

Asymptotic behavior of solutions to first-order mean field games

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Keywords: Mean Field Games, Asymptotic behavior of solutions, Non-Coercive Hamilton-Jacobi equations.

Abstract: The analysis of the ergodic behavior of solutions to Hamilton-Jacobi-Bellmann equations has a long history going back to the seminal paper by [Lions, P.-L., Papanicolaou, G. and Varadhan, S.R.S]. Since this work, the subject has grown very fast and when the Hamiltonian is of Tonelli type a large number of results have been proved. However, few results are available if the Hamiltonian fails to be Tonelli, i.e., the Hamiltonian is neither strictly convex nor coercive with respect to the momentum variable. More recently, such problems have been investigated in case of first-order mean field games, i.e., when the Hamilton-Jacobi equation is coupled with a Fokker-Plank equation that describes the evolution of players.

In this talk, I will present some recent results obtained in collaboration with Piermarco Cannarsa and Pierre Cardaliaguet concerning the long time-average behavior of solutions to the classical mean field game system and also of solutions to the mean field game system with control of acceleration. Notice that, the first belongs to the Tonelli framework and the second to the non-Tonelli framework, previously described. I will present the results obtained but I will particularly emphasize the relevant differences between the two settings which lead the way to new interesting problems in mean field games but also in pure optimal control.

References:

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