Impact & legacy of the 2010 paper on self-force and EOB

T. Damour, Physical Review D **81**, 024017 (2010)

Leor Barack
University of Southampton

Plan

- Self-force landscape as at 2010
- The 2010 paper and the 6 self-force ‘handles’
- Accomplished calculations and follow-ons
- Reflection

Self-force landscape as at 2010

- 1997 1st-order equation of motion (2nd order not until 2012)
- Practical calculation methods for EMRI setups.
  Mode-sum regularization.
  Gauge problem.
- Scalar-field implementations and other toy problems
- 2007 First calculations of *gravitational* self-force in EMRI orbits
- First comparisons with PN results: 2008 (redshift) 2009 (ISCO shift)

PHYSICAL REVIEW D **81,** 024017 (2010)

Gravitational self-force in a Schwarzschild background and the effective one-body formalism

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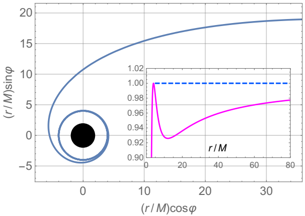
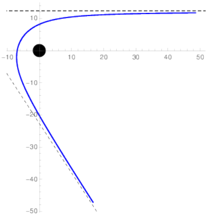
(Received 29 October 2009; published 19 January 2010)

We discuss various ways in which the computation of conservative gravitational self-force (GSF) effects on a point mass moving in a Schwarzschild background can inform us about the basic building blocks of the effective one-body (EOB) Hamiltonian. We display the information which can be extracted from the recently published GSF calculation of the first-GSF-order shift of the orbital frequency of the last stable circular orbit, and we combine this information with the one recently obtained by comparing the EOB formalism to high-accuracy numerical relativity data on coalescing binary black holes. The information coming from GSF data helps to break the degeneracy (among some EOB parameters) which was left after using comparable-mass numerical relativity data to constrain the EOB formalism. We suggest various ways of obtaining more information from GSF computations: either by studying eccentric orbits, or by focusing on a special zero-binding zoom-whirl orbit. We show that logarithmic terms start entering the post-Newtonian expansions of various (EOB and GSF) functions at the fourth post-Newtonian level, and we analytically compute the first logarithm entering a certain, gauge-invariant ''redshift'' GSF function (defined along the sequence of circular orbits).

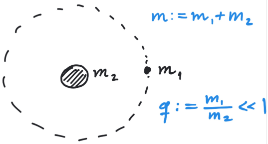
Damour's 6 self-force 'handles'

1. ISCO frequency
2. Perisatron advance of slightly eccentric orbits
3. ADM-like E and J using 2nd-order self-force (note: pre- 1st law of binaries)
4. Critical value of J for a Zero-binding-energy zoom-whirl orbit (ZEZO)
5. Frequency of the innermost bound (unstable) circular geodesic
6. Scatter angle in hyperbolic encounters





1. ISCO frequency (Schwarzschild)

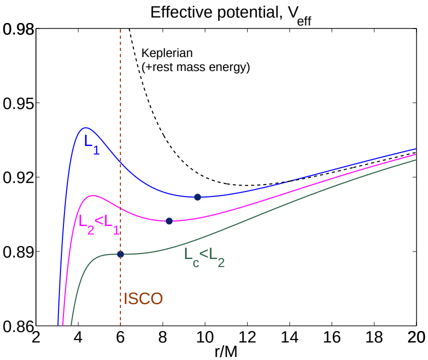


Effective radial force:

$$m_1 \ddot{r} = -\frac{m_1}{2} V'_{\rm eff} + F_{\rm self}(\propto q^2)$$

$$V'_{\rm eff} = 0 = V''_{\rm eff} \quad \Rightarrow \quad r_{\rm isco} = 6m_2 + O(q).$$

$$\Rightarrow m\Omega_{\rm isco} = 6^{-3/2}\left[1 + q\, C_\Omega + O(q^2)\right]$$



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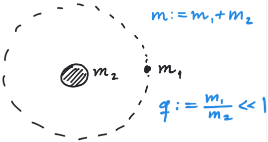

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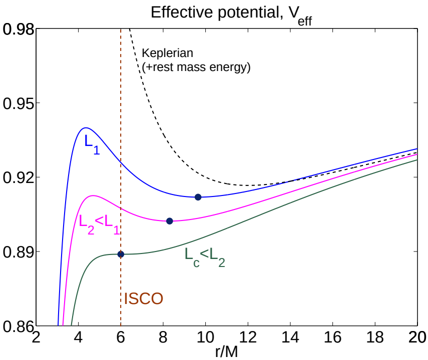


$$C_\Omega = 1.2513(6) \qquad \text{(LB \& Sago 2009)}$$

$$= 1.25101546(5) \qquad \text{(Akcay, LB, Damour \& Sago 2012)}$$

$$C_\Omega^{(3PN)} = 1.434913\ldots$$

ISCO frequency in EOB

(mass rescaled) EOB Hamiltonian near circularity:

$$\begin{aligned} H^2_{\rm eff}(u,p_r;j;\nu) &= A(u;\nu)\left(1+j^2u^2\right)+O(p_r^2) \\ A(u;\nu) &= 1-2u+\nu a(u)+O(\nu^2) \end{aligned}$$

ISCO identified by looking for inflection point of $H^2_{\rm eff}$:

$$u_{\rm ISCO}=\frac{1}{6}\left[1+\nu\left(a(1/6)+\frac{1}{3}a'(1/6)+\frac{1}{18}a''(1/6)\right)+O(\nu^2)\right]$$

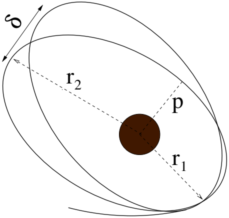
from which one can read

$$C_\Omega^{\rm EOB}=1-\sqrt{8/9}+\frac{3}{2}a(1/6)+\frac{1}{4}a'(1/6)+\frac{1}{12}a''(1/6).$$

$\Rightarrow$ Constraint on the PN expansion coefficients in $a(u)=a_3u^3+a_4u^4+a_5u^5+\cdots$.

2. Perisatron advance of slightly eccentric orbits

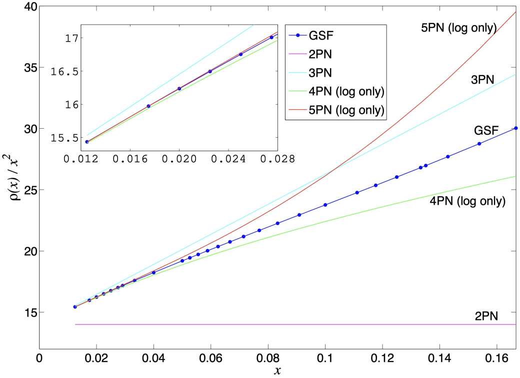
$$\left(\frac{\omega_r}{\Omega}\right)^2 = \left(1+\frac{\delta}{2\pi}\right)^2 = 1-6x+\nu\rho(x)+O(\nu^2)$$



$x := (m\Omega)^{2/3}$

ω_r : radial frequency

δ : advance per radial period



(LB, Damour & Sago 2010)

Periastron advance in EOB

From the EOB radial equation of motion obtain

$$\omega_r^2 = \frac{1}{2}u_0^2\bar{D}(u_0;\nu)\left[A(u;\nu)(1+j^2u^2)\right]''_{u_0}$$

Writing $A(u;\nu) = 1 - 2u + \nu a(u) + O(\nu^2)$
and $\bar{D}(u;\nu) = 1 + \nu\bar{d}(u) + O(\nu^2)$, this gives

$$\left(\frac{\omega_r}{\Omega}\right)^2 = 1 - 6u + \nu\left[(1-6u)\bar{d}(u) + a(u) + 2ua'(u) + \frac{1}{2}u(1-2u)a''(u)\right] + O(\nu^2).$$

from which one identifies

$$\rho^{\rm EOB}(x) = 4x\left(1 - \frac{1-2x}{\sqrt{1-3x}}\right) + (1-6x)\bar{d}(x) + a(x) + xa'(x) + \frac{1}{2}x(1-2x)a''(x).$$

- $\rho^{\rm EOB}(x) = \rho^{\rm SF}(x)$ constrains *functional* form of $O(\nu)$ EOB potentials — a coupled ODE for $a(x)$ and $\bar{d}(x)$ (with boundary conditions from PN)
- Several new constraints on the PN expansion coefficients of $a(x)$ and $\bar{d}(x)$.

Analytical GSF-PN

GSF can be calculated **analytically**, in a PN framework

Bini, Damour, Geralico 2013–

Shah *et al.* 2014–

Kavanagh, Ottewill, Wardell, Hopper 2015–

$$\begin{aligned}
\rho(x) = {} & 14x^2 + \left(\frac{397}{2} - \frac{123}{16}\pi^2\right)x^3 + \left(\frac{5024}{15}\gamma - \frac{215729}{180} + \frac{2512}{15}\ln(x) + \frac{2916}{5}\ln(3) + \frac{1184}{15}\ln(2) + \frac{58265}{1536}\pi^2\right)x^4 \\
& + \left(\frac{27824}{35}\ln(2) - \frac{6325051}{800} + \frac{1135765}{1024}\pi^2 - \frac{202662}{35}\ln(3) - \frac{22672}{7}\gamma - \frac{11336}{7}\ln(x)\right)x^5 \\
& + \frac{199876}{315}\pi x^{11/2} \\
& + \left(\frac{4990303259}{589824}\pi^2 - \frac{256727518799}{6350400} + \frac{435213}{20}\ln(3) + \frac{3606884}{945}\gamma - \frac{37648124}{945}\ln(2) + \frac{1803442}{945}\ln(x)\right. \\
& \left. - \frac{7335303}{32768}\pi^4 + \frac{9765625}{2268}\ln(5)\right)x^6 \\
& - \frac{1429274}{225}\pi x^{13/2}
\end{aligned}$$

(6.5PN) $+\cdots$ [BDG 2016]

Analytical GSF-PN

$$
\begin{aligned}
&+\Bigg(-\frac{3725312}{1575}\ln(2)^2-\frac{419921875}{6048}\ln(5)-\frac{3744144}{175}\gamma\ln(3)-\frac{3744144}{175}\ln(2)\ln(3)+\frac{253952}{15}\zeta(3)\\
&-\frac{13586432}{1575}\ln(x)\gamma-\frac{230019793907682883}{440082720000}-\frac{1872072}{175}\ln(3)\ln(x)-\frac{20598784}{1575}\ln(2)\gamma+\frac{12659060941523}{1238630400}\pi^2\\
&+\frac{681396625634}{5457375}\gamma+\frac{229716339147}{2156000}\ln(3)-\frac{3396608}{1575}\ln(x)^2+\frac{1823766172754}{5457375}\ln(2)-\frac{10299392}{1575}\ln(2)\ln(x)\\
&+\frac{471044952937}{251658240}\pi^4+\frac{340698312817}{5457375}\ln(x)-\frac{1872072}{175}\ln(3)^2-\frac{13586432}{1575}\gamma^2\Bigg)x^7\\
&+\frac{18719967989}{1455300}\pi x^{15/2}\\
&+\Bigg(-\frac{82814168955181}{132432300}\ln(2)+\frac{36686848}{441}\ln(x)\gamma+\frac{1920044921875}{4036032}\ln(5)+\frac{226912947151462749941 9}{176209121088000}\\
&+\frac{148969692}{1225}\ln(3)^2+\frac{36686848}{441}\gamma^2+\frac{103653376}{3675}\ln(2)\gamma+\frac{9171712}{441}\ln(x)^2+\frac{297939384}{1225}\gamma\ln(3)\\
&-\frac{948480}{7}\zeta(3)+\frac{297939384}{1225}\ln(2)\ln(3)+\frac{678223072849}{23166000}\ln(7)+\frac{148969692}{1225}\ln(3)\ln(x)\\
&+\frac{51826688}{3675}\ln(2)\ln(x)-\frac{6517218707007553}{55490641920}\pi^2-\frac{1442495323220011}{3972969000}\ln(x)\\
&-\frac{557542163367261}{392392000}\ln(3)-\frac{1444834607367211}{1986484500}\gamma-\frac{2049476608}{11025}\ln(2)^2-\frac{626168320805261}{5368709120}\pi^4\Bigg)x^8
\end{aligned}
$$

(8PN) $+\cdots$ [BDG 2016]

Analytical GSF-PN

$$
\begin{aligned}
&+\left(-\frac{3725312}{1575}\ln(2)^2-\frac{419921875}{6048}\ln(5)-\frac{3744144}{175}\gamma\ln(3)-\frac{3744144}{175}\ln(2)\ln(3)+\frac{253952}{15}\zeta(3)\right.\\
&-\frac{13586432}{1575}\ln(x)\gamma-\frac{230019793907682883}{440082720000}-\frac{1872072}{175}\ln(3)\ln(x)-\frac{20598784}{1575}\ln(2)\gamma+\frac{12659060941523}{1238630400}\pi^2\\
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&\left.+\frac{471044952937}{251658240}\pi^4+\frac{340698312817}{5457375}\ln(x)-\frac{1872072}{175}\ln(3)^2-\frac{13586432}{1575}\gamma^2\right)x^7\\
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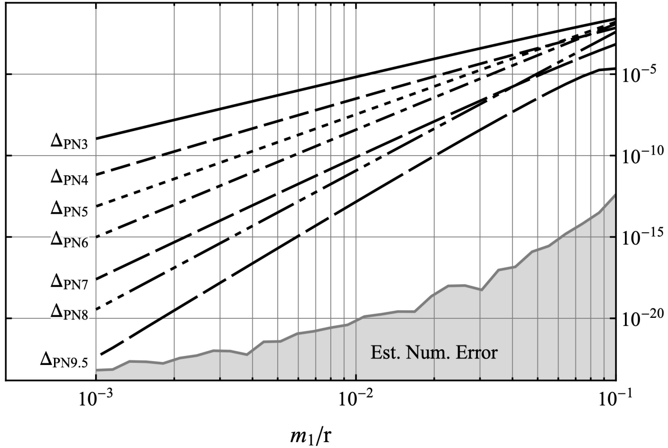
&+\frac{148969692}{1225}\ln(3)^2+\frac{36686848}{441}\gamma^2+\frac{103653376}{3675}\ln(2)\gamma+\frac{9171712}{441}\ln(x)^2+\frac{297939384}{1225}\gamma\ln(3)\\
&-\frac{948480}{7}\zeta(3)+\frac{297939384}{1225}\ln(2)\ln(3)+\frac{678223072849}{23166000}\ln(7)+\frac{148969692}{1225}\ln(3)\ln(x)\\
&+\frac{51826688}{3675}\ln(2)\ln(x)-\frac{6517218707007553}{55490641920}\pi^2-\frac{1442495323220011}{3972969000}\ln(x)\\
&\left.-\frac{557542163367261}{392392000}\ln(3)-\frac{1444834607367211}{1986484500}\gamma-\frac{2049476608}{11025}\ln(2)^2-\frac{626168320805261}{5368709120}\pi^4\right)x^8
\end{aligned}
$$

(9.5PN!)

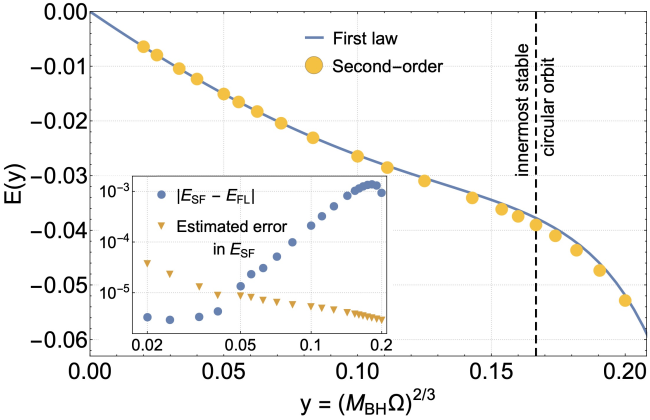
[BDG 2016]

Analytical GSF-PN



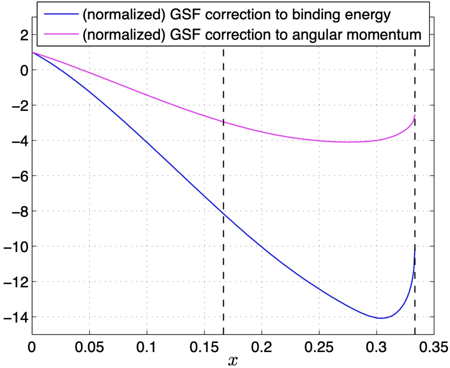
van de Meent 2016

3. Binding energy from 2nd-order SF



(Pound, Wardell, Warburton, Miller 2020)

Binding energy from Detweiler's redshift (via the 1st law)



(Akcay, LB, Damour, Sago 2012)

(Le Tiec, Barausse, Buonanno 2012)

$$E_{\rm bind} = \left(\frac{1-2x}{\sqrt{1-3x}} - 1\right) + \nu E_{\rm SF} + O(\nu^2)$$

$$E_{\rm SF} = \tfrac{1}{2} z_{\rm SF}(x) - \tfrac{x}{3} z'_{\rm SF}(x) + \alpha(x)$$

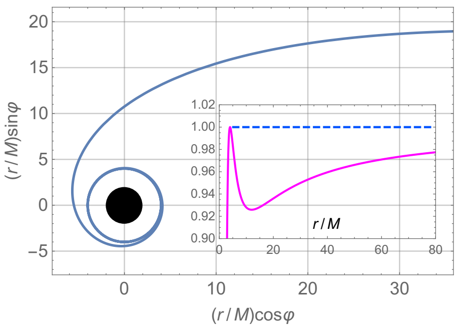
$$z(x) = 1/u^t = \sqrt{1-3x} + \nu z_{\rm SF} + O(\nu^2)$$

ISCO shift from minimization of $E_{\rm bind}$:

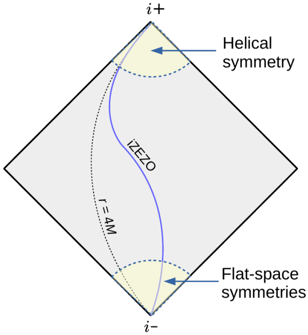
$$C_\Omega = \tfrac{1}{2} + \tfrac{1}{4\sqrt{2}}\left[\tfrac{1}{3} z''_{\rm SF}\left(\tfrac{1}{6}\right) - z'_{\rm SF}\left(\tfrac{1}{6}\right)\right]$$

- Handle on EOB potentials down to the light ring
- Extension of 1st-law to eccentric orbits [Le Tiec 2015] later allowed GSF calibration of $Q(u;\nu)$ from $\langle z_{\rm SF}\rangle_{\rm orb}$ [Akcay & van de Meent 2016 +]

4+5. J and Ω of the Zero-binding-Energy Zoom-whirl Orbit



(LB, Colleoni, Damour, Isoyama, Sago 2019)



- δJ and $\delta\Omega$ obtained by integrating the self-force along the orbit
- Direct handle on sub-ISCO EOB potential without 1st-law intermediation
- Unambiguous identification of the 1st-law's energy and angular momentum as Bondi-type quantities.

4+5. J and Ω of the Zero-binding-Energy Zoom-whirl Orbit

LB, Colleoni, Damour, Isoyama & Sago 2019

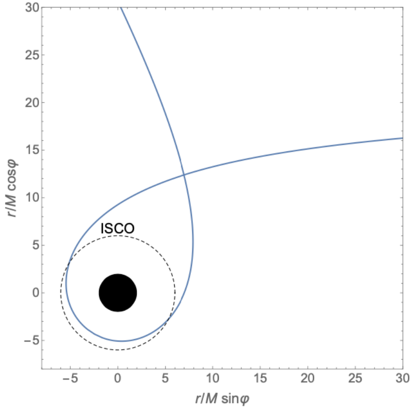
Frequency of the asymptotic circular orbit:

$$\begin{aligned} \Omega/\Omega_0 &= 1 + 0.5536(2)q && \text{direct self-force calculation} \\ &= 1 + 0.32q && \text{EOB (2010 estimation)} \\ &= 1 + 0.553603030(1)q && \text{EOB using 1st law} \end{aligned}$$

Angular momentum of the asymptotic circular orbit:

$$\begin{aligned} J/J_0 &= 1 - 0.304(3)q && \text{direct self-force calculation} \\ &= 1 - 0.288(80)q && \text{EOB (2010 estimation)} \\ &= 1 - 0.30467428782(6)q && \text{EOB using 1st law} \end{aligned}$$

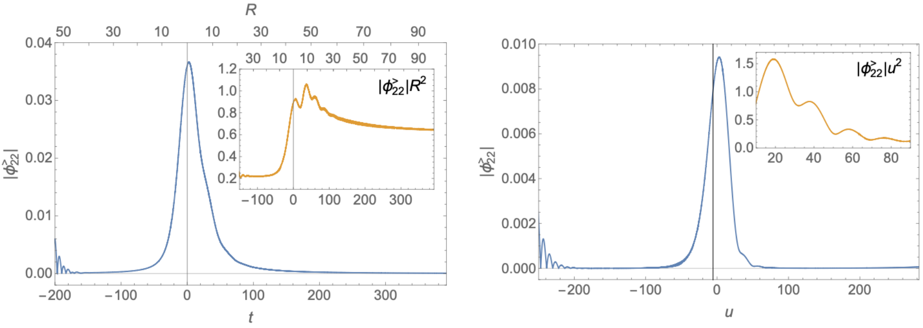
6. Scatter angle in hyperbolic encounters



- $\chi(E_{\rm COM}, b) := \varphi_{\rm out} - \varphi_{\rm in} - \pi$ defined unambiguously (even with radiation).
- Can probe down to light ring ($r = 3m_2$)

- (Damour 2016) New way of calibrating EOB, using post-Minkowskian χ info; established a new dictionary.
- Intense interest, new participants: QCD Amplitudes, EFT
- (Damour 2019) Knowledge of χ through $O(q)$ [1GSF] determines the full conservative dynamics to 4PM order. 2GSF would extend that to 6PM!
- 1GSF results give the full $O(q)$ piece of χ ("all PM orders")

Ongoing self-force calculations...



Long & LB (2021)

3 independent methods/codes:

- Long & LB (time domain, scalar field and gravity)
- Whittall & LB (frequency domain, scalar field)
- O'tool, Ottewill & Wardell (time domain, scalar field)

Reflection

2010 paper started a new programme—one still going strong—that has led to significant improvement in waveform models for GW detectors.

New ideas:

- Conservative self-force as handle on strong-field potential
- Completely new way of calibrating EOB potentials in the strong field; new dictionary using gauge-invariant physical quantities.
- Use symmetric mass ratio ν in lieu of q, foreseeing remarkable utility of self-force expansion at all mass ratios.