Numerical methods for optimal sampling times problems and applications to the Hill model in biomechanics

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Abstract

In the series of articles [1, 3, 5], we develop various methods to solve an optimal sampling times problem. These methods are based on mathematical analysis and numerical computations on force-fatigue models which predict the muscular response to a pulses train.

In this framework, the control lives in a finite dimensional space. We investigate different methods to compute the optimal pulses train, which can be expressed as a finite dimensional optimization problem whose objective function is obtained by integrating ordinary differential equations in presence of nonlinearities and singularities and the optimization variables are the sampling times.

First, discretizing the state, we obtain a finite dimensional non-linear optimizaton problem which leads to the so-called direct method. Second, the geometric structure of the optimal solution can be characterized by first order necessary optimality conditions adapting the needle-like variation used to derive the Pontryagin Maximum Principle in the Mayer case. A third method is dedicated to a real time prototype for muscular reinforcement, we approximate the input-output mapping by a finite dimensional function exploiting the concavity of the Hill-Mentens-Michaelis functions involved the Hill model and the piecewise structure of the problem.

References

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