Supervised Learning for Kinetic Consensus Control

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Abstract: Social behaviours can be seen as the result of a suitable combination of population interactions and external influences. How to successfully condition the population towards a desired purpose (e.g. consensus) is a fascinating question, whose answer is being widely researched.

The formulation of such a problem in an dynamic optimization framework ensures the availability of control synthesis methods, which nonetheless come with the huge drawback of the curse of dimensionality. The problem reads as the minimization of a cost functional subject to individual-based interaction dynamics, thus its solution easily becomes unfeasible to compute as the number of agents in the population grows. The natural way of circumventing this is using a multiscale approach working with the population density instead of its microscopic state. For a number of $N \to \infty$ interacting agents, this leads to a mean field formulation of the control problem. Although mean field optimal control problems are designed to be independent of the number of agents, they are computationally feasible only for moderate intrinsic dimensions d of the agents' state space [1].

In this talk, we study the solution to this problem using kinetic equations, i.e. through the quasi-invariant limit of binary interactions as an approximation of the mean field PDE, governing the dynamics of the probability distribution of the agent population. The need for an efficient solver of the binary interaction problem motivates the use of a supervised learning approach. As a model, we rely on a gradient-augmented feedforward neural network for the value function of the binary control problem, which is trained by mean of synthetic data generated from Pontryagin's maximum principle[3] or state-dependent Riccati equations[2]. The effectiveness of such approach is assessed through extensive numerical tests.

References:

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