A Lagrange-Galerkin method for a class of continuity equations and applications to first order mean field games

- Elisabetta Carlini (University la Sapienza)
- Francisco Silva (Université de Limoges)
- **Ahmad Zorkot** (Université de Limoges)

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Abstract: We propose a Lagrange-Galerkin scheme for the following continuity equation

$$\begin{cases} \partial_t m + \operatorname{div}(bm) = 0 & \text{in } (0,T) \times \mathbb{R}^d, \\ m(0) = \mu_0, \end{cases} \tag{1}$$

where $T > 0$, $\mu_0 \in L^2(\mathbb{R}^d)$ with compact support, and the vector field b is continuous and bounded. Using $\mathbb{P}_0$ element in the discretization, we prove the mass conservation, the consistency and the L^2-stability of the scheme.

Then we prove that the distance, in a suitable metric space, between solutions to the Lagrange-Galerkin and its area weighting version (see [2]) tends to zero as the discretization parameters tend to zero. We show that the area weighting version of the Lagrange-Galerkin scheme coincides with the scheme proposed in [1]. This allows us to show convergence of the latter in general state dimension and for non-smooth vector fields, extending importantly the results in [1]. Finally, we couple this scheme with a semi-Lagrangian discretization of the Hamilton-Jacobi-Bellman equation to obtain a scheme to approximate solutions of first order Mean Field Games systems.

References:

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