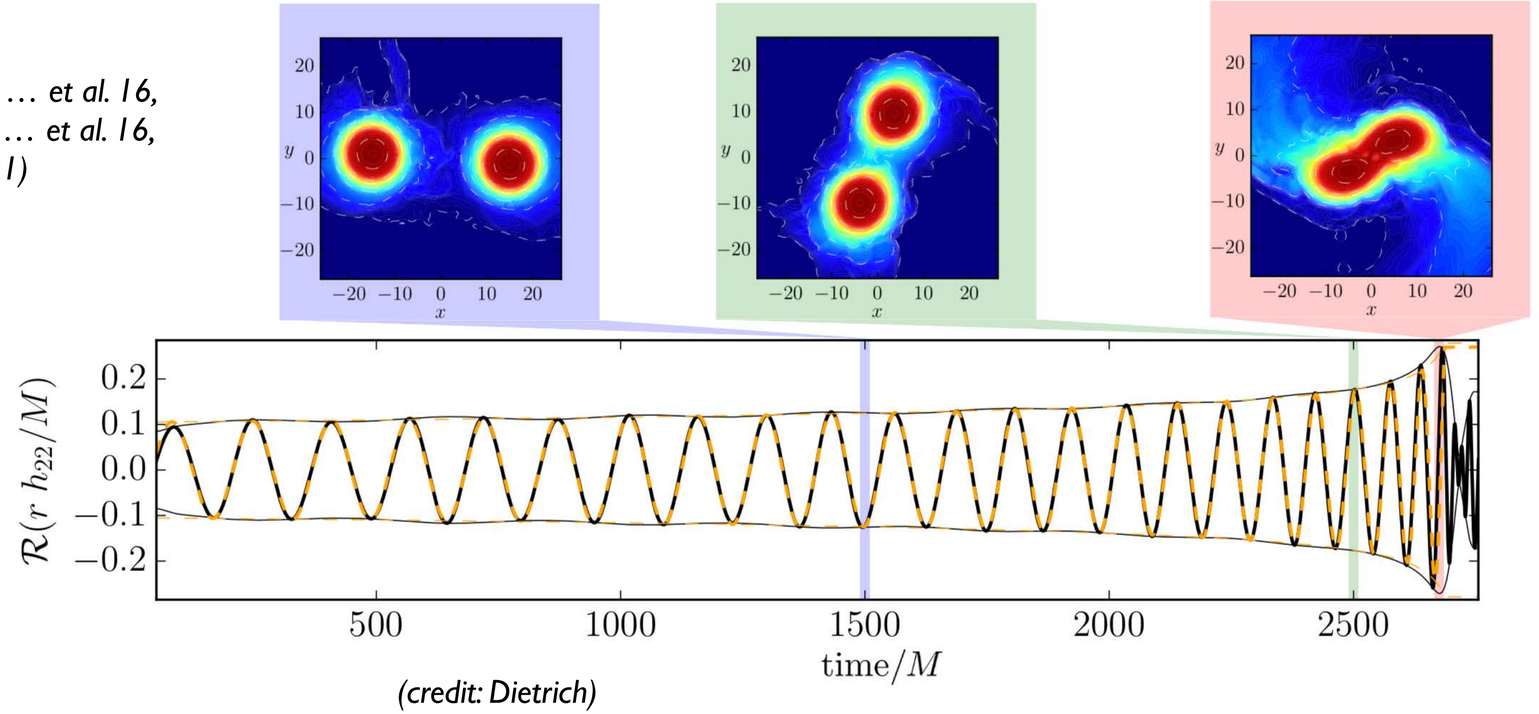
EOBNR Waveforms for Binary Neutron Stars

- Strong-field effects **in presence of matter in EOB**

$$A(r) = A_{\nu}(r) + A_{\rm tides}(r)$$



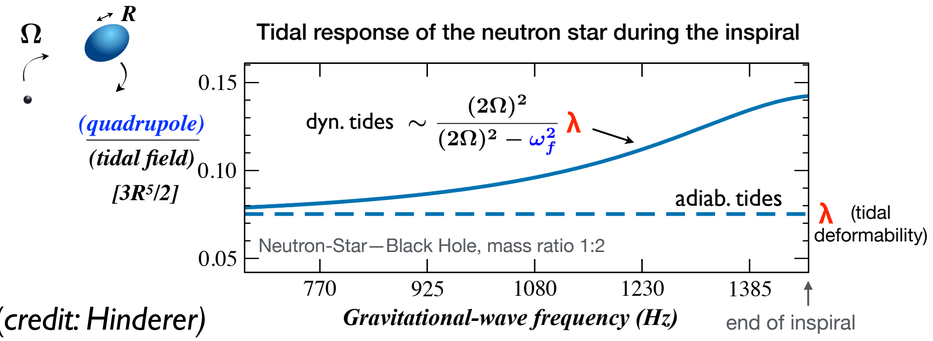
(Hinderer, …AB … et al. 16, Steinhoff, … AB … et al. 16, Steinhoff et al. 21)



(credit: Dietrich)

NR
SEOBNRT

- **Dynamical tides**



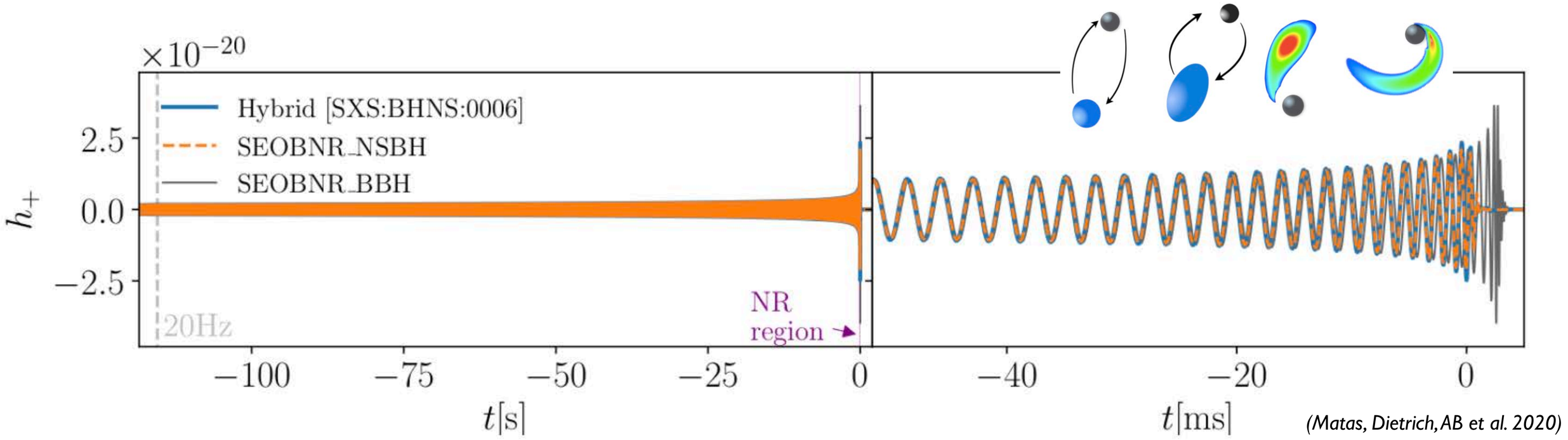
(credit: Hinderer)

- **BNS waveforms** were used to infer properties **of GW170817 & GW190425**.

(see also Damour 1983, Flanagan & Hinderer 08, Binnington & Poisson 09, Vines et al. 11, Damour & Nagar 09, 12, Bernuzzi et al. 15, Dietrich et al. 17-19, Nagar et al. 18, Gamba et al. 20)

EOBNR Waveforms for Neutron-Star—Black-Hole Binaries

- Synergy between **analytical** and **numerical work** is **crucial.**



(Matas, Dietrich, AB et al. 2020)

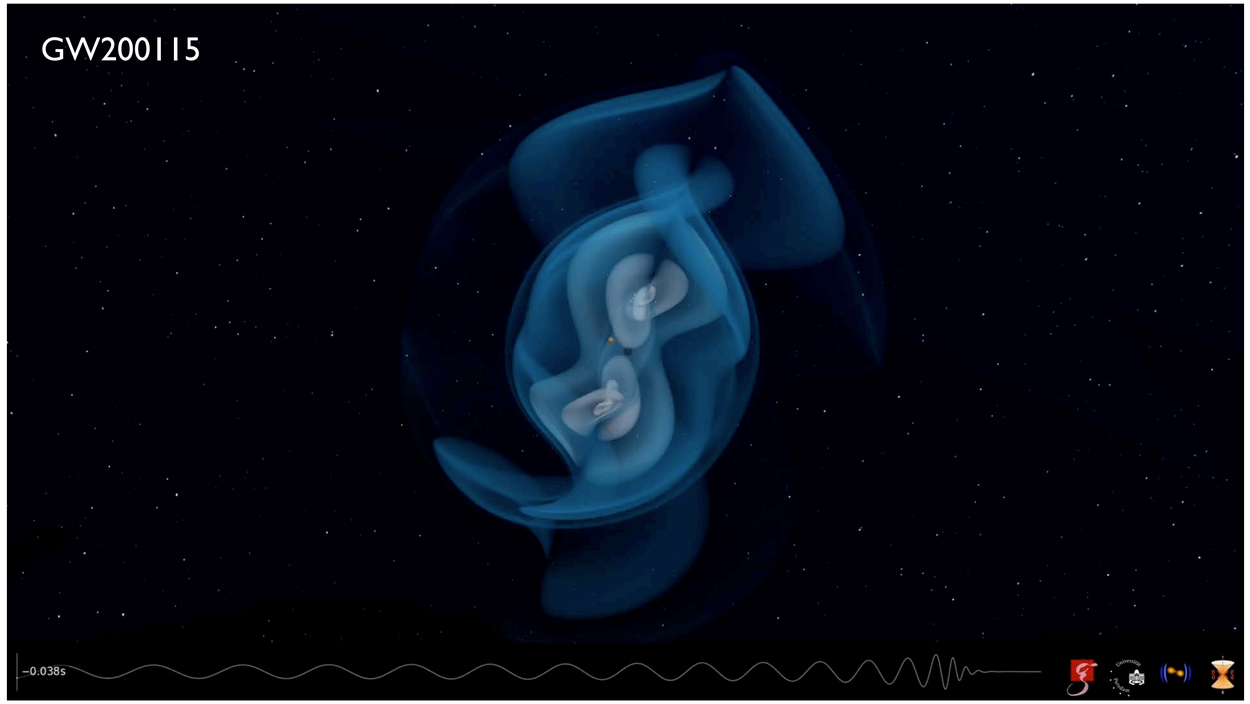
(see also Thompson et al. 2020)

(see also Lackey et al. 14, Pannarale et al. 15, 16, Pürrer et al. 17, Chakravarti et al. 17)

- So far, **NSBH waveforms** were used **to infer properties** of **GW190814 and two NSBHs discovered during O3.**

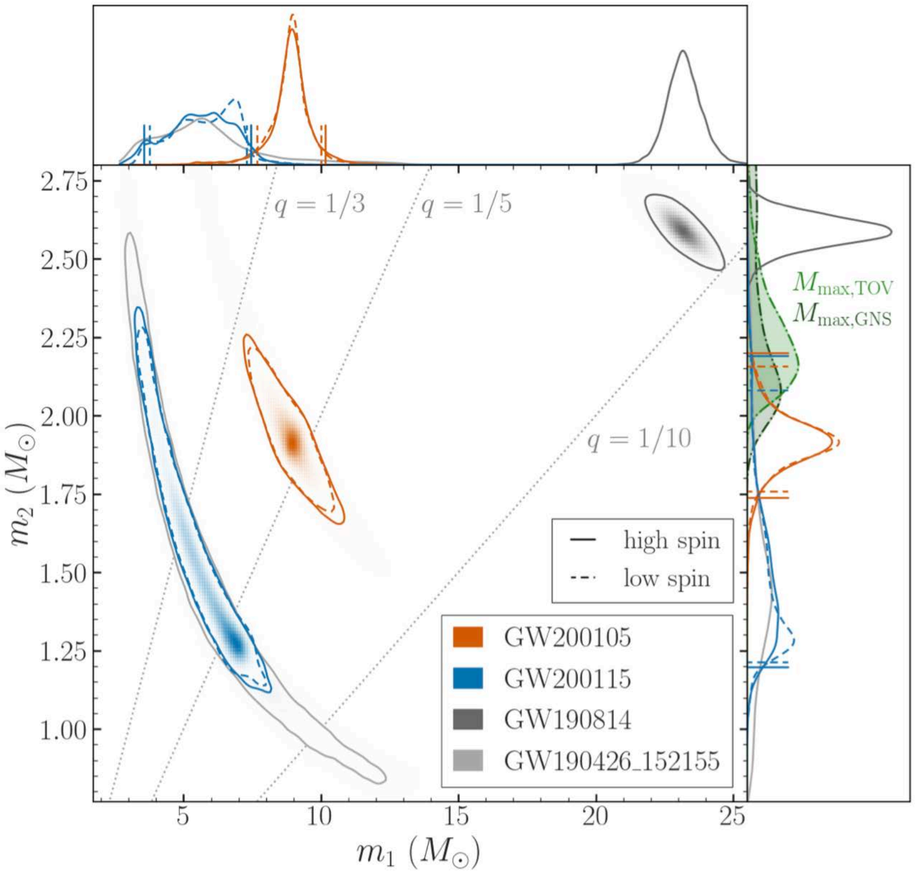
GW200105 & GW200115: two BHs swallowing the NS whole

- First **robust detection** of a mixed binary.



(credit: Chaurasia, Dietrich, Fischer, Ossokine & Pfeiffer)

(Abbott et al. APJ 915 (2021))



Tests of GR with LIGO-Virgo's Observations: Inspiral & Merger

- BBHs/BNS/NSBH **rapidly varying orbital periods** allow us to **probe GW phase**.

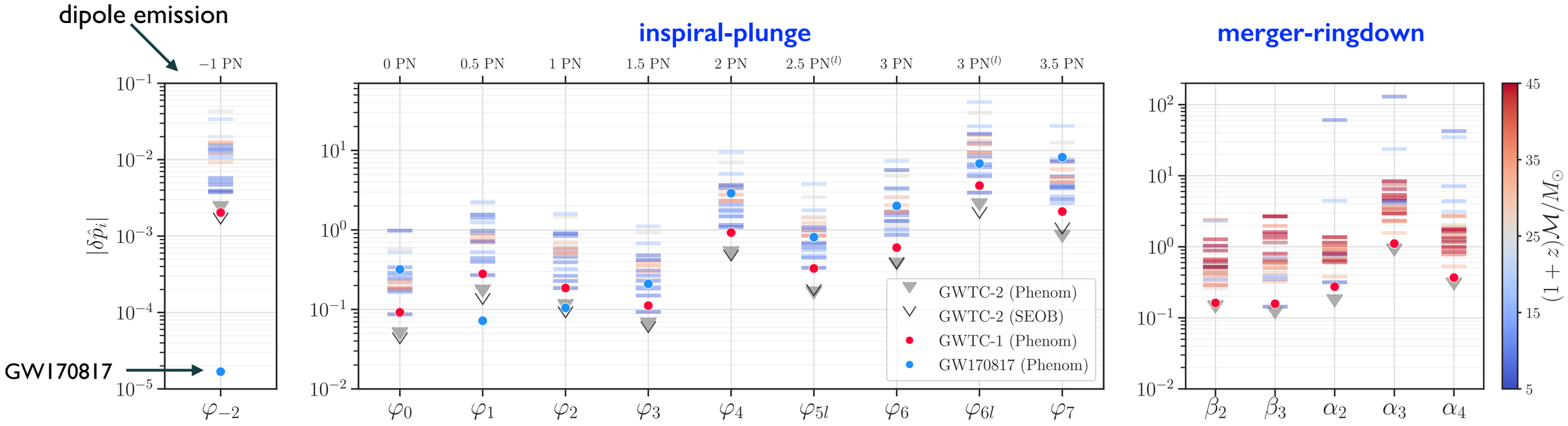
- Inspiral(-Plunge):

(Blanchet & Sathyaprakash 1995, Arun et al. 06 , Mishra et al. 10, Yunes & Pretorius 09, Li et al. 12)

$$h_{\rm GW}(f) = \mathcal{A}_{\rm GW}(f)\, e^{i\varphi_{\rm GW}(f)} \qquad v = (\pi M f)^{1/3}$$

$$\varphi_{\rm GW}(f) = \varphi_{\rm ref} + 2\pi f t_{\rm ref} + v^{-5}\left[\sum_{n=-2}^{7} \varphi_n^{(\rm GR)}(1+\delta\hat{\varphi}_n)v^n + \sum_{n=5}^{6} \varphi_{n\,\ell}^{(\rm GR)}(1+\delta\hat{\varphi}_{n\,\ell})v^n \log v\right]$$

- **PN parameters** describe: **tails** of radiation due to backscattering, **spin-orbit** and **spin-spin** couplings.



(Abbott et al. arXiv:2010.14529)

Tests of GR with LIGO-Virgo's Observations: Remnant Properties

- In General Relativity, **remnant object** resulting from coalescence of two astrophysical BHs **is a perturbed Kerr BH**.

- The remnant **BH relaxes** to its stationary Kerr state **by emitting quasi-normal modes (QNMs).**
  (Vishveshwara 70, Press 71, Chandrasekhar et al. 75)

- The QNM's **frequencies and decay times** only depend on **BH's mass and spin** (no-hair conjecture).
  (Israel 69, Carter 71; Hawking 71, Bardeen 73)

- The **no-hair conjecture can be disproved** if more than one QNM is observed.
  (Dreyer et al. 2004, Berti et al. 2006, Gossan et al. 2012, Meidam et al. 2014)

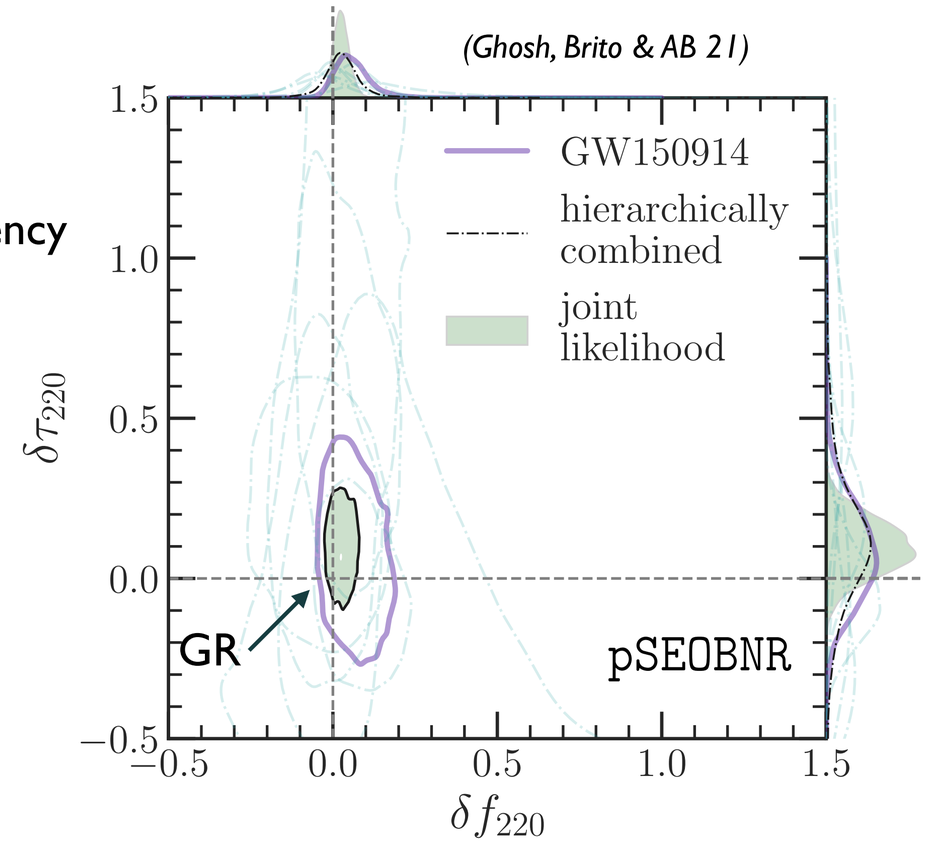
- **Inspiral-merger-ringdown** waveform model **with parameterized QNM**'s frequency and decay time (pSEOBNR):
  (Brito, AB & Raymond 18, Ghosh, Brito & AB 21)

$$f_{\ell m 0} = f^{\rm GR}_{\ell m 0}(1+\delta\hat{f}_{\ell m 0}) \qquad \delta\hat{f}_{220} = 0.02^{+0.09}_{-0.09}$$

$$\tau_{\ell m 0} = \tau^{\rm GR}_{\ell m 0}(1+\delta\hat{\tau}_{\ell m 0}) \qquad \delta\hat{\tau}_{220} = 0.13^{+0.42}_{-0.40}$$

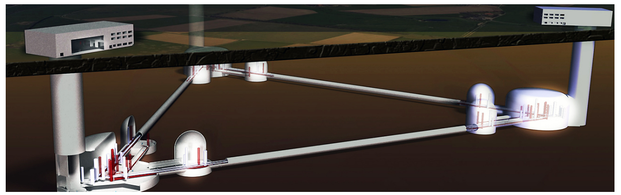
- These measurements are already **informing us on nature** of **dark object (light-ring, horizon,** etc), **e.g., remnant compactness is** $\sim 99\,\%$ **of BH compactness.** *(Cardoso et al. 18-20, Völkel et al. 20, Maggio et al. 20)*

(Abbott et al. PRD 103 (2021) 12, 122002)

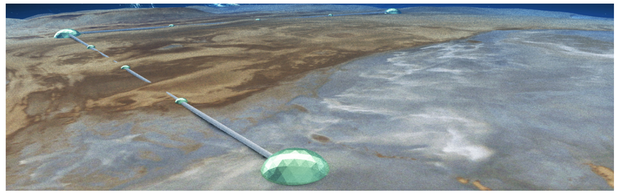


Gravitational-Wave Landscape in 2030-2040 on the Ground and in Space

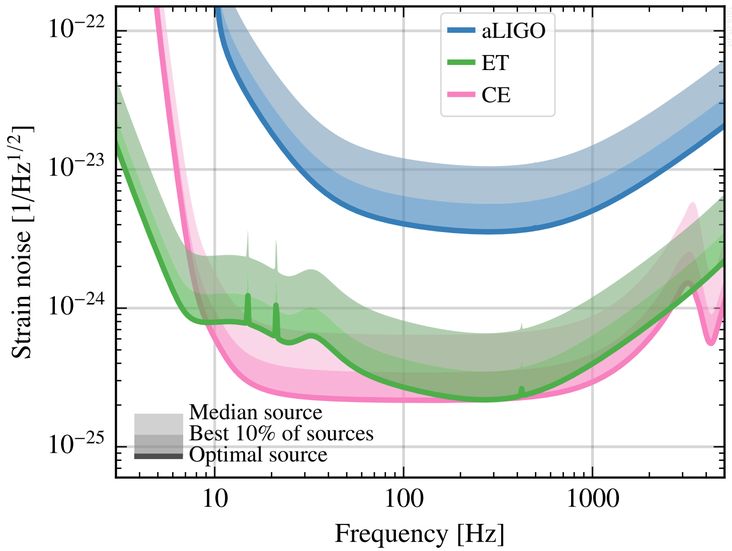
Einstein Telescope

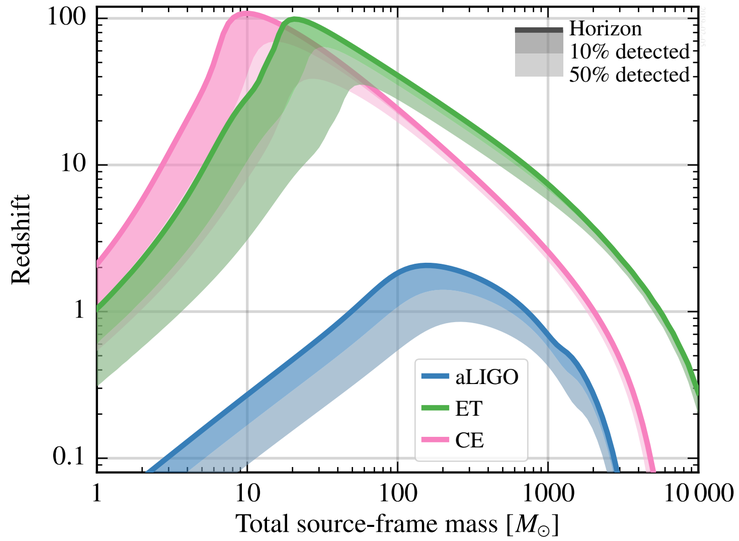


Cosmic Explorer

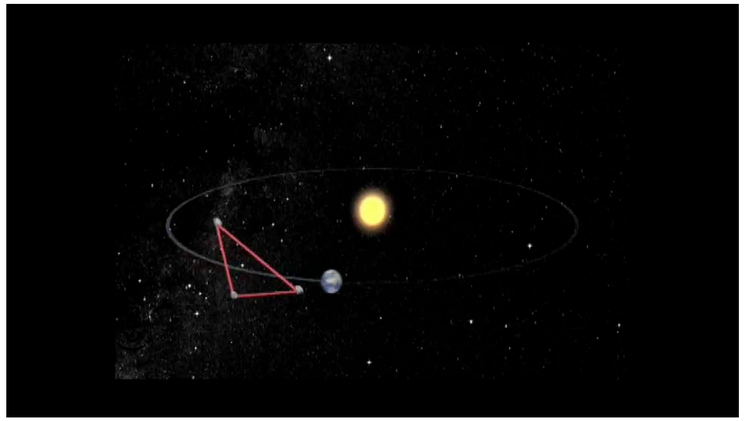


(3G Science-Case Report 21)

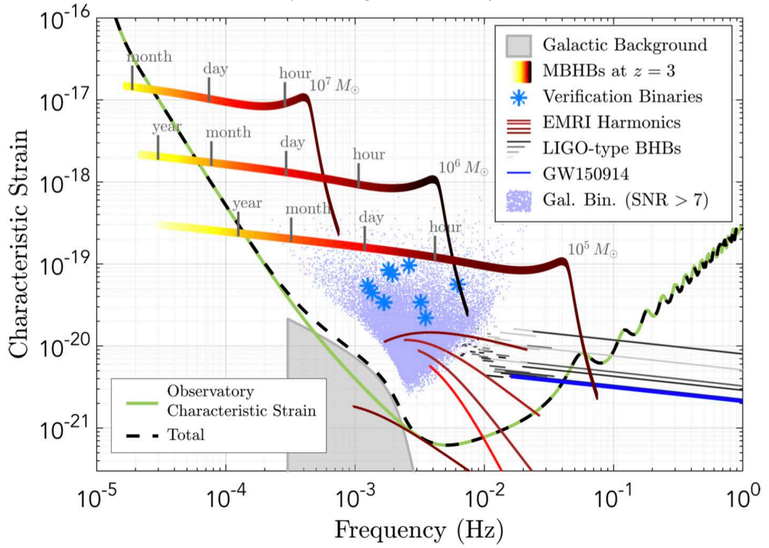




Laser Interferometer Space Antenna (LISA)



(Audley et al. 17)



- 3G and LISA will **observe binaries with SNR of 10^3! More accurate waveforms** are needed.

Summary & Outlook

- **Observing** gravitational waves and **inferring astrophysical/physical information** hinges on our **ability** to make **precise predictions** of **two-body dynamics** and **gravitational radiation.**

- **Successful interplay** between analytical and numerical relativity.

- I did not review many more applications and developments of EOB formalism, including **extension to scattering**, **interface with gravitational self-force and post-Minkowskian expansion**, **comparisons with gauge-invariant quantities** from NR simulations, and **extension to the state-of-the-art post-Newtonian** theory, etc.

- **Bright future:** next few years (decades) will bring **hundreds (thousands) more binary observations, with diverse properties at much higher SNRs**, probing fundamental and subatomic physics, dark matter, and cosmological model of our Universe.

- **To address open questions** and **take full advantage** of **discovery potential** in next years (and decades), we need to **continue to develop highly-accurate waveforms** that **cover** the **entire parameter space** and **include all physical effects** (higher modes, matter, spin precession, eccentricity, deviations from GR, etc.).

The “Astrophysical and Cosmological Relativity” Division

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A Wonderful Adventure in Gravitation at IHES

I am very grateful to Thibault for having raised my interest in the subject of gravitational waves from binary systems while at IHES, and for many fruitful and enjoyable discussions since then.