Interior point methods for solving Pareto eigenvalue complementarity problems

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Abstract: The area of complementarity problems (CP) has received great attention over the last few decades due to their various applications in engineering, economics, and sciences. Since the pioneering work by Lemke and Howson, who showed that computing a Nash equilibrium point of a bimatrix game can be modeled as a linear complementarity problem, the theory of CP has become a useful and effective tool for studying a wide class of problems in numerical optimization. On the other hand, Eigenvalue Complementarity Problems (EiCP) (also known as cone-constrained eigenvalue problems) form a particular subclass of complementarity problems that extend the classical (linear algebra) eigenvalue problems. EiCP appeared for the first time in the study of static equilibrium states of finite dimensional mechanical systems with unilateral frictional contact [4], and since then it has been widely studied both theoretically and numerically. Applications of EiCP were found in many fields such as the dynamic analysis of structural mechanical systems, vibro-acoustic systems, electrical circuit simulation, signal processing, fluid dynamics, contact problems in mechanics. Mathematically speaking, solving EiCP consists in finding a real number $\lambda \in \mathbb{R}$ and a corresponding nonzero vector $x \in \mathbb{R}^n \setminus \{0\}$ such that the following condition holds

$$K \ni x \perp (\lambda x - Ax) \in K^*, \tag{1}$$

where K is a closed convex cone in $\mathbb{R}^n$, $\perp$ indicates the orthogonality, and K^* stands for its positive dual cone, which is defined by $K^* = \{y \in \mathbb{R}^n : \langle y, x\rangle \geq 0 \quad \forall x \in K\}$.
Such scalar λ and vector x are respectively called eigenvalue and eigenvector of (1). The Pareto eigenvalue problem is given when $K = \mathbb{R}^n_+$.

In this talk, we propose to solve Pareto eigenvalue problems by using interior-point methods. Precisely, we focus the study on an adaptation of the Mehrotra Predictor Corrector Method (MPCM) and a Non-Parametric Interior Point Method (NPIPM) [5]. We compare these two methods with two alternative methods, with the first one being a semismooth approach called the Lattice Projection Method (LPM) [3], and the second one being a smoothing method called the SoftMax Method (SM). Extensions of MPCM and NPIPM methods to solve quadratic pencil eigenvalue problems under conic constraints are also discussed. Finally, we mention using NPIPM and MPCM to deal with the Inverse Pareto Eigenvalue Problem (IPEP) which is the matter of constructing a matrix $A \in \mathcal{M}_n(\mathbb{R})$ in which its set of Pareto eigenvalues contains a prescribed set of distinct reals.

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