Systems of variational inequalities with interconnected obstacles associated to infinite horizon optimal switching problems

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Abstract: This paper studies a system of m variational inequalities with interconnected obstacles in infinite horizon associated to optimal multi-modes switching problems. Our main result is the existence and uniqueness of a continuous solution in viscosity sense, for that system. The proof of the main result strongly relies on the connection between the systems of variational inequalities and reflected backward stochastic differential equations (RBSDEs) with oblique reflection, which will be characterized through a Feynman-Kac's formula. The main feature of our system of infinite horizon RBSDEs is that its components are interconnected through both the generators and the obstacles.

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