Large-scale nonconvex optimization: randomization, gap estimation, and numerical resolution

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Abstract: We address a large-scale nonconvex optimization problem, involving an aggregative term. This term can be interpreted as the sum of the contributions of N agents to some common good, with N large. Our setting is similar to the one investigated by Wang in [1]. We introduce a randomized relaxation for this problem, and prove that the relaxation gap decreases with the number of agents N, independently of the dimension of the aggregative term. We give a stochastic method to construct an approximate minimizer of the original problem, given an approximate solution of the randomized problem. With the help of McDiarmid's concentration inequality [4], we quantify the probability of success of the method. The Frank-Wolfe (FW) algorithm [2] is used to solve the randomized problem, and it allows to decompose this large-scale problem to N individual sub-problems at each iteration. A sub-linear convergence rate is obtained for the FW algorithm. In order to handle the memory overflow problem possibly caused by the FW algorithm, we propose a stochastic Frank-Wolfe (SFW) algorithm, which ensures the convergence in both expectation and probability senses. Numerical experiments on a mixed-integer quadratic program and a multi-agent optimal control problem illustrate the efficiency of the method.

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