Second-order cone programming for rolling friction contact mechanics with high accuracy computation

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Abstract: Our aim is to perform numerical solutions of an optimization model derived from the problem of unilateral contact between solid bodies with rolling friction [1]. The model is a second-order cone programming (SOCP) problem with a convex quadratic objective function and conical constraints modeling the rolling friction phenomenon. We are working on two models: convex and non-convex. The convex model will be mainly presented in this talk.

The solver is an implementation of the primal-dual interior-point (IP) algorithm. The implemented method is the Mehrotra predictor-corrector algorithm [2] extended to SOCP [3]. The first difficulty is that the rolling friction cone departs from the standard Lorentz cone in that it is not self-dual. This prevents a direct application of the Jordan algebra usually used in the framework of IP methods. We then show how to transform the original model in order to reformulate the problem with symmetric cones.

The algorithm uses Nesterov and Todd [4] scaling technique to guarantee the non-singularity and the symmetry of the linear system that is solved at each iteration of the IP algorithm. Unfortunately, this scaling technique introduces ill-conditioning when the iterates go to the solution. This prevents us computing accurate solutions by means of some well-known solvers as SPDT3 [3] or the Matlab built-in function `coneprog` [5]. We will show how we overcame these numerical difficulties by detailing some parts of our C implementation. The results of numerical experiments will be presented to validate our approach.

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