

Convergence in relative error for the fast diffusion equation

Friday, November 19, 2021 9:45 AM (30 minutes)

In this talk, I will discuss the asymptotic behavior of solutions to the fast diffusion equation. It is well known that non-negative solutions behave for large times as the Barenblatt (or fundamental) solution, which has an explicit expression. In this setting, I will introduce the Global Harnack Principle (GHP), precise global pointwise upper and lower estimates of non-negative solutions in terms of the Barenblatt profile. This can be considered as the non-linear counterpart of the celebrated Gaussian estimates for the linear diffusion equations. I will characterize the maximal (hence optimal) class of initial data such that the GHP holds by means of an integral tail condition. To the best of our knowledge, analogous issues for the linear heat equation do not possess such clear answers; only partial results are known.

As a consequence, I will provide rates of convergence towards the Barenblatt profile in entropy and in stronger norms such as the uniform relative error. These estimates were fundamental in obtaining a constructive stability result in Gagliardo-Nirenberg-Sobolev inequalities. The results are based on joint work with Matteo Bonforte, Jean Dolbeault, and Bruno Nazaret.

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