Black holes and wormholes in semiclassical gravity

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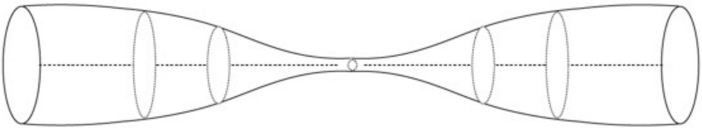
Wormholes as black hole mimickers

Replace black hole with a wormhole

$$-g_{tt} \to -g_{tt} + \epsilon^2 \,, \quad \epsilon^2 \ll 1$$

$$ds^2_{wh} = -(g(r) + \epsilon^2)dt^2 + g(r)^{-1}dr^2 + r^2 d\omega_d^2$$

Thibault and SS (2005)



Some properties:

- there is no event horizon
- it is replaced by a wormhole throat
- $t_{throat} \sim \epsilon t_\infty$
- a new time scale:

$$t_H \sim r_+ \ln 1/\epsilon$$

Choosing $\epsilon \sim e^{-S_{BH}}$ one has that t_H is of the order of the Hawking evaporation time

For observation times $t \ll t_H$ no difference with true black holes

Generalizations to stationary (rotating) case

Bueno, Cano, Goelen, Hertog and Vernocke (2017)

In this talk:

I demonstrate that wormholes of exactly this type appear quite naturally in the semiclassical gravity for a class of quantum states

Classical black holes

- Black hole solutions in Einstein theory of gravity $W_{cl} = -\frac{1}{16\pi G}\int R$

- Sufficiently large black holes remain to be solutions if added any finite number of local higher curvature terms

Quantum fields

Different states (vacua) for a quantum field:

Hartle-Hawking state: stress-energy tensor is regular at horizon, contains thermal radiation at infinity, describes black hole in thermal equilibrium with Hawking radiation

Boulware state: stress-energy tensor is singular at horizon, vanishing at infinity

Unruh state: stress-energy tensor is regular only at the future horizon, and there is a thermal flux of radiation at future null infinity, describes the process of black hole evaporation

"Quantum Black holes" as solutions to some "quantum" gravitational theory

- What happens to black hole horizons? Are there still solutions with horizons? New features due to possible non-locality of action?
- How the answer on previous question depends on the choice of the quantum state? What are the back-reacted geometries for Hartle-Hawking and Boulware states?

Previous works of York, Sanchez-Lousto, Zaslavsky, Emparan-Kaloper-Fabbri, ..

Semiclassical Gravity: background metric is classical while the matter is quantum

Integrating out the quantum fields one ends up with a modified gravitational action

$$W_{sc}[g] = W_{cl}[g] + W_Q[g]$$

where W_{cl} is classical gravitational action and W_Q is induced by quantum fields

In general W_Q is non-local and very complicated functional of metric

It can be represented as an expansion in curvature. In 4 dimnsions

$$W_Q = \int \gamma_i(-\Box_2)\mathcal{R}_1\mathcal{R}_2(i) + \Gamma_i(-\Box_1, -\Box_2, -\Box_3)\mathcal{R}_1\mathcal{R}_2\mathcal{R}_3(i) + O(\mathcal{R}^4)$$

works of Barvinsky, Vilkovisky, Gusev, Zhytnikov (1987-1997)

Rather difficult to deal with...

Two dimensions: things are simplified

For a 2d conformal field theory the quantum action is know to be the Polyakov action

$$W_Q[g] = \frac{c}{96\pi} \int R \frac{1}{\Box} R$$

This makes the two-dimensional gravitational models quite attractive as toy models to address the quantum black holes and the information problem

Callan, Giddings, Harvey and Strominger (1992) and many publications since then

In the rest of my talk:

- 2d dilaton gravity model

  based on-going research with Yohan Potaux and Deb Sarkar

- 4d quantum CFT

  based on earlier work with Clément Berthiere and Deb Sarkar

2D dilaton gravity

In two dimensions one needs to introduce an extra field (dilaton) to define dynamical gravity

String inspired 2d dilaton gravity

$$I_0 = \frac{1}{2\pi} \int_M d^2x \sqrt{-g} e^{-2\phi} (R + 4(\nabla\phi)^2 + 4\lambda^2)$$

Witten (1991), Mandal, Sengupta, Wadia (1991)

Trace of gravitational equations implies that $R = -2\Box\phi$

Trace free gravitational equations $\nabla_\mu \nabla_\nu \phi = \frac{1}{2} g_{\mu\nu} \Box\phi$

imply Killing vector $\xi_\mu = \epsilon_\mu^{\ \nu} \partial_\nu \phi$

General solution is static

$$ds^2 = -g(x)dt^2 + \frac{1}{g(x)} dx^2$$

$$g(\phi) = 1 - ae^{2\phi} \, , \quad \phi = -\lambda x$$

Describes a 2d black hole with Hawking temperature $T_H = \frac{\lambda}{2\pi}$, mass $M = \frac{a\lambda}{\pi}$ and entropy $S_{cls} = 2e^{-2\phi_h} = 2a$

Quantum CFT on a 2d black hole background

Local form of Polyakov action

$$I_1 = -\frac{\kappa}{2\pi}\int d^2x\left(\frac{1}{2}(\nabla\psi)^2 + \psi R\right), \quad \kappa = \frac{N}{24}$$

Equation for ψ: $\Box\psi = R$

Equations of motion in dilaton gravity: $R = -2\Box\phi$

Relation to dilaton: $\psi = -2\phi + w$, $\Box w = 0$

For static metric $R = -g''(x)$ so that $w(x) = \frac{C}{g(x)}$, where C is constant

As we will see in a moment w (or constant C) contains information about the choice of quantum state

Renormalized stress-energy tensor on the (fixed) dilaton black hole background

Energy density: $T^0_0 = \frac{\kappa}{\pi}[3\lambda^2 g(x) - 2\lambda^2 + \frac{C(C+4\lambda)}{4g(x)}]$

At infinity ($x \to \infty$): $T^0_0 = \frac{\kappa}{4\pi}(C + 2\lambda)^2$

Hartle-Hawking state (regular at horizon): $C = 0$ or $C = -4\lambda$

At infinity ($x \to \infty$) $T^0_{0\,HH} = \frac{\kappa\lambda^2}{\pi} = \frac{\pi}{6}NT^2$ (thermal Hawking radiation)

At horizon $x \to x_h$: $T^0_{0\,HH} = -\frac{2\kappa\lambda^2}{\pi}$

Boulware state: $C = -2\lambda$

$T^0_{0\,B} = 0$ at infinity (no radiation) and is singular at horizon ($g(x_h) = 0$)

In general: a *family of states* parametrised by C

$$T^0_0(C_1) - T^0_0(C_2) = \frac{\kappa}{4\pi g(x)}(C_1 - C_2)(C_1 + C_2 + 4\lambda)$$

Equivalence between C_1 and $C_2 = -(C_1 + 4\lambda)$

Boulware state is symmetric under this map

2d RST model

To address the issue of back-reaction we consider RST model

$$I = I_0 + I_1 + I_2 \,, \quad I_2 = -\frac{\kappa}{2\pi}\int d^2x\sqrt{-g}\,\phi R$$

Russo, Susskind, Thorlacius (1992)

A combination of dilaton and $g^{\mu\nu}T_{\mu\nu}=0$ ($T_{\mu\nu}=T^{(0)}_{\mu\nu}+T^{(1)}_{\mu\nu}+T^{(2)}_{\mu\nu}$) leads to

$$(R+2\Box\phi)(\kappa-2e^{-2\phi})=0$$

One solution is de Sitter spacetime with constant dilaton,

$$R=-2\lambda^2\,, \;\; 2\phi=-\ln\frac{\kappa}{2}=\mathrm{const}$$

The other is with non-constant dilaton with same relations as in classical case

$$R=-2\Box\phi\,, \;\; \psi=-2\phi+w\,, \quad \Box w=0$$

This simplifies integration of field equations

Quantum-corrected black hole (HH state)

Hartle-Hawking state: $w = 0$, $\psi = -2\phi$ trace-free gravitational equations reduce to

$$\nabla_\mu \nabla_\nu F(\phi) = \frac{1}{2} g_{\mu\nu} \Box F(\phi)\,, \quad F(\phi) = \phi - \frac{\kappa}{4} e^{2\phi}$$

This implies (as in classical case) Killing vector $\xi_\mu = \epsilon_\mu^{\ \nu} \partial_\nu F(\phi)$.
General solution is

$$ds^2 = -g(x) dt^2 + \frac{1}{g(x)} dx^2\,, \qquad -\lambda x = F(\phi)$$

$$g(\phi) = g_{HH}(\phi) = 1 + \kappa \phi e^{2\phi} - a e^{2\phi} \qquad\qquad SS(1995)$$

Curved metric at asymptotic infinity ($\phi \to -\infty$) $g_0(\phi) = 1 + \kappa \phi e^{2\phi}$ is due to the presence of thermal radiation (Minkowski spacetime is not a solution)

Curvature singularity is now at finite value of $\phi = \phi_{cr}$: $F'(\phi_{cr}) = 1 - \frac{\kappa}{2} e^{2\phi_{cr}} = 0$

For $a > a_{cr} = \frac{\kappa}{2}(1 - \ln \frac{\kappa}{2})$ there exists a horizon at $\phi_h < \phi_{cr}$ with same Hawking temperature $T_H = \frac{\lambda}{2\pi}$ as in classical case

There exists another asymptotically flat solution ("beyond singularity") for $\phi > \phi_{cr}$ with a horizon at $\phi'_h > \phi_{cr}$

Solutions with naked singularity for $a < a_{cr}$

Solution for a general quantum state

A general $C-$state: $\psi = -2\phi + w, \qquad w'(x) = \frac{C}{g(x)}$

The corresponding static asymptotically flat solution is

$$ds^2 = -g(x)dt^2 + \frac{1}{g(x)}dx^2 \,, \quad \frac{dx}{d\phi} = h(\phi)$$

$$g(x) = 2\lambda e^{2\phi} Z(\phi) \,, \quad h(\phi) = e^{2\phi} Z'(\phi)$$

$$Z + A(C) \ln Z = \frac{1}{2\lambda} e^{-2\phi} g_{HH}(\phi)$$

$$g_{HH}(\phi) = 1 + \kappa \phi e^{2\phi} - a e^{2\phi}$$

Here $A(C) = \frac{\kappa}{16\lambda^3} C(C + 4\lambda)$. Boulware state: $C = -2\lambda$ and hence $A = -\frac{\kappa}{4\lambda} < 0$

When $A(C) = 0$ the 2d space-time is the quantum-corrected black hole corresponding to the Hartle-Hawking state

This spacetime describes a wormhole for $A(C) < 0$ and a naked singularity for $A(C) > 0$

Back-reacted geometry for Boulware state in more detail

$$g_B(x) = 2\lambda e^{2\phi} Z(\phi)$$

$$Z - \frac{\kappa}{4\lambda} \ln \frac{Z}{\kappa/4\lambda} = \frac{1}{2\lambda} e^{-2\phi} g_{HH}(\phi)$$

Left hand side is a positive function, it has minimum at $Z_{min} = \frac{\kappa}{4\lambda}$, the value of dilaton $\phi = \phi_{min}$ corresponds to horizon value in HH state for a shifted mass, $(a - \kappa/2)$, $\phi_{min} = \phi_h(a - \kappa/2)$.

The minimal value for $g_B(\phi)$ is non-zero, $\min g_B(x) = \frac{\kappa}{2} e^{2\phi_{min}}$, it can be expressed in terms of entropy of classical black hole, $S_{BH}(a) = 2e^{-2\phi_h}$ (for simplicity consider a large black hole $a \gg \kappa/2$)

$$\min g_B(x) = \frac{\kappa}{S_{BH}(a)}$$

This is value of $-g_{tt}$ at the throat of the wormhole

Flat space (linear dilaton vacuum, $\phi = -\lambda x$, $g_B(x) = 1$) is now exact solution, it corresponds to $a = \frac{\kappa}{2} \ln \frac{\kappa}{2}$

4d quantum CFT

based on work with Deb Sarkar and Clément Berthiere (2017)

Universality at horizon

$$ds^2 = g(r)dt^2 + e^{2\phi(r)}g^{-1}(r)dr^2 + r^2 d\omega_d^2$$

$$g(r) = \frac{4\pi}{\beta}(r - r_+) + O(r - r_+)^2\,, \quad \phi(r) = O(r - r_+)$$

Optical metric

$$ds^2 = g(z)ds^2_{opt}\,, \quad ds^2_{opt} = dt^2 + dz^2 + R^2(z)d\omega_d^2$$

$$g(z) \sim e^{-4\pi z/\beta} + \ldots\,, \quad R^2(z) \sim e^{4\pi z/\beta} + \ldots \quad z \to \infty$$

Optical spacetime is product space $S_1^\beta \times M_3$

Near horizon M_3 is hyperbolic space H_3 of radius $\beta/2\pi$

Horizon as a minimal surface

It is well known that any horizon is a minimal surface

The opposite is also true. A simple illustration:

$$ds^2 = \Omega^2(\rho)dt^2 + d\rho^2 + r^2(\rho)d\omega_d^2$$

$\Omega^2 = g_{tt}$ and ρ is geodesic radial coordinate

Einstein equations:

$$2rr'' + r'^2 - 1 = 0$$
$$\Omega(r'^2 - 1) + 2rr'\Omega' = 0$$

If 2-sphere at $\rho = \rho_+$ has minimal area $r'(\rho_+) = 0$ then $\Omega(\rho_+) = 0$ and this sphere is a horizon

Before we start: general form of 4-metric we shall consider

$$ds^2 = \Omega^2(z)\left(dt^2 + N^2(z)dz^2 + R^2(z)d\omega_2^2\right) , \quad \Omega(z) = e^{\sigma(z)}$$

Useful choices of coordinates (gauge fixing):

- Black hole horizon

$$N(z) = 1 , \quad \sigma(z) = -2\pi z/\beta + \dots , \quad R(z) \simeq r_+ e^{2\pi z/\beta}$$

apriori β and r_+ are not related

- Minimal sphere

$$N(z) = 1/\Omega(\rho) , \quad z = \rho , \quad R(\rho) = r(\rho)/\Omega(\rho)$$

$r(\rho)$ is geometric radius of sphere

- Consider backreaction from quantized scalar, gauge and fermion fields on non-quantized geometric background.

- Non-perturbative handle is due to the study of conformal anomaly.

  Fradkin-Tseytlin '84, Dowker-Schofield '90, Mazur-Motola '01

- Two important contributions: due to anomaly and due to optical metric

Gravitational action

$$W_{grav} = -\frac{1}{16\pi G_N}\int R[G] + \Gamma[G]$$

$\Gamma[G]$ is quantum effective action, result of integrating out quantum fields

we represent $G_{\mu\nu} = e^{2\sigma} g_{\mu\nu}$, quantum effective action transforms as

$$\Gamma[e^{2\sigma}g] = -\frac{a}{16\pi^2}\int \sigma C^2 + \frac{b}{16\pi^2}\int \sigma E - \frac{2b}{16\pi^2}\int (2\tilde{G}^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma + 2\Box\sigma(\nabla\sigma)^2 + (\nabla\sigma)^4) + \Gamma_0[g]$$

$\tilde{G}^{\mu\nu} = R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R$ is Einstein tensor

$$C^2 = Riem^2 - 2Ricci^2 + \frac{1}{3}R^2\,, \quad E = Riem^2 - 4Ricci^2 + R^2$$

$$a = \frac{1}{120}(n_0 + 6n_{1/2} + 12n_1)\,, \quad b = \frac{1}{360}(n_0 + 11n_{1/2} + 62n_1)$$

Effective action on optical metric:

$$\Gamma_0 = \Gamma[S_1^\beta \times M_3] = -\frac{\pi^2}{90\beta^3} c_H \int_{M_3} 1 + \frac{\lambda_H}{144\beta} \int_{M_3} R_{M_3}$$

$$c_H = n_0 + \frac{7}{2} n_{1/2} + 2n_1 \,, \quad \lambda_H = n_{1/2} + 4n_1$$

- exact result if $M_3 = H_3$
- we dropped higher curvature (non-local) terms
- general structure discussed by Gusev and Zelnikov '98

Horizons in SG

Variations of $W_{grav}[\sigma(z), N(z), R(z)]$ w.r.t. $\sigma(z)$, $N(z)$ and $R(z)$ give semiclassical gravitational equations

Some observations: $E(g_{optical}) = 0$ and $C^2(g_{optical}) \to C^2(S_1 \times H_3) = 0$ as $z \to \infty$

Variation w.r.t. $N(z)$ will produce divergent (as $z \to \infty$) terms. These terms will come from derivatives of σ in b-anomaly and from Γ_0, the divergence is due to divergent volume density on M_3

$$
\begin{aligned}
\delta_N W_{grav} &= (360b - 2c_H - 10\lambda_H)\,\frac{\pi^2}{180\beta^4}\,r_+^2 e^{4\pi z/\beta} \\
&= -\left(n_0 + 6n_{1/2} - 18n_1\right)\frac{\pi^2}{180\beta^4}\,r_+^2 e^{4\pi z/\beta}\,.
\end{aligned}
$$

- What we consider appears to be Boulware vacuum
- Curiously, the equations are satisfied for $\mathcal{N} = 4$ SYM theory (have to look at subleading terms)
- For generic set of fields divergent term is there so that no static solutions with horizons in SG!

Minimal sphere

We look for solutions with a turning point: $r'(\rho) = 0$ and $\Omega'(\rho) = 0$ at $\rho = \rho_+$ such that $r'' > 0$ and $\Omega'' > 0$

Such a solution is parametrized by values of r and Ω at turning point

r is the radius of classical horizon

Values of second derivatives r'' and Ω'' are determined by r and Ω via gravitational equations

Additionally, there arise consistency conditions on possible values of Ω provided r can be arbitrary

Variation w.r.t. $N(z)$ takes the form at the turning point

$$(\Omega r r'' - r^2\Omega'')^2 = y^2\Omega^2 \, , \quad y^2 = 1 - \frac{r^2}{\kappa \bar{a} \ln \Omega^{-1}}\left(\frac{\gamma\kappa r^2}{\beta^4\Omega^4} - \frac{\lambda\kappa}{\beta^2\Omega^2} + 1\right)$$

$\kappa = 8\pi G_N$, $\bar{a} = a/12\pi^2$, $\gamma = c_H\pi^2/90$, $\lambda = \lambda_H/72$

Condition that $y^2 \geq 0$ restricts possible values of Ω!

For simplicity consider $\lambda_H = 0$ (only scalars)

Then condition $y^2 \geq 0$ is equivalent to condition

$$\Omega^4 \ln \frac{\Omega_0}{\Omega} \geq \frac{\gamma r^4}{\bar{a}\beta^4} \,, \quad \Omega_0 = e^{-\frac{r^2}{\kappa \bar{a}}}$$

Notice that $\frac{r^2}{\kappa \bar{a}}$ is proportional to Bekenstein-Hawking entropy $S_{BH} = 8\pi^2 r^2/\kappa$ of classical black hole

It immediately follows that

$$0 < \Omega < \Omega_0 = e^{-\frac{r^2}{\kappa \bar{a}}} \sim e^{-S_{BH}/a}$$

Conditions $r'' > 0$ and $\Omega'' > 0$ impose extra constraints on possible values of Ω.

In classical limit $\bar{a} \rightarrow 0$ so that $\Omega_0 = 0$ and the throat becomes horizon!

Conclusions

- *Back-reacted geometry of the Boulware state in semiclassical gravity is a wormhole*
- *Minimal non-zero value of* $-g_{tt}$ *is bounded by the classical BH entropy:* $1/S_{BH}$ *in two dimensions and* $e^{-S_{BH}}$ *in four dimensions*
- *As 2d examples show there may be infinitely many quantum states represented by horizon free back-reacted geometries*
- *It is possible that the Hartle-Hawking state is the only state whose back-reacted geometry has a horizon*
- *Experimentally hard to measure such small deviations*

One of remaining questions

Are we sure that the black holes around us are not actually wormholes?

Thank you for your attention!

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