OPTIMAL NONPERMANENT CONTROL PROBLEMS ON TIME SCALES.

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The Pontryagin Maximum Principle (PMP in short) is a fundamental result of optimal control theory established in the mid-1950s which provides first-order necessary optimality conditions for control systems described by ordinary differential equations. Soon afterwards and even nowadays, the PMP has been adapted to many situations: for control systems of different natures, with various constraints, etc. In the classical version of the PMP, the control is assumed to be *permanent*, in the sense that the value of the control is authorized to be modified at any real time. As a consequence, in numerous problems, achieving the optimal trajectory requires a permanent modification of the value of the control. However such a request is not feasible in practical situations, neither for human beings, nor for mechanical or numerical devices. For this reason, piecewise constant controls (also called *sampled-data controls* in the literature), whose number of authorized modifications is finite, are widely used in Automatic and Engineering. Sampled-data controls constitute a first example of nonpermanent controls. Another example comes from control systems whose trajectories cross noncontrol areas (such as a mobile phone or a GPS device passing under a tunnel). In order to encompass several different situations of nonpermanent controls, we will use the mathematical tools of *time scale calculus*, which allow furthermore to deal simultaneously with continuous-time and discrete-time control systems.

In this talk I will start by presenting a version of the PMP that can handle optimal nonpermanent control problems on time scales [1]. Numerous properties are well known in the continuous-time literature for optimal permanent controls (such as the continuity of the corresponding Hamiltonian function, or the saturation of the control constraint set in the case of an affine Hamiltonian function, etc.). In this talk we will discuss the preservation (or not) of these properties when we consider optimal nonpermanent controls on time scales. This discussion will also be based, among others, on the recent related works [2, 5].

Finally I will present a recent version of the PMP for optimal nonpermanent control problems on time scales in the presence of inequality state constraints [4]. In that context the PMP involves an adjoint vector which is (only) of bounded variation, and thus which may present a pathological behavior (singular part and/or discontinuity jumps). In the continuous-time and piecewise constant control setting, we provide in [3] quite unrestrictive assumptions under which the adjoint vector has no singular part and (only) a finite number of discontinuity jumps which moreover occur exactly at the times of the corresponding time partition. This behavior presents several benefits from the numerical point of view for the implementation of shooting methods.

References

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